Correlation-Based Tuning of a Restricted-Complexity Controller for an Active Suspension System

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Abstract

A correlation-based controller tuning method is proposed for the "Design and optimization of restricted-complexity controllers" benchmark problem. The approach originally proposed for model following is applied to solve the disturbance rejection problem. The idea is to tune the controller parameters such that the closed-loop output be uncorrelated with the measured disturbance. Since perfect decorrelation between the closed-loop output and the disturbance is not attainable with a restricted-complexity controller, the cross-correlation of these two signals is minimized. This is done iteratively using stochastic approximation. A frequency analysis of the tuning criterion allows dealing with control specifications expressed in terms of constraints on the sensitivity functions. Application to the active suspension system of the Automatic Laboratory of Grenoble (LAG) provides a 2nd-order controller that meets the control specifications to a large extent.

Keywords: Restricted-complexity controller; controller tuning; correlation approach; active suspension.

1 Introduction

The design of restricted-complexity linear controllers has drawn wide attention in the control community. Low-order controllers are usually preferred in industry because the controller size may be limited by hardware and/or computational requirements. Moreover, simple controllers are easier to implement, maintain and understand. There are several ways of arriving at reduced-order controllers [1]. In the first type of approaches, a high-order controller is first designed and an optimization procedure used to minimize a norm of the error between the full-order and reduced-order controllers. Information regarding the plant model and the control specifications needs to be considered in the controller reduction procedure [3, 9]. Another approach consists of deriving a reduced-order model of the plant on the basis of which the controller is designed. However, the controller-design step should consider the unmodeled dynamics in order to ensure robustness [16]. A third type of approaches solves an optimal control problem directly for the reduced-order controller [10, 2].

Furthermore, the parameters of a restricted-complexity controller can also be tuned using data collected under closed-loop operation. The gradient of the criterion can be computed using additional experiments performed on the real system, i.e. without using the model of the plant (so-called model-free approaches) [4]. Alternatively, the gradient can also be estimated using an approximate model of the plant [15, 7]. In all these approaches, since the plant model is not used explicitly for control design, the controller order is not linked directly to the plant order.

Recently, a correlation approach for iterative controller tuning has been proposed to address the model-following problem [7, 6, 8]. Instead of minimizing an LQG-like control criterion, this approach tries to decorrelate the closed-loop output error (the difference between the achieved and designed closed-loop outputs) from the excitation signal. The controller parameters, which are solutions of a correlation equation involving instrumental variables, are computed iteratively using the Newton-Raphson algorithm. In [6], this method was compared with the closely related IFT approach and applied successfully to a magnetic suspension system. The convergence of the controller parameters towards the solution of the correlation equation in the presence of noise and modeling errors was studied in [7]. Since perfect decorrelation is not possible in the context of restricted-order controller design, it is natural to reformulate the design criterion as the minimization of the cross-correlation function between the closed-loop output error and the reference signal [8]. A frequency analysis of the proposed criterion has indicated that the tuning algorithm minimizes the integral of the difference between the achieved closed-loop transfer function and the reference model, this difference being weighted by the square of the reference signal spectrum.

In this paper, the correlation approach is adapted for tuning restricted-order controllers that need to reject disturbances in certain frequency regions. Furthermore, a stopping condition based on the statistical properties of the criterion is proposed. The correlation approach is applied to solve the disturbance rejection problem for the benchmark proposed in this Special Issue of European Journal of Control [14]. The benchmark problem involves the design of the simplest controller capable of ensuring good disturbance rejection for an active suspension system. The control specifications are stated in terms of constraints on the sensitivity functions. Although the procedure used in this paper does not consider specifications in the frequency domain explicitly, these can be met thanks to the frequency analysis performed in [8]. The two-norm of the cross-correlation function is minimized using the extended instrumental variables method.

The main advantage of the proposed tuning approach is that, asymptotically, the controller parameters are *not affected by noise*. Though, for the regulation problem, IFT and the correlation-based approach lead to similar results when the measurement noise is small, this is no longer true with a significant amount of noise. Indeed, IFT provides a trade-off between closed-loop performance and noise reduction, while closed-loop performance is the only objective in the correlation-based approach since it does not attempt to minimize the effect of measurement noise.

The paper is organized as follows. Section 2 briefly presents the correlation approach for the regulation problem and introduces a stopping condition for the iterative algorithm. A frequency analysis of the tuning criterion is presented in Section 3. Section 4 describes the application of this approach to the benchmark problem. Concluding remarks are given in Section 5.

2 The Correlation Approach

The correlation-based controller-tuning approach was originally developed for the model-following problem [7, 6, 8]. The idea is to tune the controller parameters such that the output error between the closed-loop system and the reference model be uncorrelated with the reference signal. This way, the closed-loop output is forced to follow as closely as possible the desired one, and this independently of the noise characteristics of the plant. The controller parameters are solutions of a correlation equation involving instrumental variables. This solution is computed iteratively using the Newton-Raphson algorithm.

2.1 Regulation Problem

In this paper, the case of a restricted-order controller is considered. Since perfect decorrelation is not possible, the two-norm of the cross-correlation function will be minimized. The correlation approach is applied to the regulation problem depicted in Fig. 1. Note that this problem can be considered as a model-following problem with the reference model equal to zero. Let the measured output of the plant be described as:

$$y(t) = G(q^{-1})u(t) + F(q^{-1})v_1(t) + v_2(t) \qquad (1)$$

where q^{-1} is the backward-shift operator, $u(t)$ the plant input, $v_1(t)$ a measured disturbance, $v_2(t)$ a zero-mean measurement noise independent of $v_1(t)$, $G(q^{-1})$ and $F(q^{-1})$ LTI SISO discrete-time transfer operators. The signals $y_p(t)$ and $p(t)$ denote the plant output and the output of the disturbance model F, respectively.

The plant is controlled by the controller $K(q^{-1})$:

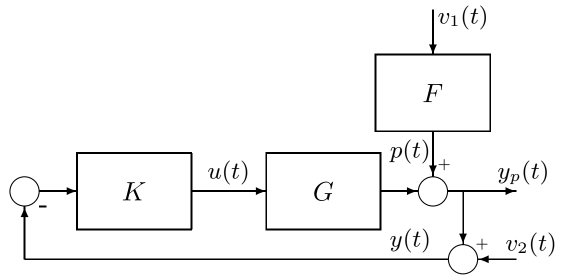

Figure 1: Controlled plant with the measured disturbance v_1 and the measurement noise v_2

$$K(q^{-1}) = R(q^{-1})/S(q^{-1}) \qquad (2)$$

where

$$\begin{aligned} R(q^{-1}) &= r_0 + r_1 q^{-1} + \cdots + r_{n_R} q^{-n_R} \\ S(q^{-1}) &= 1 + s_1 q^{-1} + \cdots + s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1}) \end{aligned}$$

The controller output can be expressed in regression form as:

$$u(t) = -S^*(q^{-1})u(t-1) - R(q^{-1})y(t) = \phi^T(\rho, t)\rho \qquad (3)$$

with the regressor vector $\phi(\rho, t)$ and the vector of controller parameters ρ, both of dimension n_ρ, defined as:

$$\begin{aligned} \phi^T(\rho, t) &= [-u(t-1) \cdots - u(t-n_S), \\ &\qquad -y(t) \cdots - y(t-n_R)] \qquad (4) \\ \rho^T &= [s_1 \cdots s_{n_S}, r_0 \cdots r_{n_R}] \qquad (5) \end{aligned}$$

2.2 Tuning Criterion

The objective is to tune the controller parameters such that the feedback controller exactly compensates the effect of the measured disturbance at the plant output. In other words, the measured output should ideally contain only the measurement noise that is uncorrelated with $v_1(t)$. Evidently, with a low-order causal controller, perfect decorrelation of $y(t)$ and $v_1(t)$ is not possible. Therefore, it is natural to formulate the design objective as the minimization of some norm of the cross-correlation function of these two signals.

Let the correlation function $f(\rho)$ be defined as follows:

$$f(\rho) = E\{y(\rho, t)\zeta(t)\} \qquad (6)$$

where $E\{\cdot\}$ is the mathematical expectation and $\zeta(t)$ a vector of instrumental variables that are correlated with the disturbance signal and independent of the measurement noise. The instrumental variables are defined as:

$$\zeta^T(t) = [v(t+n_z), v(t+n_z-1), \ldots, v(t), \ldots, v(t-n_z)] \qquad (7)$$

where

$$v(t) = W(q^{-1})v_1(t) \qquad (8)$$

with $W(q^{-1})$ being a linear generic filter and n_z a sufficiently large integer number. Then, the tuning objective can be defined as the minimization of the following criterion:

$$J(\rho) = ||f(\rho)||_2^2 = f^T(\rho)f(\rho) = \sum_{\tau=-n_z}^{n_z} R_{yv}^2(\tau) \quad (9)$$

where $||\cdot||_2$ represents the two-norm and $R_{yv}(\tau)$ is the cross-correlation function between the filtered disturbance $v(t)$ and the closed-loop output $y(\rho, t)$:

$$R_{yv}(\tau) = E\{y(\rho, t)v(t-\tau)\} \quad (10)$$

Hence, the control parameter vector ρ^* is given by:

$$\rho^* = \arg\min_{\rho} J(\rho) \quad (11)$$

2.3 Iterative Procedure

Since this problem cannot be solved analytically, a numerical method is considered. The vector ρ^* is solution of the following gradient equation:

$$J'(\rho) = 2f^T(\rho)\frac{\partial f(\rho)}{\partial \rho} = 0 \quad (12)$$

This problem can be solved by the Robbins-Monro procedure using the following iterative formula [12]:

$$\rho_{i+1} = \rho_i - \gamma_i \left[Q(\rho_i)\right]^{-1} \left[J'(\rho_i)\right]^T \quad (13)$$

where γ_i is a scalar step size and $Q(\rho_i)$ is a positive definite matrix. Under the assumption of boundedness of the signals in the loop, and with a step size tending to zero appropriately fast, this scheme converges to a local minimum as the number of iterations tends to infinity [8].

The gradient of the criterion involves the expectations of signals that are unknown and should be replaced by their estimates from closed-loop data. Let the correlation function be estimated as:

$$\hat{f}(\rho) = \frac{1}{N}\sum_{t=1}^{N} y(\rho, t)\zeta(t) \quad (14)$$

where N is the number of data points. Then, the gradient of the criterion can be expressed as:

$$\hat{J}'(\rho_i) = 2\hat{f}^T(\rho_i)\,\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\,\left.\frac{\partial y(\rho, t)}{\partial \rho}\right|_{\rho_i} \quad (15)$$

An accurate value of the gradient cannot be computed because the derivative of $y(\rho, t)$ with respect to ρ is unknown. However, an unbiased model-free estimation can be obtained using one additional closed-loop experiment as is done in the IFT approach for the case of one-degree-of-freedom controller [4]. Note that the gradient could also be computed from a plant model, which could be identified under open-loop or closed-loop operation, using the following expression [6]:

$$\frac{\partial y(\rho, t)}{\partial \rho} \approx \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})S(q^{-1}) + \hat{B}(q^{-1})R(q^{-1})}\phi^T(\rho, t) \quad (16)$$

where $\hat{B}/\hat{A}$ is the identified plant model.

In order to improve the convergence speed, $Q(\rho_i)$ in Eq. (13) can be chosen as an approximation of the Hessian of the criterion (Gauss-Newton direction):

$$\hat{Q}(\rho_i) = \left(\left.\frac{\partial \hat{f}(\rho)}{\partial \rho}\right|_{\rho_i}\right)^T \left.\frac{\partial \hat{f}(\rho)}{\partial \rho}\right|_{\rho_i} + \lambda I \quad (17)$$

where the parameter λ should be chosen so as to ensure positive definiteness of the matrix $\hat{Q}(\rho_i)$.

2.4 Stopping Condition

Inspired by the cross-correlation test for model validation available in the field of identification [13, 11], a stopping condition for the iterative algorithm is provided in this subsection.

Let denote the estimate of the cross-correlation between $y(t)$ and $v(t)$ based on N data points by:

$$\hat{R}_{yv}(\tau) = \frac{1}{N}\sum_{t=1}^{N} y(t)v(t-\tau) \quad (18)$$

It is shown in [11] that, for $N \to \infty$, the sequence of random variables $\sqrt{N}\hat{R}_{yv}(\tau)$ converges in distribution to the normal distribution with zero mean and covariance matrix P, i.e.

$$\sqrt{N}\hat{R}_{yv}(\tau) \in As\mathcal{N}(0, P) \Rightarrow \sqrt{\frac{N}{P}}\hat{R}_{yv}(\tau) \in As\mathcal{N}(0, 1) \quad (19)$$

where

$$P = \sum_{k=-\infty}^{\infty} R_y(k)R_v(k) \quad (20)$$

with $R_y(k)$ and $R_v(k)$ being the autocorrelation of $y(t)$ and $v(t)$, respectively.
Consider the following lemma taken from [13]:

Lemma 1. *Let $x \in As\mathcal{N}(m, P)$ be of dimension n. Then $(x-m)^T P^{-1}(x-m) \in As\chi^2(n)$.*

With this lemma, Eq. (19) gives:

$$\frac{N}{P}\sum_{\tau=-n_z}^{n_z} \hat{R}_{yv}^2(\tau) \in As\chi^2(2n_z + 1) \quad (21)$$

Thus, if $\chi_\alpha^2(2n_z + 1)$ denotes the α-level of the $\chi^2(2n_z + 1)$ distribution, the iteration could be stopped when the criterion (9) is statistically not different from zero with the confidence level α:

$$\hat{J}(\rho) = \sum_{\tau=-n_z}^{n_z} \hat{R}_{yv}^2(\tau) \leq \frac{\hat{P}}{N}\chi_\alpha^2(2n_z + 1) \quad (22)$$

where

$$\hat{P} = \sum_{k=-n_z}^{n_z} \hat{R}_y(k)\hat{R}_v(k)$$

Therefore, if the computed value of $\hat{J}$ falls outside the confidence region, the iteration should be continued. The stopping condition (22) can also show whether the selected controller order is appropriate. If the iterative procedure does not succeed in meeting the stopping condition after a large number of iterations, one should consider increasing the order of the controller. On the contrary, reaching the test threshold "too early" indicates that the order of the controller may be reduced.

3 Frequency Analysis of the Tuning Criterion

In this section, the frequency characteristics of the achieved closed-loop system are analyzed. An asymptotic frequency-domain equivalent of the tuning criterion (9) is derived using the relationship between the cross-correlation and the spectral density functions.

When n_z tends to infinity, applying Parseval's formula to Eq. (9) leads to:

$$\lim_{n_z \to \infty} J(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_{yv}(\omega)|^2 d\omega \qquad (23)$$

where $\Phi_{yv}(\omega)$ is the cross-spectral density between $y(\rho, t)$ and $v(t)$.

Moreover, the closed-loop output can be expressed as:

$$y(\rho, t) = \mathcal{S}_{yp}(q^{-1}, \rho)(F(q^{-1})v_1(t) + v_2(t)) \qquad (24)$$

where $\mathcal{S}_{yp}(q^{-1}, \rho)$ is the output sensitivity function of the closed-loop system defined as:

$$\mathcal{S}_{yp}(q^{-1}, \rho) = \left(1 + K(q^{-1}, \rho)G(q^{-1})\right)^{-1} \qquad (25)$$

Thus, from Eqs. (8) and (24), and using the fact that $v(t)$ and $v_2(t)$ are independent, the cross-spectral density $\Phi_{yv}(\omega)$ reads:

$$\Phi_{yv}(\omega) = \mathcal{S}_{yp}(e^{-j\omega}, \rho)F(e^{-j\omega})W^{-1}(e^{-j\omega})\Phi_v(\omega) \qquad (26)$$

where $\Phi_v(\omega)$ is the spectrum of the filtered disturbance signal $v(t)$. Finally, using $\Phi_{yv}(\omega)$ of Eq. (26) in Eq. (23) gives:

$$\lim_{n_z \to \infty} J(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\mathcal{S}_{yp}(e^{-j\omega}, \rho)|^2 |F(e^{-j\omega})|^2 \times |W(e^{-j\omega})|^2 \Phi_{v_1}^2(\omega) d\omega \qquad (27)$$

with $\Phi_{v_1}(\omega) = |W^{-1}(e^{-j\omega})|^2 \Phi_v(\omega)$ being the spectrum of $v_1(t)$. This equation indicates that the criterion based on the correlation approach is not affected by the noise signal $v_2(t)$. Furthermore, when $W(q^{-1}) = 1$ and $v_1(t)$ is white noise with variance 1, the tuning algorithm tries to minimize the magnitude of the sensitivity function $\mathcal{S}_{yp}$ in the frequency regions where $|F(e^{-j\omega})|$ is large.

4 Application to an Active Suspension System

The objective in the benchmark problem is to design a reduced-complexity controller for the active suspension system of LAG [14]. The block diagram of the active suspension system is presented in Fig. 2.

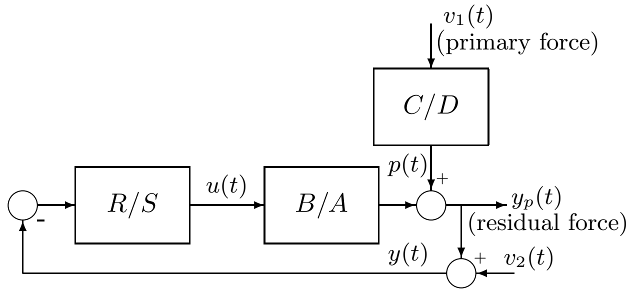

Figure 2: Block diagram of the active suspension system

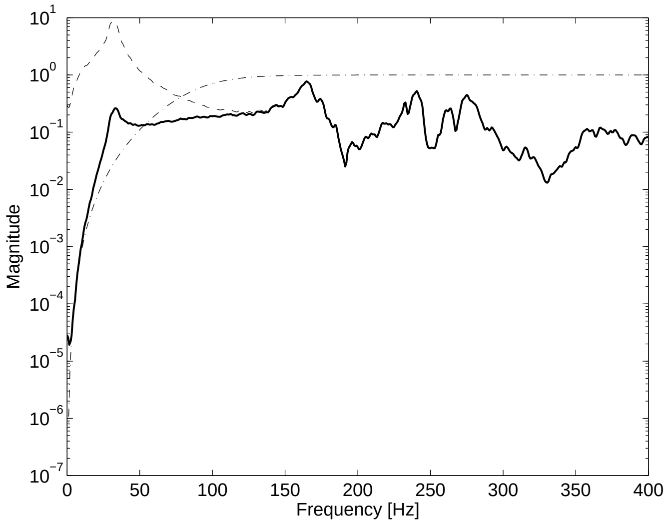

Figure 3: Weighting of $\mathcal{S}_{yp}$: Spectrum of the primary path $\Phi_p(\omega) = |C(e^{-j\omega})/D(e^{-j\omega})|\Phi_{v_1}(\omega)$ (dashed), magnitude of $|W(e^{-j\omega})|$ (dash-dot), and aggregate weighting $|W(e^{-j\omega})|\Phi_p(\omega)$ (solid)

The system is excited by the primary force $v_1(t)$ generated by a computer-controlled shaker. The transfer function C/D between the primary and the residual forces is called the primary path. The disturbance signal $p(t)$ is the output of the primary path. The output $y(t)$ is the measured voltage corresponding to the residual force $y_p(t)$. The non-parametric model of the primary path shows that there are several vibrational modes, with the first mode at 31.47 Hz and the second mode around 160 Hz being the most important ones (dashed line in Fig. 3).

The control input $u(t)$ drives a piston that can modify the residual force. The secondary path is defined as the transfer function B/A between the control input and the residual force.

The design objective is to compute a low-order linear discrete-time controller $R(q^{-1})/S(q^{-1})$ that minimizes the residual force around the first and second vibrational modes of the primary path while trying to distribute the amplification over higher frequencies. The control

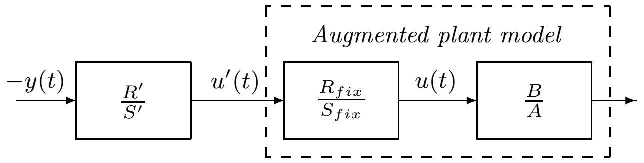

Figure 4: Incorporating fixed terms in the secondary path

specifications are expressed as constraints on the output sensitivity function $\mathcal{S}_{yp}$ and input sensitivity function $\mathcal{S}_{up} = \mathcal{S}_{yp}R/S$. In addition, since the controller gain should be zero at the Nyquist frequency, the term $R_{fix}(q^{-1}) = 1 + q^{-1}$ is incorporated in the controller.

The tuning procedure is modified as follows in order to include the fixed terms R_{fix} and S_{fix} in R and S, i.e. $R = R'R_{fix}$ and $S = S'S_{fix}$: The secondary path model B/A is augmented with the fixed terms R_{fix} and S_{fix} (Fig. 4). Then, $u(t)$ in (4) is replaced by the input of the augmented plant, $u'(t) = \frac{S_{fix}}{R_{fix}}u(t)$. The estimate of the gradient (16) is calculated by replacing $\hat{B}$, $\hat{A}$, R and S with $\hat{B}R_{fix}$, $\hat{A}S_{fix}$, R' and S', respectively. Finally, R' and S' are computed using the iterative algorithm and later multiplied by the fixed terms to obtain the controller polynomials R and S.

Experiments performed on the real suspension system showed that the plant varies slightly with time, thereby reducing the convergence rate of the algorithm. Because of this and the fact that the number of real-time experiments available in this benchmark study is limited, a high-order model of the secondary path (available from the benchmark web site [14]) is used to simulate the secondary path B/A and generate the data needed in this "data-driven" controller tuning procedure. The same model is also used to compute the gradient $\hat{J}'(\rho_i)$ in (15) and Hessian $\hat{Q}(\rho_i)$ in (17). Moreover, open-loop experimental data are available from the benchmark web site, where $v_1(t)$ is a PRBS generated by a 10-bit shift register with data length $N = 20000$ and the measured signal $y(t)$ corresponds to $p(t) + v_2(t)$. Thus, the model C/D of the primary path is not involved in the controller tuning procedure.

The following controller structure is used:

$$K(q^{-1}) = \frac{(r_0 + r_1 q^{-1})(1 + q^{-1})}{1 + s_1 q^{-1} + s_2 q^{-2}} \tag{28}$$

A 2nd-order controller is chosen since it corresponds to the lowest order still capable of meeting the benchmark specifications. All the parameters of the controller are initialized to zero except for $r_0 = 0.0025$. It is also verified that the initial controller $K^0 = r_0(1 + q^{-1})$ stabilizes the closed-loop system.

Considering the spectrum of the primary path (dashed line in Fig. 3) and choosing $W(q^{-1}) = 1$ in Eq. (27), it is evident that the algorithm will reduce the sensitivity function $\mathcal{S}_{yp}$ mainly around the first resonant mode. However, in order to accentuate the higher frequencies, the vector of instrumental variables is filtered by a 3rd-order high-pass Butterworth filter with the cut-off frequency of 100 Hz:

$$W(q^{-1}) = \frac{0.4459 - 1.3377q^{-1} + 1.3377q^{-2} - 0.4459q^{-3}}{1 - 1.459q^{-1} + 0.9104q^{-2} - 0.1978q^{-3}}$$

The magnitude plot of this filter is presented as the dash-dot line in Fig. 3, while the aggregate weighting $|C(e^{-j\omega})/D(e^{-j\omega})||W(e^{-j\omega})|\Phi_{v_1}(\omega)$ of $\mathcal{S}_{yp}$ in Eq. (27) is shown as the solid line. The length of the instrumental variables vector should be larger than the number of controller parameters to be tuned but much smaller than the data length. Here, $n_z = 28$ is chosen.

A local optimum is reached after 8 iterations. In all iterations, the initial step size $\gamma_i = 1$ is used. When the tuning procedure provides a controller that destabilizes the closed-loop system (which is readily verified if a plant model is available), the step size is simply divided by 2. If a plant model is not available, the stability test based on the Vinnicombe gap between two successive controllers can be performed using the closed-loop data [5]. Note that destabilizing controllers were frequently found due to the fact that the underlying open-loop plant has several oscillatory modes.

Fig. 5 shows the output and input sensitivity functions $\mathcal{S}_{yp}$ and $\mathcal{S}_{up}$ before tuning (dash-dot), after 3 iterations (dashed), and after 8 iterations (thick solid line) along with the constraints (thin solid line) provided in the benchmark problem. The resulting controller reduces $\mathcal{S}_{yp}$ considerably around the first and second resonant modes without violating the constraints on $\mathcal{S}_{up}$.

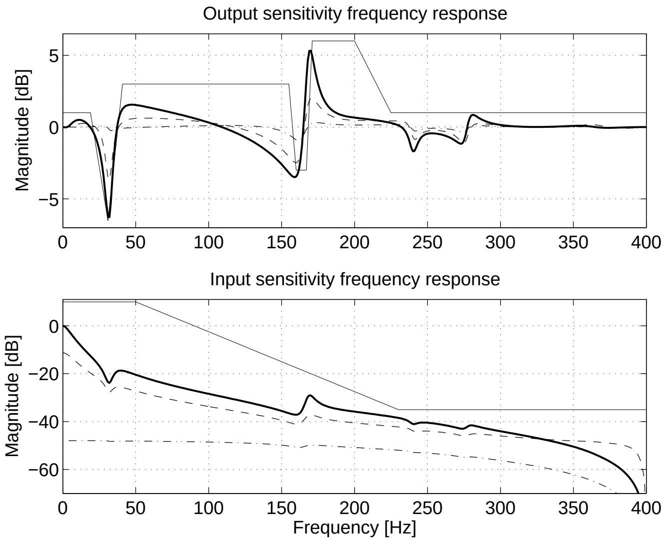

Figure 5: Output and input sensitivity functions of the closed-loop system: before tuning (dash-dot), after 3 iterations (dashed), after 8 iterations (thick solid line), and constraints (thin solid line)

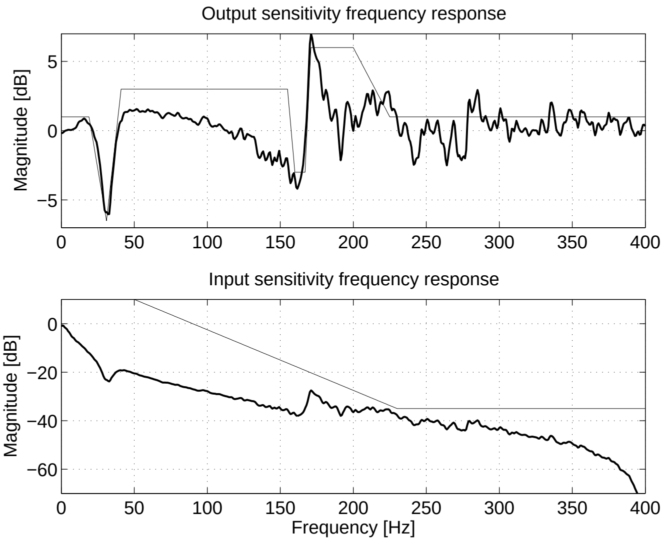

Figure 6: Closed-loop output and input sensitivity function estimated from data collected on the experimental set-up with the final controller (thick line), and constraints (thin line)

The controller obtained in simulation is implemented on the experimental set-up and Fig. 6 shows the corresponding sensitivity functions. The output sensitivity function estimated by spectral analysis slightly violates the constraints at some frequencies. This can be explained by the fact that the model used to generate the data needed for controller design does not describe the experimental system very well around those frequencies. Nevertheless, satisfactory experimental results are obtained using the 2nd-order controller.

5 Conclusions

This paper presents an application of the iterative correlation-based controller tuning scheme to the "Design and optimization of restricted-complexity controllers benchmark". With the assumption that the disturbance signal can be measured, it has been shown that reducing the correlation between the disturbance signal and the output of the closed-loop system is an appropriate objective for tuning restricted-complexity controllers. This approach can also be used with systems where the disturbance cannot be measured but there is the possibility of injecting a known test signal. Though the proposed controller-tuning algorithm uses data collected in the time domain, a frequency analysis indicates how to handle the control specifications expressed in terms of constraints on the sensitivity functions. The resulting restricted-order controller provides satisfactory performance both in simulation and real-time implementation for the active suspension system of LAG.

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