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On the Brutsaert temperature roughness length model for sensible heat flux estimation

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Abstract. The scalar roughness length for temperature, z_{0h}, is necessary to estimate the sensible flux from atmospheric surface layer similarity theory in conjunction with skin temperature measurements. A theoretical relationship for z_{0h} as a function of the roughness Reynolds number z_{0+} which was developed by *Brutsaert* [1975] for rough-bluff surfaces is often used to link infrared skin temperature measurements to atmospheric temperature measurements. Measurements of sensible heat flux and temperature at two semiarid sites are used to evaluate and test the temperature roughness length model coefficients. Consideration of the measurement error is important to derive an accurate set of coefficients. These new field-based coefficients correct for some of the underprediction of sensible heat flux at high flux rates that occurred with past formulations.

1. Introduction

The *Monin and Obukhov* [1954] mean similarity model for temperature may be written

$$T_s - T_a = \frac{H}{ku_*\rho c_p}\left[\ln\left(\frac{z_h}{z_{0h}}\right) - \Psi_h(\zeta)\right], \qquad (1)$$

where H is the sensible heat flux, z_h is the measurement height for temperature in the surface layer of the atmospheric boundary layer (ABL), k $(=0.4)$ is von Karman's constant, u_* $(=(\tau_0/\rho)^{1/2})$ is the friction velocity, τ_0 is the surface shear stress, ρ is the air density, c_p is the specific heat of air, T_s and T_a are the surface and air temperatures, respectively, z_{0h} is the roughness length for temperature, and Ψ_h is the similarity function for temperature. The similarity function depends on the dimensionless variable ζ, defined as z_h/L, where L is the Obukhov length,

$$L = \frac{-\rho u_*^{\,3}}{kg\left[\left(\frac{H}{T_a c_p}\right) + 0.61E\right]}, \qquad (2)$$

E is the evaporation, and g is the gravitational acceleration. The roughness length for temperature, z_{0h}, is simply an integration constant defined by the Monin-Obukhov similarity theory which quantifies the height above the surface ($z = 0$) where the temperature is taken to be equal to T_s. Note that z_{0h} is the surface intercept of the atmospheric surface layer temperature profile in the same way that the momentum roughness length z_{0m} is the zero velocity intercept for the surface layer velocity profile,

$$\bar{u} = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_{0m}}\right) - \Psi_m(\zeta)\right], \qquad (3)$$

where $\bar{u}$ is the mean wind speed, z is the measurement height, and Ψ_m is the similarity function for momentum.

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Equation (1) provides a useful approach for obtaining sensible heat flux estimates when remotely sensed surface temperatures are available and z_{0h} is known a priori [*Kustas et al.*, 1989; *Sugita and Brutsaert*, 1996; *Sugita and Kubota*, 1994]. However, because of its definition as an integration constant, there is no direct way to measure the temperature roughness height. Although it is assumed that for fully rough flow the momentum roughness length is a constant equal to the surface roughness length, z_0, roughness lengths for scalars are not the same as the roughness length for momentum [*Brutsaert*, 1982]. This is because near the wall momentum is transferred by both diffusion and pressure gradients, while scalars are transferred from the surface by diffusion only.

It is usually theorized that for bluff-rough surfaces, scalar roughness lengths depend only on the surface shearing parameterized by the roughness Reynolds number, z_{0+} $(=(u_*\, z_0)/\nu$, where ν is the kinematic viscosity), and the surface geometry. (*Nikuradze* [1933] was perhaps the first to present roughness lengths as a function of roughness Reynolds number.) A model for z_{0h} as a function of z_{0+} can be developed by matching the profile of temperature in the interfacial sublayer with the dynamic sublayer profile [*Brutsaert*, 1975, 1982]. The profile in the interfacial sublayer (or bottom of the atmospheric surface layer (ASL)) is assumed to be of the form

$$T_s - T_i = \frac{H}{ku_*\rho c_p}\Phi_{0h}, \qquad (4)$$

where T_i is the temperature at z_i, the top of the interfacial sublayer, and Φ_{0h} is a function of the Prandtl number of air, z_i, and the surface geometry. Although it is possible to derive the Φ_{0h} function for a known geometry and flow field in the laboratory [e.g., *Hosni et al.*, 1991], this approach is not feasible for field use where the geometry of the surface cannot be known in such detail. Following Brutsaert, the unknown function Φ_{0h} is eliminated by defining an interfacial Stanton number

$$St_0 = \frac{H}{u_*\rho c_p(T_s - T_i)}, \qquad (5)$$

Table 1. Formulations for Interfacial Transfer Coefficients in Literature

Reference	$St_0^{-1} - Cd_0^{-1/2}$
Dipprey and Sabersky [1963]	$10.25 z_{0+}^{0.20} Pr^{0.44} - 8.48$
Owen and Thomson [1963]	$2.40 z_{0+}^{0.45} Pr^{0.8}$*
Sheriff and Gumley [1966]	$7.78 z_{0+}^{0.199} - 4.65$
Brutsaert [1975]	$7.3 z_{0+}^{1/4} Pr^{1/2} - 5$

*Owen and Thomson do not use a value for the drag coefficient.

where $\Phi_{0h} = k\ St_0^{-1}$ and

$$T_s - T_i = \frac{H}{ku_*\rho c_p}(k\ St_0^{-1}). \quad (6)$$

Using the logarithmic temperature profile for the dynamic sublayer (i.e., the lower region of the ASL),

$$T_i - T_h = \frac{H}{ku_*\rho c_p}\ln\left(\frac{z_h}{z_i}\right), \quad (7)$$

combined with (6) gives

$$T_s - T_h = \frac{H}{ku_*\rho c_p}\left[kSt_0^{-1} + \ln\left(\frac{z_h}{z_i}\right)\right]. \quad (8)$$

The unknown height, z_i, is eliminated using

$$\ln(z_i) = kCd_0^{-1/2} + \ln(z_{0m}) \quad (9)$$

where $Cd_0^{-1/2}[=\bar{u}_i/u_*]$ is the drag coefficient at the top of the interfacial sublayer. Substituting (9) into (8),

$$T_s - T_h = \frac{H}{u_*\rho c_p}\left[St_0^{-1/2} - Cd_0^{-1/2} + \frac{1}{k}\ln\left(\frac{z_h}{z_{0m}}\right)\right], \quad (10)$$

and taking the dynamic sublayer equation for T_h,

$$T_s - T_h = \frac{H}{ku_*\rho c_p}\ln\left(\frac{z_h}{z_{0h}}\right), \quad (11)$$

we arrive at

$$z_{0h} = z_{0m}\exp\left[(-k)(St_0^{-1/2} - Cd_0^{-1/2})\right], \quad (12)$$

where $St_0^{-1} - Cd_0^{-1/2}$ is the interfacial transfer coefficient for heat.

To close this equation, a parameterization of the terms of the interfacial transfer coefficient in terms of known variables is needed. It is assumed that the Stanton number is a function of the roughness Reynolds number, z_{0+}, the Prandtl number of the fluid, Pr, and a geometric coefficient C_R,

$$St_0 = C_R z_{0+}^{-\beta} Pr^{-\gamma}, \quad (13)$$

so that in general form (12) can be written

$$z_{0h} = z_{0m}\exp\left[(-k)(C_R^{-1} z_{0+}^{\beta} Pr^{\gamma} - Cd_0^{-1/2})\right]. \quad (14)$$

Numerous sets of the four empirical coefficients C_R, β, γ and $Cd_0^{-1/2}$ in this relationship have been developed from various studies; some are presented in Table 1 (see also table 4.2 of *Brutsaert* [1982] for equivalent relations for $Da_0^{-1} - Cd_0^{-1/2}$, the interfacial transfer coefficient for water vapor). The values for $\beta(=1/4)$ and $\gamma(=1/2)$ in the *Brutsaert* [1975] equation are derived from an eddy renewal model of the heat transfer process. In this model, *Danckwerts*' [1951] assumption of a time-independent renewal rate is used, so that the age distribution of the eddies is exponential, allowing an analytic solution.

The results of using these expressions to estimate heat transfer and evaporation in atmospheric flows over bluff-rough surfaces have been mixed. For example, Brutsaert's proposed relationship, given in Table 1, has often been used [e.g., *Parlange and Katul*, 1992b; *Katul and Parlange*, 1993] to calculate scalar roughness heights for the calculation of sensible heat and evaporation. However, *Hignett* [1994] found a large amount of scatter of the measured value of the ratio $\ln(z_0/z_{0h})$ around the line predicted by Brutsaert's model. When the models for z_{0h} presented in Table 1 were used to predict H from (1) using the experimental data collected for this study, the calculated H from all four of the models underestimated the sensible heat flux compared to the measured sensible heat ($\rho c_p \overline{w'T'}$ where w is the vertical wind velocity and the primes indicate fluctuations). The slopes of the regression line forced through the origin for the predictions were found to be close to 0.80 for all four tabulated models. This suggests that coefficients for the interfacial heat transfer relationship given in (14) derived from analyzing the field data themselves could provide a better means of predicting sensible heat at another field site. One reason for the discrepancy between the theory and experiment in the field is that the calibration of past models for the drag coefficient Cd_0 has been based on laboratory measurements, which may not necessarily scale well to the geometries and flows found at the land-atmosphere interface. Alternately, the assumptions in the eddy-renewal model may not be true. If the renewal rate of the eddies were not time-independent, the age distribution would not be exponential, in which case the values of 1/4 and 1/2 for the coefficients β and γ would not be perfectly accurate. In this paper, in using a large number of high-quality field measurements, we seek to quantify the parameters in the interfacial transfer coefficient for heat. The parameters derived from a set of measurements taken at one site (the Campbell Tract, University of California, Davis) were tested against measurements of sensible heat flux taken at another separate location (Owens Valley, southern California).

2. Experiment

The hydrometeorological data used in the analysis were taken during two experiments during the summer of 1994. The first experiment took place at the University of California–Campbell Research Tract, at Davis, California, while the second was conducted on the dry bed of Owens Lake, in Owens Valley, California. Because the data set from the Campbell Tract was larger than that taken at Owens Valley, the Campbell Tract data were used to determine the transfer coefficient parameters, while the Owens Valley data set was used to test the efficacy of the parameters in predicting sensible heat.

The section of the Campbell Tract used in the experiment is an unvegetated field of uniform bare soil, approximately 500 m × 500 m, oriented along the north-south and east-west axes. Sprinkler irrigation allowed uniform wetting of the field over an area approximately 160 m × 160 m. The instruments used in the experiment were situated in the northeast corner of the field in order to provide as much fetch as possible for the prevailing southwest winds. Three irrigations were performed during the course of the summer. It has been found in previous investigations that the fetch length of the Campbell Tract is sufficient for full development of the boundary layer [*Katul and Parlange*, 1992; *Parlange and Katul*, 1992a]. The irrigations, which took place at night, were uniformly distributed over the surface (see *Katul and Parlange* [1993] for calculated distribution), and the soil surface was saturated for at least a day after the irrigation. The soil is Yolo silt loam.

The second experiment took place on the dry, unvegetated bed of Owens Lake. There were two sites used on the Owens Lake bed; upwind fetch at both Owens Lake sites was on the order of 10 km, with virtually no soil mounds or debris. There were no hydrometeorological differences between the two sites at Owens Lake; the two sites were chosen for a separate groundwater study. The Owens Lake bed is very arid, and a salt crust on the soil surface prevents much evaporation [*Albertson et al.*, 1995]. There was no measurable precipitation recorded during the experiment, and skies were generally clear, although a large cloud formation went over the experimental site during the afternoon of the second day of measurements.

The same suite of instruments was used at both the Campbell Tract and the Owens Lake sites. Instruments used consisted of a Gill Instruments three-dimensional (3-D) sonic anemometer to measure $u_*(=(\overline{-u'w'})^{1/2})$ and mean wind speed $\bar{u}$, a Campbell Scientific eddy correlation system (a sonic anemometer with a fine-wire thermocouple and a krypton hygrometer) to measure $E(=\rho\overline{w'q'_a}$ where q_a is the air humidity) and $H(=\rho c_p\overline{w'T'})$, an Everest infrared temperature sensor (which was placed at a 45°) to measure T_s, and a Vaisala relative humidity and temperature probe to measure q_a and T_a. To estimate energy budget closure, net radiation was measured with a REBS Q-7 net radiometer, and soil heat flux (G) was measured with two soil heat flux plates buried at approximately 0.5 cm depth. The heights of the atmospheric instruments are listed in Table 2. Measurements of air and surface temperature, relative humidity, net radiation, and soil heat flux were collected at 1 Hz and stored as 20-min averages. The eddy correlation station measurements of H and E were taken at 10 Hz and were saved at the same 20-min intervals as the micrometeorological measurements. The q signal from the krypton hygrometer was corrected for both attenuation of the krypton light by oxygen and the nonzero mean vertical velocity $\bar{w}$ present under unstable density stratification (the "Webb correction" [*Webb et al.*, 1980]). The 3-D sonic anemometer was operated at 21 Hz. The entire signal was saved and later broken into 20-min blocks, from which average u_* and $\bar{u}$ were calculated. The Obukhov length L was calculated for a given 20-min period using the measured u_* from the 3-D sonic, and H and E from the eddy correlation system.

3. Results and Discussion

3.1. Regression Models

As mentioned before, since the data were available from two separate sites, it was decided to use the data set collected at the Campbell Tract to determine the coefficients found in (14), which could then be tested against the data collected in Owens Valley. Because there are four unknown parameters (C_R, β, γ, and $Cd_0^{-1/2}$) in (14), a large number of different fitting schemes can be developed. We considered two different regression models that were designed to test different coefficients of (14). (Alternately, we could optimize a merit function for C_R, β, and γ, but given the significant amount of noise in the data, it is a difficult task to get statistically significant results.)

Table 2. Instrument Heights

	Campbell Tract	Owens Lake Site 1	Owens Lake Site 2
3-D sonic anemometer	0.85, 1.50*	2.67	2.71
Eddy correlation instruments	0.95	1.74	1.80
Infrared temperature sensor	1.30	1.80	1.84
Air temperature and relative humidity	0.85	1.52	1.09
Net radiation	1.30	1.75	1.83

Heights given in meters.
*The 3-D sonic was at 0.85 m until Julian date 194; on this date it was raised to 1.50 m.

3.1.1. Model 1. The first regression model is based on the argument that the existing values for β and γ are the most strongly grounded in theory [*Brutsaert*, 1975] and that the drag coefficient $Cd_0^{-1/2}$ is probably the least well known parameter. Hence the drag coefficient was regressed against the measured sensible heat flux; this should yield a constant value for Cd_0, which can then be used with the theoretical values $\beta = 1/4$ and $\gamma = 1/2$ from *Brutsaert* [1975].

3.1.2. Model 2. The second regression was designed to find the values of the dimensionless group $C_R\,Pr^{\gamma}$ and β which gave the best fit relationship of the Stanton number St_0 as a function of z_{0+}, while the drag coefficient was held at the value of 5. The possibility that the theoretical values for β and γ are not strictly correct exists if the assumption of time-independent eddy renewal rate does not hold. The desirability of testing how well the previously used values for these parameters match the observations is shown in Figure 1. This figure presents a sensitivity analysis of the calculated sensible heat flux for a given change in a single parameter (either $C_R\,Pr^{\gamma}$, β, or $Cd_0^{-1/2}$) from the values used in the *Brutsaert* [1975] model, with the other two parameters held constant. As can be seen, the calculated value of H changes more for a given percentage change in β or the dimensionless group $C_R\,Pr^{\gamma}$ than for the drag coefficient. This indicates that for accuracy in sensible heat flux prediction, it is most important to know the values of β and $C_R\,Pr^{\gamma}$.

In the laboratory experiments it is possible to investigate the values of the geometric constant and γ separately by performing experiments with fluids with different Prandtl numbers. Since our atmospheric observations deal only with air, we are unable to do this. The results are reported therefore in terms of the combined dimensionless quantity $C_R\,Pr^{\gamma}$, and no attempt is made to derive an independent value for γ. We will later relax this restriction and use the theoretical value of $\gamma = 1/2$ in order to compare the resulting value of the empirical geometric factor C_R against the previously proposed value of 7.3.

Before attempting any fitting procedure, it is instructive to note the great deal of scatter in the values of $\ln(St_0)$ versus $\ln(z_{0+})$ calculated from the field measurements (Figure 2) as compared with any of the equivalent plots from the laboratory studies (cf. Figure 14 of *Dipprey and Sabersky* [1963], Figure 3 of *Owen and Thomson* [1963], and Figure 16 of *Sheriff and Gumley* [1966]). The calculation of the Stanton number was done using (5), after determining T_i from (1), with a value of $7.5z_0$ assumed for z_i. This value for the height of the interfacial sublayer follows directly from the assumption that $Cd_0^{-1/2} = 5$. Similar scatter can be seen in the graph of $Cd_0^{-1/2}$ versus H (see Figure 3). This difference is due in large part to the great care the laboratory researchers were able to

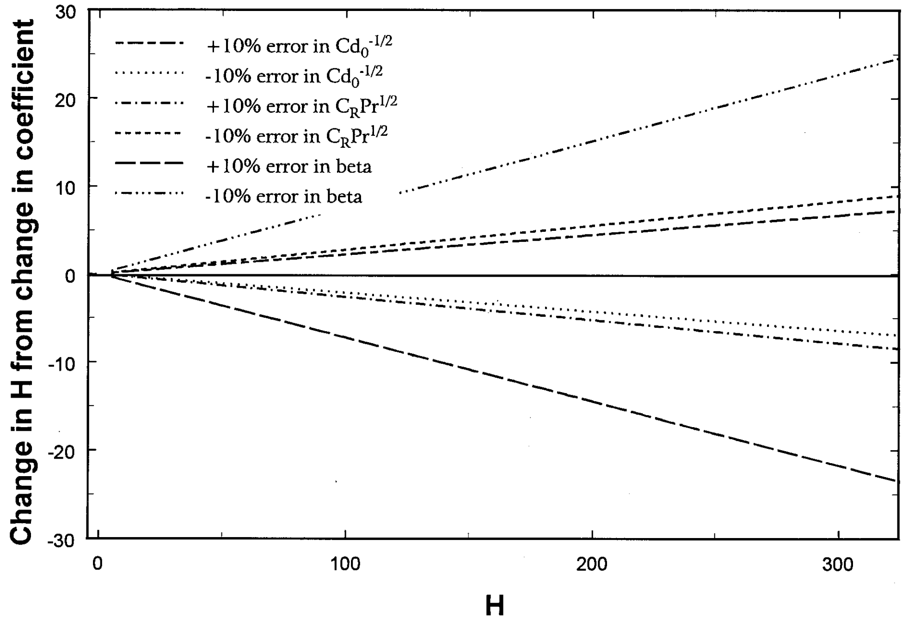

Figure 1. Sensitivity of estimated H to error in parameters $C_R Pr^{1/2}$, β, and $Cd_0^{-1/2}$ were each separately changed from their respective assumed values of $7.3 \times 0.71^{1/2}$, 1/4 and 5, and the difference between H predicted from the new values from H predicted from the original values was calculated.

take in running their experiments and in their ability to replicate exactly the experimental conditions so that results could be averaged to reduce noise (as several of the researchers state they did). Although the laboratory researchers do not explicitly describe their regression procedures, it seems apparent that only simple unweighted regression needed to be used because of the low noise level in the laboratory measurements. We are unable to manipulate experimental conditions when studying environmental flows, so experimental noise needs to be treated in an alternate way than signal averaging. Our analysis needs to take into account the certainty we have of how accurate a given measured variable is, based on instrument response. For variables such as St_0 and z_{0h}, which are calculated from measured variables such as H and u_*, we need an analysis of the propagation of error of the measured variable to the calculated ones.

To briefly illustrate the importance proper weighting of the data has on the derived coefficients and the resulting prediction of H, we refer again to Figure 3, which shows the results of the best fit for $Cd_0^{-1/2}$ with and without weighting of the

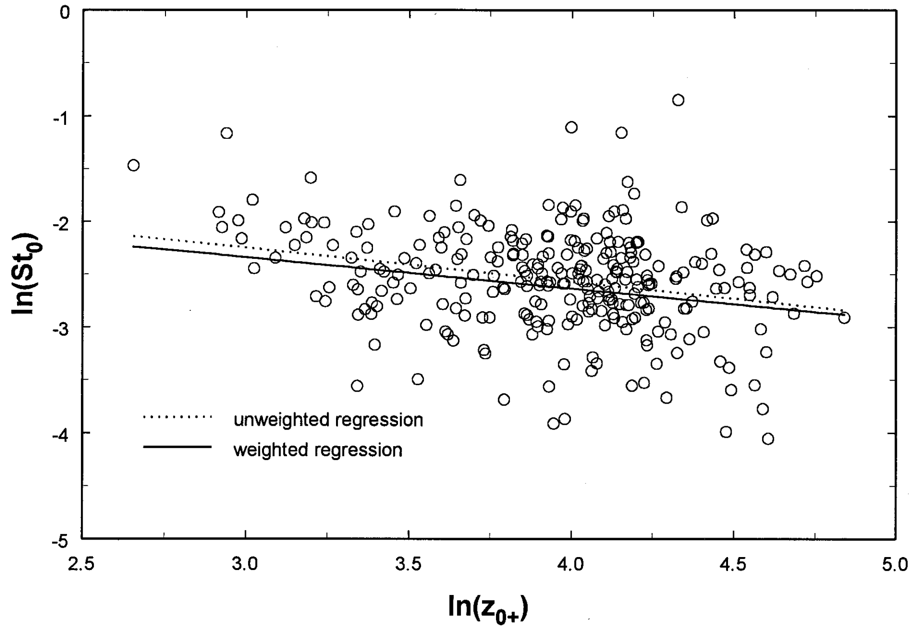

Figure 2. Model 2: ln (St_0) versus ln (z_{0+}), calculated from Campbell Tract measurements with best fit lines for both weighted (solid line) and unweighted (dotted line) regression.

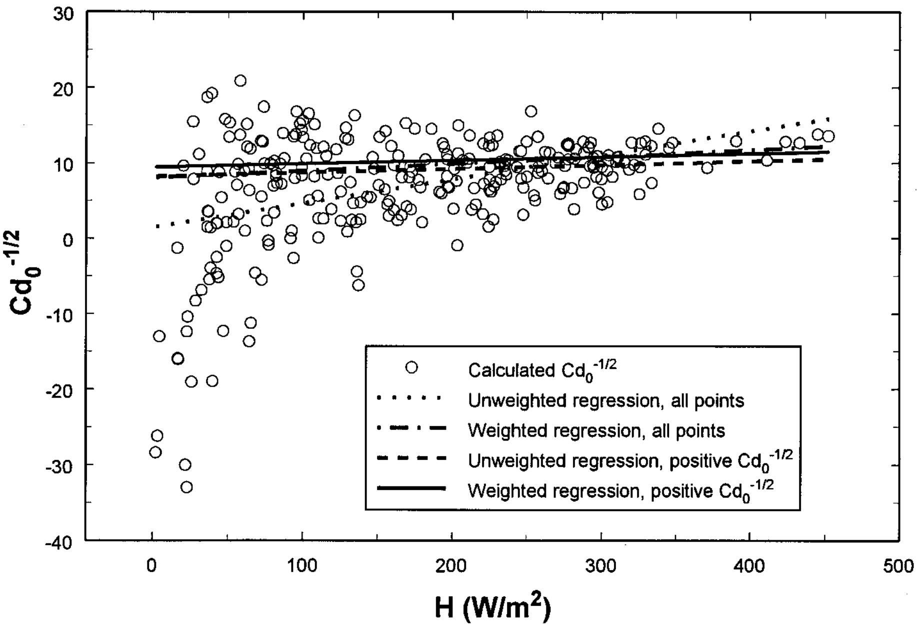

Figure 3. Model 1: $Cd_0^{-1/2}$ calculated from Campbell Tract measurements versus measured $H(= \rho c_p \overline{w'T'})$ with best fit lines for both weighted and unweighted regression. Results for regressions in which all points are used and in which only positive values of $Cd_0^{-1/2}$ are used are presented.

data points (the regression model is discussed in more detail below.) We would expect the unstable drag coefficient to be a constant with respect to H. It can been seen, however, that there is significant scatter, especially at low values of the sensible heat, where there are even negative values of the drag coefficient. Although $Cd_0^{-1/2}$ cannot be less than 0, the negative values arise because the errors of the measured quantities (H, T_s, T_a, u_*) have combined in such a manner as to cause the error in the calculated $Cd_0^{-1/2}$. From the regression results presented in Table 3, we see that the unweighted regression is strongly affected by the spurious negative values of the calculated drag coefficient present at low measured H. On the other hand, the weighted regression yields a nearly constant positive value for $Cd_0^{-1/2}$, which is expected.

3.2. Error Analysis

The goal of the error analysis was to determine what was the level of uncertainty, expressed as the standard deviation, for each point used in the fitting of the two z_{0h} models. Because the variables used in the two regression models are not directly measured but are instead derived from measured quantities, it is necessary to use the error propagation equation to express the standard deviation of the derived variables in terms of standard deviations of measured variables [*Bevington and Robinson*, 1992]. For a derived variable x which is a function of measured variables u and v, the error propagation equation is

$$\sigma_x^2 \cong \sigma_u^2 \left(\frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v} \right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u} \right) \left(\frac{\partial x}{\partial v} \right), \quad (15)$$

where σ_x is the error in variable x, σ_u is the error in variable u, σ_v is the error in variable v, and σ_{uv} is the covariance of the error between variables u and v. The approximation relationship is specified because higher-order terms of a Taylor series are dropped. It is obvious that in order to calculate the standard deviations of the derived variables, expressions for the measurement error of the measured variables are needed. This section therefore contains two parts: (1) calculation of the error estimates in the measured quantities H, T_h, T_s, and u_* and (2) calculation of the error estimates in the derived variables (ln (St_0), ln (z_{0+}), ln (z_{0h}), $Cd_0^{-1/2}$, etc.) using the measured variables' error estimates.

It is important to make clear that σ as used in this section refers to the standard deviation of the measurements of a variable, which arises from the impossibility of exact measurements. This standard deviation of the measurements is conceptually different from the standard deviation of a fluctuating variable, which exists apart from any measurement. For example, the ratio of the horizontal wind fluctuations divided by the mean horizontal wind, $\overline{(u'^2)^{1/2}}/\bar{u}$, is the turbulence intensity, which is a characteristic of the flow field at a given time. However, the standard deviation of the measured mean horizontal wind as defined in this section is an instrument-dependent quantity. A sonic anemometer would give a more accurate measurement than a cup anemometer, and would have a smaller σ_u. Likewise, the quantity $\overline{u'^2}/u_*$ is generally constant in the boundary layer [*Stull*, 1988]. The quantity σ_{u_*}/u_*, however, will not be constant, since the measurement error in the friction velocity σ_{u_*} is a function of the measurement

Table 3. Regression Statistics of Model 1, $Cd_0^{-1/2} = a + b * H$

$Cd_0^{-1/2}$ regression	a	b	Standard Error of a	Standard Error of b
Unweighted, all points	1.48	3.18e-2	8.39e-1	4.20e-3
Weighted, all points	8.05	9.19e-3	7.01e-1	2.76e-3
Unweighted, positive $Cd_0^{-1/2}$ points only	8.23	5.01e-3	5.49e-1	2.60e-3
Weighted, positive $Cd_0^{-1/2}$ points only	9.42	4.55e-3	7.22e-1	2.83e-3

$Cd_0^{-1/2}$ should be a constant with respect to H.

errors of the wind velocity components, which are not constants.

Following Wyngaard (1973), the relative error in the measurement of sensible heat by eddy correlation was estimated to be

$$\frac{\sigma_H}{H} = \left(\frac{z_h}{1200u} \left[\frac{1}{u_*^2} - 1 \right] \right)^{0.5}. \tag{16}$$

This expression gave an average relative error in H of 11%. Because u_*^2 was calculated as $-\overline{u'w'}$, the negative of the sample covariance of the u and w velocity signals, the relative error in friction velocity, σ_{u_*}/u_*, was not easy to quantify. On the basis of the manufacturer's estimate of the relative error for wind speed measurements of up to 3% for a 10-s reading, the error in an individual measurement of u or w at a measurement rate of 21 Hz may be as high as 43%; but if we write

$$\sigma_{u_*^2}^2 = \sigma_{u'}^2(-\overline{w'})^2 + \sigma_{w'}^2(-\overline{u'})^2, \tag{17}$$

we find the error in the measurement of u_*^2 is 0, since the sum of the fluctuations for a variable is zero. Instead, we chose to estimate the relative error in u_* by

$$\frac{\sigma_{u_*}}{u_*} = \left(\frac{\sigma_u^2}{u^2} + \frac{\sigma_w^2}{w^2} \right)^{0.5}, \tag{18}$$

where w was taken to be the average of the magnitude of the vertical velocity readings. Equation (18) gave an average relative error in u_* of just under 12%. This is comparable to the 14% relative error in u_* estimated by W. E. Eichinger et al. (Surface fluxes in the equatorial Pacific Ocean, submitted to *Journal of Climate*, 1996) for their readings which were made by the alternative method of applying a drag coefficient and measuring u by means of a cup anemometer. For T_h and T_s, the manufacturer's specifications of $\sigma_{T_s} = 0.5°\text{C}$ for the IRT sensor $\sigma_{T_h} = 0.4°\text{C}$ for the air temperature sensor were used.

In our error estimates of the derived quantities, we neglected an error analysis of the similarity function Ψ_h and ignored covariances between measured variables. The similarity functions are imperfectly known, so it is not clear how much information could be added by attempting an analysis of how much uncertainty they contributed to estimates of the scalar roughness heights. The Businger-Dyer similarity functions were used in the analysis in this study. The analysis was also done with the three-sublayer similarity model proposed by *Brutsaert* [1992]; the results did not differ from those derived using the Businger-Dyer functions, and only these results are presented. The covariance of measurement error between any two variables is often neglected on the assumption that the covariance is negligible compared to the individual measurement error [*Bevington and Robinson*, 1992]. For our experiment, quantifying the covariance between variables such as H and u_* is akin to describing the turbulent flow at a level of resolution beyond the scope of this paper. Although H, T_h, and T_s were measured with three separate instruments so that the covariance in measurement error between H and ΔT is reduced somewhat, one would still expect the correlation of errors in H and ΔT to approach 1; since the heat flux is positively correlated with the temperature gradient between the surface and air, time periods where it is difficult to measure the temperature gradient because it is small are also periods when it is difficult to measure the heat flux by eddy correlation. This means that the covariance $\sigma_{H\Delta T}$ approaches $\sigma_H\sigma_{\Delta T}$. However, including this covariance in the error propagation equation would not change the conclusions described below, and the estimates given were regarded as conservative.

For ln (St_0) the error was calculated by

$$\sigma_{\ln(St_0)} = \left(\frac{\sigma_H^2}{H^2} + \frac{\sigma_{u_*}^2}{u_*^2} + \frac{\sigma_{\Delta T}^2}{\Delta T^2} \right)^{1/2}, \tag{19}$$

where for $\ln(St_0)$, $\Delta T = T_s - T_i$, with T_i calculated as described above. For an estimate of $\sigma_{\Delta T}^2$, we have

$$\sigma_{\Delta T}^2 = \sigma_{T_s}^2 + \sigma_{T_i}^2, \tag{20}$$

where we calculate $\sigma_{T_i}^2$ as

$$\sigma_{T_i} = \left(\frac{\sigma_{T_h}^2}{T_i^2} + \frac{\sigma_H^2}{H^2} + \frac{\sigma_{u_*}^2}{u_*^2} \right)^{1/2} T_i, \tag{21}$$

where the factor T_i appears below $\sigma_{T_h}^2$ because of the additive appearance of T_h in the equation for T_i. For z_{0+}, the error was estimated to be

$$\sigma_{z_{0+}} = \left(\frac{\sigma_*^2}{u_*^2} + \frac{\sigma_{z_0}^2}{z_0^2} \right)^{1/2} z_{0+}, \tag{22}$$

where δ_{z_0} was the constant standard error in the z_0 as given above. For ln (z_{0+}), this yielded

$$\sigma_{\ln(z_{0+})} = \frac{\sigma_{z_{0+}}}{z_{0+}}. \tag{23}$$

Since the regression model that estimated $Cd_0^{-1/2}$ used fixed values for C_R, β, and γ, these same assumed values were used in the error estimate calculation for the drag coefficient. The error of $Cd_0^{-1/2}$ was estimated to be

$$\sigma_{Cd_0^{-1/2}} = \left[(7.3Pr^{1/2})^2 \frac{1}{16} \frac{\sigma_{z_{0+}}^2}{z_{0+}^{3/2}} + \left(\frac{\sigma_H^2}{H^2} + \frac{\sigma_{u_*}^2}{u_*{}^2} + \frac{\sigma_{\Delta T}^2}{\Delta T^2} \right) \cdot \left(\frac{\Delta T(u_*\rho c_p)}{H} \right)^2 \right]^{1/2}. \tag{24}$$

The estimates of the error in $Cd_0^{-1/2}$ and ln (St_0) are plotted against H in Figures 4 and 5, respectively. There is a definite trend towards lower reliability in the quantities at lower H, pointing out the need for a weighted regression. Including the covariance of H and ΔT would have increased the error of the lower flux periods more than it increased the error of the higher flux periods and would have altered the magnitude of the weights somewhat but not the fact that the measurements of the higher flux points are more accurate.

3.3. Data Analysis

Only periods when the mean wind was blowing from a direction ±30° from the centerline of the 3-D sonic anemometer were used; additionally, any time periods when measured H or LE was less than 0 and periods with turbulent intensity (σ_u/u) greater than 0.5 were also rejected so that the number of time periods used for analysis was reduced from 552 to 280. For these time periods, z_{0+} was calculated under the assumption that for fully rough flow (flow with a roughness Reynolds number greater than 2), the momentum roughness height z_{0m}

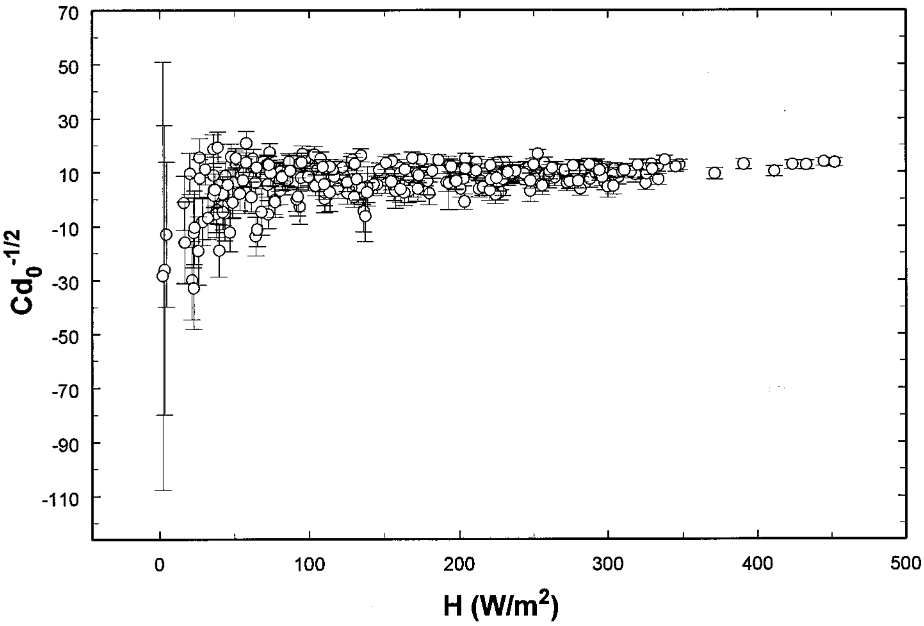

Figure 4. Calculated $Cd_0^{-1/2}$ as a function of measured sensible heat flux. The error bars mark the uncertainty in each data point estimated by equation (24).

is equivalent to the roughness height z_0 of the surface. A value for z_{0m} was determined by a regression of a scatter plot of u/u_* against z/L for 126 points where $|z/L| < 0.1$ [cf. *Kohsiek et al.*, 1993, section 4.2; *Parlange and Brutsaert*, 1989]. The roughness height of the Campbell Tract was found to be 0.004 m, with a standard error of 0.0003 m; for the Owens Lake bed the roughness height was 0.0035 m, with a standard error of 0.0004 m. Since all periods observed had fully rough conditions, these values were used for z_0 in all subsequent analysis. As discussed earlier, displacement height for both bare surfaces has been taken to be 0 [*Parlange and Katul*, 1992b; *Albertson et al.*, 1995].

As noted above, the regression was performed under the assumption that there was noise present in both the dependent and independent variables. The merit function for a weighted regression (least-squares fitting of a straight line $y = a + bx$ to data where there is measurement error in both x and y is a nonlinear function of the slope b) is

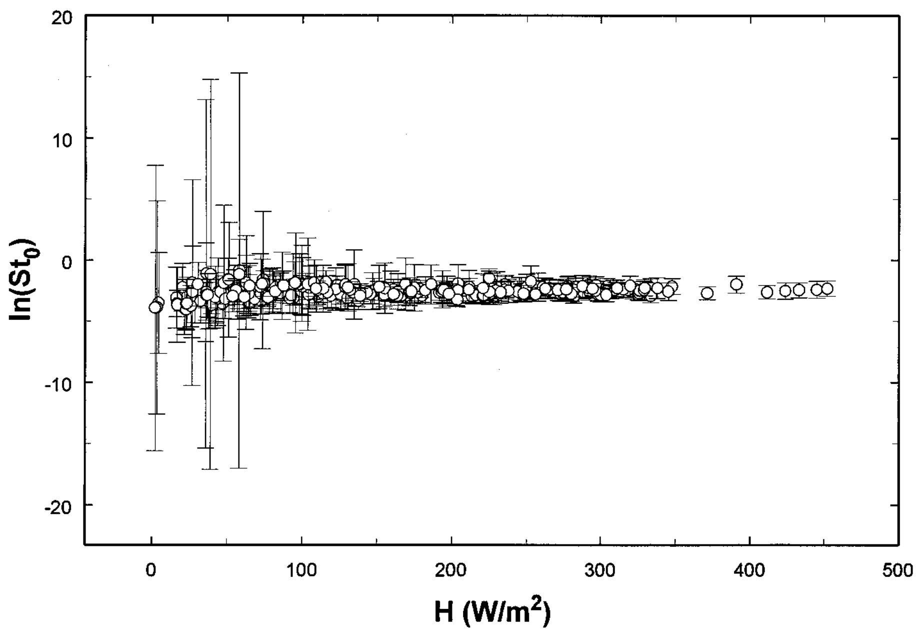

Figure 5. Calculated ln (St_0) as a function of measured sensible heat flux. The error bars mark the uncertainty in each data point estimated by equation (19).

Table 4. Weighted Regression Statistics for Model 2

	$\ln(St_0) = \ln(C_R Pr^\gamma) + \beta \ln(z_{0+})$
$\ln(C_R Pr^\gamma)$	-1.46
β	-2.47×10^{-1}
Standard Deviation of $\ln(C_R Pr^\gamma)$	2.86×10^{-1}
Standard Deviation of β	6.94×10^{-2}

$$\chi^2(a, b) = \sum_{i=1}^{n} \frac{(y_i - a - bx_i)^2}{\sigma_{y_i}^2 + b^2\sigma_{x_i}^2}, \tag{25}$$

so direct minimization by taking derivatives (as would be done in an unweighted regression where the merit function does not have b in the denominator) with respect to a and b will not work for the slope. Instead, a nonlinear root finder [*Press et al.*, 1992] is used to minimize χ^2. For model 1 this means minimizing the expression

$$\chi^2 = \sum_{i=1}^{n} \frac{(Cd_{0i}^{-1/2} - a - bH_i)^2}{\sigma_{Cd_{0i}^{-1/2}}^2 + b^2\sigma_{H_i}^2}, \tag{26}$$

where we expect b to be small, since the drag coefficient should not be affected by the sensible heat flux. For model 2, the expression for χ^2 is

$$\chi^2 = \sum_{i=1}^{n} \frac{(\ln(St_0)_i - a - b\ln(z_{0+})_i)^2}{\sigma_{\ln(St_0)_i}^2 + b^2\sigma_{\ln(z_{0+})_i}^2} \tag{27}$$

where $a = \ln(C_R Pr^\gamma)$ and $b = \beta$.

Results for the regressions for the various coefficients are presented in Figures 2 and 3; the accompanying regression statistics are in Tables 3 and 4. The predicted sensible heat fluxes for the coefficients from the weighted regression are shown in Figures 6 and 7. For all three cases, (1) was used to calculate H, with z_{0h} determined from the various regression coefficients. For the $Cd_0^{-1/2}$ model a value of $Cd_0^{-1/2} = 9.5$ was used; this was the average value of the drag coefficient when the regression equation $Cd_0^{-1/2} = 9.42 + 0.00455 \times H$ was averaged over the observations of H. The statistics for an unweighted regression of the predicted against the measured H, with the intercept forced through the origin, are given

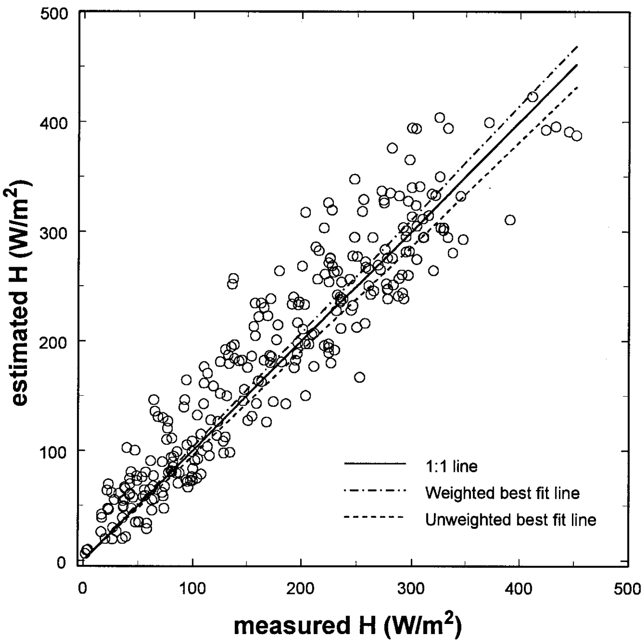

Figure 7. H estimated by the $\ln(St_0)$ regression (model 2) versus measured $H(= \rho c_p \overline{w'T'})$ at the Campbell Tract.

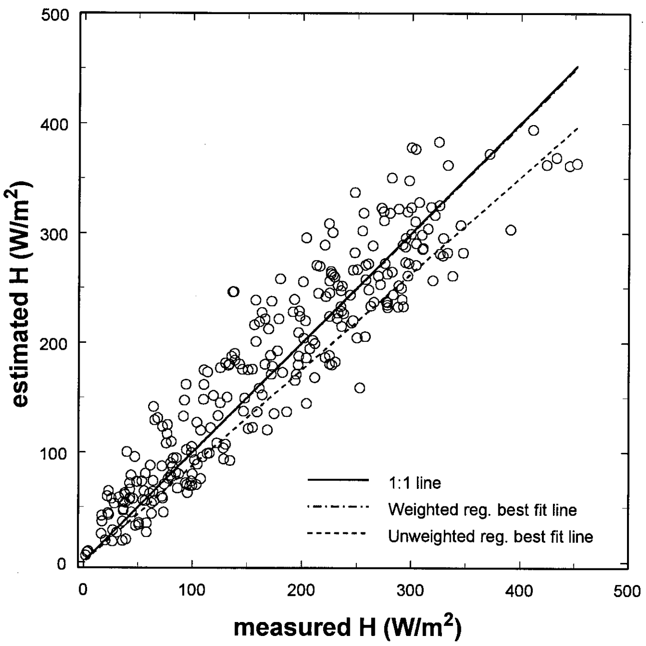

Figure 6. H estimated by the $C_{d0}^{-1/2}$ regression (model 1) versus measured $H(= \rho c_p \overline{w'T'})$ at the Campbell Tract. The weighted regression line is collinear with the one-to-one line.

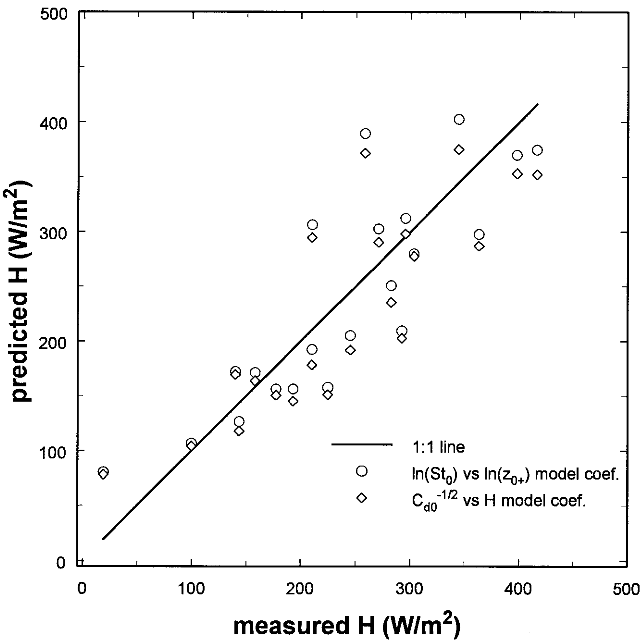

Figure 8. H estimated using the coefficients from the two regression models versus measured $H(= \rho c_p \overline{w'T'})$ at Owens Lake bed.

Table 5. Comparison of Sensible Heat Flux Calculated Using Parameters From the Different Regression Models to Derive z_{0h} Against Measured Sensible Heat ($\rho c_p \overline{w'T'}$) for the Campbell Tract Data

Regression Model Used	Slope of Best-Fit Line	r^2	Standard Error of Estimated H	Standard error of Best-Fit Slope
Model 1: $Cd_0^{-1/2}$ versus H	1.00	0.85	37.4	0.01
Model 2: ln (St_0) versus ln (z_{0+})	1.03	0.86	38.4	0.01

The best-fit line of the predicted values versus measured values is forced through the origin.

in Table 5 for both models. The r^2's for predictions of both models are very close because both models used the same measured input (air and surface temperature, friction velocity, etc.); more-important measures of the difference between the models are the slope of the fitted line and the standard error in the H prediction. As can be seen, adjusting the drag coefficient (model 1) gives the best predictions, although both models are relatively close to one another; the presence of a few points of high measured H are what cause the slope of the ln (St_0) − ln (z_{0+}) model (model 2) to perform more poorly. The standard error of the predicted H for both models is under 40 W/m^2, which is well within the resolution of measurement error for sensible heat. Also shown on the graphs are the best fit lines for the same models using the coefficients derived by unweighted regression. In all cases the improvement in the slope of the best fit line to the predictions can be seen; the standard error of the predictions for both the weighted and unweighed coefficients was similar.

3.4. Test With Owens Valley Data

The coefficients given by the regression of the Campbell Tract data were tested against the measurements taken at the dry Owens Lake bed. Results for the 21 periods out of 81 suitable for analysis (the same criteria as were applied to the Campbell Tract data were used) are shown in Figure 8 and Table 6. The scatter around the one-to-one line is larger than in the Davis experiment, resulting in a smaller r^2; however, the behavior in the mean (slope of best fit line) is good for both sets of coefficients.

4. Conclusions

Using the conceptual framework developed by *Brutsaert* [1975], the meteorological measurements taken at the Campbell Tract provide coefficients for the Brutsaert model of z_{0h} that adequately describe sensible heat flux both at the Campbell Tract and at the Owens Lake site. The field-based empirical coefficients for the relationship between the Stanton number and the roughness Reynolds number are slightly different than the ones based on laboratory measurements that Brutsaert initially derived, as is the value estimated for the drag coefficient from the field measurements. The parameters presented in this study show the greatest improvement for sensible heat flux estimation for periods of high sensible heat flux. A simple change in the value of $Cd_0^{-1/2}$ from 5 (Brutsaert's original value) to 9.5 used with Brutsaert's original values for the parameters of the relationship between St_0 and z_{0+}, that is,

$$z_{0h} = z_{0m} \exp\left[(-k)(7.3 z_{0+}^{1/4} Pr^{1/2} - 9.5)\right], \tag{28}$$

provided the best results. This is not surprising since $Cd_0^{-1/2}$ is the parameter least well established by theory. The other regression models also gave coefficients that provided good estimates of sensible heat when compared to eddy-correlation flux measurements. The regression of ln (St_0) versus ln (z_{0+}) provides an equation which is of the same form as Brutsaert's result. The equation resulting from this regression is

$$z_{0h} = z_{0m} \exp\left[(-k)(4.31 z_{0+}^{0.247} - 5)\right], \tag{29}$$

or if for purposes of comparison with Brutsaert's model for the interfacial transfer coefficient for heat shown in Table 1 we set $C_R\, Pr^{1/2} = 4.31$, then

$$z_{0h} = z_{0m} \exp\left[(-k)(5.11 z_{0+}^{0.247} Pr^{1/2} - 5)\right]. \tag{30}$$

It was very important to perform an analysis of the information value of each measurement prior to the regression; otherwise, measurements taken during periods of low sensible heat flux were given undue weight. This error analysis is one of the reasons for the improvement of the field-based coefficients at predicting H during periods of high flux. Measurements during high sensible heat flux periods were more accurate than measurements taken during low flux periods, and when this relative worth of individual measurements was used in the regression, relationships which better estimated sensible heat fluxes were derived.

The regression results given by the ln (St_0) − ln (z_{0+}) model for the power of the roughness Reynolds number and the geometric coefficient and the results for $Cd_0^{-1/2}$ merit discussion together. The difference between the field geometric coefficient of 5.11 and the value of 7.3 given by Brutsaert's regression of laboratory data may show the effect of the dif-

Table 6. Comparison of Sensible Heat Flux Calculated Using Parameters From the Different Regression Models to Derive z_{0h} Against Measured Sensible Heat ($\rho c_p \overline{w'T'}$) for the Owens Valley Data

Regression Model Used	Slope of Best-Fit Line	r^2	Standard Error of Estimated H	Standard Error of Best-Fit Slope
Model 1: $Cd_0^{-1/2}$ versus H	0.93	0.69	52.0	0.04
Model 2: ln (St_0) versus ln (z_{0+})	0.97	0.70	54.2	0.05

The best-fit line of the predicted values versus measured values is forced through the origin.

ferent scales of motion and different geometries present in the field or may simply be because the value of the $Cd_0^{-1/2}$, which was fixed at 5, was too low. The theoretical value of 1/4 for β, the power of the roughness Reynolds number, is essentially the same as the regressed value of 0.247, supporting the original eddy-renewal assumptions. Since there is less theoretical justification for the original value for $Cd_0^{-1/2}$ of 5 than the values of 1/4 for β and 1/2 for γ, it may preferable to use a value of 9.5 for $Cd_0^{-1/2}$, although the resulting predictions of sensible heat flux are roughly the same. Finally, the interfacial-sublayer/dynamic-sublayer matching model of Brutsaert for bluff-rough surfaces shows evidence of having captured the essential physics of the process and works reasonably well when used with either the geometric coefficient-Prandtl number value and z_{0+} power or the value of the drag coefficient derived from the measurements at the Campbell Tract.

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