A prediction model for reflection on varnished metallic plates

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Abstract

Several models predict the reflectance of a rough metallic surface. They are however not adapted to the surfaces used in the packaging industry, such as boxes made of printed metallic plates. For printing purposes, metallic surfaces need to be varnished. Light reflection properties are therefore modified. We propose methods which adapt existing reflectance models to varnished surfaces. We also present a correction capable of predicting the reflectance even if incident light is not collimated, i.e. not composed of parallel rays.

Introduction

A ray of light arriving on a perfectly smooth surface is reflected in a single direction according to Snell's laws, called *specular* direction. When the surface is rough, it is scattered in every direction of the space with an intensity varying according to the direction of incidence, the direction of reflection, and the roughness profile. In the first part of this paper we give some definitions in order to characterize roughness and to describe the scattering phenomenon.

In the early sixties, Beckmann [1] studied electromagnetic wave reflections on curved and rough surfaces, in order to describe radar wave reflections on sea waves. He defined two kinds of roughness: periodical and random. He obtained some general equations, including interference phenomena, applicable to a large range of surfaces. However, these equations were computationally too complicated to be used for a simple prediction model.

In 1967, Torrance and Sparrow [2] simplified considerably the problem by proposing a model based only on geometric optics, which combines works both in applied physics and computer graphics. Blinn [3] (1977) and Cook and Torrance [4] (1981) took up this model and gave some further details concerning its components. As we will see in the second part, the Cook-Torrance model predicts reflectance of randomly rough surfaces, when roughness is isotropic and when the metal is in direct contact with air.

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Nevertheless, metallic surfaces used in the printing industry are often stroked, i.e. the metallic roughness structure has a privileged orientation. Furthermore, they are varnished in order to ensure ink adherence, and the incident light rays are not always strictly parallel. We present an extension of the Cook-Torrance model in order to take into account the orientation depending roughness and the varnish effect. We also extend it to non-collimated incident light.

Definitions

Roughness

Although roughness is an intuitive concept, we can find in the literature [5] statistic tools allowing to characterize it, assuming that the profile of the surface is made of a succession of straight segments (see Figure 1.)

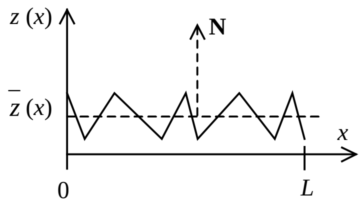

Figure 1. Profile of a rough surface

Several characteristic terms can be defined, such as:
- the standard deviation of the height σ:

$$\sigma = \left(\lim_{L\to\infty} \frac{1}{L} \int_0^L \left(z-\bar{z}\right)^2 dx \right)^{\frac{1}{2}} \tag{1}$$

with

$$\bar{z} = \lim_{L\to\infty} \frac{1}{L} \int_0^L z(x)\, dx \tag{2}$$

- the standard deviation of the segments slope m:

$$m = \left(\lim_{L\to\infty} \frac{1}{L} \int_0^L \left(\frac{dz}{dx} - \bar{z}'\right)^2 dx \right)^{\frac{1}{2}} \tag{3}$$

with

$$\bar{z}' = \lim_{L\to\infty} \frac{1}{L} \int_0^L \frac{dz}{dx}\, dx \tag{4}$$

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The parameter *m* is zero when the surface is perfectly smooth. The interest of this parameter is that it can be obtained directly from scattering measurements [5].

Angular configuration

The scattering of a ray on a rough surface is characterized by three unit vectors called **N**, **L** and **E,** with **N** the normal of the mean surface, **L** the direction of incoming light, and **E** a direction of observation. We also define a vector **H**, bisector of **L** and **E** :

$$\mathbf{H} = \frac{\mathbf{L} + \mathbf{E}}{\| \mathbf{L} + \mathbf{E} \|} \quad (5)$$

We can also describe each direction with angles (see Figure 2): let us call ψ the angle of incidence, θ the angle of observation and φ the azimuth angle of observation. α and ψ', for which we find in [2] an analytical expression as a function of ψ, θ and φ, are angles relative to vector **H**. The cosine of each angle is equal to the dot product of its two unit vectors.

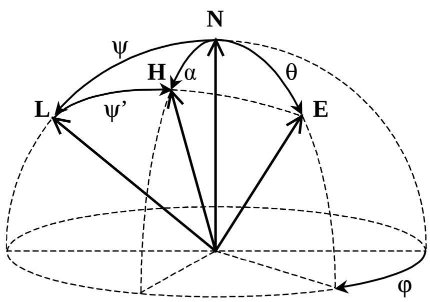

Figure 2. Angular configuration of the reflection

Bi-directional reflection

The light reflected on a rough surface is scattered in every direction of the half-sphere. We describe the scattering phenomenon by relating incoming and reflected light at a point of the surface: we call *Bi-directional Reflectance Distribution Function* (or *BRDF* for short), the ratio between the energy reflected in a direction (θ, φ) and the one coming from direction ψ:

$$BRDF_\psi\ (\theta, \varphi) = \frac{dN_r\ (\psi, \theta, \varphi)}{N_i\ (\psi)\ \cos\psi\ d\omega} \quad (6)$$

where dN_r is the radiance reflected through a spherical surface of infinitesimal area $d\omega$ and N_i is the irradiance on the surface.

The Cook-Torrance model

The Cook-Torrance model is based on the microfacets model of Torrance and Sparrow [2], which assumes that a rough surface can be represented by a set of randomly oriented mirror-like microfacets (same profile as shown on Figure 1). It allows to calculate the BRDF of a surface according to five parameters: angle of incidence ψ, angles of observation θ and φ, roughness parameter *m*, and relative index of refraction of the interface $\underline{N}$, if the following conditions are verified:

- the roughness does not depend on the azimuth orientation of the surface (isotropic roughness);
- the characteristic dimensions of roughness are far larger than the wavelength (σ >> λ);
- incident and observed beams are collimated (rays are parallel);
- roughness is purely random (no periodicity).

The BRDF is given by a combination of three terms: the first one is the microfacets orientation distribution function, noted *D*, the second one is the geometrical attenuation function, noted *G*, and the third one is the function of the attenuation due to absorption of energy by metal, noted *F*. The resulting formula is:

$$BRDF_\psi\ (\theta, \varphi) = \frac{D(\alpha, m)\, G(\psi, \theta)\, F(\psi, \underline{N})}{\cos\theta\ \cos\psi} \quad (7)$$

The denominator includes two terms for normalization of the lighted and observed areas, which are proportional to the cosine of respectively the incidence angle and the observation angle.

More details on the components *D*, *G* and *F* are given in the next paragraphs.

Microfacets orientation distribution function, term *D*

Since the microfacets are assumed to be plane mirrors, they reflect light according to Snell's laws. That means that a ray coming from a direction **L** is reflected in a direction **E** by a microfacet whose normal is **H** (vector bisector between **L** and **E**). We can express the scattering function as a function of the orientation of microfacets, given by their normal **H**, or the angle α they make with the mean surface (recall that α is the angle between a vector **H** and the normal **N** of the mean surface).

This function is a statistical distribution, noted *D*, which depends on roughness (parameter *m*) and α. Blinn [3] proposed to fit it as a gaussian, but Cook and Torrance extracted from Beckman's equations the following expression :

$$D(\alpha, m) = \frac{e^{-\left(\frac{\tan\alpha}{m}\right)^2}}{m^2\ \cos^4\alpha} \quad (8)$$

Attenuation due to absorption of light, term *F*

The term *F* is an attenuation factor taking into account the fact that only a ratio of incident light is reflected. Indeed, when a ray of light spreading in a medium 1 (index of refraction n_1) arrives at the interface of a medium 2 (index of refraction n_2), a ratio of energy is refracted into the medium 2 and spreads in the direction η given by the third Snell's law [6]:

$$\sin\psi = \underline{N} \sin\eta \quad (9)$$

where ψ is the incidence angle, and $\underline{N}$ the relative index of refraction of the interface. $\underline{N}$ is in the general case a complex number defined by:

$$\underline{N} = \frac{n_2}{n_1} = n_0 + i\,\kappa_0 \qquad (10)$$

When $\underline{N}$ is complex, sin η is also complex, and makes no sense. In fact, the light is not refracted but absorbed. This happens here, since medium 2 is a metal, whose index of refraction $\underline{n} = n + i\,\kappa$ is a complex number. Note that both n and κ are wavelength-dependant. We can find tables [6] giving their values for different wavelengths and several types of metal. The medium 1 is air. Therefore, the index of refraction n_1 is close to 1 and $\underline{N}$ is equal to $\underline{n}$.

Then F is given by Fresnel's formulae, and depends on the angle of incidence on a microfacet ψ', the relative index of refraction $\underline{N}$ and the polarization of the light. Let us note $F_{//}$ and $F_{\perp}$ the terms corresponding respectively to parallel and perpendicular polarizations. Fresnel's formulae are:

$$F_{//} = \left| \frac{\sin\,(\psi' - \eta)}{\sin\,(\psi' + \eta)} \right|^2 \qquad (11)$$

and

$$F_{\perp} = \left| \frac{\tan\,(\psi' - \eta)}{\tan\,(\psi' + \eta)} \right|^2 \qquad (12)$$

Another form of (11) and (12) is presented in [7], where only real components appear:

$$F_{//} = \frac{a^2 + b^2 - 2a\cos\psi' + \cos^2\psi'}{a^2 + b^2 + 2a\cos\psi' + \cos^2\psi'} \qquad (13)$$

and

$$F_{\perp} = F_{//}\,\frac{a^2 + b^2 - 2a\,\sin\psi'\tan\psi' + \sin^2\psi'\tan^2\psi'}{a^2 + b^2 + 2a\,\sin\psi'\tan\psi' + \sin^2\psi'\tan^2\psi'} \qquad (14)$$

where a and b are defined as:

$$2a^2 = \sqrt{\left(n_0^2 - \kappa_0^2 - \sin^2\psi'\right)^2 + 4\,n_0^2\kappa_0^2} + \left(n_0^2 - \kappa_0^2 - \sin^2\psi'\right) \qquad (15)$$

$$2b^2 = \sqrt{\left(n_0^2 - \kappa_0^2 - \sin^2\psi'\right)^2 + 4\,n_0^2\kappa_0^2} - \left(n_0^2 - \kappa_0^2 - \sin^2\psi'\right) \qquad (16)$$

For natural light, the resulting term F is the mean value of $F_{//}$ and $F_{\perp}$:

$$F = \frac{1}{2}\,(F_{//} + F_{\perp}) \qquad (17)$$

The value of F is between 0 and 1 and is 0 when the light is totally absorbed. Note that it is always 1 for grazing angles, independently of the wavelength or the metal.

Attenuation due to surface geometry, term *G*

A rough surface has some little gaps in which the light cannot enter, or parts where light is absorbed after several reflections, yielding an attenuation. In the Cook-Torrance model, this phenomenon is taken into account by the term G. The attenuation coefficient corresponds to the ratio of light that is not reflected, because it is shadowed by an adjacent microfacet. Either the incident light (*shadowing*), or the reflected light (*masking*) is blocked, depending on angles of incidence and observation, and on the orientation of microfacets. Blinn [3] gives the following relation, made of three terms relative respectively to no attenuation, masking and shadowing:

$$G = \min\left(1,\ 2\,\frac{\cos\alpha\,\cos\theta}{\cos\psi'},\ 2\,\frac{\cos\alpha\,\cos\psi}{\cos\psi'}\right) \qquad (18)$$

When we measure the BRDF of a metallic rough surface, we observe that the direction of maximal intensity is not the specular direction but another one situated a few degrees below. This phenomenon is called *off-specular peak*, predicted by Torrance and Sparrow's term G. The Cook-Torrance model is known [8] to yield physically plausible results, but it is too restrictive for our purpose. We need to predict reflectance of stroked and varnished surfaces. In the next sections, we take into account the strokes and varnish effects by an extended reflectance prediction model.

Anisotropy

The Cook-Torrance model requires the roughness to be isotropic: the roughness term m must be the same at all surface orientations. This is not the case for *brushed* metallic surfaces, where the roughness m depends on the direction β of the strokes.

As proposed by Ward [9], we consider that anisotropy can be modeled by two parameters: $m_{//}$ the roughness in direction of the strokes and $m_{\perp}$ the roughness perpendicularly to them. For other orientations, m is interpolated according to an ellipse. Therefore, the resulting function $m(\beta)$ is the polar expression of an ellipse with $m_{//}$ as the small axis and $m_{\perp}$ as the large axis:

$$m(\beta) = \frac{m_{//}\,m_{\perp}}{\sqrt{m_{\perp}^2\cos^2\beta + m_{//}^2\sin^2\beta}} \qquad (19)$$

Varnish effects

Until now, the metallic surface was assumed to be directly in contact with air. The presence of a varnish between the metal and the air produces two phenomena that change the reflection properties. Firstly, the varnish reflects and refracts incident light as well as outgoing reflected light. Secondly, absorption by the metal must be corrected because the relative index of refraction is modified. We assume that the whole surface is covered by a perfectly plane coat of varnish as shown in figure 3.

We apply the Cook-Torrance model to predict scattering, but it is necessary to make the three modifications presented below.

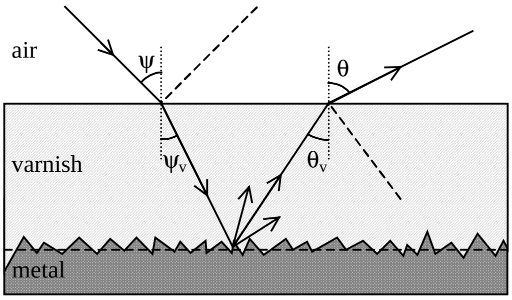

Figure 3. Spreading and scattering of light on a varnished rough metallic surface.

Refractions at the interface air-varnish

The incident ray arrives on the varnish surface with an angle ψ. When entering the varnish, the light is refracted and arrives on the metallic surface with an angle ψ_v. Similarly, a ray observed in the air at an angle θ has been refracted by the surface at an angle θ_v. The Cook-Torrance model is then applied with ψ_v and θ_v as angles of incidence and observation. ψ_v as θ_v are linked to ψ and θ by Snell's law:

$$\sin\psi = n_v \sin\psi_v \tag{20}$$

$$\sin\theta = n_v \sin\theta_v \tag{21}$$

where n_v is the varnish index of refraction, typically close to 1.5. Note that the azimuth angle of observation φ remains unchanged.

Reflections at the interface air-varnish

When passing through the interface between air and varnish, the light is partially reflected, which we consider as an attenuation. We call *specular reflection* (respectively *internal reflection*) the reflection occurring in air (resp. in varnish) and we note R_s (resp. R_i), the ratio of reflected light. R_s and R_i are given by the Fresnel's formulae (13), (14) and (17):

For R_s,

$$a = \sqrt{n_v^2 - \sin^2\psi} \quad \text{and} \quad b = 0 \tag{22}$$

For R_i,

$$a = \sqrt{\frac{1}{n_v^2} - \sin^2\theta_v} \quad \text{and} \quad b = 0 \tag{23}$$

At every incidence angles ψ, Eq. (22) has a solution since n_v is larger than 1. On the contrary, Eq. (23) has a solution only when $\sin\theta_v$ is lower than $1/n_v$, i.e. the internal reflection is total ($R_i = 1$) when θ_v is larger than the limit angle θ_{lim}:

$$\theta_{lim} = \sin^{-1}(1/n_v) \tag{24}$$

$1 - R_s$ represents the attenuation of light entering the varnish and $1 - R_i$ the attenuation of light exiting the varnish.

The typical value for n_v is 1.5. At normal incidence ($\psi = \theta_v = 0$), R_s and R_i are equal to 0.04. R_s is equal to 0.09 for $\psi = 60°$ and 1 for $\psi = 90°$. $\theta_{lim} = 41.8°$.

Absorption at interface varnish-metal

Since the metal is not in contact with air but with varnish, we must change the relative index of refraction for calculating the term F of the Cook-Torrance model. The expression of $\underline{N}$ is now:

$$\underline{N} = \frac{n}{n_v} = \frac{n}{n_v} + i\,\frac{\kappa}{n_v} \tag{25}$$

We only consider the case of natural light. If light is polarized, we must take care of the change of polarization occurring at the interface air-varnish [6]. Then eq. (7) becomes:

$$BRDF_\psi(\theta,\varphi) = (1 - R_s)(1 - R_i)\frac{D(\alpha_v, m)\,G(\psi_v,\theta_v)\,F\left(\psi_v, \frac{n}{n_v}\right)}{\cos\theta\cos\psi} \tag{26}$$

Point light source

The definition of BRDF requires incident light to be collimated, which is not the case in most applications. In this paragraph, we consider the extreme case of a point source that emits light uniformly in every directions.

Figure 4 shows the different parameters that we use to describe the non-collimated light effect. A surface sample, with a normal **N** and a center C, is illuminated from a point source L situated at a distance d from the surface, through a solid angle Ω. The ray of light LC arrives on C with an incidence angle ψ_c, and is scattered according to a BRDF that we note $BRDF_c(\theta, \varphi)$. For other rays LP arriving on a point P of the surface, the incidence angle is ψ, the azimuth angle made with the ray LC is Φ, and the corresponding BRDF is noted $BRDF_{\psi,\Phi}(\theta, \varphi)$. We call H the projection of L on the surface and x the distance HP.

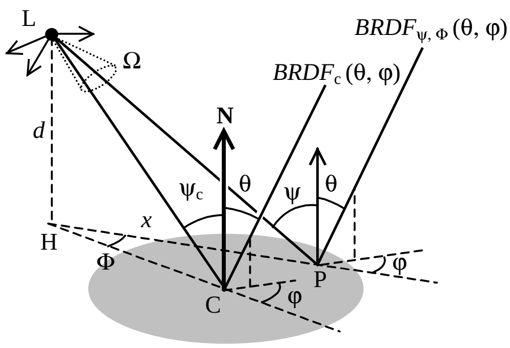

Figure 4. Surface illuminated by a point source.

The intensity of light received at a point P depends on the distance between this point and the source, according to Cauchy's distribution function [10]:

$$I(x) = \frac{I_0}{\pi}\cdot\frac{d}{d^2 + x^2} \tag{27}$$

where I_0 is the total intensity emitted by the source. Replacing x by its expression in function of ψ and d,

$$x = d\tan\psi \tag{28}$$

then we obtain I as a function of ψ, depending on the parameter d:

$$I(\psi) = \frac{I_0}{\pi d} \cdot \frac{1}{1 + \tan^2 \psi} \qquad (29)$$

Note that I is a bi-dimensional function, even if it is a constant in respect to Φ. Therefore, we write $I(\psi, \Phi)$ to recall that I is a function of two angular parameters.

An observer looking at the surface sample receives light from each point P of the surface, with an intensity given by the summation of all $BRDF_{\psi,\Phi}$ weighted by the incident intensity I. We call $BRDF_g(\theta, \varphi)$ the intensity received by the observer at direction (θ, φ). Then we have:

$$BRDF_g(\theta, \varphi) = \iint_\Omega BRDF_{\psi,\Phi}(\theta, \varphi)\ I(\psi, \Phi)\ d\psi\ d\Phi \qquad (30)$$

If the source is far enough or the surface sample area is small, we can make a first approximation: in the angular range Ω, all $BRDF_{\psi,\Phi}$ are the same function, rescaled according to the incident intensity $I(\psi, \Phi)$ and centered around their specular direction ($\theta = \psi$, $\varphi = 0$). Thanks to this assumption, we calculate only one BRDF. Figure 5 illustrates the rotation and rescale transformations applied to one elementary $BRDF_{\psi,\Phi}$.

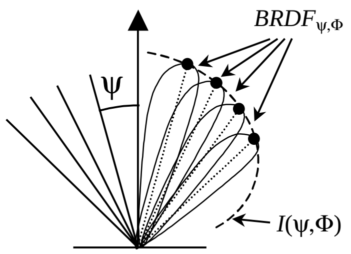

Figure 5. Rotation and rescale of an elementary $\mathrm{BRDF}_{\psi,\Phi}$.

Note that the configuration of the curves in figure 5 is characteristic of a convolution: if we choose $BRDF_c$ as the reference function, we obtain for each ψ of Ω:

$$BRDF_{\psi,\Phi}(\theta, \varphi) = BRDF_c(\theta + \psi_c - \psi, \varphi - \Phi)\ I(\psi, \Phi) \qquad (31)$$

With relations (30) and (31), we reach to the following bi-dimensional convolution:

$$BRDF_g(\theta, \varphi) = BRDF_c(\theta + \psi_c, \varphi) \underset{\theta,\varphi}{*} I(\theta, \varphi) \qquad (32)$$

An even simpler approximation can be made by assuming that I is constant within the range of incidence angles. This is nearly true when the distance from the source to the surface sample is far larger than the sample diameter.

Measurement results

We realized three measures of BRDF within the incidence plane ($\varphi = 0$). The surface sample was lighted by a point source. The three measures were made at three angular positions of the point source in respect to the center of the surface sample: $\psi_c = 17°$, $\psi_c = 44°$ and $\psi_c = 63°$. But, since the light was not collimated, the surface sample was lighted by rays having the following related ranges of incidence angles: [14.2°, 19.8°], [43.4°, 44.7°] and [62.8°, 63.2°].

The predictions given by the Cook-Torrance model and by the extended model were compared with goniometric measurements. Results are presented by the six plots of figure 6. For the prediction, we used the following parameters: roughness value $m = 0.03$, varnish index of refraction $n_v = 1.5$, metal complex index of refraction $\underline{n} = 2.29 + 3.37\ i$ (value given for iron with a wavelength of 650 nm [6]).

Conclusion

The presented model extending the Cook-Torrance model gives a satisfying prediction of reflectance, for a varnished surface illuminated by a point source at an incidence angle lower than 45°. However, the prediction is not exact at grazing angles. For such angles, more investigations and more measurements need to be carried out in order to complete the prediction methods.

Nevertheless, this adapted model can be a good starting point for spectral reflectance predictions on varnished metallic surfaces.

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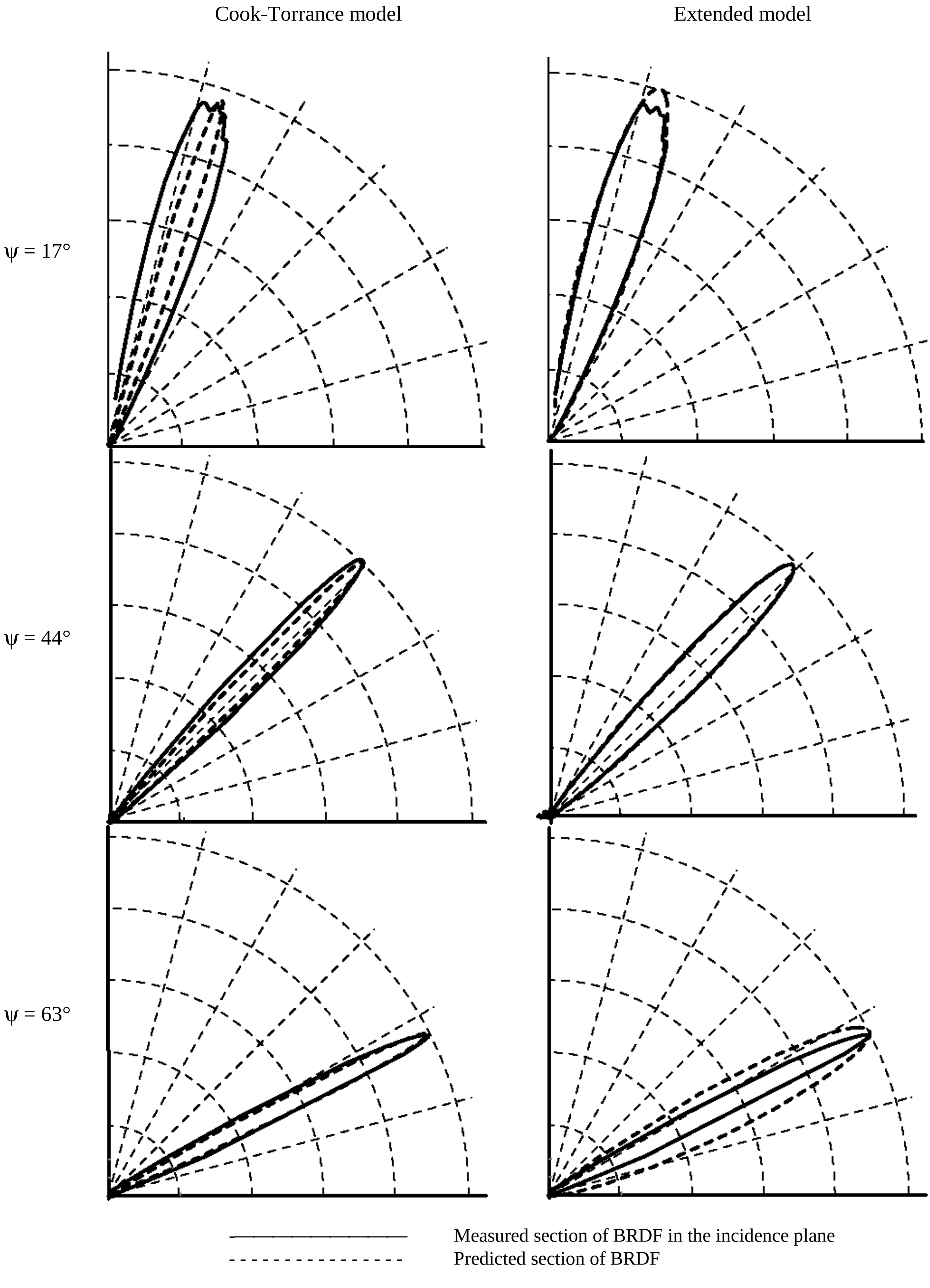

Figure 6. Measurements (fat line) of the BRDF of a varnished metallic surface, at different angles of incidence, prediction (dotted line, left) according to the Cook-Torrance model and prediction (dotted line, right) according to our extended model.