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The Accuracy on the Parameter Estimation of an Odometry System of a Mobile Robot *

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Abstract

In this paper the odometry error of a mobile robot with a synchronous drive system is modeled by introducing four parameters characterizing its systematic and non-systematic components (translational and rotational). A strategy in order to simultaneously estimate the model parameters is suggested. This strategy only requires to measure the change in the orientation and in the position between the initial and the final configuration of the robot related to suitable robot motions. In other words it is unnecessary to know the actual path followed by the robot. The proposed strategy is illustrated by discussing the accuracy on the parameters estimation both in an indoor and outdoor environment.

Key Words: Robot Navigation, Odometry, Localization

1 Introduction

Determining the odometry errors of a mobile robot is very important both in order to reduce them, and to know the accuracy of the state configuration estimated by using encoder data. Odometry errors can be both systematic and non-systematic.

In a series of papers Borenstein and collaborators [1, 2, 3, 4, 5, 6, 16] investigated on possible sources of both kind of errors. A review of all the types of these sources is given in [6]. In the work by Borenstein and Feng [5], a calibration technique called UMBmark test has been developed to calibrate for systematic errors of a two wheel robot. This method has been used by other authors [7]. Goel, Roumeliotis and Sukhatme [8] used another calibration procedure to compensate systematic errors. They referred to the differential drive mobile robot Pioneer AT. Finally, Roy and Thrun [14] suggested an algorithm that uses the robot's sensors to automatically calibrate the robot as it operates. In a series of papers Borenstein ([6] and reference therein) suggested also a method, called IPEC (Internal Position Error Correction), to improve the accuracy of the odometry data by reducing the effect of the non-systematic errors. Experimental results showed that the accuracy achieved with the IPEC method was one to two orders of magnitude better than that one of systems based on conventional dead-reckoning.

Many investigations have been carried out on the odometry error from a theoretical point of view. Wang [15] and Chong and Kleeman [7] analyzed the non-systematic errors and computed the odometry covariance matrix Q for special kind of the robot trajectory. Kelly [9] presented the general solution for linearized systematic error propagation for any trajectory and any error model. Martinelli [10], [11] and [12] derived general formulas for the covariance matrix Q and also suggested a strategy to estimate the model parameters characterizing the odometry error. This strategy is based on the evaluation of the mean values of some quantities (called *observables*) which depend on the model parameters and on the chosen robot motion.

In this paper we introduce two new observables in order to improve the accuracy on the model parameters estimation. The odometry error model is the same as in [10] and it is here summarily discussed in Sect. 2. In Sect. 3 we introduce the two new observables. On the basis of our odometry error model we analytically compute the mean values and the variances of the observables, which depend on the model parameters and on the considered robot motion. In Sect. 4 we explicitly compute the observables for a simple robot motion. Evaluating the observables for this robot motion is a possible strategy in order to simultaneously estimate the model parameters. This strategy only requires to measure the change in the orientation and in the position between the initial and the final configuration of the robot related to the considered robot motion. In other words it does not require to know the actual path followed by the robot. The proposed strategy is illustrated in Sect. 5 where the accuracy

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on the parameters estimation, by adopting the two introduced observables, is discussed both in an indoor and outdoor environment.

2 The odometry error model

We consider a mobile robot with a synchronous drive system. Assuming a two-dimensional world, we can define the robot configuration with respect to a world-coordinate frame W by the vector $X = [x, y, \theta]^T$, containing its position and orientation. The robot configuration estimated by odometry measurements is different from the actual configuration X because of the odometry errors. In order to compute the global odometry error related to a given robot motion we approximated the trajectory with N small segments. We firstly model the elementary error related to a single segment. Then we compute (next sections) the cumulative error on the global path. Finally we take the limit value when $N \to \infty$.

We introduce the following assumptions about the actual motion:

- the robot moves straight along each given segment whose length, measured by the encoder sensor, is always $\overline{\delta\rho} = \frac{\overline{\rho}}{N}$;
- the angle $\widehat{\delta\theta}_i$ between the actual orientations related to the $(i+1)^{th}$ and the i^{th} segment and the actual translation $\widehat{\delta\rho}_i$ covered during the same step are gaussian random variables;
- the random variable $\widehat{\delta\rho}_i$ is independent of the random variable $\widehat{\delta\theta}_i$. Moreover $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\rho}_j$ $(i \neq j)$ and $\widehat{\delta\theta}_i$ is independent of $\widehat{\delta\theta}_j$.

We therefore can write:

$$\widehat{\delta\rho}_i \sim N(\overline{\delta\rho}(1 + E_T), \sigma^2_{\delta\rho}) \tag{1}$$

$$\widehat{\delta\theta}_i \sim N(\overline{\delta\theta_i} + E_R\overline{\delta\rho}, \sigma^2_{\delta\theta}) \tag{2}$$

where $\overline{\delta\theta_i}$ is the angle between the orientations related to the $(i+1)^{th}$ and the i^{th} segment measured by the encoder sensor, $E_T\overline{\delta\rho}$ and $E_R\overline{\delta\rho}$ represent the systematic components of the error and $\sigma^2_{\delta\theta}$ and $\sigma^2_{\delta\rho}$ are directly related to the rolling conditions and are assumed to increase linearly with the traveled distance, i.e.:

$$\sigma^2_\theta = K_\theta \overline{\rho} \tag{3}$$

and

$$\sigma^2_\rho = K_\rho \overline{\rho} \tag{4}$$

The odometry error model here proposed is based on 4 parameters. Two of them (E_R, E_T) characterize the systematic components while the other two (K_θ, $K\rho$) characterize the non-systematic components. Clearly, these parameters depend on the environment where the robot moves.

We want to remark that a statistical treatment of the non-systematic component assumes the environment homogeneous on large scale. Therefore, the expressions we are deriving in the next sections hold if the robot moves on regions larger than the scale beyond it the environment can be considered homogeneous.

3 The Observables

The observables are measurable quantities related to a given robot motion. Their mean values depend on the parameters E_R, E_T K_θ and K_ρ and therefore estimating the observables is a possible strategy to estimate the model parameters.

Let consider a given robot motion and let suppose to repeat it n times. The robot motion is always the same in the world coordinate frame of the odometry system. In [11] the observables $Obs_{\hat{x}} = \frac{1}{n}\sum_{j=1}^{n} x_j$ and $Obs_{\hat{y}} = \frac{1}{n}\sum_{j=1}^{n} y_j$ were introduced and their statistical properties were derived. x_j and y_j are the position change respectively along the $\hat{x}$-axis and $\hat{y}$-axis between the initial and the final configuration related to the j^{th} robot motion.

We now introduce the following new observable:

$$Obs_{\hat{\theta}} = \frac{1}{n}\sum_{j=1}^{n} l_j \tag{5}$$

where l_j is the position change along the $\hat{\theta}$-axis (fig. 1).

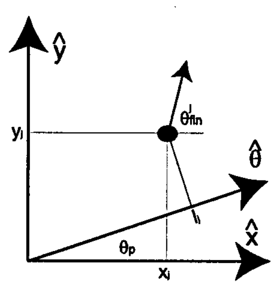

Figure 1: The robot configuration after the j^{th} robot motion and the projections of its position along the $\hat{x}$-axis (x_j), $\hat{y}$-axis (y_j) and $\hat{\theta}$-axis (l_j).

Clearly when $\hat{\theta}$ coincides with $\hat{x}$ we obtain $Obs_{\hat{x}}$ and analogously for $\hat{y}$. Let θ_p be the angle between the $\hat{\theta}$-axis and the $\hat{x}$-axis. From the fig. 1 it is easy to obtain for the j^{th} robot motion $l_j = cos\theta_p\ x_j + sin\theta_p\ y_j$ and therefore:

$$Obs_{\hat{\theta}} = cos\theta_p\ Obs_{\hat{x}} + sin\theta_p\ Obs_{\hat{y}} \tag{6}$$

$Obs_{\hat{\theta}}$ coincides with $Obs_{\hat{x}}$ whose robot motion has an initial orientation θ_{in} decreased by θ_p.

The computation of its variance follows from the formula

$$\sigma^2_{Obs_{\hat{\theta}}} = cos^2\theta_p\ \sigma^2_{Obs_{\hat{x}}} + sin^2\theta_p\ \sigma^2_{Obs_{\hat{y}}} + 2\ sin\theta_p\ cos\theta_p\ \times$$

$$\times(< Obs_{\hat{x}}Obs_{\hat{y}} > - < Obs_{\hat{x}} >< Obs_{\hat{y}} >) \tag{7}$$

and therefore requires to compute the mist product $< Obs_{\hat{x}}Obs_{\hat{y}} >$. This mean value is given in [11].

In the next section we analytically compute the optimal values of θ_p (i.e. the optimal projection axis) in order to maximize the dependence of $< Obs_{\hat{\theta}} >$ on the model parameters. Since the dependence on the parameter K_θ is very weak we introduce the following observables:

$$Obs_{\hat{x}\theta} = \frac{1}{n(n-1)} \sum_{jj'}^{n} x_j\Delta_j - x_j\Delta_{j'} \tag{8}$$

$$Obs_{\hat{y}\theta} = \frac{1}{n(n-1)} \sum_{jj'}^{n} y_j\Delta_j - y_j\Delta_{j'} \tag{9}$$

where Δ_j is the angular difference between the initial and the final orientation ($\Delta_j = \theta^j_{fin} - \theta_{in}$).

Previous observables were also discussed in [12] where their statistical properties were derived. We finally introduce the following observable:

$$Obs_{\hat{\theta}\theta} = \frac{1}{n(n-1)} \sum_{jj'}^{n} l_j\Delta_j - l_j\Delta_{j'} \tag{10}$$

where l_j is still the position change along the $\hat{\theta}$-axis. We have the following relation:

$$Obs_{\hat{\theta}\theta} = cos\theta_p\ Obs_{\hat{x}\theta} + sin\theta_p\ Obs_{\hat{y}\theta} \tag{11}$$

From this expression it is possible to compute its mean value and its variance.

4 The optimal projection axis

The two observables $Obs_{\hat{\theta}}$ and $Obs_{\hat{\theta}\theta}$ depend on the angle θ_p between the $\hat{x}$ and $\hat{\theta}$-axis. The aim of this section is to find the optimal values of θ_p in order to maximize the accuracy on the parameter estimation reachable with these observables and the following simple robot motion: the robot moves straight forth and back k times in order to cover a fixed distance $\bar{p} = 2kl$ (measured by the encoder sensor). The initial orientation of the robot is assumed to be $\theta_{in} = \theta_0 = 0$. To compute the mean values and the variances of the previous observables for this robot motion it is very useful to introduce the following complex quantity:

$$z = \frac{K_\theta l}{2} + iE_R l \tag{12}$$

This complex quantity characterizes the rotational components of the odometry error. In particular its real part contains the non-systematic component while the imaginary part the systematic one. It is possible to obtain (see [11], [12])

$$< Obs_{\hat{x}} > -i < Obs_{\hat{y}} >= (1 + E_T)lf(z) \tag{13}$$

$$< Obs_{\hat{x}\theta} >= -K_\theta(1 + E_T)l^2 Im\left\{\frac{\partial f}{\partial z}\right\} \tag{14}$$

$$< Obs_{\hat{y}\theta} >= -K_\theta(1 + E_T)l^2 Re\left\{\frac{\partial f}{\partial z}\right\} \tag{15}$$

where $f(z) = \frac{(1-2e^{-z}+e^{-2z})(e^{-2zk}-1)}{z(e^{-2z}-1)}$

From the equation (13) we see that the real part of $(1 + E_T)lf(z)$ gives the mean value of $< Obs_{\hat{x}} >$ and the opposite of the imaginary part the mean value of $< Obs_{\hat{y}} >$.

From equations (6) and (13) we obtain

$$< Obs_{\hat{\theta}} >= (1 + E_T)\ l\ |f(z)|\ cos(\theta_p + \varphi) \tag{16}$$

From equations (11), (14) and (15) we obtain

$$< Obs_{\hat{\theta}\theta} >= (1 + E_T)\ l\ \left|\frac{\partial f}{\partial z}\right|\ sin(\theta_p + \varphi') \tag{17}$$

where φ and φ' are respectively the phases of the complex quantities $f(z)$ and $\frac{\partial f}{\partial z}$.

In order to find the optimal projection axis we have to find the values of θ_p maximizing the derivatives of the previous mean values with respect to the model parameters. Concerning $< Obs_{\hat{\theta}} >$ we obtain from equation (16)

$$\left|\frac{\partial < Obs_{\hat{\theta}} >}{\partial E_T}\right| = l\ |f(z)|\ |cos(\theta_p + \varphi)| \tag{18}$$

$$\left|\frac{\partial < Obs_{\hat{\theta}} >}{\partial E_R}\right| = (1+E_T)l^2 \left|\frac{\partial f}{\partial z}\right| |sin(\theta_p + \varphi')| \quad (19)$$

and finally

$$\left|\frac{\partial < Obs_{\hat{\theta}} >}{\partial K_\theta}\right| = (1+E_T)\frac{l^2}{2} \left|\frac{\partial f}{\partial z}\right| |cos(\theta_p + \varphi')| \quad (20)$$

The previous three equations give therefore the following optimal values of θ_p:

$$\theta_{p,E_T} = i\pi - \varphi \qquad i = ... -2, -1, 0, 1, 2... \quad (21)$$

$$\theta_{p,E_R} = \frac{(2i+1)\pi}{2} - \varphi' \qquad i = ... -2, -1, 0, 1, 2... \quad (22)$$

and

$$\theta_{p,K_\theta} = i\pi - \varphi' \qquad i = ... -2, -1, 0, 1, 2... \quad (23)$$

As we will show in the next section $< Obs_{\hat{\theta}} >$ depends very weakly on K_θ. On the other hand $< Obs_{\hat{\theta}\theta} >$ depends very weakly on E_R and E_T. Here we only consider the derivative of $< Obs_{\hat{\theta}\theta} >$ with respect to K_θ. We obtain from equation (17):

$$\left|\frac{\partial < Obs_{\hat{\theta}\theta} >}{\partial K_\theta}\right| = \left|\frac{\partial < Obs_{\hat{\theta}} >}{\partial E_R} - \left|\frac{\partial^2 f}{\partial z^2}\right| \times \right.$$

$$\left. \times (1+E_T)K_\theta \frac{l^3}{2} sin(\theta_p + \varphi'')\right| \quad (24)$$

where φ'' is the phase of the complex quantity $\frac{\partial^2 f}{\partial z^2}$.

Regarding the computation of the variances we use the general expressions given in section 3. We only need to compute $\sigma^2_{Obs_{\hat{x}}}$, $\sigma_{Obs_{\hat{x}}Obs_{\hat{y}}}$ and $\sigma^2_{Obs_{\hat{y}}}$ related to the considered robot motion. The computation of these quantities can be carried out starting from the general expressions given in the section 3 and is similar (more troublesome) to the computation of $< x > -i < y >$ given in [12].

Our strategy consists of the estimation of the mean values of the observables for the considered robot motion. The advantage of this strategy is that we only need to consider the initial and the final configuration of the real robot motion. In the next section we discuss the accuracy reachable on the parameter estimation through this strategy.

Env.	$E_R(\frac{deg}{m})$	$K_\theta(\frac{deg^2}{m})$	E_T	$K_\rho(m)$
Ind.	-0.20	0.010	-0.020	$4.0\ 10^{-5}$
Out.	-0.20	1.0	-0.020	$4.0\ 10^{-3}$

Table 1: The model parameters used to characterize the indoor and the outdoor environment

5 Discussion

We discuss the accuracy on the parameters estimation reachable by adopting the proposed observables $Obs_{\hat{\theta}}$ and $Obs_{\hat{\theta}\theta}$ as a function of the angle θ_p and the model parameters. We consider several values of the model parameters. In particular we change the non-systematic parameters. Indeed, K_θ and K_ρ strongly depend on the environment. In particular the parameters in table 1 represent typical values of the parameters for an indoor environment ([11]) and for an outdoor environment like asphalt.

We do not consider here the parameter K_ρ. In [11] we introduced an observable depending on this parameter and in [12] and [13] we discussed the accuracy on its estimation through that observable.

We assume that the errors associated to the measurement of the change in position and orientation between the initial and final configuration are smaller than the variances of the observables (with $n = 1$ in the case of $Obs_{\hat{\theta}}$) and therefore the accuracy in the parameter estimation only depends on the observable variances.

Figures 2-5 show the accuracy on the parameters estimation vs the parameter θ_p in an indoor and outdoor environment. We set $n = 20$ (the number of motion repetition), $l = 5\ m$ and $k = 1$. Therefore the considered experiment requires to move the robot along a global distance equal to $2kl \times n = 200\ m$. Previous values are chosen on the basis of the discussion given in [11] and [12], where we showed that the accuracy on the parameters E_R, E_T and K_θ decreases with k. Moreover the accuracy increases of course with the distance traveled by the robot but the improvement of the accuracy is not significative for distances larger than $200\ m$.

The error on the estimation of a given model parameter (for example E_R) using the observable Obs_i is given by $\Delta E_R = \frac{\Delta < Obs_i >}{\left|\frac{\partial < Obs_i >}{\partial E_R}\right|} = \frac{\sigma_{Obs_i}}{\left|\frac{\partial < Obs_i >}{\partial E_R}\right|}$. Therefore the relative error on the estimation of E_R (in %) using the observable Obs_i is given by $\frac{1}{E_R}\frac{\sigma_{Obs_i}}{\left|\frac{\partial < Obs_i >}{\partial E_R}\right|} \times 100$. Regarding the parameter E_T we actually consider as a relative error the quantity $\frac{\Delta(1+E_T)}{1+E_T} = \frac{\Delta E_T}{1+E_T}$, that is much smaller than $\frac{\Delta E_T}{E_T}$. Clearly, the accuracy on the

Environment	θ_{p,E_R}	θ_{p,E_T}	θ^a_{p,K_θ}	θ^b_{p,K_θ}
Indoor	88	89	178	88
Outdoor	88	86.5	178	88

Table 2: The optimal orientation of the projection axis for the indoor and outdoor environment as obtained from the equations (21)-(24). θ^a_{p,K_θ} is obtained with the observable $Obs_{\hat{\theta}}$ (equation (23)), θ^b_{p,K_θ} with $Obs_{\hat{\theta}\theta}$ (i.e. it is the value of θ_p maximizing the quantity on the right hand side of the equation (24))

estimation of a given model parameter improves as the derivative of the mean value of the considered observable with respect to that parameter increases. In section 3 we found the values of θ_p maximizing the above derivative. Actually, the optimal θ_p should minimize the ratio $\frac{\sigma_{Obs_i}}{\left|\frac{\partial <Obs_i>}{\partial E_R}\right|}$. On the other hand the variance σ_{Obs_i} weakly depends on θ_p and in fact the optimal values of θ_p from the figures 2-5 are about the values we obtain from equations (21)-(24), given in table 2.

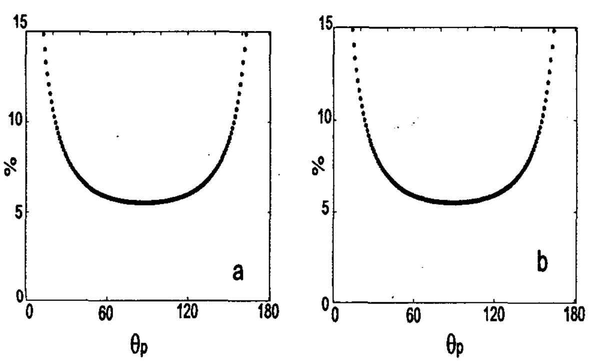

Figure 2: The relative error (in %) on the parameter E_R (a) and E_T (b) estimated by using the observable $Obs_{\hat{\theta}}$, in an indoor environment. The angle θ_p in the $x-$axis characterizes the orientation of the $\hat{\theta}$-axis as explained in section 3. The other parameters introduced in sections 3 and 4 are here set to: $n = 20$, $k = 1$, $2\ k \times l = 10\ m$.

Concerning the systematic parameter E_R we found that the optimal value of θ_p depend very weakly on the model parameters. In particular the change is not appreciable in figures 2a and 4a where the systematic parameters have a change of two orders. The best value of θ_p is very close to $90deg$ ($Obs_{\hat{\theta}} \simeq Obs_{\hat{y}}$)

Regarding E_T the value θ_p decreases by increasing the value of the systematic parameters, but the change is still very small. In any case the best value of θ_p is still close to $90deg$ (figures 2b and 4b and table 2).

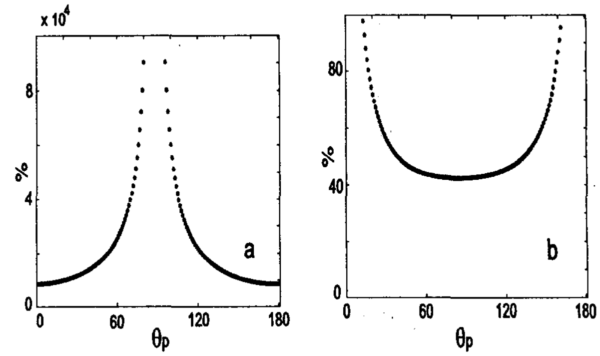

Figure 3: The relative error (in %) on the parameter K_θ estimated by using the observable $Obs_{\hat{\theta}}$ (a) and the observable $Obs_{\hat{\theta}\theta}$ (b) in an indoor environment. The angle θ_p in the $x-$axis characterizes the orientation of the $\hat{\theta}$-axis as explained in section 3. The other parameters introduced in sections 3 and 4 are here set to: $n = 20$, $k = 1$, $2\ k \times l = 10\ m$.

Concerning the non-systematic parameter K_θ (fig 3, 5) we conclude that the observable $Obs_{\hat{\theta}}$ is definitely inadequate for its estimation, especially in the indoor case (fig 3a). On the other hand by adopting the observable $Obs_{\hat{\theta}\theta}$ it is possible to reach an accuracy of about 40% (fig 3b and 5b).

6 Conclusions and Future Research

This paper extends previous work in parameter estimation for mobile robot with a synchronous drive system. In particular two new observables are introduced and the accuracy on the parameter estimation using these observables is discussed.

We are investigating in order to check the validity of the proposed model. The assumption that $\widehat{\delta\rho}_i$ is independent of $\widehat{\delta\theta}_i$ is clearly a simplified approximation. A disturbance on the robot trajectory can generate both a distance error and a dependent angle error. This assumption enables us to only consider the back and forth motion to estimate the model parameters. This robot motion can be easly implemented. On the other hand, it would be very interesting to do actual trial using robot motion with different shape.

Moreover, more sophisticated model should also take into account the error dependence on the robot velocity and acceleration.

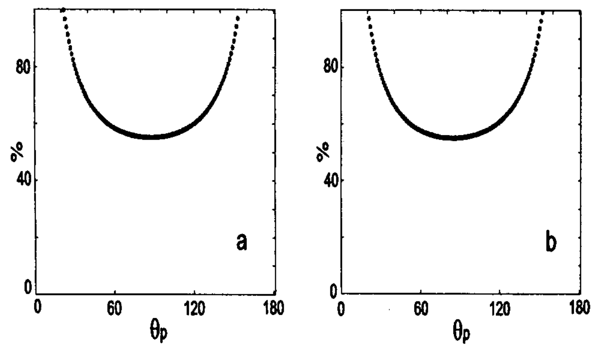

Figure 4: The relative error (in %) on the parameter E_R (a) and E_T (b) estimated by using the observable $Obs_{\hat{\theta}}$, in an outdoor environment. The angle θ_p in the $x-$axis characterizes the orientation of the $\hat{\theta}$-axis as explained in section 3. The other parameters introduced in sections 3 and 4 are here set to: $n = 20$, $k = 1$, $2\ k \times l = 10\ m$.

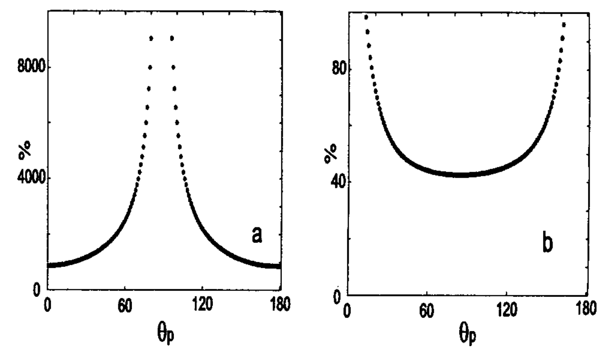

Figure 5: The relative error (in %) on the parameter K_θ estimated by using the observable $Obs_{\hat{\theta}}$ (a) and the observable $Obs_{\hat{\theta}\theta}$ (b) in an outdoor environment. The angle θ_p in the $x-$axis characterizes the orientation of the $\hat{\theta}$-axis as explained in section 3. The other parameters introduced in sections 3 and 4 are here set to: $n = 20$, $k = 1$, $2\ k \times l = 10\ m$.

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