ON DESCRIBING FLUID FLOW IN POROUS MEDIA

P.J. Hensley[1] and D.A. Barry[2]

SYPNOSIS

This paper presents a discussion on equations used for describing fluid flow in porous media. Engineers have traditionally used Darcy's Law to describe fluid motion through a soil body. However, the recent involvement of geotechnical engineers in environmental problems concerning, for example, hazardous waste disposal, has demanded that consideration be given to the subsurface migration of variable density and viscosity fluids through both saturated and partially saturated soils. Under these circumstances, Darcy's Law is still valid only when these variations are incorporated. Alternative equations that describe the motion of a variable density and viscosity fluid through a partially saturated soil are presented in the paper. The use of these equations is illustrated by an example concerning waste flow from a lined tailings pond. The relevance of the equations to geotechnical centrifuge modelling is also discussed.

BACKGROUND

In recent times, geotechnical engineers and researchers have become increasingly involved in the design, modelling and analysis of environmental engineering problems. Research, such as that presented by, for example, Cividini and Gioda, 1989, Aiban and Znidarcic, 1989, Gordon et al, 1989, Chapuis, 1990 and Cooke and Mitchell, 1991a, serves to illustrate the contribution that can be made by geotechnical engineers towards the sensible design of landfill liners, tailings embankments and effective containment measures for waste. However, these papers also serve to highlight a long-standing problem, namely, what is the appropriate equation to employ when describing the relative motion i.e. the motion of the fluid relative to the (possibly) moving porous medium, of a fluid though a soil body? Resolution of this problem will allow members of the geotechnical community to communicate more effectively with other professionals working in the field of environmental engineering.

DARCY'S LAW

The above-mentioned authors have used Darcy's Law to describe the relative

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[1] University Research Fellow, Department of Civil and Environmental Engineering, The University of Western Australia, Perth, Australia.

[2] Senior Lecturer, Centre for Water Research, The University of Western Australia, Perth, Australia.

motion of a fluid through a soil body. Recall that for laminar flow, and a homogeneous, isotropic medium, Darcy's Law simply states that fluid flux is proportional to the gradient in fluid potential.

The generalisation of Darcy's Law for a homogeneous anisotropic porous medium is given by (e.g., Bear and Verruijt, 1987)

$$v = -K \cdot \nabla\phi \qquad (1)$$

where v is the specific discharge vector (volume of fluid flow per unit time per unit area), with components v_x, v_y, v_z in the x, y and z directions, respectively, K is a matrix of the form (Bear, 1972)

$$K = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \quad \text{with} \quad \begin{matrix} K_{xy} = K_{yx} \\ K_{xz} = K_{zx} \\ K_{yz} = K_{zy} \end{matrix} \qquad (2)$$

ϕ is the piezometric head at any point (x, y, z), and $-\nabla\phi$ is the hydraulic gradient with components $-\partial\phi/\partial x$, $-\partial\phi/\partial y$ and $-\partial\phi/\partial z$ in the x, y and z directions, respectively.

The piezometric head, ϕ, at any point (x, y, z) is defined by (Freeze and Cherry, 1979)

$$\phi = z + \frac{P_f}{\rho_f\, g} \qquad (3)$$

where z, the elevational head, represents the potential energy per unit weight of fluid at that point, and $P_f/\rho_f g$, the fluid pressure head, represents the pressure energy per unit weight of fluid at that point.

Note that in many cases, the hydraulic gradient $-\nabla\phi$ is denoted by the symbol i, and the specific discharge, v, is termed the Darcy seepage velocity. Furthermore, the generalisation of Darcy's Law for one-dimensional fluid flow is often expressed as the 'throughput'

$$Q = AKi \qquad (4)$$

where A is the cross-sectional area of soil through which flow is taking place.

It is clear, that the generalised form of Darcy's Law (equation 1), relates the specific discharge of a fluid along any principal direction of an anisotropic porous medium, to the head loss per unit length along that direction and a coefficient of proportionality, K. Equation (1) also demonstrates that, in anisotropic media, the specific discharge is not, in general, in the same direction as the gradient in piezometric head.

The coefficient of proportionality K, normally termed the 'hydraulic conductivity' of the medium (see for example, Rose, 1966, Bear 1972, Greenkorn, 1983 and Wentz, 1989) expresses the ease at which a fluid is transmitted through a soil body. It is a coefficient that depends upon the properties of the soil body itself, namely the shape, packing and arrangement of the solid grains (and pores) within a soil matrix, the percentage of the soil pore space connected by fluid filled channels, and the density and viscosity of the transmitted fluid. The coefficient K is therefore not a fundamental soil property.

The use of K, and hence Darcy's Law, to characterise fluid motion through a soil body, is accordingly limited to describing the transmission of constant density and viscosity fluids through soil bodies of invariant saturation.

INTRINSIC PERMEABILITY

Traditionally, geotechnical engineers have been concerned with the transmission of water through saturated soil deposits under isothermal conditions. The use of Darcy's equation to describe such fluid motion is, in the main, well founded, and so it has commonly been adopted by geotechnical engineers as a general equation for fluid motion.

However, the recent involvement of geotechnical engineers in environmental problems concerning, for example, contaminant transport and hazardous waste disposal, has demanded that consideration be given to the subsurface migration of variable density and viscosity fluids through both saturated and partially saturated soil deposits. We wish to point out that Darcy's law is still valid under these conditions only when these variations are incorporated. In fact, it is more logical to replace the coefficient of proportionality, K, with a more elementary parameter describing ease of fluid flow through a soil body. Such a parameter is given by the intrinsic permeability, k, of a porous medium.

The intrinsic permeability is a measure of the ease of fluid movement through interconnected voids in the porous matrix when all voids are completely fluid saturated (Voss, 1984). It is therefore a parameter which depends only upon the microscopic configuration of the pore space, and consequently may be considered independent of the properties of the interstitial fluid moving through if (de Marsily, 1986).

Numerous formulas relating k to selected properties of the porous matrix are presented in the literature. One empirical relation is given by (Bear and Verruijt, 1987).

$$k = c\,d^2 \tag{5}$$

where d is the effective grain size of the medium and c is a dimensionless coefficient, varying between 4.5 × 10^{-6} for a clayey sand to 14 × 10^{-6} for pure sand (Bear 1979). This equation is more commonly known as Hazen's approximation by geotechnical engineers (Scott, 1980).

A further example is given by the theoretically derived Kozeny-Carman equation (Scott, 1980)

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[2] The tortuosity of the pores is defined as L_e/L, that is the actual distance, L_e, a fluid particle must travel through a porous medium of length L.

$$k = \frac{1}{k_o k_T S_s^2} \quad \frac{n^3}{(1-n)^2} \tag{6}$$

where k_o is a factor depending on the shape of the soil pores, k_T is a factor depending on the *tortuosity*[2] of the pores, S_s is the specific surface area of the solid matrix and n is the porosity of the soil. Carman (1937) suggested a value of 5 for the product $k_o k_T$.

Equations (5) and (6) demonstrate that k is a geometric property of the medium, and serve to illustrate the dependence of k upon various characteristics of the solid matrix.

For Cartesian Co-ordinates (x, y, z), the intrinsic permeability may be expressed as a three dimensional matrix, k. By dimensional analysis it can be related to the hydraulic conductivity, K, through the expression

$$K = \frac{k \rho_f g}{\mu_f} \tag{7}$$

where ρ_f and μ_f denote the density and viscosity of the interstitial fluid, respectively.

GENERAL EQUATIONS FOR FLUID FLOW

Substituting Equations (3) and (7) into Equation (1), and re-arranging, gives the following equation of motion for saturated fluid flow

$$v = -\frac{k}{\mu_f} \left(\nabla P_f + \rho_f g \nabla z\right) \tag{8}$$

Equation (8) equates the driving force per unit volume of fluid, $-(\nabla P_f + \rho_f g \nabla z)$, to viscous drag at the fluid-particle interface, $v \mu_f k^{-1}$ (Bear and Verruijt, 1987). This equation thereforegives some **fundamental insight** into motion of fluid through a porous medium; such insight is not given by Equation (1).

For unsaturated flow, it is necessary to replace the intrinsic permeability with a parameter, k_{fe}, which describes the effective permeability of the medium to the fluid as a function of the fluid saturation level, S_f (Bear, 1972). Due to hysteresis, the effective permeability frequently displays different values at the same fluid saturation level during wetting and drying.

For convenience, the effective permeability of the medium to the fluid can be decomposed into (de Marsily, 1986)

$$k_{fe} = k \, k_{fr} \tag{9}$$

where k_{fr} is the relative permeability to fluid flow at a given level of fluid saturation. The dimensionless parameter k_{fr} is often assumed independent of direction and insensitive to hysteresis during wetting and drying (Voss, 1984).

The counterpart of Equation (8) for unsaturated flow is therefore given by

$$v_f = -\frac{kk_{fr}}{\mu_f}\;(\nabla P_f + \rho_f g \nabla z) \tag{10}$$

Equation (10) is capable of describing the specific discharge of a variable density and viscosity fluid through both saturated and unsaturated zones of soil. As such, this equation is more generally applicable than Equation (1).

Lastly, we advise that the use of 'specific discharge' to describe fluid motion is not a particularly worthwhile notion in environmental engineering problems where the transport of contaminants is considered. The concept of specific discharge is relevant when the quantity of water flowing through a given surface is of interest. However, specific discharge does not give a true indication of the relative velocity of fluid motion through a porous medium, as its value is always less than the true average fluid velocity, u_f. The latter quantity is thus more appropriate when water quality is of primary concern.

The relative motion of a fluid through a porous medium of effective porosity n, is therefore often more instructively described by v_f/nS_f, or

$$u_f = -\frac{kk_{fr}}{\mu_f n S_f}\;(\nabla P_f + \rho_f g \nabla z) \tag{11}$$

where the product nS_f characterises the effective area for fluid flow.

ILLUSTRATIVE EXAMPLE

Consider the measurement of hydraulic conductivity at 25°C in a homogeneous, saturated clay soil using a water permeant, Fig. 1. A hydraulic gradient of $-\nabla\phi = -1$ is set through the sample, and the vertical hydraulic conductivity of the soil is found to be $K_z = 1 \times 10^{-8}$m/s.

The same clay soil is used to construct a liner for a waste tailings pond, figure 2. Underdrainage installed beneath the liner recovers any leachate that has seeped through the liner for re-processing. An estimate of the flow rate through the liner (throughput) is necessary to establish the volume capacity of the re-processing plant. Furthermore, an estimate of the time it would take for a particle of waste to travel through the liner is also needed. The second estimate is required because the concentration of waste placed in the tailings pond periodically varies, and an evaluation of time lapsed between a change of waste concentration in the pond, and a change of liquor concentration in the base drainage system, is wanted for re-processing purposes. Initially, the estimate need only consider the transport of waste by vertical advection.

Provided that the tailings waste has a density and viscosity that is close to that of water at 25°C, Equation (1) can be used to calculate the vertical specific discharge

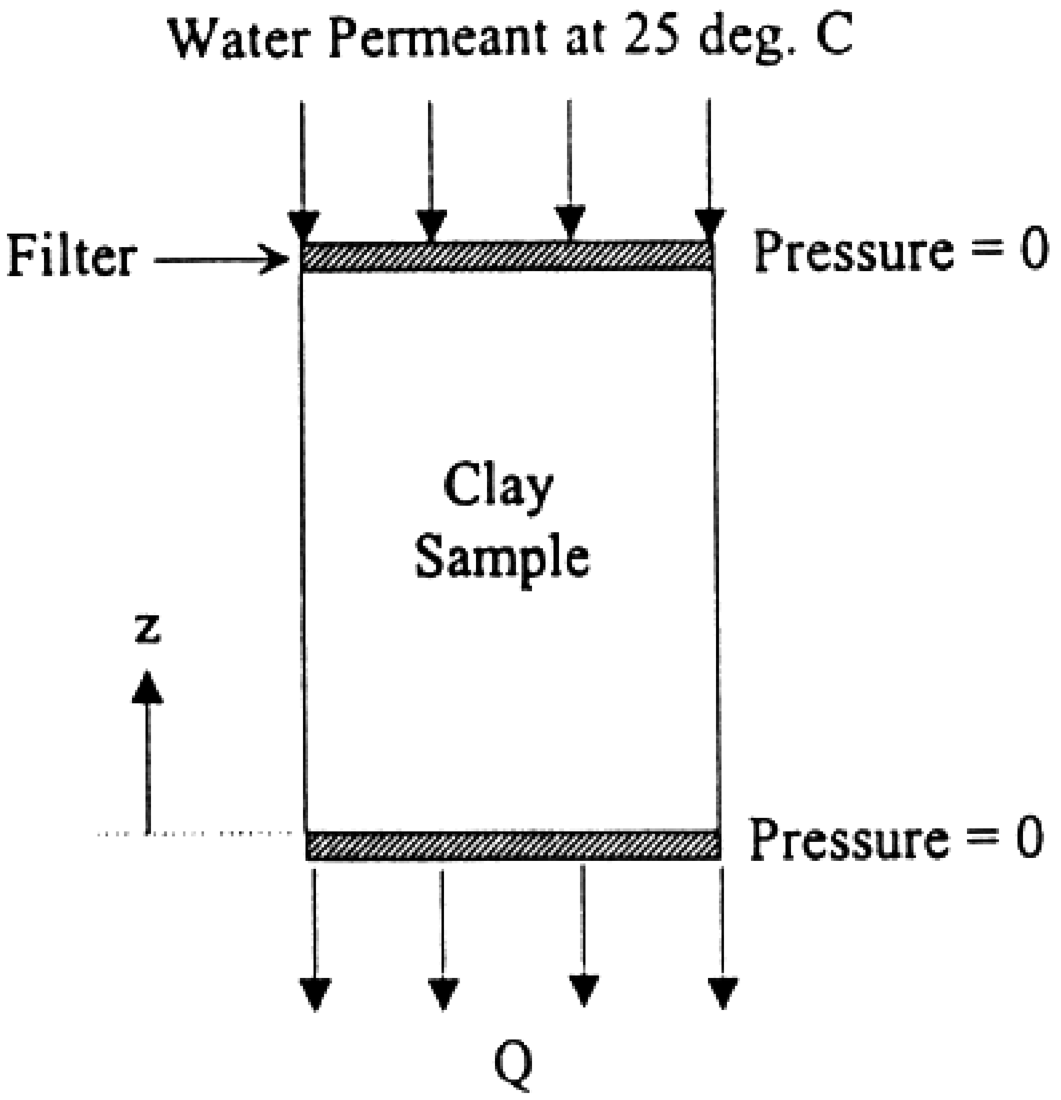

Fig. 1 Hydraulic Conductivity Measurement in Laboratory

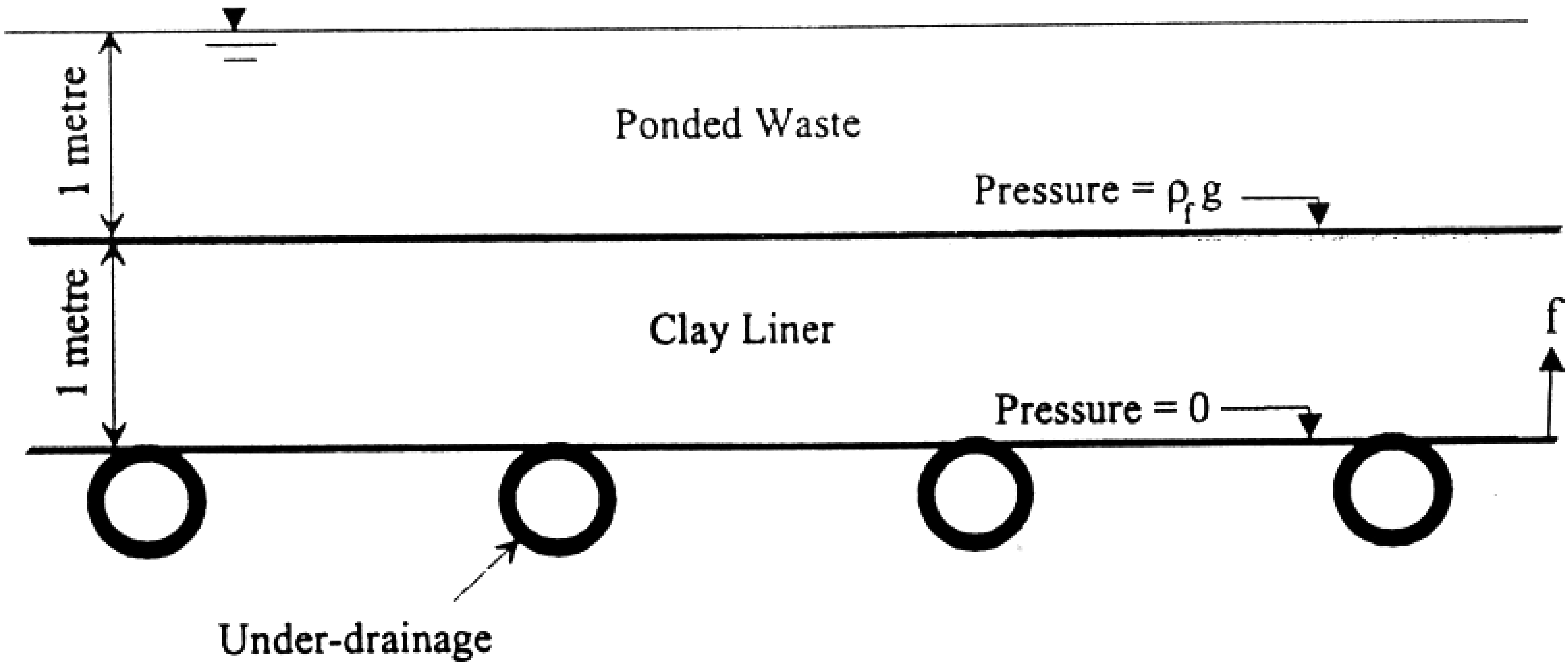

Fig. 2 Hypothetical Problem

(Darcy seepage velocity) of the waste. Recall that the specific discharge is simply the volume of fluid flow per unit area per unit time. The vertical throughput of waste through the liner is thus provided by the product of the vertical specific discharge and the plan area of the liner (refer to Equation 4). For the problem under consideration, the vertical specific discharge is given by $v_{fz} = -2 \times 10^{-8}$m/s : the minus sign merely indicates that the discharge occurs in the opposite direction to $+z$ (refer to Fig. 2).

The traditional application of Darcy's Law to an environmental engineering problem is therefore able to supply information regarding the quantity of waste transmitted through a soil system. However, this approach does not furnish information regarding the true velocity of flow through such a system. Accordingly, the calculated specific discharge can be used to estimate the daily volume of waste recovered from the tailings drainage system. However, we are not in a position to estimate the average travel time for waste through the liner system.

Consider now a situation where the impounded waste is a liquid with a density and/or viscosity that does not match that of water at 25°C. For example, the waste could be an organic liquid, such as oil or xylene, or the waste could contain heavy metals, such as zinc or copper. Alternatively, the temperature of the waste might lie above 25°C, as a result of pre-disposal operations or climatic conditions. Under these circumstances, the vertical hydraulic conductivity, K_z, measured at 25°C in the laboratory using a water permeant, is clearly inappropriate as a parameter to describe the ease at which the waste would pervade the soil liner[3].

We could chose to overcome this problem by repeating our hydraulic conductivity experiment at a representative temperature using the correct permeant. This would enable us to identify a further value for the vertical hydraulic conductivity, that we could symbolise by K'_z. (Note that it is now apparent that hydraulic conductivity is not a fundamental parameter, because we can identify more than one value for the conductivity of a single soil). As before, the value of K'_z could be used in Equation (1) to obtain an estimate for the vertical specific discharge, and subsequently the throughput of waste through the liner. However, we would still not be in a position to estimate the average transport time for waste through the liner.

The problems outlined above can be surmounted if we adopt the intrinsic permeability, k, as a fundamental parameter, and use Equations (10) and (11) as general expressions for fluid flow through the soil liner.

Let us first consider the case where the waste has a density and viscosity close to that of water at 25°C. Through Equation (7), we can ascertain that the vertical intrinsic permeability of the clay is $k_{fz} = 1.02 \times 10^{-15} \mathrm{m}^2$ [4]. The liner is saturated with the waste fluid, so the dimensionless parameter k_{fr} is unity. The pressure gradient through the liner, ∇P_f, is 9810 kg/(m^{2}s^2), and the product $\rho_f g \, \nabla z$ is 9810 kg/(m^{2}s^2). The effective porosity of the liner is $n = 0.4$: we will assume that this value was determined from a representative core sample of the liner.

By inserting the above values into Equations (10) and (11), we calculate that the vertical specific discharge is $v_{fz} = -2 \times 10^{-8}$m/s, and the vertical average fluid velocity is $u_{fz} = -5 \times 10^{-8}$m/s. As before, we can establish the flow rate of waste through the liner from the product of v_{fz} and the liner plan area. However, we are now also in a position to conclude that it would take approximately $1/5 \times 10^{-8}$s

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[3] We know this instinctively as we understand, for example, that under the same bydraulic gradient, a viscous fluid such as oil would permeate through a soil at a slower rate than would water.

[4] It has been assumed that the waste has a density $\rho_f = 1000 \, \mathrm{kg/m^3}$ and a viscosity $\mu_f = 1 \times 10^{-3}$ kg/(ms).

(~230 days) for a waste particle to travel through the 1m thick liner.

As a further example, let us consider a case where the impounded waste has a density $\rho_f = 1220 \text{ kg/m}^3$ and a viscosity $\mu_f = 2.5 \times 10^{-3} \text{ kg/(ms)}$[5]. The vertical intrinsic permeability of the clay remains as $k_{fr} = 1.02 \times 10^{-15} \text{m}^2$, the effective porosity of the liner remains as $n = 0.4$, and the dimensionless parameter k_{fr} remains as unity. However, the vertical pressure gradient across the liner, ∇P_f, and the product $\rho_f g \nabla z$ are now both given by $11968 \text{ kg/(m}^2\text{s}^2)$.

Substitution of the above values into Equations (10) and (11), yields $v_{fz} = -0.98 \times 10^{-8} \text{m/s}$ and $u_{fz} = -2.4 \times 10^{-8} \text{m/s}$, respectively. We may thus estimate that the discharge of the more dense and viscous fluid through the liner is approximately half that of water at 25°C. Furthermore, we can gauge that the latter waste would take an average of 480 days to travel through the liner, as opposed to 230 days, for a waste with properties analogous to those of water at 25°C.

Our calculations have also informed us that, although the higher density of the latter waste increased the driving force through the liner, the high viscosity of the waste actually afforded a greater viscous drag. We have thus gained some practical insight into waste transport through the liner.

The calculations performed above related to saturated fluid flow in a soil medium. For an unsaturated system, the relationship between k_{fr} and the degree of fluid saturation must be determined. This could be done independently in the laboratory. For our hypothetical example, the level of fluid saturation in the field could be ascertained from a representative core sample of the liner. Appropriate values of k_{fr} and S_f could then be used in Equations (10) and (11), to calculate the vertical specific discharge and the average fluid velocity through the liner, respectively.

Our calculations for a saturated soil have shown that we can use the intrinsic permeability to investigate the flow of an assortment of fluids through a soil, without having to determine the hydraulic conductivity of each permeant. We have assumed that the intrinsic permeability is a fundamental soil parameter. However, the reader should be aware that certain concentrated organic liquids, such as ethanol and acetone, may actually amend the permeability of the media that they infiltrate (Brown and Daniel, 1988).

FLUID FLOW IN GEOTECHNICAL CENTRIFUGE MODELS

Researchers have long recognised that, for a given porous medium, the local fluid velocity in a reduced scale centrifuge model of 1/N experiencing a relative centrifugal field of Ng, is N times higher than the corresponding fluid velocity in the full scale structure.

$$u_{f_m} = N\, u_{f_p} \tag{12}$$

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[5] These values are similar to those of waste fluid from alumina production (Randolph et al. 1991).

where subscripts m and p refer to model and prototype, respectively.

Many researchers attribute the increase in local fluid velocity, u_{fm}, to an N-fold increase in the local hydraulic gradient, $-\nabla\phi$, acting within the centrifuge model (see for example, Goodings, 1979 and Jessberger and Stone, 1991). This assumption presupposes that the Darcy coefficient, K, is a material constant for a given soil permeant.

Other researchers recognise the strict dependence of K upon the unit weight of permeant (refer to Equation 7), and attribute the increase in local fluid velocity to an N-fold increase in K (see for example Carghill and Ko, 1983 and Tan and Scott, 1985). This assumption presumes similitude of the hydraulic gradient between model and prototype.

In the past, disparity between scaling relationships for modelling fluid flow under enhanced gravities has led to a certain amount of debate and confusion (Taylor, 1987 and Craig, 1991). We suggest that any ambiguity in defining these relationships can be avoided if Equation (11) is used to characterise the fluid flow.

In a reduced scale model experiencing a relative centrifugal field of Ng, both pressure gradients and gravity forces are increased by a factor N. Under these conditions, the driving force per unit volume of fluid, $-(\nabla P_f + \rho_f g \nabla z)$, is N times higher in the centrifuge model than the corresponding prototype, and viscous drag at the fluid-particle interface, $v\mu_f\,(kk_{fr})^{-1}$, must therefore be increased by the same factor N.

If the same soil and fluid are used in both the centrifuge model and prototype, equality between model and prototype will be achieved for the effective permeability to partially saturated flow, kk_{fr}[6] (Cooke and Mitchell, 1991b). Under these conditions, the N-fold increase in viscous drag at the fluid-particle interface is the result of an N-fold increase in the fluid specific discharge, v_f, thus intimating an N-fold increase in the local seepage velocity, u_f. Of course, this relationship only holds true if laminar flow conditions are preserved in the centrifuge model.

This approach corroborates the scaling law given by Equation 12, and clearly identifies the relationships which form the basis for this law.

If similitude between soil microstructure, fluid viscosity or fluid saturation level is not achieved between model and prototype, the N-fold increase in viscous drag at the fluid-particle interface will not result in an N-fold increase in the specific discharge. This fact is commonly exploited by researchers who wish to model dynamic events in a geotechnical centrifuge (Whitman, 1984, Lee and Schofield, 1988). During dynamic testing, similarity requires that the soil particle velocity, u_s, is equivalent in both model and prototype (Schofield, 1981). As a consequence, similarity must also be maintained between fluid velocities in model and prototype, in order to ensure correct relative motion of fluid in the centrifuge model. This criterion can be met by representing the prototype interstitial fluid with a model permeant of viscosity $\mu_{fm} = N \mu_{fp}$. Under these conditions, the N-fold increase in viscous drag is realised entirely by the N-fold increase in fluid viscosity, and local fluid velocities in the centrifuge model remain equivalent to local velocities in the prototype.

Fluid movement in porous media where fluid density varies spatially may be driven both by differences in fluid pressure and by variations in fluid density (Voss, 1984). Equation (11) illustrates that, in a reduced scale model under a centrifugal field of Ng, the mobilising forces for pressure and density driven flow are increased proportionally by the same factor, N. The correct balance between these forces is difficult to achieve outside a centrifugel field, unless a full scale field experiment is carried out. Equation (11) therefore also serves to highlight the value in centrifuge modelling of environmental engineering problems : this attribute is not apparent from equation (1).

CONCLUSIONS

Engineers have traditionally used Darcy's Law to describe fluid motion through a soil body. However, the recent involvement of geotechnical engineers in environmental problems concerning, for example, hazardous waste disposal, has demanded that consideration be given to the subsurface migration of variable density and viscosity fluids through both saturated and partially saturated soils. Under these circumstances, Darcy's Law is still valid only when these variations are incorporated. Alternative expressions, that characterise the motion of a variable density and viscosity fluid in a partially saturated soil, were presented as Equations (10) and (11) in the Paper. The majority of other disciplines working in the environmental engineering field use these equations as general expressions for describing specific discharge and fluid velocity, respectively. If geotechnical engineers are to be effective in communicating and generalising their work across interdisciplinary barriers, we suggest that they too accept these equations as their standard. Of course, one can specialise these equations as permitted by the physical conditions under consideration.

Finally, Darcy's coefficient K is often referred to, somewhat confusingly, as 'soil permeability'. Equation (7) shows clearly that it is both a fluid and a porous medium property. It appears logical to reserve the term 'soil permeability' for the porous medium property, and the term 'hydraulic conductivity' for K. This distinction is common in all disciplines dealing with flow in natural porous media, and we recommend that it is also adopted by geotechnical engineers.

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[6] It should be noted that equality is only achieved for this parameter if microscopic lengths, such as particle size, are not scaled between model and prototype.

LIST OF SYMBOLS

A	cross-sectional area for fluid flow, L^2
c	dimensionless coefficient
d	effective grain size, L
g	acceleration due to gravity, LT^{-2}
i	hydraulic gradient
k	intrinsic permeability, L^2
k	intrinsic permeability tensor, L^2
k_{fe}	effective permeability tensor, L^2
k_{fr}	relative permeability
k_o	shape factor
k_T	tortuosity factor
K	hydraulic conductivity (Darcy's coefficient), LT^{-1}
K	hydraulic conductivity tensor, LT^{-1}
L	length of medium, L
L_e	actual distance travelled by fluid particle through medium, L
n	porosity
N	centrifuge scaling factor
m	model
p	prototype
P_f	fluid pressure, $ML^{-1}T^{-2}$
S_f	fluid saturation level
S_s	specific surface area, L^{-1}
u_f	true average fluid speed, LT^{-1}
u_f	true average fluid velocity, LT^{-1}
u_s	soil particle velocity, LT^{-1}
v	specific discharge vector, LT^{-1}
z	elevational head, L
μ_f	viscosity of fluid, $ML^{-1}T^{-1}$
ρ_f	density of fluid, ML^{-3}
ϕ	piezometric head, L
∇	del operator, L^{-1}

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