

# On the joint source and channel coding of atomic image streams

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**Abstract**— This paper presents an error resilient coding scheme for atomic image bitstreams, as generated by Matching Pursuit encoders. A joint source and channel coding algorithm is proposed, that takes benefit of both the flexibility in the image representation, and the progressive nature of the bitstream, in order to finely adapt the channel rate to the relative importance of the bitstream components. An optimization problem is proposed, and a fast search algorithm determines the best rate allocation for given bit budget and loss process parameters. Simulation results show that the unequal error protection is quite efficient, even in very adverse conditions, and it clearly outperforms simple FEC schemes.

## I. INTRODUCTION

The problem of image transmission over error-prone channels can generally only be efficiently addressed by joint source and channel coding approaches, where source rate is traded against channel protection to optimize the end-to-end average image quality. The separation principle enounced by Shannon indeed does not hold in practical applications with delay or complexity constraints, and optimal approaches are inevitably based on joint compression and protection strategies.

Image signals are non-stationary by nature, and most of the coding schemes produce coded representations with non-equivalent elements. Some parts of the bitstream are thus more important than others, which naturally yields to the concept of unequal error protection. Unequal error protection has been widely studied in the recent years (see for example [1]–[3]), and researchers have proposed optimal channel coding strategies for different, generally scalable, compression schemes based on orthogonal transforms.

This paper proposes to investigate the joint source and channel coding of streams built on atomic expansions of image signals, in particular those based on Matching Pursuit. Coding approaches that use redundant signal representations have recently gained interest in the research community. In addition to an improved approximation rate, they present interesting properties in terms of flexibility, sparsity and scalable nature of the signal representation. The strategy proposed in this paper proposes to take benefit from the flexibility of Matching Pursuit streams, and to derive an optimal channel rate allocation, finely adapted to the importance of the independent

bitstream elements. An end-to-end quality optimization problem is derived, and a fast search algorithm is proposed, that jointly optimizes the number of Matching Pursuit elements to be transmitted, along with their respective channel protection. Interestingly enough, the proposed coding strategy may be seen as a form of Multiple Description Coding, similar, for example, to the scheme proposed in [4], where each data packet can be perceived as a different description of the image signal. Finally, the proposed unequal error protection strategy is shown to outperform basic error resiliency schemes, and offers graceful degradations of the image quality, even in very adverse channel conditions.

The paper is organized as follows. Sec. II briefly presents the source coding scheme used in this paper, which is based on a Matching Pursuit algorithm. The joint source and channel coding scheme is presented in Sec. III, and the related channel rate optimization problem is described in Sec. IV. Sec. V presents simulation results that highlight the performance of the optimized Unequal Error Protection strategy, and Sec. VI finally concludes the paper.

## II. MATCHING PURSUIT IMAGE CODING

Recent studies have shown the potential of novel representation methods, that target the efficient approximation of natural images, generally dominated by edge-like characteristics [5], [6]. Algorithms based on redundant expansions have also been shown to provide very good approximation properties. In the same time, they present numerous advantages in terms of flexibility and adaptivity [7], even if there is often a price to pay in terms of encoding complexity. In contrast to orthogonal transforms, overcomplete expansions of signals are indeed not unique. The number of feasible decompositions is infinite, and finding the best solution under a given criteria is a NP-complete problem. Matching Pursuit (MP) is one of the sub-optimal approaches that greedily approximates the solution to this complex problem. It iteratively decomposes any function  $f$  in the Hilbert space  $\mathcal{H}$  in a possibly redundant dictionary of functions called *atoms* [8]. Let  $\mathcal{D} = \{g_\gamma\}_{\gamma \in \Gamma}$  be such a dictionary with  $\|g_\gamma\| = 1$  and  $\Gamma$  represents the set of possible indices. The function  $f$  is first decomposed as follows :

$$f = \langle g_{\gamma_0} | f \rangle g_{\gamma_0} + \mathcal{R}f, \quad (1)$$

where  $\langle g_{\gamma_0} | f \rangle g_{\gamma_0}$  represents the projection of  $f$  onto  $g_{\gamma_0}$  and  $\mathcal{R}f$  is a residual component. Since all elements in  $\mathcal{D}$  have by

definition a unit norm, it is easy to see from eq. (1) that  $g_{\gamma_0}$  is orthogonal to  $\mathcal{R}f$ , and this leads to

$$\|f\|^2 = |\langle g_{\gamma_0} | f \rangle|^2 + \|\mathcal{R}f\|^2. \quad (2)$$

To minimize  $\|\mathcal{R}f\|$ , one must choose  $g_{\gamma_0}$  such that the projection coefficient  $|\langle g_{\gamma_0} | f \rangle|$  is maximum. The pursuit is carried out by applying iteratively the same strategy to the residual component. After  $M$  iterations, one has the following decomposition for  $f$  :

$$f = \sum_{n=0}^{M-1} \langle g_{\gamma_n} | \mathcal{R}^n f \rangle g_{\gamma_n} + \mathcal{R}^M f, \quad (3)$$

where  $\mathcal{R}^M$  is the residual of the  $M^{\text{th}}$  step with  $\mathcal{R}^0 f = f$ . Similarly, the energy  $\|f\|^2$  is decomposed into :

$$\|f\|^2 = \sum_{n=0}^{M-1} |\langle g_{\gamma_n} | \mathcal{R}^n f \rangle|^2 + \|\mathcal{R}^M f\|^2. \quad (4)$$

The approximation error decay rate in Matching Pursuit has been shown to be bounded by an exponential. In other words, the decay of the residue norm is faster than an exponential decay curve whose rate depends on the dictionary only. From [9], there exists a decay parameter  $\lambda > 0$  such that for all  $M \geq 0$

$$\|\mathcal{R}^M f\| \leq 2^{-\lambda M} \|f\|. \quad (5)$$

The decay rate can be written as [9]

$$2^{-\lambda} = (1 - \alpha^2 \beta^2)^{\frac{1}{2}}, \quad (6)$$

where  $\beta$  is the redundancy factor and  $\alpha \in (0, 1]$  is driven by the search strategy.

The image coder used in this paper is similar to the MP coder presented in [7]. The dictionary of atoms  $g_{\gamma}$  is built on anisotropic refinement of wavelet-like functions. Quantization and arithmetic coding are then applied to the coefficients and atom parameters. Note that the resulting MP bitstream presents very interesting properties that can be exploited in the joint source and channel coding scheme. First, following eq. (5), the stream is progressive. Second, atoms are totally independent, which allows to avoid error propagation within the bitstream. Finally, the order of the atoms is irrelevant, and the decoder can reconstruct the decoded image regardless of the atom numbers.

### III. JOINT SOURCE AND CHANNEL CODING

In practical applications with limited delay, and non-stationary channels, the separation theorem enounced by Shannon does not hold anymore. In this context, only joint source and channel coding can lead to minimal end-to-end distortion. In the presence of channel loss, the encoder needs to trade-off source rate against channel protection, in order to optimize the end-to-end quality. Under a fixed bit budget constraint, the sender may choose to send only a subset of the atoms generated by the Matching Pursuit encoder, and to protect them with efficient channel coding.

The end-to-end distortion is then composed of the source distortion  $\|\mathcal{R}^M f\|^2$ , that is driven by the number  $M$  of encoded atoms, and the distortion generated by the potential

loss of atoms. In other words, the average total distortion can be written as:

$$D = \|\mathcal{R}^M f\|^2 + \sum_{n=0}^{M-1} \|c_n\|^2 p_n, \quad (7)$$

where  $c_n = \langle g_{\gamma_n} | \mathcal{R}^n f \rangle$  is the atom coefficient, and  $p_n$  represents the probability of loosing the atom  $g_{\gamma_n}$ . From eq. (4), the distortion can further be expressed as:

$$D = \|f\|^2 + \sum_{n=0}^{M-1} \|c_n\|^2 (p_n - 1). \quad (8)$$

Note that, without loss of generality, the quantization error has been neglected for the sake of clarity. On the one hand, the error in a priori quantization schemes is included in the source rate distortion  $\|\mathcal{R}^M f\|^2$ . On the other hand, optimal a posteriori quantization schemes induce the same quantization error for any coefficient, which can easily be factored in eq. (8).

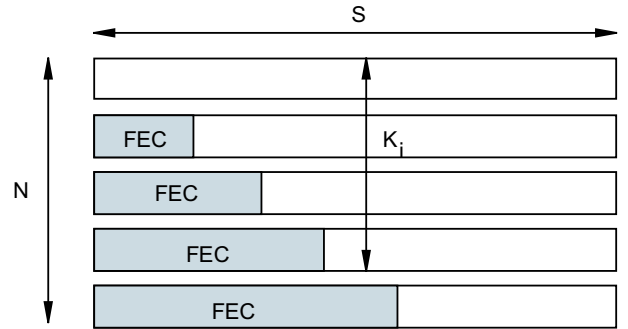


Fig. 1. Unequal Error Protection Scheme.

The progressive Matching Pursuit bitstream is segmented in  $N$  packets of  $S$  coded atoms, as illustrated in Figure 1. Without loss of generality, we assume that all Matching Pursuit atoms are coded with the same number of bits<sup>1</sup>. The atoms, sorted along the decreasing magnitude of their coefficients, are distributed according to a simple round robin strategy in the successive packets. As illustrated in Figure 1, Forward Error Correction (FEC) is then applied column-wise across the  $N$ -packet block, using a systematic code, like a Reed-Solomon code. In column  $i$ ,  $k_i$  atoms are protected with a channel rate  $\frac{k_i}{N}$ . Since all atoms do not have the same importance, unequal error protection is naturally applied to the series of atoms, in order to increase the chance to recover the most important atoms. Note that the unequal error protection scheme used in this work is similar to the method proposed in [4] in the context of Multiple Description Coding.

Recall that a FEC code with channel rate  $\frac{k_i}{N}$  is able to recover up to  $N - k_i$  erasures. If  $N - k_i$  packets at maximum are lost, the channel protection is able to recover the  $k_i$  Matching Pursuit atoms in column  $i$ . We consider here a packet erasure channel, that can be modelled by the widely accepted Gilbert model. It is a simple two-state Markov chain allowing to capture the first order correlation of the loss

<sup>1</sup>This assumption has been verified in a first approximation on several MP bitstreams.

process. The loss probability on the channel is denoted  $\pi$ , and the average size of burst of losses is represented by  $\alpha$ . For a given packet loss probability and average burst length on the transmission channel, the loss probability  $p_n$  of losing the atom  $n$  is therefore directly driven by the channel rate chosen for this atom (see [10] for details). In order to guarantee optimal quality, the channel rate has to be finely adapted to the atom importance in the Matching Pursuit expansion. The optimization of the joint source and channel coding strategy is presented in the next section.

#### IV. CHANNEL RATE OPTIMIZATION

##### A. Optimization Problem

The joint source and channel coding problem becomes equivalent to jointly optimizing the number of atoms, and the channel rate for each of these atoms, under a fixed bit budget. In other words, with the average distortion from eq. (8), and the packetization scheme proposed here-above, the optimization problem can be written as:

$$\min_{\{k_i\}} D = \max_{\{k_i\}} \sum_{i=1}^S \sum_{n=1}^{k_i} \|c_{K_i+n}\|^2 (1 - p_{K_i+n}(k_i)), \quad (9)$$

under the bit budget constraint  $\sum_{i=1}^S k_i \leq NS$ . The cumulative value  $K_i = \sum_{j=1}^{i-1} k_j$  represents the number of atoms that has been packetized in the first  $k_{i-1}$  columns of the matrix presented in Fig. 1 (with  $K_1 = 0$ ). The channel rate allocation is further defined as  $\vec{k}^j = [k_1^j \dots k_S^j]$ , and the corresponding energy in the reconstructed image is written as:

$$E(\vec{k}^j) = \sum_{i=1}^S \sum_{n=1}^{k_i^j} \|c_{K_i^j+n}\|^2 (1 - p_{K_i^j+n}(k_i^j)). \quad (10)$$

##### B. Fast Search Algorithm

In order to solve the channel rate optimization problem, a fast search algorithm is proposed, that takes benefit of the progressive nature of the Matching Pursuit bitstream. The search algorithm is illustrated in Algorithm 1.

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#### Algorithm 1 Fast search algorithm

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 $k_i^0 \leftarrow N, \forall i \in [1..S]$ 
 $j \leftarrow 0$ 
repeat
   $j \leftarrow j + 1$ 
   $\mathcal{I}^j = \left\{ \iota \mid ((\iota = 1) \text{ or } (k_{\iota-1}^{j-1} < k_{\iota}^{j-1})) \text{ and } (k_{\iota}^{j-1} > 1) \right\}$ 
   $E(\vec{k}^j) \leftarrow E(\vec{k}^{j-1})$ 
  for all  $\iota \in \mathcal{I}^j$  do
     $\kappa_i \leftarrow k_i^{j-1}, \forall i \neq \iota$ 
     $\kappa_{\iota} \leftarrow k_{\iota}^{j-1}$ 
    if  $E(\vec{\kappa}) > E(\vec{k}^j)$  then
       $k^j \leftarrow \kappa$ 
       $E(\vec{k}^j) \leftarrow E(\vec{\kappa})$ 
    end if
  end for
until  $E(\vec{k}^j) = E(\vec{k}^{j-1})$ 

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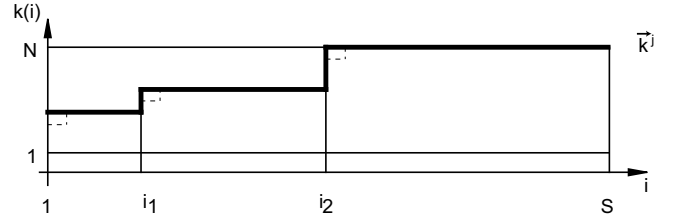


Fig. 2. Representation of the solution  $k^j$  in the iterative search strategy.  $\mathcal{I}^j = \{1, i_1, i_2\}$ .

The fast search algorithm iteratively looks for the best channel rate allocation, starting from the initial allocation  $k_i^0 = N, \forall i \in [1..S]$ . Since the bitstream is progressive,  $k_i^j$  can only be non-decreasing with the column order  $i$ , as represented in Figure 1. At each iteration  $j$  of the search algorithm, the allocation  $\vec{k}^j$  that maximizes the energy  $E(\vec{k}^j)$  is retained. The possible candidates at iteration  $j$  are limited to the subset of channel rate allocations  $\vec{\kappa}$  that are equal to the solution  $\vec{k}^{j-1}$ , except for the column  $i = \iota$ , where  $k_{\iota}^j = k_{\iota}^{j-1} - 1$ . The columns  $\iota$  in which the the rate allocation differ are either the first column ( $\iota = 1$ ), or the columns that represent a change in the channel rate, i.e.,  $k_{\iota-1}^{j-1} < k_{\iota}^{j-1}$ , under the condition that  $k_{\iota} > 1$ . The strategy is illustrated in Figure 2, where the solution at iteration  $j + 1$  can only be different from  $\vec{k}^j$  in  $i$  equal to 1,  $i_1$  or  $i_2$ . The search algorithm stops when none of the candidates at iteration  $j + 1$  improves the average energy  $E(\vec{k}^j)$ . Finally, it can be noted that the computation of the average energy  $E(\vec{\kappa})$  is quite fast, since at most  $N$  atoms change their respective contribution between successive iterations of the search algorithm.

#### V. SIMULATION RESULTS

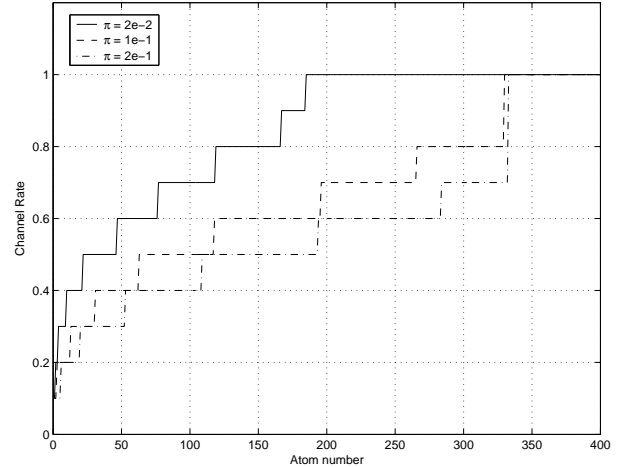


Fig. 3. Channel rate per atom, for different loss ratios  $\pi$  ( $\alpha = 2, N = 10$  and  $S = 120$ ).

This section presents simulation results of the optimal joint source and channel coding strategy proposed here-above. Figure 3 illustrates the channel rate allocation for the different atoms of the Matching Pursuit stream, in different channel conditions. The total rate is set to 10 packets of 120 atoms

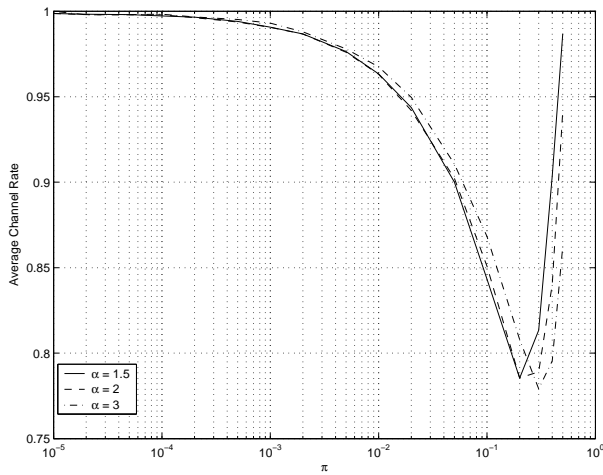


Fig. 4. Average Channel Rate versus packet loss ratio  $\pi$ , for different loss characteristics  $\alpha$  ( $N = 10$  and  $S = 120$ ).

(i.e., approximately half a MTU), which corresponds to a bit budget of 45 kbits. As expected, the channel rate increases with the atom order, and larger FEC protection is applied for the most important atoms. The channel rate also decreases when the packet loss ratio  $\pi$  increases, since obviously more protection is needed when transmission conditions worsen. Figure 4 shows the influence of the channel average burst length  $\alpha$  on the average channel rate allocation. For low loss ratios, the protection is more important for very bursty channel loss processes. Due to the limit of FEC in bursty loss conditions, the channel rate however increases for bursty channel conditions, at high loss rates. Since loss often cannot be recovered in these conditions, the optimal joint source and channel coding prefers to increase the number of atoms to be sent, in order to augment the benefit due to correctly received packets.

Figure 5 illustrates the influence of the packet size on the performance of the coding strategy. For a fixed total bit budget, small packets allow for larger FEC blocks (i.e., larger values of  $N$ ), and thus for better error resilience at low and medium values of the packet loss ratio  $\pi$ . For very high loss ratios however, the bursty nature of the loss process highlights the limits of FEC protection, even for large FEC blocks.

The performance of the Unequal Error Protection strategy is finally compared to the behavior of a simple Equal Error Protection (EEP) method in Figure 6. The EEP simply consists in distributing the atoms in the different packets following a round-robin strategy, in order to balance the importance of the packets. FEC packets are then added to the data packets, depending on the channel characteristics. In contrary to the UEP strategy, packets are either data packets, or FEC packets, which limits the possibility to finely adapt the channel rate to the atom importance, in EEP. As expected, the UEP strategy outperforms the EEP scheme for all packet loss rates  $\pi$ . More surprisingly, it can be seen also that the EEP strategy is in general very inefficient. It is slightly better than a strategy without any channel coding only for medium loss rates.

Figure 7 proposes visual comparisons of the decoded images in both UEP and EEP joint source and channel coding

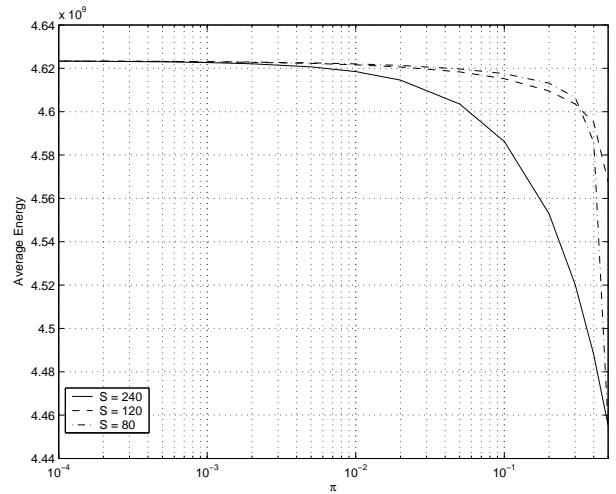


Fig. 5. Average Energy  $E_m$  versus packet loss ratio  $\pi$ , for different packet size ( $\alpha = 2$ ,  $N S = 45\text{kbits}$ ).

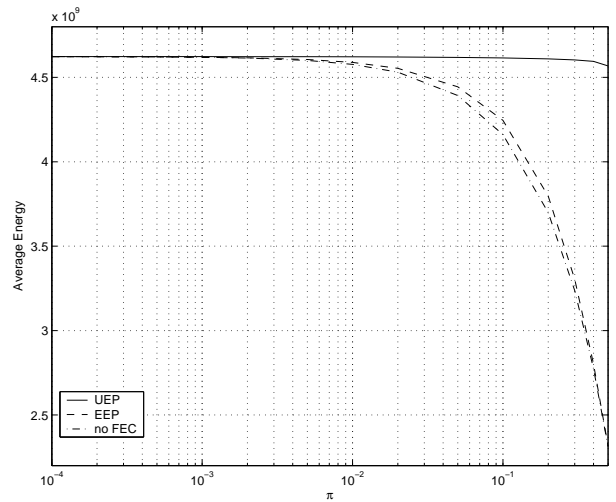


Fig. 6. Average Energy  $E_m$  versus packet loss rate  $\pi$ , for different channel coding strategies ( $\alpha = 2$ ,  $N = 10$ ,  $S = 120$ ).

strategies. If only one loss affects the bitstream transmission, both schemes behaves similarly since the coding strategy for  $\pi = 0.1$  allows for recovering the packet erasure. However, when two packet losses affect the transmission, EEP can recover only one lost packet, or even none of them when the channel protection has been underestimated. It can be seen however that even in these very adverse conditions, the UEP scheme is able to recover most of the bitstream energy. The decoded image stays of very good quality, even in the very poor conditions where two packets out of 10 are lost, while the expected loss ratio was actually smaller (i.e.,  $\pi = 0.01$ ).

## VI. CONCLUSIONS

Joint source and channel coding of bitstreams built on atomic image representations has been discussed in this paper. Such approaches jointly optimize the number of atoms to be coded for a given bit budget constraint, and their respective channel protection that depends on the atom importance, and the channel state. An Unequal Error Protection algorithm

(a) UEP,  $\pi = 0.1$ , one loss.(b) UEP,  $\pi = 0.1$ , two losses.(c) UEP,  $\pi = 0.01$ , two losses.(d) EEP,  $\pi = 0.1$ , one loss.(e) EEP,  $\pi = 0.1$ , two losses.(f) EEP,  $\pi = 0.01$ , two losses.

Fig. 7. Decoded image for different loss patterns, for different coding strategies ( $N = 10$ , total bit budget of 45 kbits).

has been proposed as a solution to a channel rate allocation optimization problem. It has been shown to outperform basic channel coding strategies, and to offer graceful degradation of image quality, even in very poor channel conditions. We are currently working on the application of Unequal Error Protection to Matching Pursuit video streams [11], in taking benefit from an increased flexibility in signal representation, compared to state-of-the-art coders.

## VII. ACKNOWLEDGMENTS

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