Estimating the Odometry Error of a Mobile Robot during Navigation

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Abstract. This paper addresses the problem of the odometry error estimation during the robot navigation. The robot is equipped with an external sensor (like laser range finder). Concerning the systematic error an augmented Kalman Filter is introduced. This filter estimates a vector state containing the robot configuration and the parameters characterizing the systematic component of the odometry error. It uses encoder readings as inputs and the readings from the external sensor as observations. The estimation of the non-systematic component is carried out through another Kalman Filter where the observations are obtained by two subsequent robot configurations provided by the previous augmented Kalman Filter. Both synchronous and differential drive systems are considered.
Key Words: Robot Navigation, Kalman Filter, Odometry, Autocalibration

1   Introduction

Determining the odometry errors of a mobile robot is very important both in order to reduce them, and to know the accuracy of the state configuration estimated by using encoder data.

Odometry errors can be both systematic and non-systematic. In a series of papers Borenstein and collaborators [3, 4, 5, 6, 7, 8, 18] investigated on possible sources of both kind of errors. A review of all the types of these sources is given in [8]. In the work by Borenstein and Feng [7], a calibration technique called UMBmark test has been developed to calibrate for systematic errors of a mobile robot with a differential drive. Larsen et al. [11, 12] suggested an algorithm that uses the robot's sensors to automatically calibrate the robot as it operates. In particular, they introduced an augmented Kalman filter (AKF) which simultaneously estimates the robot configuration and the parameters characterizing the systematic odometry error. This filter uses encoder readings as inputs and vision measurements as observations. They referred to a mobile robot with a differential drive system.

Many investigations have been carried out on the odometry error from a theoretical point of view. Wang [17] and Chong and Kleeman [9] analyzed the non-systematic errors and computed the odometry covariance matrix Q for special kind of the robot trajectory. Kelly [10] presented the general solution for linearized systematic error propagation for any trajectory and any error model. Martinelli [14] derived general formulas for the covariance matrix and also suggested a strategy to estimate the model parameters for a mobile robot with a synchronous drive system. This strategy is based on the evaluation of the mean values of some quantities (called *observables*) which depend on the model parameters and on the chosen robot motion.

In this paper we suggest a method to estimate both systematic and non-systematic odometry error of a mobile robot, during navigation. Concerning the systematic component, we adopt the same AKF introduced by Larsen et al. [11, 12] by considering also the case of a synchronous drive. Concerning the non-systematic parameters, we introduce a new filter (the Observable Filter, OF) where the state to be estimated contains the parameters characterizing the non-systematic error and the observations are provided by the observables as defined in [14] and which can be evaluated by knowing two subsequent robot configurations.

In Section 2 we introduce the models adopted to characterize the odometry error for mobile robot with both synchronous and differential drive. In Section 3 we summarize the AKF introduced by Larsen et al. [11, 12]. The new filter (OF) is presented in Sect. 4. In Section 5 we show some results obtained through simulations. Finally, some conlcusions are given in Sect. 6

2   The odometry error model

We consider two different drive system: synchronous and differential. Concerning the former we adopt the same model introduced in [14] whereas for the latter we adopt a simple model similar to the one introduced by Chong-Kleeman ([9]). In the next subsections we briefly summarize these odometry error models.

2.1 Synchronous Drive

In the synchronous drive system each wheel is capable of being driven and steered. Let denote with $\delta\rho_i$ and $\delta\theta_i$ respectively the robot translation and rotation in the ith time step with respect to a global world-coordinate frame. Because of the odometry errors these values differ from the encoder readings. The model here adopted assumes that $\delta\rho_i$ and $\delta\theta_i$ are random variables, uncorrelated, with gaussian distribution. In particular their mean values are given by the encoder readings corrected for the systematic error. It is assumed that the systematic errors (both in translation and rotation) increase linearly with the distance traveled by the robot. Therefore,

$$\overline{\delta\rho}_i = \delta_\rho \delta\rho_i^e \qquad \overline{\delta\theta}_i = \delta\theta_i^e + E_\theta \delta\rho_i^e \tag{1}$$

where $\delta\rho_i^e$ and $\delta\theta_i^e$ are the encoder readings respectively for the robot translation and rotation, and δ_ρ and E_θ characterize the systematic errors. Finally, it is also assumed that the variances increase linearly with the distance traveled by the robot. We therefore can write:

$$\delta\rho_i = \overline{\delta\rho}_i + \nu_i^\rho \qquad \delta\theta_i = \overline{\delta\theta}_i + \nu_i^\theta \qquad \nu_i^\rho \sim N(0, K_\rho|\delta\rho_i^e|) \qquad \nu_i^\theta \sim N(0, K_\theta|\delta\rho_i^e|) \tag{2}$$

The odometry error model here presented is based on 4 parameters. Two of them (δ_ρ, E_θ) characterize the systematic components whereas the other two (K_ρ, K_θ) characterize the non-systematic components. Clearly, these parameters depend on the environment where the robot moves.

2.2 Differential Drive

A simple way to characterize the odometry error for a mobile robot with a differential drive system is obtained by modeling separately the error in the translation of each wheel [9]. The actual translation of the right/left wheel related to the ith time step is assumed to be a gaussian random variable satisfying the following relation:

$$\delta\rho_i^{R/L} = \overline{\delta\rho}_i^{R/L} + \nu_i^{R/L} \qquad \overline{\delta\rho}_i^{R/L} = \delta\rho_i^{eR/L}\delta_{R/L} \qquad \nu_i^{R/L} \sim N(0, K_w|\delta\rho_i^{eR/L}|) \tag{3}$$

In other words, both $\delta\rho_i^R$ and $\delta\rho_i^L$ are assumed gaussian random variables, whose mean values are given by the encoder readings (respectively $\delta\rho_i^{eR}$ and $\delta\rho_i^{eL}$) corrected for the systematic errors (which are assumed to increase linearly with the distance traveled by each wheel), and whose variances also increase linearly with the traveled distance. Moreover, it is assumed that $\delta\rho_i^R$ and $\delta\rho_i^L$ are uncorrelated. With respect to the Chong-Kleeman model, only one parameter (K_w) is here adopted to characterize both the variances for the right and left wheel. The robot translation and rotation are given by the following relations:

$$\delta\rho_i = \frac{\delta\rho_i^R + \delta\rho_i^R}{2} \qquad \delta\theta_i = \frac{\delta\rho_i^R - \delta\rho_i^L}{d\delta_d} \tag{4}$$

where d is the estimated distance between the wheels and δ_d characterizes the uncertainty on this estimation. Clearly, the robot translation and rotation are correlated accordingly to the equations (3-4). The odometry error model here proposed is based on 4 parameters. Three of them (δ_R, δ_L and δ_d) characterize the systematic components whereas the last one (K_w) characterizes the non-systematic components.

In Sect. 3 and 4 we suggest a strategy to simultaneously estimate all these parameters during the robot navigation.

3 Systematic Parameters Estimation during Navigation

In order to estimate the parameters characterizing the systematic error (both for synchronous and differential drive) we adopt the same AKF introduced by Larsen et al. [11, 12] for the differential drive. This filter estimates a state (the augmented state) containing the robot configuration and the systematic parameters, through an extended Kalman filter (EKF).

Let denote with X the robot configuration ($X = [x, y, \theta]^T$) and with X_a the augmented state. We have, respectively for the synchronous and differential drive

$$X_a = [x, y, \theta, \delta_\rho, E_\theta]^T \qquad X_a = [x, y, \theta, \delta_R, \delta_L, \delta_d]^T$$

The state X evolves accordingly to the dynamical equation $X_{i+1} = f(X_i, U_i)$ where $U_i = [\delta\rho_i, \delta\theta_i]^T$ for the synchronous drive and $U_i = [\delta\rho_i^R, \delta\rho_i^L]^T$ for the differential drive. The observation at the ith time step depends on the current robot configuration and it is assumed to be affected by an error w_i with a gaussian distribution, zero-mean and covariance matrix $R_i = < w_i w_i^T >$

$$z_i = h(X_i, w_i) \tag{5}$$

The dynamical equation for the augmented state X_a is given by the equation:

$$X_{ai+1} = f_a(X_{ai}, U_i) \tag{6}$$

The function f_a, restrictly to the first three components, is obtained directly from the function f including the dependence on the systematic parameters in the input U_i; concerning the last components (two for the synchronous drive and three for the differential drive) f_a is the identity function since there is no evolution in time for the error parameters.

In order to obtain the EKF equations for the augmented state (i.e. the equations of the AKF), it is necessary to compute the Jacobian F_a of the function f_a with respect to X_a and the Jacobian G_a of the function f_a with respect to the vector ν, which is $[\nu^\rho, \nu^\theta]^T$ in the synchronous drive (eq. (2)) and $[\nu^R, \nu^L]^T$ in the differential drive (eq. (3)):

$$F_a = \nabla_{X_a} f_a|_{X_a(i|i), \overline{U}_i} \quad G_a = \nabla_\nu f_a|_{X_a(i|i), \overline{U}_i}$$

where $X_a(i|i)$ is the state estimated at the previous time step and $\overline{U}_i$ is the mean value of the vector U_i previously defined. The computation of these matrix can be found in [11, 12] for the differential drive and can be easily carried out for the synchronous drive. Once these matrix are known it is possible to implement the AKF by applying the standard equations of the EKF ([2, 13]).

4 Non-Systematic Parameters Estimation during navigation

The non-systematic parameters cannot be evaluated following the previous method. Indeed, by including in the augmented state the non-systematic parameters, the Kalman gain related to these parameters is null.

The idea we suggest here is based on the observables defined in [14]. The observables are random variables related to a given robot motion whose properties (mean value and variance) depend on the parameters characterizing the odometry error and on the robot trajectory in the odometry reference frame. It is possible to evaluate the observable mean value only by knowing the actual initial and final configuration. We build another kalman filter where the state to be estimated contains the non-systematic parameters and the observation are directly provided by the observable mean value estimation obtained from two subsequent robot configuration estimations obtained from the AKF. Let denote with K the vector containing the non-systematic parameters. The dynamical and observational equations are:

$$K_{i+j} = f_K(K_i) = K_i \qquad z_{i+j}^{Obs} = m^{Obs}(K_{i+j}) + w_{i+j}^{Obs} \tag{7}$$

(we use $i + j$ instead of $i + 1$ to remark that the frequency of this second filter is not necessarily the same of the previous one). $m^{Obs}(K_{i+j})$ is the mean value of the chosen observable computed with the non-systematic parameters at the $(i + j)^{th}$ time step, and w_{i+j}^{Obs} is a zero-mean random variable whose covariance matrix contains both the covariance matrix of the chosen observable and the error in the robot configuration estimated by the AKF (i.e. the matrix $P_a(i|i)$ and $P_a(i+j|i+j)$), since the observable mean value is estimated from two subsequent robot configuration estimations obtained from the AKF and these estimations are affected by the error given by the matrix P_a.

In order to introduce the adopted observable we define the following quantities. Let ΔX^e, ΔY^e and $\Delta \theta^e$ the displacements respectively in the x-axis, y-axis and orientation between the $(i + j)^{th}$ and i^{th} time step as evaluated by the odometry corrected for the systematic errors by using the systematic parameters estimated by the AKF at the $(i + j)^{th}$ time step. Moreover, we denote with ΔX, ΔY and $\Delta \theta$ the same displacements as evaluated by the AKF. The observable we adopt is:

$$z^{Obs} = \left[(\Delta X - \Delta X^e)^2 + (\Delta Y - \Delta Y^e)^2, \ (\Delta \theta - \Delta \theta^e)^2 \right]^T \tag{8}$$

The mean value of the second component of this observable can be computed without approximation for any trajectory followed by the robot between the $(i + j)^{th}$ and i^{th} time step [14]. Concerning the first component the same property holds only for the synchronous drive. However, even in this case we show here the result obtained by approximating the trajectory by an arc of circumference for the sake of simplicity [15]. In the next subsection we compute the mean value of this observable for the synchronous drive. Concerning the differential drive we adopt a simpler observable consisting only of the second component of the previous observable, $z^{Obs} = (\Delta \theta - \Delta \theta^e)^2$.

4.1 Synchronous Drive

It is possible to define the robot trajectory by giving the orientation as a function of the curve length. In the synchronous drive both the orientation and the curve length are directly estimated by the odometry. We obtain for the increments in the orientation and translation between the $(i+j)^{th}$ and i^{th} time step respectively $\Delta\theta^e = \sum_{k=i}^{i+j} \delta\theta_k^e$ and $\Delta\rho^e = \sum_{k=i}^{i+j} \delta\rho_k^e$. Moreover, we obtain for the mean value of the observable in (8), ([15])

$$< z^{Obs} >= m^{Obs}(K) = \left[K_\rho \Delta\rho^e + 2(\delta_\rho \Delta\rho^e)^2 \left(\Re\{F(z)\} - \frac{1 - cos(\Delta\theta^e)}{(\Delta\theta^e)^2} \right), \ K_\theta \Delta\rho^e \right]^T \tag{9}$$

where $F(z) = \frac{z-1+e^{-z}}{z^2}$ and $z = \frac{K_\theta \Delta\rho^e}{2} + i \left(E_\theta + \frac{\Delta\theta^e}{\Delta\rho^e} \right) \Delta\rho^e$. We do not report here the computation of the covariance matrix. It can be carried out following similar computation as described in [14].

4.2 Differential Drive

From the equations (3-4) it is easy to obtain the mean value and the variance of the observable $z^{Obs} = (\Delta\theta - \Delta\theta^e)^2$ ([16]):

$$< z^{Obs} >= m^{Obs}(K) = \frac{K_w(\Delta\rho^{eR} + \Delta\rho^{eL})}{(d \ \delta_d)^2} \qquad Cov_{Obs} = \frac{2K_w^2}{d^4 \ \delta_d^4} \left(\Delta\rho^{eR} + \Delta\rho^{eL} \right)^2 \tag{10}$$

where $\Delta\rho^{eR} = \sum_{k=i}^{i+j} \left| \delta\rho_k^{eR} \right|$ and $\Delta\rho^{eL} = \sum_{k=i}^{i+j} \left| \delta\rho_k^{eL} \right|$.

The state estimated at the ith time step by the OF is, respectively for the synchronous and differential drive:

$$K = [K_\rho, K_\theta]^T \qquad K = K_w \tag{11}$$

The equations of the filter are the equations of the EKF. Clearly, the matrix $F = \nabla_K f_K$ is the identity and the matrix G is the zero-matrix since the dynamical equation in (7) is not affected by any error. The matrix H (i.e. the jacobian of the observational equation with respect to the state estimated by the filter) is, respectively for the synchronous and differential drive, the jacobian of the function in equation (9) and in equation (10) with respect to the state K in (11). Finally, the matrix R (i.e. the error matrix of the observable when the state K is known) is given by the sum of the covariance matrix of the observable (Cov_{Obs}) plus the error matrix which takes into account the errors in the used configuration estimations both at $(i+j)^{th}$ and i^{th} time step ($\left[\nabla_X z_{i+j}^{Obs} \right] [P(i+j|i+j) + P(i|i)] \left[\nabla_X z_{i+j}^{Obs} \right]^T$, where the matrix P is the submatrix of P_a containing the covariance of the robot configuration X). Observe that in the most of cases the function $m^{Obs}(K)$ is linear in K (second component in the synchronous drive and in the differential drive). However, the kalman filter is still not optimal since the distribution of w^{Obs} is not gaussian.

5 Results

We simulate a mobile robot moving in an environment consisting of a square with side measure $10m$. Therefore, the map consists of four straight line and it is a priori known. The external sensor is simulated through a function which provides the distance of the map lines from the actual robot configuration. In particular, at each time step, 36 distances are provided yielding a $10deg$ angular resolution. An error source is introduced by adding at each distance a gaussian random variable, zero-mean, and whose variance is equal to $(3cm)^2$. The random variables corresponding to different distances are independent. The errors in the encoder readings are obtained by introducing gaussian random variables accordingly to the models described in the sections 2.1 and 2.2. The AKF introduced in section 3 is used to estimate the robot configuration (x, y, θ) and the systematic parameters (δ_ρ and E_θ for the synchronous drive and δ_R, δ_L and δ_d for the differential drive). The systematic parameters are initialized in order to have a null systematic error ($\delta_\rho = 1$, $E_\theta = 0$ and $\delta_R = \delta_L = \delta_d = 1$). The non-systematic parameters are initialized at a value which differs from the actual one by a factor 100 (we both considered the cases of smaller and larger initial value obtaining similar results). Table 1 shows the values of the adopted actual parameters.

We simulated the same robot motion (a circumference with radius equal to $5m$) 100 times. The length of each robot motion is about $30m$. The error on the estimated robot configuration at each time step is about $1cm$ for

$\delta_\rho = 1.1$	$E_\theta = 1\frac{deg}{m}$	$K_\rho = (1.0e-3)m$	$K_\theta = 1\frac{deg^2}{m}$	$\delta_R = 1.1$	$\delta_L = 0.9$	$\delta_d = 1.1$	$K_w = (2.5e-4)m$

Table 1. The actual systematic and non-systematic model parameters for the synchronous and differential drive

the position and $1deg$ for the orientation (and this is consistent with the experimental results obtained in our laboratory [1]). Finally, the filter frequency is set to the same value (1 cycle per cm) for both the AKF and the OF.
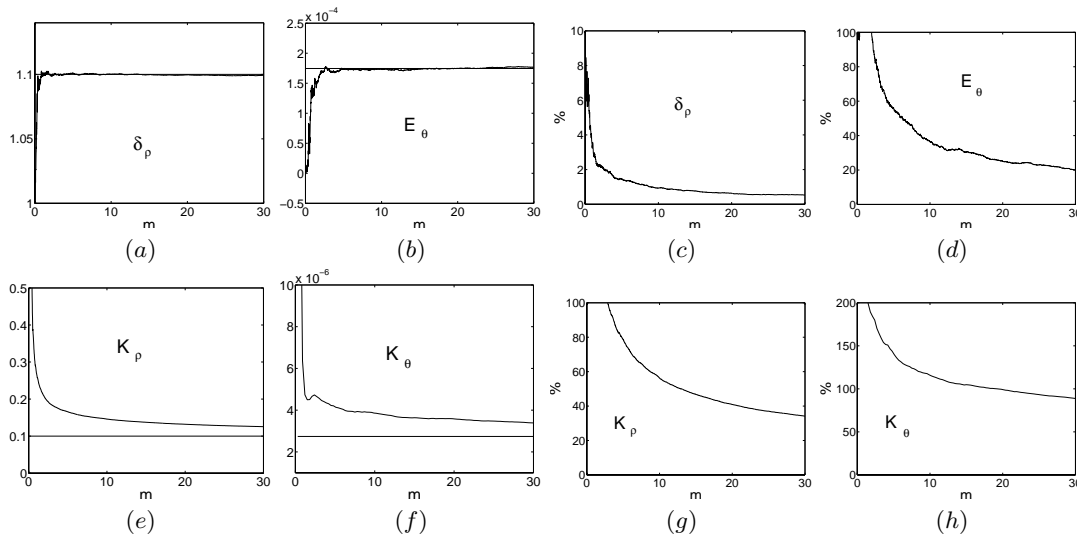

Fig. 1. Simulation results for the synchronous drive. The units adopted to represent the model parameters are rad for angle and cm for length

Fig. 1 shows the results related to the synchronous drive. Fig. 1a and 1b display the mean values of δ_ρ and E_θ at each time step i (δ_{ρ_i} and E_{θ_i}). These mean values are plotted vs the traveled distance (in m). These values are obtained from the 100 simulated robot motion (for instance, concerning the former, $\delta_{\rho_i} = \sum_{sim=1}^{100} \delta_{\rho_{isim}}$). Fig. 1c and 1d display the accuracy on the previous parameter estimations (in %) (for instance $\frac{\Delta\delta_{\rho_i}}{\delta_\rho} \times 100\%$, where $\Delta\delta_{\rho_i} = \sqrt{\frac{1}{100}\sum_{sim=1}^{100}(\delta_{\rho_{isim}} - \delta_\rho)^2}$). Figures 1e-1h show the results related to the non-systematic parameters. The frequency of the OF is the same as for the AKF (i.e. $j = 1$ in the equations (7-9)). Fig. 1e and 1f show the obtained mean values of K_ρ and K_θ at each time step. Fig. 1g and 1h show the accuracy on the previous estimated parameters in %.

Fig. 2 show the results related to the differential drive. We plot the same quantities as in the previous case. Fig. 2a-f concern the systematic parameters and fig. 2g and 2h concern the non-systematic parameter K_w.

We can conclude that it is possible to reach good accuracy on the parameter estimation by moving the mobile robot along quite short distances ($30m$)

6   Conclusions and Future Research

A new filter, the OF, was introduced for the estimation of the non-systematic odometry error during the robot navigation. This filter is based on the Observables (introduced in a previous work [14]) which provide the observations for an EKF which estimates a state containing the parameters characterizing the non-systematic odometry error. When this new filter is used together with the AKF (introduced by Larsen et al. [11, 12] and here extended to the case of a mobile robot with a synchronous drive) the simultaneous estimation of the systematic and non-systematic odometry error can be carried out during the robot navigation. Both cases of synchronous and differential drive were considered and the performance of the proposed method was successfully tested through simulations. We are implementing the proposed strategy on a real mobile platform.
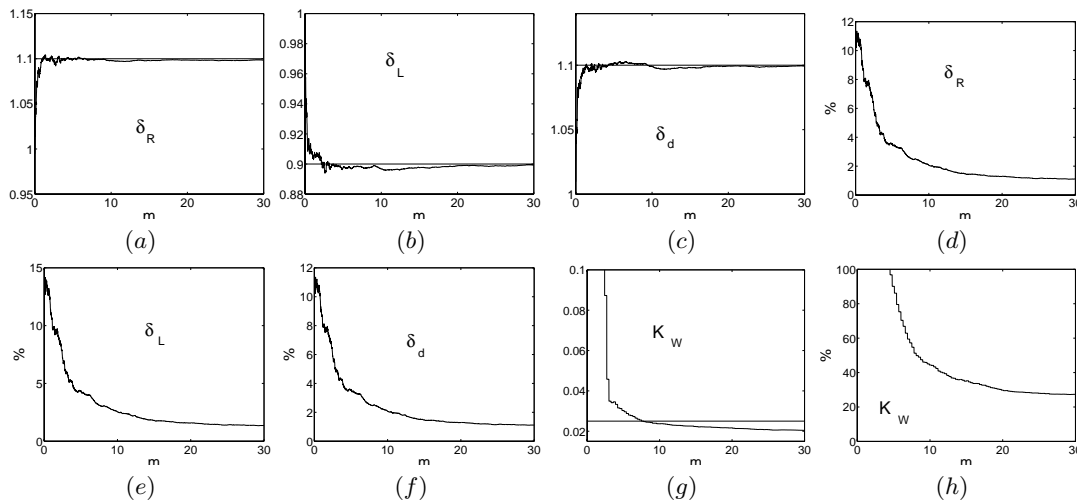
Fig. 2. Simulation results for the differential drive. The units adopted to represent the model parameters are *rad* for angle and *cm* for length

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