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CONSTITUTIVE MODELLING OF UNSATURATED SANDY SILT

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ABSTRACT: The mechanical behaviour of a sandy silt is characterised experimentally in saturated and unsaturated states. First, these test data are used to examine the performance of a constitutive model developed by Alonso *et al*. (1990) for partially saturated soils. The corresponding parameters are determined and predictions based on this model are compared with the experimental data. Then a new approach based on the Disturb State Concept (DSC) is proposed to incorporate some typical features of unsaturated soils to obtain better predictions.

1. INTRODUCTION

Until 1990, strength and volumetric behaviours of unsaturated soils had been modelled separately (Fredlund *et al*., 1978, Lloret & Alonso, 1985). The first works taking into account both aspects were based on an extension of the theories developed for saturated soils. In 1990 Alonso *et al.* proposed a constitutive model using the critical state concept to couple strength and volumetric response in unsaturated soils. This model is used in this paper to simulate experimental data of an unsaturated sandy silt.

In the first part, a brief presentation of the experimental characterisation of the behaviour of the material is given. Then the determination of the constitutive parameters for the model is explained. Using the model, predictions are made for some stress paths and compared with experimental values. The last section proposes a new approach based on the Disturb State Concept to predict the unsaturated soil behaviour.

2. EXPERIMENTAL CHARACTERISATION

The authors have undertaken a research on an unsaturated sandy silt, which involves rheological characterisation performing both extensive testing procedures under different stress paths and numerical simulations. Part of the experimental results are used in this paper.

2.1 *Material and apparatus*

The studied soil is a washing sandy silt from the region of Sion (Switzerland). Its characteristics are given in Table 1.

Table 1: Characteristics of the Sion silt

w_L(%)	w_P(%)	I_P	%<2μm	%>60μm	γ_s(kN/m^3)
25	17	8	8	20	27.4

The sample preparation is explained in Laloui *et al.*, (1995).

Standard experimental apparatus were adapted to run unsaturated tests with the air-pressure technique: triaxial cells, oedometers and Richards cells (pressure plate extractors). This technique consists in imposing air-pressure at the top of the sample, while the pore water-pressure is measured or imposed at the bottom, where a special high air entry value ceramic is located (for details see Laloui *et al.*, 1997). The suction s is then obtained as the excess of pore air-pressure u_a to water-pressure u_w:

$$s = u_a - u_w \tag{1}$$

2.2 *Solicitation paths*

The different types of experimented stress paths are summarised by Figure 1. σ_1 and σ_3 are the principal vertical and lateral stresses. Samples are submitted to mechanical loading at different initial suctions and to hydric loading (variation of suction) at different initial mechanical stresses.

In the triaxial apparatus, samples are first

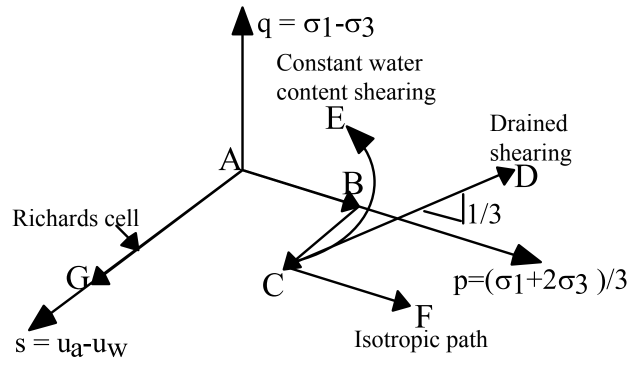

Fig. 1: Hydro-mechanical stress paths

consolidated to a given confining pressure (A-B) in saturated conditions. This should erase all the mechanical history of the sample as the confining pressure is always higher than the preconsolidation pressure of the fabricated soil. Then, in an unsaturated test, samples are submitted to an hydric path (B-C): the suction increases with the application of u_a, while keeping u_w equal to the atmospheric pressure. Finally, when the equilibrium is reached (point C), several stress paths are analysed separately:

1- C-F : *"isotropic"* condition. This path is obtained by increasing the confining pressure p while keeping both u_a and u_w constant (thus allowing free flow of air and water).

2- C-E : *"constant water content shearing"* condition. This path is driven in water undrained conditions while maintaining a constant air-pressure u_a. The water-pressure u_w is measured during the shearing, so that the suction is known.

3- C-D: *"drained shearing test"*. It consists in increasing the deviatoric stress while keeping both u_a and u_w constant.

In the Richards cell, the sample is submitted to an hydric path (A-G) without any "mechanical" stress.

3. ALONSO *et al*. CONSTITUTIVE MODEL

3.1 Introduction to the model

The constitutive law proposed by Alonso *et al.* is based on the modified Cam-Clay model. It incorporates many features of unsaturated soils. This model is formulated within the framework of hardening plasticity using two independent sets of stress variables: the excess of total mean pressure over air pressure $p^* = p-u_a$ (defined as the net mean pressure) and the suction s. The detailed description of the model is given in Alonso *et al.* (1990).

3.1.1 Parameters associated with isotropic behaviour (loading-collapse yield curve)

– $\lambda(0)$: compressibility coefficient for the saturated state along virgin loading
– κ: compressibility coefficient along elastic (unloading-reloading) stress paths

When the samples are unsaturated, two more parameters are needed to describe the evolution of the compressibility λ with the suction:

$$\lambda(s)=\lambda(0)\ [(1-r)\exp(-\beta s)+r] \tag{2}$$

- r: establishes the minimum value of the compressibility coefficient for high values of suction
- β: controls the rate of increase in stiffness with suction
- p^c is the reference stress

3.1.2 Parameters associated with changes in suction

- λ_s: compressibility coefficient for increments of suction across virgin states
- κ_s: compressibility coefficient for changes in suction within the elastic region

3.1.3 Parameters associated with changes in shear stress and shear strength

- ν: Poisson's ratio
- M: the slope of the critical state line
- k: parameter that controls the increase in cohesion with the suction.

3.2 Determination of the parameters

The determination of the parameters for saturated and unsaturated states is explained in the following sections.

3.2.1 Parameters associated with isotropic behaviour

$\lambda(0)$ and κ are deduced from Figure 2 showing an isotropic consolidation test of the saturated silt. This silt shows an increase in mechanical compressibility $\lambda(0)$ with the mean pressure for the experimented stress range. As a result, two compressibilities (slopes in the e-lnp diagram) are chosen for two different ranges of stresses: $\lambda_1(0)=0.032$ for a mean pressure p between 100 and 400 kPa and $\lambda_2(0)=0.047$ for p higher than 400 kPa. The unloading slope κ is around $\kappa=0.007$.

The parameters r and β are deduced from unsaturated oedometric tests at different constant suctions (Charlier *et al.*, 1997). Figure 3 shows the calibration of parameters for the compressibility decrease with the suction as the model suggests. The best fit was find with r=0.65 and $\beta=0.005\ kPa^{-1}$. The experimental data also show that the unloading slope κ can be considered as independent of the suction, as proposed in the model.

The last term associated with the isotropic stress conditions is the reference stress p^c. It is chosen as equal to 20 kPa.

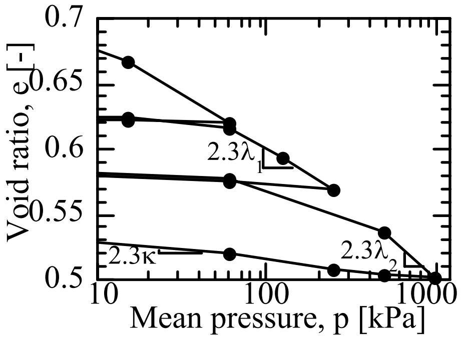

Fig. 2: Consolidation in saturated conditions

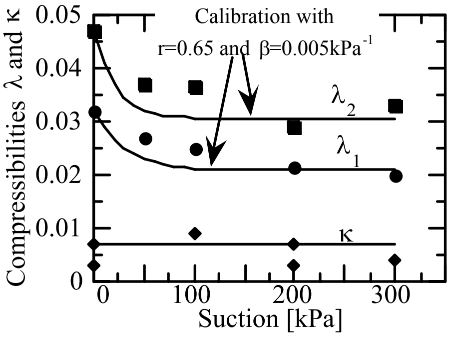

Fig. 3: Evolution of the compressibilities with the suction

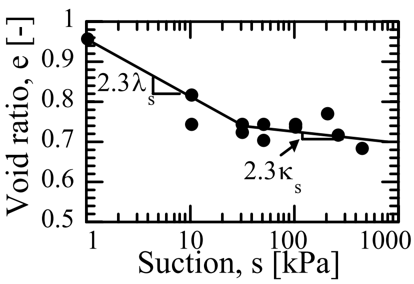

Fig. 4: Hydric compressibilities

3.2.2 Parameters associated with changes in suction

λ_s and κ_s are deduced from the Richards cell tests on the drying paths (A-G). Figure 4 shows the evolution of the void ratio vs. suction.

We can notice that the suction has the same type of influence as a mechanical stress, but with different compressibility slopes: λ_s=0.06 and κ_s=0.009. As long as the sample is almost saturated (s<30kPa, see Laloui *et al*., 1997) the void ratio decreases highly with suction.

3.2.3 Parameters associated with changes in shear stress and shear strength

The Poisson's ratio is deduced from the unloading

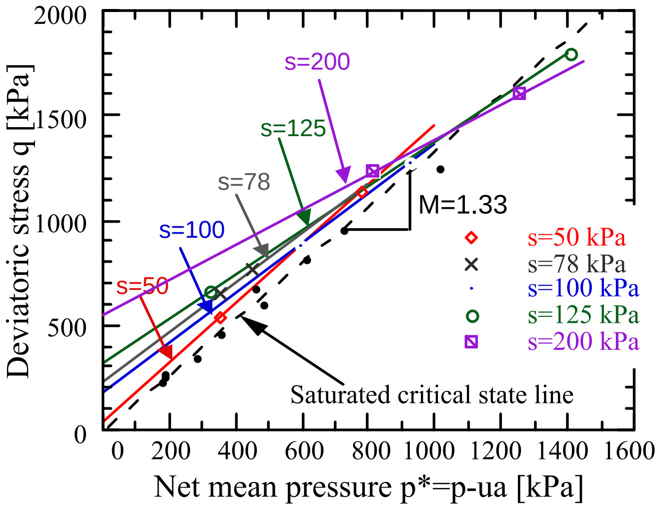

Fig. 5: Evolution of the critical state line with the suction

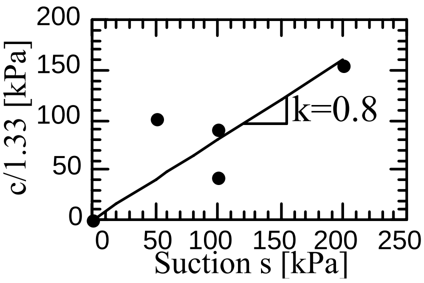

Fig. 6: Determination of the parameter k from the experimental results.

slopes of the deviatoric saturated triaxial tests: ν is taken as equal to 0.4.

To find the parameters M and k we use the critical state points from the shearing tests (paths C-D and C-E) in the p* vs. q (deviatoric stress) plane. Figure 5 shows a decrease of the slope of the critical state line M with the suction. Simultaneously the cohesion c (value of the deviatoric stress at p*=0) increases with the suction. Alonso *et al.* assume that the slope of the critical state line is independent of the suction s. Only the cohesion increases with the suction.

The slope M is equal to 1.33 in saturated conditions. As the model does not take in account an increase of M, the cohesion c is calculated for each critical point with the slope of the critical state line in saturated conditions:

$$c = q - 1.33 * (p - u_a) \tag{3}$$

Figure 6 shows the points resulting from this assumption for the drained tests. The parameter k is chosen to be equal to 0.8.

The application of the model requires of course, in addition to the constitutive parameters, information on the initial state of stress, suction and void ratio of each test.

3.3 Comparison of model predictions with *experimental data*

Two types of stress paths were predicted with the Alonso *et al* model: isotropic path (C-F) and drained shearing path (C-D).

3.3.1 Isotropic path

Figure 7 shows the predictions of the model for the isotropic behaviour of the Sion sandy silt at different saturation states. Note that on the figure the volumetric strain represents the water volume change divided by the total volume.

At small suctions (100 kPa) the observed and predicted behaviours do not match very well. The more the suction increases, the more the predictions approach the experiment. At a suction of 100 kPa, the model predicts an elastic domain up to a mean pressure of about 500 kPa, what is not observed in the experiment. However the observed and calculated slopes are not very different. Nevertheless the predicted volume variations are underestimated because of this elastic domain.

For higher suctions (s=200 and 280 kPa), the

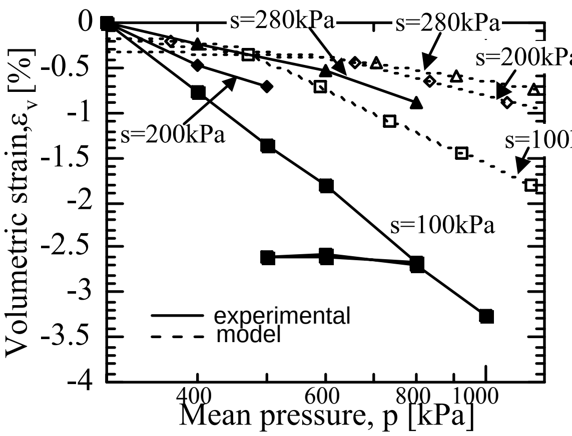

Fig. 7: comparison of the numerical predictions (Alonso *et al*. model) and the experimental data.

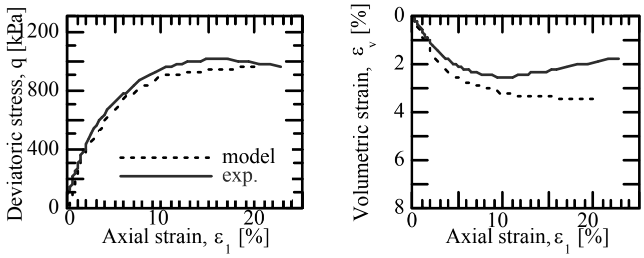

Fig.8:Drained shearing test, σ_3=400 kPa, s=0 kPa

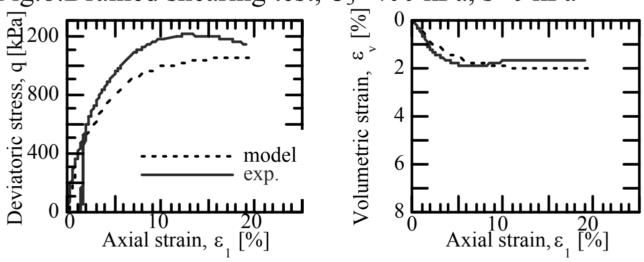

Fig.9:Drained shearing test, σ_3=450 kPa, s=50 kPa

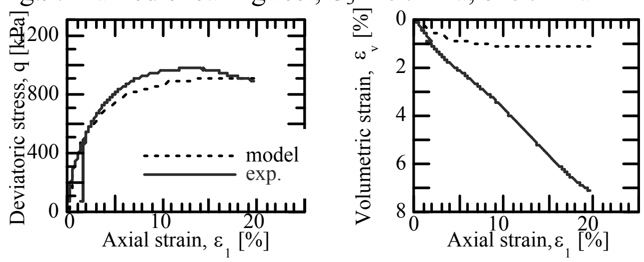

Fig.10:Drained shearing test, σ_3=400 kPa, s=100 kPa

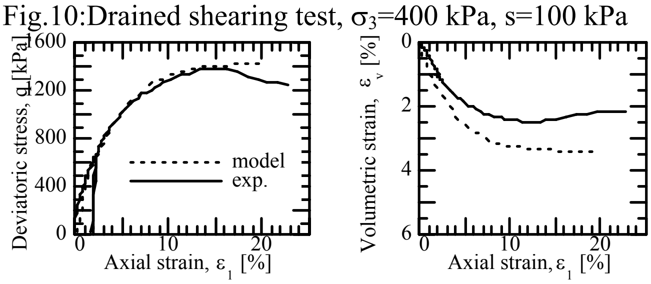

Fig.11:Drained shearing test, σ_3=600 kPa, s=0 kPa

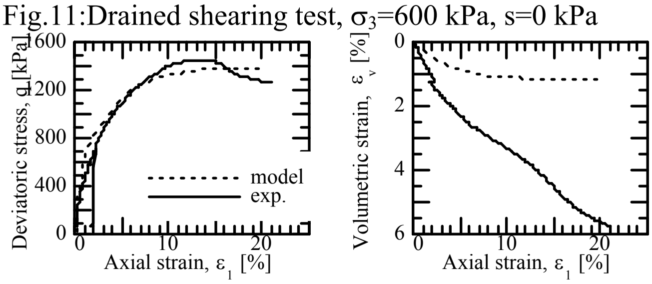

Fig.12:Drained shearing test, σ_3=600 kPa, s=100 kPa

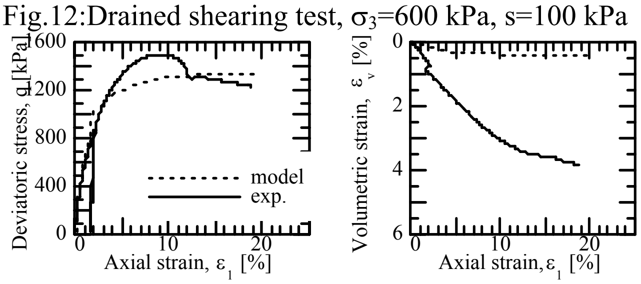

Fig.13:Drained shearing test, σ_3=600 kPa, s=200 kPa

elastic domain of the model increases. The experience also shows a pseudoelastic phase up to mean pressures of about 600 kPa. In these conditions, with close slopes, the volumetric predictions are still underestimated but qualitatively better.

3.3.2 Drained shearing path

Figures 8 to 13 compare the predictions and the experimental data for different confining pressures and suctions (path C-D). The volumetric strain still represents the water volume change divided by the total volume.

Figure 8 and 11 show the saturated state. The model reproduces the experimental data very well in the deviatoric plane - axial strain versus deviatoric stress. The predictions of the model in the volumetric plane - axial strain versus volumetric strain - are quantitatively good.

Figure 9 shows an unsaturated test at a low suction (s=50 kPa) and quite high degree of saturation (Sr≈0.90). The model predicts well the observed behaviour in this partially saturated case.

Figures 10, 12 and 13 show the results of drained shearing tests at higher suctions (100 and 200 kPa). As before, the predicted ultimate resistance is in good accordance with the test results in the deviatoric plane. In the volumetric plane, the experimental data show a typical feature of unsaturated silts (see Maâtouk, 1993). The water content of the sample continuously decreases without a clear stabilisation level. Due to the fact that the constitutive model does not include this phenomenon, the predicted volume change is much less than the experimental value and does not show any continuous increase of the water volumetric strain.

Note that for the stress-strain behaviour, the experimental results show first an increase of strength and then a loss of resistance like a brittle failure (see Figure 13). This behaviour was already observed, with no significant consequences, for smaller suctions (Figures 10 & 12). The model of Alonso *et al.* is not able to predict such phenomenon which tends to be more pronounced for higher suctions.

However this model is applicable to the principal paths (for qualitative responses in isotropic conditions and for resistance and volume predictions in deviatoric paths) when the soil is partially saturated ($0.9 \leq Sr \leq 1$ as a first approximation).

4. NEW APPROACH BASED ON THE DSC

The use of a general concept in the framework of the Disturb State Concept (DSC) (Desai, 1994) is proposed to model the particular volumetric behaviour of the unsaturated soils and the loss of strength in the stress-strain relationship due to suction. The disturb state concept was first presented by Desai and co-workers. We present here first steps of the use of this approach in the modelling of unsaturated soils (Desai *et al.*, 1996).

The DSC is based on the idea that a deforming material element can be treated as a mixture of two constituent parts in the relative intact (RI) and fully adjusted (FA) states, referred to as reference state. During external loading, the material experiences internal changes in its microstructure due to a self-adjustment process, and as a consequence, the initial RI state transforms continuously to the FA state. The observed stress σ^a is defined as:

$$\sigma^a = (1 - D)\sigma^i + D\,\sigma^c \tag{4}$$

where σ^i is the RI stress, σ^c the FA stress and D the disturbance function ($0 \leq D \leq 1$).

The FA state is considered as the stress state with D=1. It corresponds to the saturated state.

Figure 14 shows schematically the first choices made for the modelling of the unsaturated behaviour of the silt.

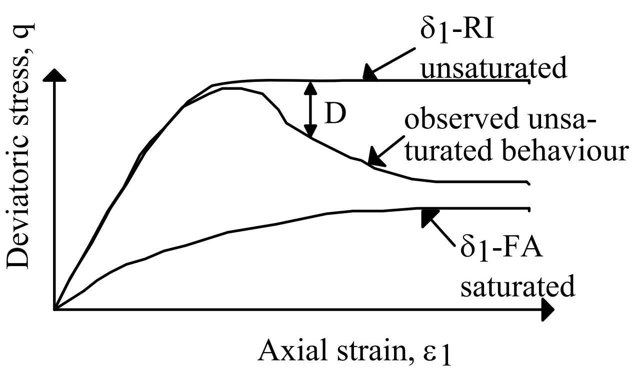

Fig.14: DSC concept for unsaturated soil

4.1 *Modified δ_1-model*

To represent the relative intact soil behaviour, we modified the nonassociative Hierarchical Single Surface Model δ_1-HISS (Desai, 1994). The expanding yield surface F is defined by:

$$F \equiv J^*_{2D} - \left[- \alpha (J^*_1)^n + \gamma (J^*_1)^2\right](1 - \beta\, \bar{S}_r)^{-0.5} \tag{5}$$

where the material parameters are explicit functions of the suction.

Expressions in Eq. (5) are the following:

$J^*_{2D} = J_{2D} / p_a^2$, J_{2D} is the second invariant of the deviatoric stress tensor, t_{ij};

$J^*_1 = J_1 / p_a$, J_1 is the first invariant of the net stress tensor $J_1=3(p-u_a)$;

p_a = atmospheric pressure constant;

γ and β are the ultimate parameters;

$\bar{S}_r$ is the stress ratio $= \sqrt{27/2}\; J_{3D} \cdot J_{2D}^{-3/2}$, J_{3D} is the third invariant of the deviatoric stress tensor t_{ij};

α is the hardening function defined as:

$$\alpha = \frac{a_1}{\xi^{\eta_1}} \tag{6}$$

where a_1 and η_1 are the hardening parameters and ξ is the trajectory of total plastic strains given by

$$\xi = \int (d\varepsilon^p_{ij} d\varepsilon^p_{ij})^{1/2} \tag{7}$$

n is the phase change parameter related to the state of stress at which transition from compaction to dilation occurs or at which the change in the volume vanishes.

The plastic potential function is defined in terms of F and a correction function introduced through α. It requires a nonassociative parameter κ.

For the elasticity (here linear) two more parameters are needed: the Young's modulus E and the Poisson's ratio ν.

4.2 *RI and FA states*

The RI-state is represented by the modified δ_1-model. As a first approach, we only take into account the evolution of the three parameters γ, a_1 and n with the suction. Further, we do not change the flow rules and we keep D=0. The parameters β, η_1, κ, E and ν are assumed to be independent of the suction.

The FA-state is the saturated case. It can be seen as a particular case of the RI-state when s = 0.

The parameter evolution with the suction are determined experimentally (see Figure 15). With increasing suction, the parameter γ (which defines the slope of the ultimate envelope) increases and the hardening parameter a_1 decreases. This denotes a stiffening of the soil with increasing suction. The phase change parameter n seems also to be affected by the suction.

Three simulations are done for the RI-state (see Figure 16) with the modified δ_1-model for the same initial net mean pressure p*=400 kPa:

- saturated case: σ_3=400 kPa, s=0kPa
- small suction: σ_3=450 kPa, s=50kPa
- high suction: σ_3=600 kPa, s=200kPa

The saturated simulation is in good accordance with the experimental data in both deviatoric and volumetric plane. The test at a small suction (s=50 kPa) gives also quantitatively good results. However as for the former model, similar observations can be done (especially for the case of a high suction) for the strength and the volume variation.

To achieve the modelisation of this behaviour, it is necessary to modify the stress tensor. This can be obtained with the use of the disturbance function (D

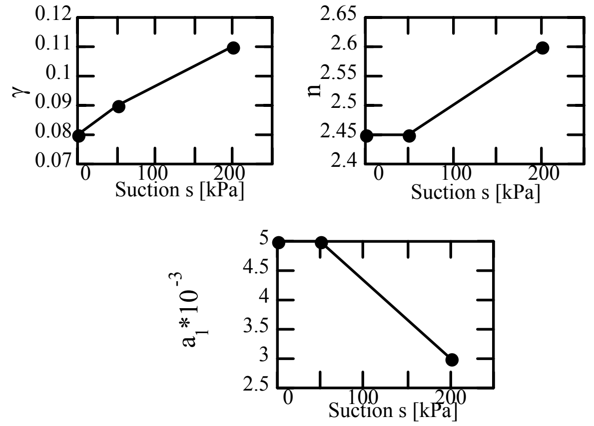

Figure 15: Evolution of the parameters γ, n and a_1 with the suction

was kept constant in the results of Figure 16). The function D will be dependent on the hardening variable (α) and the suction s:

$$D = D(\alpha, s) \tag{8}$$

This approach is actually developed.

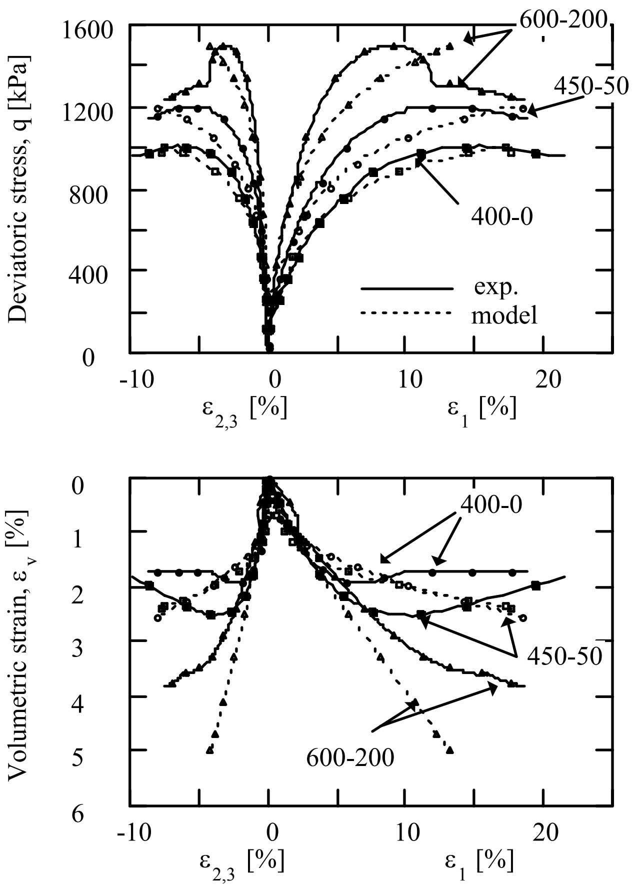

Figure 16: Simulations with the modified δ_1 -model

5. CONCLUSION

The test data obtained on a sandy silt are used to examine the performance of the constitutive model proposed in 1990 by Alonso *et al*. The model reproduces relatively well the principal characteristics in the partially saturated state ($0.9 \leq Sr \leq 1$). Under deviatoric solicitation, both predicted resistance and volumetric behaviour match the experimental results. The predictions are only qualitatively good along isotropic path. However this model is not able to predict the particular volumetric behaviour (continuous volume contraction) and the loss of strength for small degrees of saturation ($0.5 \leq Sr \leq 0.9$). To improve the constitutive approach, we present a first step towards the use of the DSC to take into account these particular features of unsaturated soils.

6. ACKNOWLEDGEMENTS

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