# Automated Parameterization and Patching of Bifurcating Vessels

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## Introduction

Recent developments in computational modeling of human arteries have opened the possibility of performing subject-specific analyses on increasingly larger numbers of subjects. This achievement will eventually lead to a better understanding of the role of geometry and hemodynamics in the initiation and development of vascular disease. The availability of data from population or longitudinal studies raises the problem of quantitatively comparing distributions of geometric and hemodynamic quantities among different models. This task is made difficult by the fact that modeled arterial segments typically comprise bifurcations and regions of high curvature.

A technique for comparing surface distributions among realistic models of the carotid bifurcation has been recently proposed in [1]. In that work, surface mesh nodes were classified as belonging to semi-automatically defined quadrilateral patches, and nodal quantities of interest averaged over each patch. This avoided node-to-node comparison and the need for registration. However, patch definition required user interaction and was thus subject to operator-variability.

In this work we present a fully automated technique for parameterization and patching of the surface of bifurcating vessels. The method is based on robust and objective schemes aimed at preserving the consistency of the parameterization over a wide range of bifurcating geometries, allowing quantitative comparison of surface distributions in presence of high anatomic variability.

## Methods

The parameterization of three dimensional surface meshes of bifurcating vessels is accomplished with a series of successive steps, summarized in the following:

**Computation of centerlines**. Centerlines are obtained with the approach presented in [2], in which a minimal action path problem is solved over an approximation of the medial axis of the surface. The resulting centerlines are smooth lines traced between the inlet and the outlets of the model (Figure 1). Each centerline point is the center of a maximal inscribed sphere of known radius. The method requires the specification of centerline endpoints, which are automatically identified from the surface mesh and classified according to their relative position.

**Definition of the bifurcation reference system**. Based on the computed centerlines, a local reference system is defined at the bifurcation, as shown in Figure 1. Four reference points are first defined on the centerlines: A,B are the points in which each centerline intersects the union of the maximal inscribed spheres defined on the other centerline; C,D are the centers of the spheres touching A,B defined on the *upstream* tracts of the respective centerlines. The bifurcation origin O is then defined as the barycenter of the reference points weighted with the squared radii of the associated spheres. The bifurcation normal, which defines the bifurcation plane, is defined as the normal of the polygon ABDC computed at O, and the bifurcation up-normal as the normalized sum of vectors **AC** and **BD**. At this point, the model can eventually be registered to an absolute reference system.

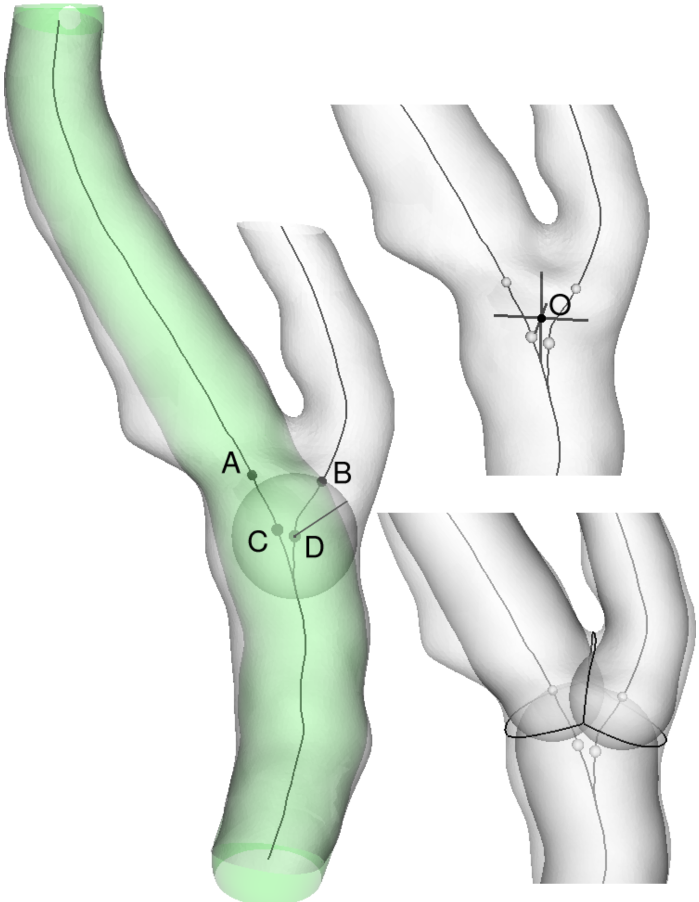


Figure 1: Left: centerlines, reference points (A,B,C,D) for a realistic model of carotid bifurcation. The union of maximal inscribed balls defined on the left centerline is shown in green. Right top: bifurcation reference system (O, cross-hairs). Right bottom: definition of the splitting lines.

**Splitting of bifurcation branches**. The bifurcating vessel is then split into its three constituent vessels. The splitting scheme employs the same four reference points used in defining the bifurcation reference system. First, centerlines are split into the tracts defined from points A,B,C,D to the respective endpoints. Successively, the power distance of mesh nodes to each of the sphere unions defined by the split centerlines is computed. The splitting lines are then defined over the surface as the zero-level lines of the pairwise differences of the computed power distances (see Figure 1).

**Longitudinal parameterization**. After splitting, the individual branches are parameterized in the longitudinal direction by solving a Laplacian equation over the surface, with boundary conditions equal to 0 on the splitting lines and 1 on the inlet and outlet boundaries. Since the metric over a 3D surface is in general non-Euclidean, the Laplacian equation is written employing the Laplace-Beltrami operator. The resulting partial differential equation is approximated over the surface mesh using a finite-elements / finite-volumes discretization. The resulting linear system is solved using the BiCStab iterative method. The parameterization is finally stretched in order to reflect the centerline abscissa with respect to the bifurcation origin.

**Circumferential parameterization**. The paramterization in the circumferential direction is generated on the basis of centerline tortuosity. First a reference system (tangent, normal and binormal) is defined along the centerlines using a parallel transport approach (for which zero-torsion is imposed between adjacent reference systems) and referred to the bifurcation normal. The circumferential parameterization, ranging from 0 to $2\pi$, is then computed as the angle between the position vector of each mesh node relative to the correspondent centerline point and the centerline normal in that point.

**Flattening and patching**. Once longitudinal and circumferential parameterizations are defined, the surface of each branch is flattened and patched in the parameter space, as shown in Figure 2.

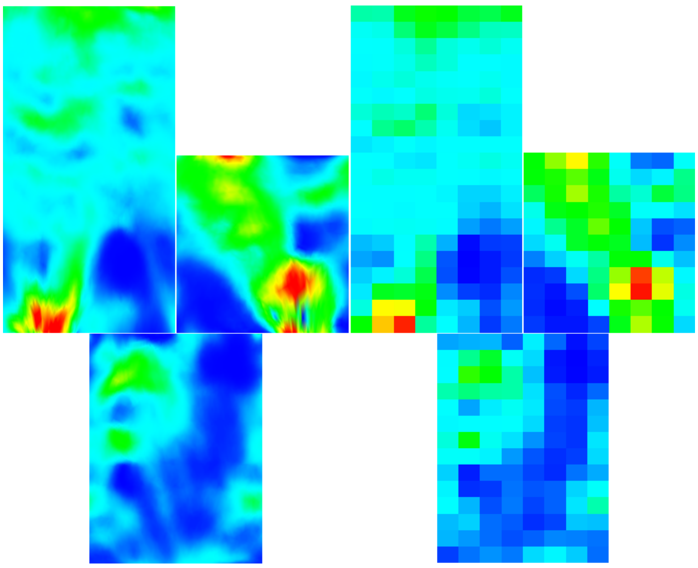

Figure 2: Left: wall shear stress (WSS) distribution on the surface of the model in Figure 1 flattened onto the parameter space. Right: WSS distribution patched in the parameter space (patch size: 1.5mm longitudinally and $\frac{\pi}{4}$ circumferentially).

## Results & Discussion

Figure 3 (left) shows the shape of patches on the model in Figure 1. The stability of patch shape and position (and henceforth of the underlying parameterization) to variability in surface geometry is demonstrated by comparison with Figure 3 (center and right).

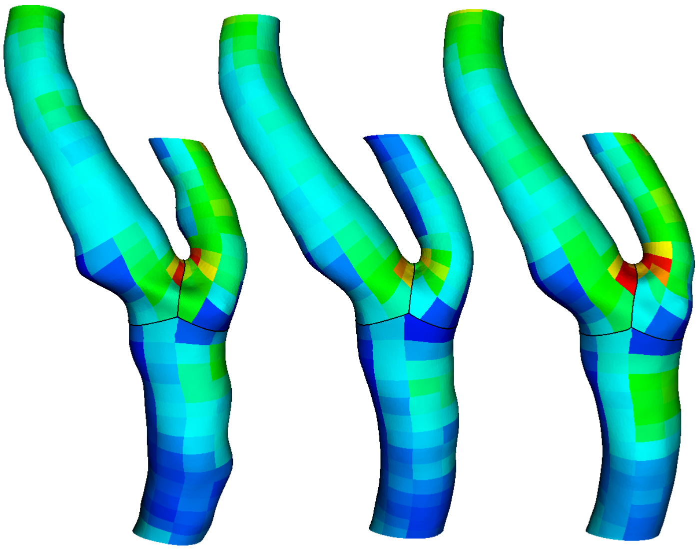

Figure 3: Left: patched surface in Figure 2 refolded in the 3D space. Center and right: result of patching models reconstructed from two additional independent acquisitions of the same subject. The models have been registered based on the bifurcation reference system. As in Figure 2, WSS distributions are shown.

The method proposed is fully automated and is based on objective criteria. Conversely to existing techniques, our method doesn't rely on the identification of the bifurcation apex, whose reconstruction accuracy is affected by imaging and segmentation artifacts. Preliminary results show how the technique is robust to changes in surface geometry and produces consistent results over a wide range of bifurcating geometries (see Figure 4). The method can also be readily extended to handle more complex configurations comprising more than one bifurcation.

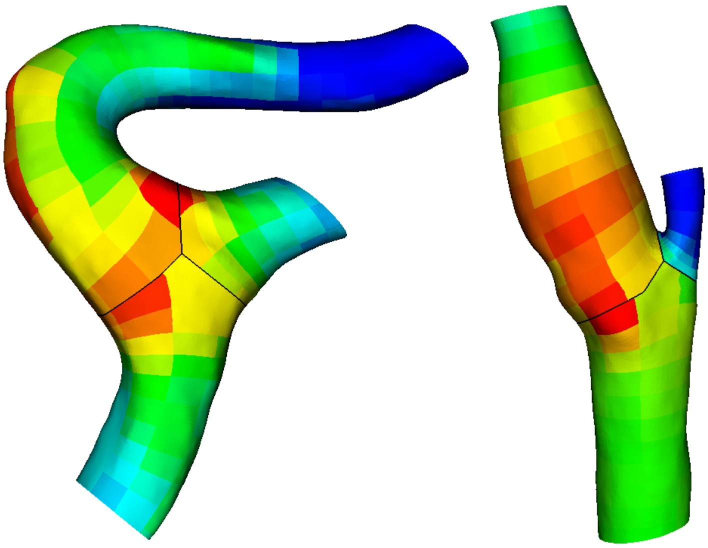

Figure 4: Result of registering, splitting and patching two extreme cases of carotid bifurcation geometry. Surface distributions of local radii are shown.

## References

1. Thomas JB et al. *Biorheology*, 2002. 39(3-4):443-448.
2. Antiga L et al. *IEEE Trans Med Img*, 2003. In press.