SLAM BASED ON QUANTITIES INVARIANT OF THE ROBOT'S CONFIGURATION

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Abstract: This paper presents a solution to the Simultaneous Localization and Mapping (SLAM) problem in the stochastic map framework for a mobile robot navigating in an indoor environment. The approach is based on the concept of the relative map. The idea consists in introducing a map state, which only contains quantities invariant under translation and rotation. In this way the landmark estimation is decoupled from the robot motion and therefore the estimation does not rely on the unmodeled error sources of the robot motion. A new landmark is introduced by considering the intersection point between two lines. Only landmarks whose position error is small are considered. In this way the intersection point is the natural extension of the corner feature. The relative state estimated through a Kalman filter contains the distances among the intersection points observed at the same time. Real experiments carried out with a mobile robot equipped with a 360^o laser range finder show the performance of the approach.

Keywords: SLAM, Sensor Fusion, Kalman Filter, Robot Navigation

1. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) requires a mobile robot to autonomously explore the environment with its on-board sensors, gain knowledge about it, interpret the scene, build an appropriate map and localize itself relative to this map. Many approaches have been proposed both in the framework of the metric and the topological navigation. A very successful metric method is the stochastic map (Smith, 1988), where early experiments (Crowley, 1989) (J.J. Leonard, 1992), have shown the quality of fully metric SLAM. However, these approaches suffer from some limitations. Firstly, they rely strongly on odometry making the global consistency of the map difficult to maintain in large environments due to the odometry drift. Furthermore, they represent the robot position with a single Gaussian distribution meaning that an unmodeled event (i.e. collision) could cause a divergence between the ground truth and the estimation, which could be unrecoverable for the system (lost situation). In order to minimize the divergence of the built map, one have to concentrate on two important points: Adopt an optimal filter (accordingly to the dynamics and the observation); Use the best statistical model to characterize the error of the adopted sensor readings. Clearly, to deal with the second remark, it is better not to use the odometry in the estimation phase if, as often happens, other more precise sen-

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sors are available with a well-known error model. The absolute map filter (AMF) (Dissanayake and Csorba, 2001), using odometry, diverges when there is even a very small, undetected systematic component. This divergence is proven through simulations in (Martinelli and Siegwart, 2004a) and through experiments on a real platform in (Martinelli and Siegwart, 2004b). Therefore, decoupling odometry from the estimation process becomes a main issue. Newman introduced a relative map and he used two filters in the estimation, called the relative map filter and the geometric projection filter ((P.M.Newman, 1999) and (P.M.Newman and H.F.Durrant-Whyte, 2001)). The second one provides a means to produce a geometrically consistent map from the relative map, by solving a set of linear constraints. Both filters are optimal since the dynamics and the observation are linear and they are based on the Kalman Filter. However, the elements used in this approach are invariant for translation only, not for rotation. The approach adopted here is to take invariant elements for both translation and rotation in order not to rely the robot motion for the estimation. Then we apply a Kalman filter for estimation, contrasting to (M.Csorba and H.F.Durrant-Whyte, 1997) and (M.C.Deans and M.Hebert, 2000), who used the same invariants in combination with a non-optimal filter. The observation, as well as the dynamic, will be linear.

A new landmark whose configuration is defined through its position and orientation is introduced by considering the intersection point between two lines. The position error of such a landmark in the robot reference is analytically derived. Only landmarks with small error are considered. Following this criterion, the intersection points are either very close to the segments generating them, or generated by segments whose error parameters can be estimated with very high accuracy (e.g. large segments). Therefore, these intersection points are the natural extension of the corner feature with the same degrees of freedom (position and orientation). The strategy adopted to extract these landmarks from a laser scan and to evaluate the error on the estimated position is illustrated in section 2. The relative state estimated through the Kalman filter contains the distances among the intersection points observed at the same time and therefore is invariant of the robot configuration (in this paper we do not estimate the relative orientations among the intersection points, i.e. we only use the position information contained in the intersection point and not its orientation). The relative filter equations are in section 3. In the sections 4 and 5 some experimental results, obtained with a mobile robot equipped with a 360^o laser range finder sensor, are shown and discussed.

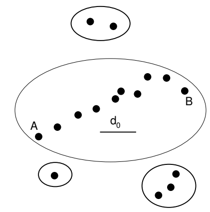
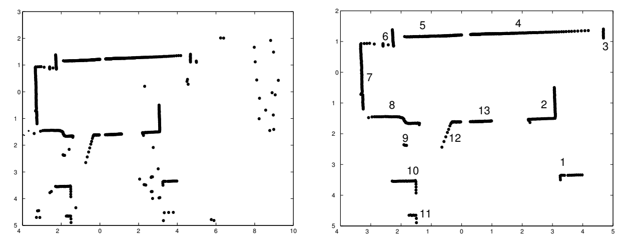


Fig. 1. The two points A and B belong to the same cluster although their distance is larger than d_0.



(a) (b)

Fig. 2. The raw laser scan (a) and the scan after the clustering step (b).

2. EXTRACTING THE INTERSECTION POINTS FROM A LASER SCAN

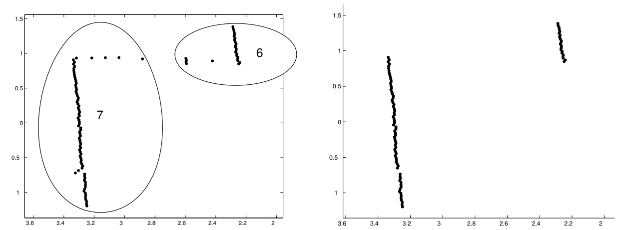
The following steps are considered in order to extract the intersection points from a laser scan:

- Clustering;
- Segmentation;
- Segment parameter estimation;
- Estimation of the intersections among all the extracted segments;

In the following subsections we detail the strategies adopted for each step.

2.1 Clustering

Fig. 2a shows a laser scan. Our first step consists in grouping all the points of the scan in clusters and in removing the small clusters. A cluster is defined through the following property: *if the Euclidean distance between two points is smaller than a given distance d_0, these two points belong to the same cluster*. Clearly, the definition of the cluster is based on the parameter d_0. Moreover, we want to remark that if two points belong to the same cluster this does not imply that their distance is smaller than d_0 as shown in Fig 1. Finally, we introduce another parameter N_0. All clusters whose number of points is smaller than N_0 are removed. Fig. 2b displays the scan after the clustering step. In this case we get 13 clusters ($d_0 = 0.2m$ and $N_0 = 6$).



(a) (b)

Fig. 3. The points in the two ellipses belong to two distinct clusters (a). The result of the segmentation step is shown in (b).

2.2 Segmentation

The aim of the segmentation step is to divide the points belonging to the same cluster in subgroups representing segments. The points in the cluster which are not in any segment are removed. We apply for each cluster the algorithm introduced by Fisher and Bolles (Fisher and Bolles, 1981). Fig. 3a and b show the results obtained in this step for two clusters.

2.3 Segment Parameter Estimation

Once we know that a given set of points represents a segment, we can estimate the parameters characterizing this segment. We put a reference frame on the segment: its origin is in the middle and its $x-$axis is along the segment. The parameters characterizing the segment are the estimated position of the reference frame origin ($\hat{x}$, $\hat{y}$) and the estimated orientation $\hat{k}$. Concerning the error, accordingly with the SPmodel for the case of a line (see (Castellanos and Tardós, 1999)), we consider only the component on the orientation and on the direction orthogonal to the segment (σ_θ^2, σ_n^2 and $\sigma_{n\theta}$).

2.4 Estimation of the intersections among all the extracted segments

Let x_{int} and y_{int} be the local coordinates of the intersection point between two segments. The aim of this step is to estimate these coordinates together with the error ($\sigma_{x_{int}}^2$, $\sigma_{y_{int}}^2$ and $\sigma_{x_{int}y_{int}}$). Moreover, when two intersection points are generated by a common segment, they are correlated and therefore we need to compute also the covariance error between the two points. In the following we provide the method adopted to the parameters related to a single intersection point. Similar computation is carried out to compute the covariance between two intersections.

By adopting the same notation introduced in the previous section we represent the two segments through the following parameters: $\hat{x}_1$, $\hat{y}_1$, $\hat{k}_1$, $\sigma_{\theta_1}^2$, $\sigma_{n_1}^2$ and $\sigma_{n_1\theta_1}$ for the first segment and $\hat{x}_2$, $\hat{y}_2$, $\hat{k}_2$, $\sigma_{\theta_2}^2$, $\sigma_{n_2}^2$ and $\sigma_{n_2\theta_2}$ for the second one. We denote with the hat the estimated quantities to distinguish them from their actual value.

The intersection point satisfies the following equations

$$(x_{int} - x_1)\ \ sin\theta_1 - (y_{int} - y_1)\ \ cos\theta_1 = 0$$

$$(x_{int} - x_2)\ \ sin\theta_2 - (y_{int} - y_2)\ \ cos\theta_2 = 0$$

where θ_1 and θ_2 are respectively the orientation of the two vectors k_1 and k_2. The previous two equations can be written in the compact form:

$$\overrightarrow{f}(x_{int}, y_{int}, x_1, y_1, \theta_1, x_2, y_2, \theta_2) = 0 \qquad (1)$$

By expanding this function at the first order we get:

$$\overrightarrow{f}(\hat{x}_{int}, \hat{y}_{int}, \hat{x}_1, \hat{y}_1, \hat{\theta}_1, \hat{x}_2, \hat{y}_2, \hat{\theta}_2) = 0 \qquad (2)$$

and

$$P_{int} = AP_{seg}A^T \qquad (3)$$

where:

- $P_{int} = \begin{bmatrix} \sigma_{x_{int}}^2 & \sigma_{x_{int}y_{int}} \\ \sigma_{x_{int}y_{int}} & \sigma_{y_{int}}^2 \end{bmatrix}$;
- $A = -F_{int}^{-1}F_{seg}$, with F_{int} and F_{seg} respectively the Jacobian of the function in (1) with respect to the intersection and the segment parameters computed in the point satisfying the equation (2);
- P_{seg} is the covariance matrix for the segments $P_{seg} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ with

$$P_{1/2} = \begin{bmatrix} \sigma_{x_{1/2}}^2 & \sigma_{x_{1/2}y_{1/2}} & \sigma_{x_{1/2}\theta_{1/2}} \\ \sigma_{x_{1/2}y_{1/2}} & \sigma_{y_{1/2}}^2 & \sigma_{y_{1/2}\theta_{1/2}} \\ \sigma_{x_{1/2}\theta_{1/2}} & \sigma_{y_{1/2}\theta_{1/2}} & \sigma_{\theta_{1/2}}^2 \end{bmatrix}$$

  where $\sigma_{x_{1/2}}^2$, $\sigma_{y_{1/2}}^2$ and $\sigma_{x_{1/2}y_{1/2}}$ are easily computed by knowing $\sigma_{n_{1/2}}^2$, $\sigma_{n_{1/2}\theta_{1/2}}$ and $\sigma_{\theta_{1/2}}^2$

The equations (2) and (3) define the position of the intersection and its covariance error matrix.

Fig. 4 shows the intersections obtained from the scan in fig 2a. We removed all the intersections whose error in the estimation is larger than a threshold. In particular, we compute the trace of the covariance error matrix (which is independent of the robot pose) and we reject all the intersections whose square root of this trace is larger than $2.5cm$.

The intersections defined in this way generalize the concept of the corner since they are usually

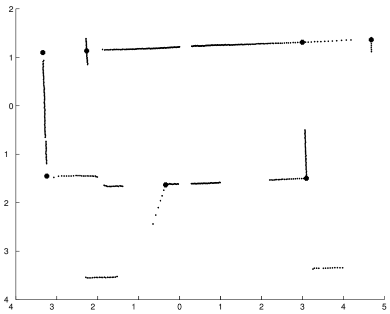


Fig. 4. The intersections obtained from the scan in fig 2a.

close to the segments generating them. In the case they are not close, they are generated by segments whose error parameters can be estimated with very high accuracy (e.g. large segments).

3. THE STRUCTURE OF THE RELATIVE MAP FILTER

The odometry can be decoupled from the estimation process by introducing a filter whose state only contains quantities invariant under translation and rotation. This is the idea characterizing the relative filter introduced here. Once the relative map has been estimated through this filter and the absolute location of a set of landmarks is known (e.g. by using the first observation) it is possible to build the absolute map. Therefore, the entire method contains two algorithms. The former estimates the relative map, the latter builds the absolute map. In the following we provide the equations to estimate the relative state. These equations are very general and can be applied to any kind of landmarks.

Let denote with d the state and with P its covariance matrix. In fig. 5a the vector d contains the marked distances between the 6 landmarks. Clearly, not all of the distances between the 6 landmarks are stored in d because not all the landmarks were observed together at the same time. At a given time step, the observation consists of a set of distances between the landmarks observed by the robot through its external sensor (fig. 5b). Of course, these distances may be already observed (i.e. can be in the vector d) or may not. Let introduce the following notation:

$$d_{old} = [u, w_{old}]^T \qquad d_{obs} = [w_{obs}, v]^T \tag{4}$$

where d_{old} is the state estimated at a given time step and d_{obs} is the observation at the same time step, containing a set of distances between the landmarks observed by the robot. u contains the distances which are not re-observed (i.e. which do not appear in the vector d_{obs}) and w_{old} contains the distances re-observed (denoted by w_{obs} in the vector d_{obs}). Finally, v contains the distances observed for the first time at the considered time step. The covariance matrix of the previous vectors are:

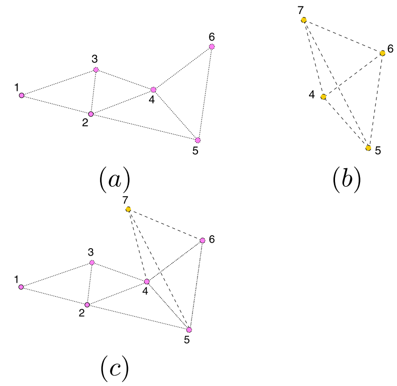


Fig. 5. Relative Map before the observation (a), the observation (b), and the relative map obtained by fusing the information coming from the old map and the observation (c). In all the three figures the map state only contains the indicated distances between the landmarks

$$P_{old} = \begin{bmatrix} P_{uu} & P_{uw} \\ P_{uw}^T & P_{ww} \end{bmatrix} \qquad P_{obs} = \begin{bmatrix} R_{ww} & R_{wv} \\ R_{wv}^T & R_{vv} \end{bmatrix} \tag{5}$$

We adopt the following notation to denote the estimated quantities, obtained by fusing the old state with the observed one (the new estimated distances are depicted in fig. 5c).

$$d_{new} = [u_{new}, w_{new}, v_{new}]^T \tag{6}$$

$$P_{new} = \begin{bmatrix} Pn_{uu} & Pn_{uw} & Pn_{uv} \\ Pn_{uw}^T & Pn_{ww} & Pn_{wv} \\ Pn_{uv}^T & Pn_{wv}^T & Pn_{vv} \end{bmatrix} \tag{7}$$

We obtain the new estimation for the state and its covariance matrix by applying the equations of the Kalman filter. Observe that the observation is linear in the state (is the identity) and therefore the Kalman filter is optimal.

$$u_{new} = u + P_{uw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{8}$$

$$w_{new} = w_{old} + P_{ww}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{9}$$

$$v_{new} = v + R_{vw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{old} - w_{obs}\right) \tag{10}$$

$$Pn_{uu} = P_{uu} - P_{uw}\left(P_{ww} + R_{ww}\right)^{-1} P_{wu} \tag{11}$$

$$Pn_{uw} = P_{uw} - P_{uw}\left(P_{ww} + R_{ww}\right)^{-1} P_{ww} \tag{12}$$

$$Pn_{uv} = 0 \tag{13}$$

$$Pn_{ww} = P_{ww} - P_{ww}\left(P_{ww} + R_{ww}\right)^{-1} P_{ww} \tag{14}$$

$$Pn_{wv} = R_{wv} - R_{ww}\left(P_{ww} + R_{ww}\right)^{-1} R_{wv} \tag{15}$$

$$Pn_{vv} = R_{vv} - R_{vw}\left(P_{ww} + R_{ww}\right)^{-1} R_{wv} \tag{16}$$

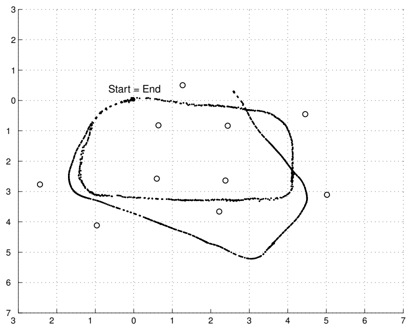


Fig. 6. The closed line and the circles represent respectively the robot trajectory and the beacons position estimated with proposed filter. The open line is the robot trajectory estimated through the AMF with the same data (odometry and laser). The real robot motion was a closed path.

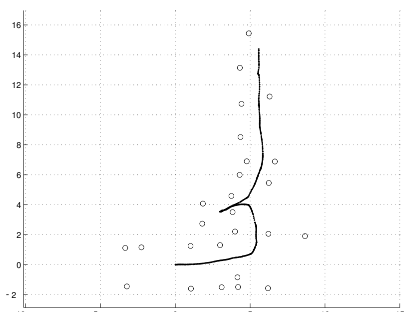


Fig. 7. The trajectory of the robot and the position of the intersection point estimated through the relative map filter

Instead of the equations (9) and (14) it is possible to use the following equations:

$$w_{new} = w_{obs} + R_{ww}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{old} - w_{obs}\right) \quad (17)$$

$$Pn_{ww} = R_{ww} - R_{ww}\left(P_{ww} + R_{ww}\right)^{-1} R_{ww} \quad (18)$$

They are derived by observing the symmetry of the filter with respect to the change "observation" $\leftrightarrow$ "old state". Observe that the coincidence of the previous equations could be easily proven also by using the inversion lemma.

4. RESULTS AND CONCLUSIONS

For the experiments, two fully autonomous mobile robots Donald Duck and the BIBA robot have been used. Both robots have the same functionality: They are equipped with wheel encoders, two 180-laser range finders and a CCD camera (not used here). The experiments are of two kinds. The first is based on reflectors, which are used as beacons. These experiments set a benchmark for the intersection approach presented in Section 2. Then the first results of the SLAM based on intersections are presented with the current limitations. In the figures, the estimated robot position is represented with a dot and the estimated landmark (beacon and intersection point) with a circle. The unities are meters in both axes. The initial robot configuration coincides with the origin of the global reference whose axes were chosen coincident with the axis of the robot at the initial time. Figure 6 concerns the results obtained with beacons. Ten beacons were placed in the environment to be used as point landmarks. The laser sensor could detect them with an accuracy of about $2cm$. The robot moved along a closed trajectory at around $20cms^{-1}$ and estimated at each time step its configuration and the position of the beacons in the environment. An error on the odometry was artificially introduced by increasing the wheel diameter of a factor equal to 2%. The same data (from laser and encoder) were used as the input for the relative map filter described in the section 3 and for the AMF. The closed line is the trajectory estimated through the first filter. Since in this case the odometry was completely decoupled by the estimation process, there is not any drift in the built map. During the experiment the odometry data were only used to solve the data association problem and not in the estimation process. In this case, we get a complete correct result if we change the value of the wheel radius by a factor equal up to 10%. For changes larger an error in associating the data arises. The open line represents the trajectory estimated with the AMF. Finally, the circles represent the position of the beacons estimated through the relative map filter. Note for comparison, that the same data with the AMF creates a map divergence.

The results related to the second kind of experiment are shown in Fig 7. The experiment is carried out in an indoor environment. The landmark here adopted is the intersection point extracted from the laser data by following the method presented in Section 2. The odometry is not well calibrated, making the experiments very challenging even without adding any further error. Moreover, the estimation process is not completely independent of the odometry since the number of common landmarks observed in subsequent observations is sometimes equal to 1 (i.e. it is not possible to locate a new landmark by only using one distance). This happened six times during the experiment shown in the figure and therefore the location of several beacons rely on the odometry. Clearly, although the estimated map depends on the odometry, the dependency is quite weak. To completely avoid this dependency, we are considering also the orientation for an intersection point, defined through the segments generating it. Then, the relative orientations among these intersection points will be estimated through a new relative filter. In this way, a new landmark can be located by knowing only the relative distance and orientation respect to only one landmark (whose absolute configuration is known).

5. CONCLUSIONS AND FUTURE RESEARCH

This paper presented an approach to solve the SLAM problem in the stochastic map framework based on the concept of the relative map. The idea consists in introducing a map state which only contains quantities invariant under translations and rotations and to carry out the estimation of this relative map in an optimal way (a Kalman filter was adopted). This is a way in order to have a decoupling between the robot and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources in the robot motion. A new kind of landmark is introduced by considering the intersection point between two segments. The position error of such a landmark in the robot reference is analytically derived. Only landmarks with small error are considered. Following this criterion, the intersection points are either very close to the segments generating them, or generated by segments whose error parameters can be estimated with very high accuracy (e.g. large segments). Therefore, these intersection points are the natural extension of the corner feature. We are considering also the orientation for an intersection point, defined through the segments generating it. We are also introducing a new relative filter estimating the distances and the relative orientations among these intersection points.

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