IMAGE SEGMENTATION MODEL USING ACTIVE CONTOUR AND IMAGE DECOMPOSITION

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ABSTRACT

This paper proposes an image segmentation model based on the active contour model, the Mumford-Shah functional and the image decomposition process. Generally speaking, the active contour model detects boundaries in images from sharp intensities variations and the Mumford-Shah model finds smooth regions from homogeneous intensities. Our model merges these two complementary approaches while considering the Four Color Theorem to globally partition any given image. We also consider the textural part lying in natural images by separating it from the geometric part, which contains the meaningful objects, to help the segmentation process. Our segmentation model is experimented with a 1-D signal and 2-D images.

1. INTRODUCTION

One of the most fundamental issues in the fields of image processing and computer vision is image segmentation. It is the basis of higher level applications such as in medical imaging. Its objective is to determine a partition of an image into a finite number of semantically important regions. This paper proposes a new image segmentation model based on the active contour/snake model and the image decomposition process. More precisely, the main contributions of this paper are as follows:

1. definition of an image segmentation method based a global minimization of the active contour model,
2. integration of the image (structures-textures) decomposition process in the segmentation process,
3. definition of a method to determine an initial condition close to the optimal solution.

2. RELATED WORKS

2.1. Image Segmentation Based on Active Contours

Image segmentation consists of identifying homogeneous semantic regions in images. One way to carry out the segmentation process is to detect the boundaries between different semantic regions. This is realized with the active contour or snake model, initially proposed by Kass-Witkin-Terzopoulos in [1] and developed by Caselles-Kimmel-Sapiro in [2] and Kichenassamy-Kumar-Olver-Tannenbaum-Yezzi in [3]. The geodesic/geometric active contour (GAC) model is a variational model which consists of finding the curve C which minimizes the following energy:

$$E_{GAC}(C) = \int_0^{L(C)} g_b(|\nabla f(C(s))|)\, ds, \qquad (1)$$

where ds is the Euclidean element of length, $L(C)$ is the length of the curve C and the function g_b is an edge indicator function that vanishes at object boundaries such as $g_b(|\nabla f|) = \frac{1}{1+\beta|\nabla f|^2}$, where f is the original image and β is an arbitrary positive constant. Hence, the energy functional (1) is actually a new length obtained by weighting the Euclidean element of length ds by the function g_b which contains information concerning the boundaries of objects [2]. The calculus of variations provides us the Euler-Lagrange equation of the functional E_{GAC} and the gradient descent method gives us the flow that minimizes as fast as possible E_{GAC} (see [2]). The evolution equation of active contour is handled with the level set method defined by Osher-Sethian [4], which efficiently solves the contour propagation problem and deal with topological changes.

Despite the many good numerical results obtained with this segmentation model and strong theoretical properties, the snake/GAC model is highly sensitive to the initial condition. Actually, the quality of the segmentation result depends a lot on the choice of the initial contour, which means that a bad initial contour can give an unsatisfactory result. To overcome this drawback, several authors introduced region-based evolution criteria into active contour energy functionals built from intensity statistics and homogeneity requirements. One of the most successful models is the active contours without edges (ACWE) model developed by Chan-Vese [5]. The ACWE model is based on the Mumford-Shah (MS) model [6] which provides an optimal piecewise smooth approximation of a given image, in other words an image made up of homogeneous intensities regions which common boundaries are sharp and piecewise regular. The MS functional is defined as follows:

$$F_{MS}(s,C) = \int_\Omega |s-f|^2 dx + \mu \int_{\Omega\setminus C} |\nabla s|^2 dx + \nu \mathcal{H}^{N-1}(C), \qquad (2)$$

where f is defined on a domain Ω, s corresponds to a piecewise smooth approximation of the original image f, C is a discontinuity set (representing the edges of s), the length of C is given by the (N-1)-dimensional Hausdorff measure $\mathcal{H}^{N-1}(C)$ (one can say that $\mathcal{H}^1$ is the length and $\mathcal{H}^2$ the area) and μ, ν are positive parameters. The first term of (2) is a fidelity term w.r.t. the given data f, the second term is a regularization term that constraints the function I to be smoothed inside the region $\Omega \setminus C$ and the last term imposes a regularization constraint on the discontinuity set C, i.e. the boundaries between smooth regions. The ACWE model proposes to minimize the Mumford-Shah functional, which is difficult to carry out in the original formulation, in the context of active contour, which is easier to realize with the calculus of variations. The ACWE model also corresponds to the piecewise constant/cartoon case of the MS functional obtained when $\mu \to \infty$. This case corresponds to the minimal partition problem, since the optimal solution is an image composed of regions of approximatively constant intensities equal to the mean value of intensities in the corresponding region. Finally, Vese-Chan considered in [7] the segmentation with the original Mumford-Shah energy (2).

As we said above, an usual problem when dealing the image segmentation with the active contour model is the local minima, which makes the initial guess critical to get satisfactory result. In a recent work, Chan-Esedoḡlu-Nikolova [8] propose an approach to overcome the limitation of local minima by determining a global minimum to the ACWE model. Inspired by this work, Bresson-Esedoḡlu-Vandergheynst-Thiran-Osher proposed in [9, 10] a model to compute a global minimum to the standard snake model. In this paper, we propose to extend the result of [9, 10] to the general case of image segmentation based on the Four Color Theorem.

2.2. Image Decomposition

Image decomposition aims at splitting the structural/geometric part and the textural part lying in images. Structural parts are represented by piecewise smooth regions which constitutes the meaningful geometric components of images. Textural parts are, roughly speaking, fine scale-details, usually with some periodicity and oscillatory nature [11]. Meyer suggests in [12] to decompose an image f into a component s belonging to the space of functions with bounded variation, BV, and a component t in the Banach space G containing signals with large oscillations s.a. textures and noise. The variational model of Meyer is as follows:

$$\min_{(s,t)\in BV\times G/f=s+t} \left\{ F_M(s,t,\lambda) = \int_\Omega |\nabla s|dx + \lambda \|t\|_G \right\}, \quad (3)$$

Since Meyer, several variational approaches, based on partial differential equations (PDEs), have been proposed to carry out the image decomposition task. In this paper, we propose to introduce the image decomposition process in the segmentation process to improve its performance. Indeed, the separation of the textural part and the geometric part will help us to segment the meaningful regions which boundaries are easily visible in the geometric part. The snake model will also help us to split the smooth part and the textural part, which can be used for other high-level processing tasks.

3. GENERAL IMAGE SEGMENTATION MODEL

We propose to compute a global minimum for the image segmentation model defined by Vese-Chan in [7]. More precisely, a global minimum for the active contour model based on the general formulation of the image segmentation method of MS is determined. The MS model is very well adapted to segment smooth regions lying in images but it does not take into account textures in its original definition. One of the motivation of our method is to consider textures in the MS model. In [7], Vese-Chan minimize the MS energy (2) using a multiphase level set approach motivated by the Four Color Theorem [13]. They use two level set functions to represent four phases (and triple junctions) and these four phases are sufficient to partition an image in a general way because each phase can be used to "color"/delimit different adjacent regions in an image according to the Four Color Theorem (see Figure 1).

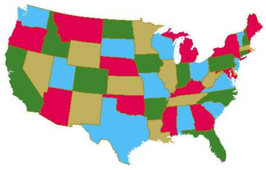

Fig. 1. Illustration of the Four Color Theorem.

The variational model introduced in [7] to approximate the general MS functional is:

$$\min_{\Omega_{C_1},\Omega_{C_2},s_{ij}} \Big\{ E_{VC}(\Omega_{C_1},\Omega_{C_2},s_{ij},\lambda,\eta) = \textstyle\sum_{m=1,2} \mathrm{Per}(\Omega_{C_m}) + \lambda \sum_{i,j=+,-} \int_{\Omega_{ij}} \Big(\eta \big(s_{ij}(x) - f(x)\big)^2 + |\nabla s_{ij}(x)|^2 \Big) dx \Big\}, \quad (4)$$

where Ω_{C_m} are two closed subsets of the image domain Ω, λ, η are two non-negative parameters, Per is the perimeter, f is the given image and functions s_{ij} represent four phases, defined in $\Omega_{ij} \subset \Omega$, to partition any given image s.t. $\Omega = \cup_{ij}\Omega_{ij}$ and $\cap_{ij}\Omega_{ij} = \emptyset$.

At this stage, we replace the L^2-norm of the fidelity term in the energy (4) by the L^1-norm to separate textures from the structural parts. The gradient-based term in (4) is unchanged to capture smooth regions. Thus, the energy (4) becomes:

$$E^1(\Omega_{C_1},\Omega_{C_2},s_{ij},\lambda,\eta) = \textstyle\sum_{m=1,2} \mathrm{Per}(\Omega_{C_m}) + \lambda \sum_{i,j=+,-} \int_{\Omega_{ij}} \Big(\eta |s_{ij} - f| + |\nabla s_{ij}|^2 \Big) dx \quad (5)$$

Minimizing (5) w.r.t. the functions s_{ij} leads to the variational problem for each function s_{ij}:

$$\min_{s_{ij}} \Big\{ \int_{\Omega_{ij}} \eta |s_{ij} - f| + |\nabla s_{ij}|^2 dx \Big\}. \quad (6)$$

A new function t_{ij} is introduced in the previous minimization problem to extract textures:

$$\min_{s_{ij},t_{ij}} \Big\{ \int_{\Omega_{ij}} \eta |t_{ij}| + \frac{1}{2\theta_s} \big(s_{ij} - (f - t_{ij})\big)^2 + |\nabla s_{ij}|^2 dx \Big\}, \quad (7)$$

where the parameter $\theta_s > 0$ is small so that we almost have $f = s_{ij} + t_{ij}$ in Ω_{ij}. Minimizing (7) w.r.t. the function s_{ij}, using the calculus of variations, leads to:

$$\begin{cases} s_{ij} - (f - t_{ij}) = 2\theta_s \Delta s_{ij} & \text{in} \quad \Omega_{ij}, \\ \frac{\partial s_{ij}}{\partial \mathcal{N}} = 0 & \text{on} \quad \partial\Omega_{ij}, \end{cases} \quad (8)$$

and minimizing (7) w.r.t. the function t_{ij} gives:

$$t_{ij} = \begin{cases} f - s_{ij} - \theta_s \eta & \text{if} \quad f - s_{ij} \geq \theta_s \eta \\ f - s_{ij} + \theta_s \eta & \text{if} \quad f - s_{ij} \leq \theta_s \eta \\ 0 & \text{if} \quad |f - s_{ij}| \leq \theta_s \eta \end{cases} \quad (9)$$

In [7], regions s_{ij} are represented by two level set function, namely ϕ_1, ϕ_2, s.t.

$$s(x) := \begin{cases} s_{++}(x) & \text{if} \quad \phi_1(x) > 0, \phi_2(x) > 0, \\ s_{+-}(x) & \text{if} \quad \phi_1(x) > 0, \phi_2(x) < 0, \\ s_{-+}(x) & \text{if} \quad \phi_1(x) < 0, \phi_2(x) < 0, \\ s_{--}(x) & \text{if} \quad \phi_1(x) < 0, \phi_2(x) > 0, \end{cases} \quad (10)$$

Thus, Energy (5) can be re-written as follows:

$$E^2(\phi_1, \phi_2, s_{ij}, t_{ij}, \lambda, \eta, \theta_s) = \sum_{m=1,2} \int_\Omega |\nabla H_\epsilon(\phi_m)| + \lambda \sum_{i,j=+,-} \int_\Omega \left(\eta |t_{ij}| + \frac{1}{2\theta_s} \big(s_{ij} - (f - t_{ij})\big)^2 + |\nabla s_{ij}|^2 \right) H(i\phi_1) H(j\phi_2) dx, \quad (11)$$

The flow minimizing the energy (11) is as follows:

$$\partial_t \phi_m = H'(\phi_m) \left\{ \text{div} \left(\frac{\nabla \phi_m}{|\nabla \phi_m|} \right) - \lambda r_m(x, s_{ij}, t_{ij}, \eta, \theta_s) \right\} \quad (12)$$

where

$$r_m(x, s_{ij}, t_{ij}, \eta, \theta_s) = \sum_{i,j=+,-} (-i)^{2-m} (-j)^{2-n} \Big(\eta |t_{ij}| + \frac{1}{2\theta_s} \big(s_{ij} - (f - t_{ij})\big)^2 + |\nabla s_{ij}|^2 \Big) H(j\phi_n) \quad (13)$$

for $m, n = 1, 2$ and $m \neq n$. If a non-compactly supported smooth approximation of the Heaviside function is chosen, the steady state solution of the gradient flow (11) is the same as:

$$\partial_t \phi_m = \text{div} \left(\frac{\nabla \phi_m}{|\nabla \phi_m|} \right) - \lambda r_m(x, s_{ij}, t_{ij}, \eta, \theta_s) \quad (14)$$

and this equation is the gradient descent flow of the energy:

$$E^3(\phi_1, \phi_2, s_{ij}, \lambda, \eta, \theta_s) = \sum_{m=1,2} \int_\Omega |\nabla \phi_m| + \lambda \int_\Omega r_m(x, s_{ij}, t_{ij}, \eta, \theta_s) \phi_m \, dx. \quad (15)$$

As a result of the previous developments, the following constrained minimization model is proposed to carry out the general image segmentation process:

$$\min_{0 \leq u_m \leq 1} \left\{ E_m(u_m, s_{ij}, t_{ij}, \lambda, \eta, \theta_s) = \sum_{m=1,2} \int_\Omega g_b(x) |\nabla u_m| + \lambda \int_\Omega r_m(x, s_{ij}, t_{ij}, \eta, \theta_s) u_m \, dx \right\}. \quad (16)$$

where g_b is the edge detector function in the GAC model. The energy (16) provides us a global minimization for the active contour model. The global minimization theorem is the same as in [9, 10]. Firstly, it consists of applying the coarea formula to notice that $\int_\Omega g_b(x) |\nabla(u_m = \mathbf{1}_{\Omega_{C_m}})| = \int_{C_m} g_b ds = E_{GAC}(C_m)$ as in (1). Then, for *fixed* functions $u_{n \neq m}, s_{ij}, t_{ij}$, a theorem establishes that $\mathbf{1}_{\Omega_{C_m}(\mu) = \{x : u_m(x) > \mu\}}$ for $\mu \in [0, 1]$ is a global minimizer of (16).

The snake variational model proposes in this framework is globally minimized, which is very important because a global minimization allows us to be independent of the initial condition. However, the proposed model is globally minimized only w.r.t. the snakes, represented by u_1, u_2, but *not* w.r.t. the functions s_{ij}, t_{ij}. Thus, the choice of the initial functions s_{ij}, t_{ij} is critical to get a satisfactory segmentation result. The next section proposes a fast way to compute initial s_{ij}, t_{ij} close to the optimal solution.

4. INITIAL CONDITION

We propose to determine a good initial condition for the functions s_{ij}, t_{ij} to find the optimal segmentation solution. Two steps are followed:

1. Computation of an initial smooth function s_0 and an initial texture function t_0 as follows:

$$\min_{s_0, t_0} \left\{ \int_\Omega \eta |t_0| + \frac{1}{2\theta_s} \big(s_0 - (f - t_0)\big)^2 + |\nabla s_0|^2 dx \right\}. \quad (17)$$

2. Determination of a function s_1 from s_0 with a region growing algorithm based on the Mumford-Shah model as described in [14].

The initialization process takes less than one minute for the 2-D images presented in the next section.

5. RESULTS

Firstly, our segmentation model is tested on the 1-D signal (Figure 2). Our model correctly detects the transitions between smooth regions (see small red circles). The decomposition between smooth (green curve) and textural parts (blue curve) provides us a good approximation of the original signal (black curve).

Secondly, our model is experimented on Figure 3 which is a linear combination of a smooth image, which intensities vary between $[0, 1]$, and a texture image, varying between $[-0.6, 0.6]$. The image also contains a triple junction which needs three phases to be detected. Our model is able to find the triple junction, the original smooth part and the texture part. The segmentation takes a few minutes to converge.

Finally, the image segmentation model is tested on the benchmark image *Barbara* which contains textures. The model recovers the smooth part of the given image and also the textures and several meaningful boundaries. As previously, a few minutes are needed to converge to the solution.

6. REFERENCES

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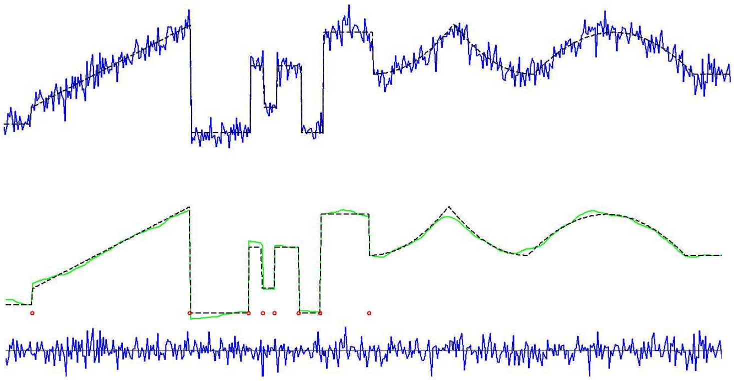
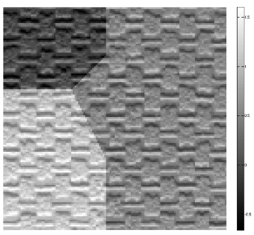
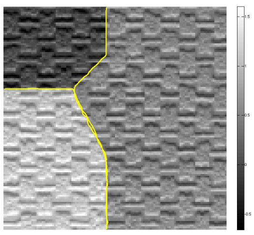


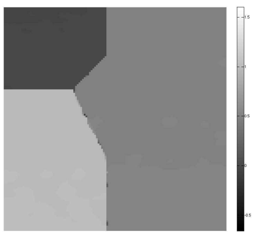
Fig. 2. Our segmentation model applied to a noisy 1-D signal. Figure (a): original signal in dark, noisy signal in blue. Figure (b): original signal in dark, active contour in red circles, smooth part in green, textural part in blue.



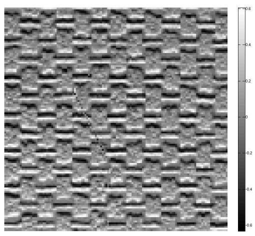
(a) Original Image.



(b) Final Active Contour/ Boundaries of (a).



(c) Smooth/Structural Part of (a).

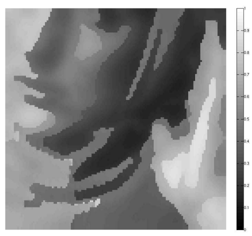


(d) Textural Part of (a).

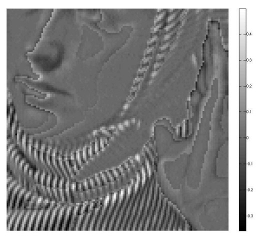
Fig. 3. Segmentation of a triple junction.

(a) Original Image.

(b) Final Active Contour/ Boundaries of (a).



(c) Smooth/Structural Part of (a).



(d) Textural Part of (a).

Fig. 4. Segmentation of *Barbara*.

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