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Generalizations of chain-dependent processes: Application to hourly precipitation

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Abstract. Stochastic models are fitted to time series of hourly precipitation amounts. These models are extensions of a form of chain-dependent process commonly fit to daily precipitation amounts. The extensions involve allowing hourly intensities to be autocorrelated and allowing the model parameters to possess diurnal cycles. These models are applied to two quite different sets of hourly precipitation data: July at Denver, Colorado, for which diurnal cycles are substantial; and January at Chico, California, for which a relatively high degree of persistence is present. The temporal aggregation properties of the hourly models (e.g., for 12-hour or daily total precipitation) are examined, and the role of the extensions in improving these properties is quantified. On this basis, it is argued that generalizations of chain-dependent processes could be competitive with, if not superior to, so-called conceptual models of the precipitation process.

1. Introduction

Much research has dealt with the stochastic modeling of gauge-based precipitation measurements, especially when aggregated to daily totals [e.g., *Todorovic and Woolhiser*, 1975]. For instance, a chain-dependent process is one particular stochastic model that is popular for time series of daily total precipitation [*Katz*, 1977a, b]. Less effort has been devoted to precipitation data on shorter timescales (e.g., hourly), with the most prevalent approach being based on so-called conceptual (or physically based) models, which involve chance mechanisms (e.g., clustering) by which "storms" arise (originated by *LeCam* [1961]). Another approach, closely related to conceptual modeling, starts with a storm and disaggregates the data through use of a hyetograph [*Huff*, 1967]. Of course, individual storms are not actually observed in practice. Instead, either the parameters of the conceptual model are indirectly estimated through temporally aggregated precipitation data [*Obeysekera et al.*, 1987], or the operational definition of a storm is simply taken to be a period of consecutive hours, say, during which measurable precipitation occurs (or some variant thereof) [*Garcia-Guzman and Aranda-Oliver*, 1993].

The direct application of conventional stochastic models, such as a chain-dependent process, to time series of hourly precipitation amounts has not been as successful. Either the assumptions that have been made to simplify the analysis are quite unrealistic, such as taking hourly intensities to be independent [*Nguyen and Rousselle*, 1981], or a structure is imposed to allow for such dependence that makes the model too complex for straightforward analysis [*Nguyen*, 1984]. *Brown et al.* [1985] presented evidence that the conditions imposed by an ordinary chain-dependent process are inappropriate for hourly precipitation.

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Nevertheless, a wealth of information is available in hourly precipitation data sets, the shortest timescale of aggregation for which observations are available with relatively long historical records at a large number of sites [*Collander et al.*, 1993]. Modeling precipitation observations directly in the form in which they are available has some advantages, especially for parameter estimation. The incompatibility of effectively continuous time conceptual models with the inherently discrete observations is avoided [*Foufoula-Georgiou and Guttorp*, 1986], although some discrete clustering models for precipitation occurrences do exist [*Foufoula-Georgiou and Lettenmaier*, 1986; *Smith*, 1987]. Moreover, from an information-theoretic point of view, it is of interest to characterize probabilistically the future evolution of the precipitation process through use of only the past and present observations. In this regard, *Elsner and Tsonis* [1993] have applied entropy measures to examine the predictability of hourly precipitation.

In the present paper, we demonstrate how a chain-dependent process can be generalized in a manner that is more realistic for sequences of hourly precipitation amounts but is still consistent with conventional models for daily precipitation totals. These extensions include allowing the hourly intensities to be dependent, by a technique based on power transformations that keeps the resultant model tractable. Also, Fourier series techniques are employed to allow for diurnal cycles in the model parameters in a parsimonious manner. This modeling approach is applied to two hourly precipitation data sets with quite different characteristics.

2. Extensions of Chain-Dependent Processes

A chain-dependent process for hourly precipitation consists of two component processes: (1) an occurrence process (i.e., the sequence of wet or dry hours) and (2) an intensity process (i.e., the sequence of precipitation amounts on wet hours). Such models were originally fitted to time series of daily precipitation amounts by *Katz* [1977a] and *Todorovic and Woolhiser* [1975].

2.1. Occurrence Process

The hourly occurrence process $\{J_t(h): h = 1, 2, \cdots, 24; t = 1, 2, \cdots\}$ is defined as

$$J_t(h) = 1 \text{ if } h\text{th hour of } t\text{th day is wet} \qquad J_t(h) = 0 \text{ otherwise,} \tag{1}$$

where a "wet hour" refers to one on which measurable precipitation occurs. We adopt the obvious conventions that $J_t(25) = J_{t+1}(1), J_t(26) = J_{t+1}(2), \cdots, J_t(0) = J_{t-1}(24), J_t(-1) = J_{t-1}(23), \cdots$. It is assumed that the process $J_t(h)$ constitutes a two-state, first-order Markov chain with transition probabilities

$$P_{ij}(h) = \Pr\{J_t(h+1) = j | J_t(h) = i\} \qquad i, j = 0, 1 \tag{2}$$

that possibly depend on the hour h. For simplicity, the transition probabilities in (2) are not allowed to vary with the day t (i.e., no seasonal cycles). A generalization to higher than first-order Markov chains is treated in section 4.

The probability that the hth hour of the day is wet (i.e., that the occurrence process takes on state 1), $\pi_1(h) = \Pr\{J_t(h) = 1\}$, is the solution to the recursion

$$\pi_1(h+1) = P_{01}(h) + \pi_1(h)[P_{11}(h) - P_{01}(h)] \qquad h = 1, 2, \cdots, 24, \tag{3}$$

with the convention that $\pi_1(25) = \pi_1(1)$. This linear system of 24 equations in 24 unknowns can be solved algebraically; alternatively, (3) can simply be iterated starting with a trial value for $\pi_1(1)$, as convergence to the solutions is very rapid. Given the $\pi_1(h)$, the first-order autocorrelation coefficient between the hth and $(h + 1)$th hours of the occurrence process, $\rho_1(h) = \mathrm{Corr}[J_t(h), J_t(h+1)]$, can be expressed as

$$\rho_1(h) = [P_{11}(h) - P_{01}(h)]\{\pi_1(h)[1 - \pi_1(h)]\}^{1/2} \cdot\{\pi_1(h+1)[1 - \pi_1(h+1)]\}^{-1/2}, \tag{4}$$

$h = 1, 2, \cdots, 24$. Diurnal cycles in the transition probabilities (2) produce corresponding cycles in the wet hour probabilities via (3) and in the autocorrelation via (3) and (4). These expressions, (3) and (4), can be derived by probabilistic arguments, through conditioning on whether the hth and $(h + 1)$th hours of the day are dry or wet. They are natural generalizations of those for a two-state, first-order Markov chain with constant transition probabilities [e.g., *Katz and Parlange*, 1993].

2.2. Intensity Process

Let $X_t(h)$ denote the precipitation amount on the hth hour of the tth day. If $J_t(h) = 1$, then $X_t(h) > 0$ and is referred to as an intensity. Again, the convention is adopted that $X_t(25) = X_{t+1}(1), X_t(26) = X_{t+1}(2), \cdots, X_t(0) = X_{t-1}(24), X_t(-1) = X_{t-1}(23), \cdots$. These hourly intensities have means and variances denoted by

$$\mu_1(h) = E[X_t(h) | J_t(h) = 1] \qquad [\sigma_1(h)]^2 = \mathrm{Var}[X_t(h) | J_t(h) = 1], \tag{5}$$

but generally possess positively skewed rather than symmetric distributions. For simplicity, the means and variances in (5) are not allowed to vary with the day t (i.e., no seasonal cycles). To allow for skewness, the intensity distribution is taken to be a power transformation of the normal. That is, given $J_t(h) = 1$,

$$X_t^*(h) = [X_t(h)]^p \qquad \text{for some } p,\ 0 < p < 1, \tag{6}$$

has a normal distribution with mean $\mu_1^*(h)$ and variance $[\sigma_1^*(h)]^2$, written $N(\mu_1^*(h), [\sigma_1^*(h)]^2)$. This power transform parameter p is assumed to be independent of the hour h.

On a daily timescale, it is usually assumed [e.g., *Katz*, 1977a] that the amounts of precipitation are conditionally independent given the occurrence process (i.e., the amounts of precipitation on consecutive wet days are independent). On an hourly timescale, it is important to allow for the possibility that the intensities are actually dependent. To do so, the power-transformed intensities within a given wet spell (i.e., a run of consecutive wet hours) are modeled as a first-order autoregressive (i.e., AR(1)) process. Suppose that two consecutive hours, say the hth and $(h + 1)$th hours of the tth day, are wet (i.e., $J_t(h) = 1$ and $J_t(h + 1) = 1$). Then a first-order autocorrelation coefficient ϕ_1^* is introduced into the transformed intensity process by

$$Z_t^*(h+1) = \phi_1^* Z_t^*(h) + \varepsilon_t(h+1),$$

where

$$Z_t^*(h+l) = [X_t^*(h+l) - \mu_1^*(h+l)]/\sigma_1^*(h+l) \qquad l = 0, 1 \tag{7}$$

and the uncorrelated error term $\varepsilon_t(h + 1)$ is $N[0, 1 - (\phi_1^*)^2]$. Note that this AR(1) process can be viewed as terminating whenever a wet spell ends and regenerating (i.e., with a new initial state) when the next wet spell starts. For simplicity, the transformed intensity autocorrelation ϕ_1^* in (7) is assumed to be independent of the hour h. Even though the hourly intensities are autocorrelated, if these hourly amounts were aggregated to daily totals, then the daily "intensities" (i.e., a wet day is defined as one in which one or more hours is actually wet) would have little or no autocorrelation, consistent with the usual assumptions for stochastic models for daily precipitation [e.g., *Katz and Parlange*, 1993].

2.3. Combined Process

Some relationships exist among the various parameters of a chain-dependent process. For instance, the unconditional mean and variance of hourly precipitation amounts $X_t(h)$ are given by

$$\mu(h) = E[X_t(h)] = \pi_1(h)\mu_1(h), \qquad [\sigma(h)]^2 = \mathrm{Var}[X_t(h)] = \pi_1(h)[\sigma_1(h)]^2 + \pi_1(h)[1 - \pi_1(h)][\mu_1(h)]^2. \tag{8}$$

These two expressions can be obtained by probabilistic arguments, through conditioning on whether or not the hth hour of the day is wet. When temporal aggregation properties are studied in sections 3 and 4, the variance of precipitation totals (e.g., over a half day or an entire day) will also be considered. Any such variance expressions would be more complex than that for an ordinary chain-dependent process, both because of diurnal cycles in the parameters and because the hourly intensities are autocorrelated.

3. Denver Hourly Precipitation

Time series of hourly precipitation at Denver, Colorado, during the month of July for the period 1949–1990 are modeled. These observations are part of an hourly precipitation data set for the United States that has been critically assessed by *Collander et al.* [1993]. Summer precipitation in this region (i.e., near the eastern edge of the Rocky Mountains) is predominantly of local convective origin. Consequently, substantial diurnal cycles in hourly precipitation statistics would be anticipated [e.g., *Wallace*, 1975]. Moreover, a relatively high degree of persistence in precipitation regimes would not necessarily be expected. *Obseysekera et al.* [1987] and *Rodríguez-Iturbe et al.* [1987, 1988] all used hourly precipitation from this region to estimate the parameters and to evaluate the performance of conceptual models.

3.1. Model Fitting

To model the diurnal cycles in the occurrence process, the logistic transformations [e.g., *Neter et al.*, 1983, pp. 361–367] of the transition probabilities, $P_{01}(h)$ and $P_{11}(h)$, are represented as cosine waves (i.e., with both phase and amplitude unknown). Such a model can be most conveniently fit in the following form:

$$\ln (P_{i1}(h)/[1 - P_{i1}(h)]) = A_i + B_i \cos [(2\pi h)/24] + C_i \sin [(2\pi h)/24] \qquad i = 0, 1. \tag{9}$$

This transformation has the convenient property of automatically constraining the smoothed transition probabilities to fall within the interval (0, 1). The unknown parameters, A_i, B_i, and C_i, in (9) are estimated by using weighted least squares (see (A2) in Appendix A), a special case of fitting a generalized linear model [*McCullagh and Nelder*, 1983]. A similar approach has been employed by *Stern and Coe* [1984] and *Woolhiser et al.* [1993] to fit seasonal cycles to the transition probabilities for Markov chain models of daily precipitation occurrences.

Likewise, the mean and standard deviation of the hourly transformed intensities, $\mu_1^*(h)$ and $\sigma_1^*(h)$, are represented as cosine waves:

$$\mu_1^*(h) = A_\mu + B_\mu \cos [(2\pi h)/24] + C_\mu \sin [(2\pi h)/24], \tag{10}$$

$$\sigma_1^*(h) = A_\sigma + B_\sigma \cos [(2\pi h)/24] + C_\sigma \sin [(2\pi h)/24].$$

The unknown parameters, A_μ, B_μ, C_μ, A_σ, B_σ, and C_σ, in (10) are estimated by using weighted least squares (see (A5) in Appendix A and also *Bell and Reid* [1993]). The method of estimating the first-order autocorrelation coefficient of the transformed intensities, ϕ_1^*, is also discussed in Appendix A. Finally, the value of the power transform parameter p is identified by using trial values, $p = 1/2, 1/4, 1/8, \cdots$, and selecting the value that minimizes Hinkley's index of symmetry [e.g., *Katz and Parlange*, 1993].

The Bayesian information criterion (BIC) is employed to identify which of the individual parameters require diurnal cycles and whether the intensities are autocorrelated [e.g., *Katz and Parlange*, 1993]. This procedure requires that the maximized likelihood function be obtained for each candidate model (see Appendix A for a description of the techniques by which these likelihood functions are calculated). For clarity in presentation, the occurrence and intensity processes are treated separately in the model identification exercise.

Table 1. Model Identification of Hourly Precipitation Occurrence Process and Parameter Estimates of Optimal Model for July at Denver ($24 \times 31 \times 42 = 31,248$ Observations)

Diurnal Cycle? P_{01}	P_{11}	Log Likelihood Function	Number of Parameters	BIC
no	no	−3244.3	2	6509.4
no	yes	−3243.3	4	6528.0
yes	no	−3021.7	4	6084.7†
yes	yes	−3020.6	6	6103.4

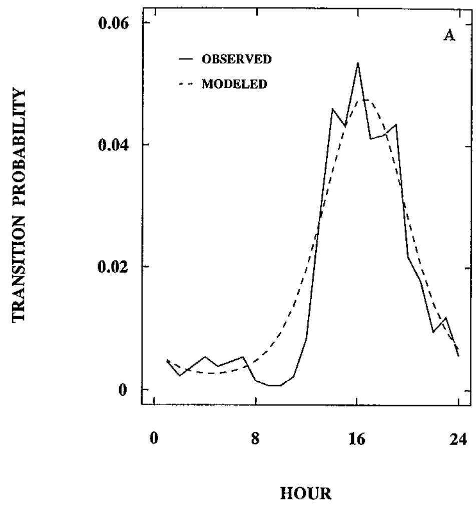
Optimal model: $P_{01}(h)$: $A_0 = -4.4414$, $B_0 = -0.5435$, $C_0 = -1.3503$; $P_{11}(h) = 0.4965$ (i.e., $A_1 = -0.0141$, $B_1 = 0$, $C_1 = 0$).
†Minimum.

Table 1 summarizes the results of applying the BIC to identify the most appropriate model of the hourly precipitation occurrence process for July at Denver (see (A4) of Appendix A). The optimal model (i.e., minimum BIC value) allows for a diurnal cycle in one transition probability $P_{01}(h)$, but not in the other $P_{11}(h)$ (Akaike's information criterion (AIC) [e.g., *Katz and Parlange*, 1993] also chooses this model as optimal). The second best model allows both transition probabilities to possess diurnal cycles and is much superior to the model with no diurnal cycles.

Also included in Table 1 are the parameter estimates for the optimal model of the hourly precipitation occurrence process. Figure 1a shows the model curve, along with the observed individual hourly values, for the transition probability $P_{01}(h)$. Because the inverse of the logistic transformation has been applied to convert (9) back into probabilities (see (A3) of Appendix A), the curve shown is no longer a cosine wave. This model curve has a range of over an order of magnitude, from a minimum probability of about 0.003 at 5 A.M. to a maximum of about 0.048 at 5 P.M. Only an apparent diurnal cycle with very small amplitude is evident in the observed hourly values for $P_{11}(h)$ (not shown).

It is more natural to interpret the diurnal cycles of the occurrence process in terms of the alternative parameters, the probability of a wet hour $\pi_1(h)$ and the hourly persistence $\rho_1(h)$. Using (3) and (4), $\pi_1(h)$ and $\rho_1(h)$ can be determined for the optimal model listed in Table 1. For the probability of a wet hour, Figure 1b shows the derived model curve along with the observed individual hourly values. Again, the model curve has a range of over an order of magnitude, from a minimum probability of about 0.006 at 7 A.M. to a maximum of about 0.083 at 6 P.M. The corresponding derived curve (not shown) for the hourly persistence $\rho_1(h)$ likewise matches the observations well, ranging from a minimum correlation of about 0.40 at 1–2 P.M. to a maximum of about 0.57 at 1–2 A.M.

Table 2 summarizes the results of applying the BIC to identify the most appropriate model of the hourly precipitation intensity process for July at Denver (see (A6) of Appendix A). To achieve approximate normality, the selected value of the power transform parameter is $p = 1/8$ in (6). This value of p was first obtained through pooling the intensities for all hours into one sample (i.e., ignoring any diurnal cycles) and subsequently checked by examining the distribution of the transformed intensities with the fitted diurnal cycles for $\mu_1^*(h)$ and $\sigma_1^*(h)$ removed. The optimal model according to the BIC (as well as for the AIC) allows for autocorrelated intensities (i.e.,



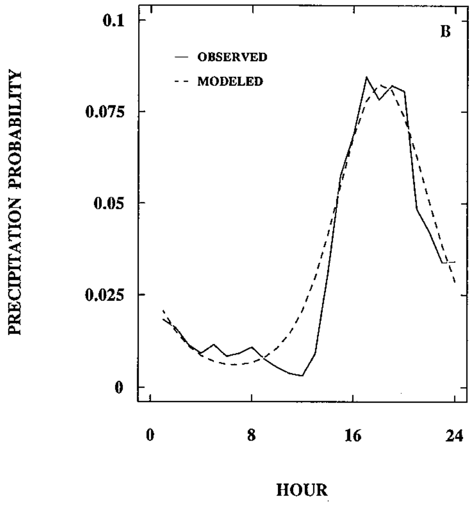


Figure 1. Observed and model probabilities for hourly precipitation occurrence process in July at Denver. (a) Transition probability $P_{01}(h)$; (b) probability of wet hour $\pi_1(h)$.

$\phi_1^* \neq 0$), as well as permitting diurnal cycles in both the mean and standard deviation of the transformed intensities, $\mu_1^*(h)$ and $\sigma_1^*(h)$. The next best models are the other two that permit $\phi_1^* \neq 0$.

Also included in Table 2 are the parameter estimates for the optimal model of the hourly precipitation intensity process. Figures 2a and 2b show the model curves, along with the observed individual hourly values, for the transformed intensity mean $\mu_1^*(h)$ and transformed intensity standard deviation $\sigma_1^*(h)$, respectively. It is more natural to interpret the diurnal cycles of the intensity process in terms of the hourly means and standard deviations, $\mu_1(h)$ and $\sigma_1(h)$, for the original, untransformed intensities. These means and standard deviations are each functions of both $\mu_1^*(h)$ and $\sigma_1^*(h)$, depending as well on the power transform parameter ($p = 1/8$ in this example). By combining the general relationship between the central and noncentral moments of a distribution [*Stuart and Ord*, 1987, pp. 72–73] with that for the central moments of a normal distribution [*Johnson and Kotz*, 1970, p. 47], the corresponding curves for the untransformed means and standard deviations can be determined from the model curves for the transformed statistics. See *Katz and Garrido* [1994] for an example of this calculation. Figure 2c includes the derived model curve for the mean intensity $\mu_1(h)$, ranging from a minimum of about 0.76 mm at 5 A.M. to a maximum of about 2.16 mm at 5 P.M., and Figure 2d includes the corresponding curve for the intensity standard deviation $\sigma_1(h)$, ranging from about 0.67 mm at 5 A.M. to about 3.28 mm at 5 P.M. It is noteworthy that the amplitude of the diurnal cycle for $\sigma_1(h)$ is even greater than that for $\mu_1(h)$. Because of transformation bias, these model curves necessarily do not match the hourly observed values as well as for the directly fit transformed intensities (Figures 2a and 2b).

Finally, the fit of the power transform distribution to the hourly precipitation intensities is illustrated, making use of a closed-form expression for the density function [*Katz and Garrido*, 1994; *Wilks*, 1993]. For the two examples (i.e., 5 and 11 P.M.) shown in Figure 3, it is evident that the high degree of positive skewness exhibited by the observations is captured reasonably well by this particular distribution. The discrepancies at the lowest possible recorded precipitation intensity value are at least partially attributable to this discretization of the rain gauge measurements.

3.2. Aggregation Properties

One way to evaluate any stochastic model for the hourly precipitation process is to study how well it fits precipitation totals over longer timescales. It is straightforward to compute aggregation properties on the basis of the hourly occurrence process, even when the transition probabilities (equation (2)) of the first-order Markov chain possess diurnal cycles. In particular, the lengths of dry and wet spells are of considerable interest and naturally reflect the presence of diurnal cycles in the transition probabilities.

Let $N_i(h)$ denote the length of a dry ($i = 0$) or wet ($i = 1$) spell starting on the hth hour (i.e., a run of consecutive dry (or wet) hours). The probability that a dry (or wet) spell starting on the hth hour lasts at least l hours can be represented as a product of transition probabilities:

$$\Pr\{N_i(h) \geq l\} = \prod_{0}^{l-2} P_{ii}(h + j) \qquad l = 2, 3, \cdots; i = 0, 1. \tag{11}$$

If the transition probability involved does not have diurnal cycles, then (11) reduces to $\Pr\{N_i(h) \geq l\} = P_{ii}^{l-1}$, $l = 2$,

Table 2. Model Identification of Hourly Precipitation Intensity Process and Parameter Estimates of Optimal Model for July at Denver (996 Wet Hours)

Diurnal Cycle? μ_1^*	σ_1^*	$\phi_1^* \neq 0$?	Log Likelihood Function, ln $(\mathrm{mm}^{1/8})^2$	Number of Parameters	BIC
no	no	no	1832.6	2	−3651.4
no	no	yes	1865.8	3	−3710.9
yes	no	no	1843.8	4	−3659.9
yes	no	yes	1876.4	5	−3718.3
yes	yes	no	1869.9	6	−3698.4
yes	yes	yes	1909.3	7	−3770.2†

Optimal model: $\mu_1^*(h)$: $A_\mu = 0.97057$ mm$^{1/8}$, $B_\mu = -0.00823$ mm$^{1/8}$, $C_\mu = -0.03733$ mm$^{1/8}$; $\sigma_1^*(h)$: $A_\sigma = 0.13537$ mm$^{1/8}$, $B_\sigma = -0.00574$ mm$^{1/8}$, $C_\sigma = -0.03714$ mm$^{1/8}$; $\phi_1^* = 0.3838$.
†Minimum.

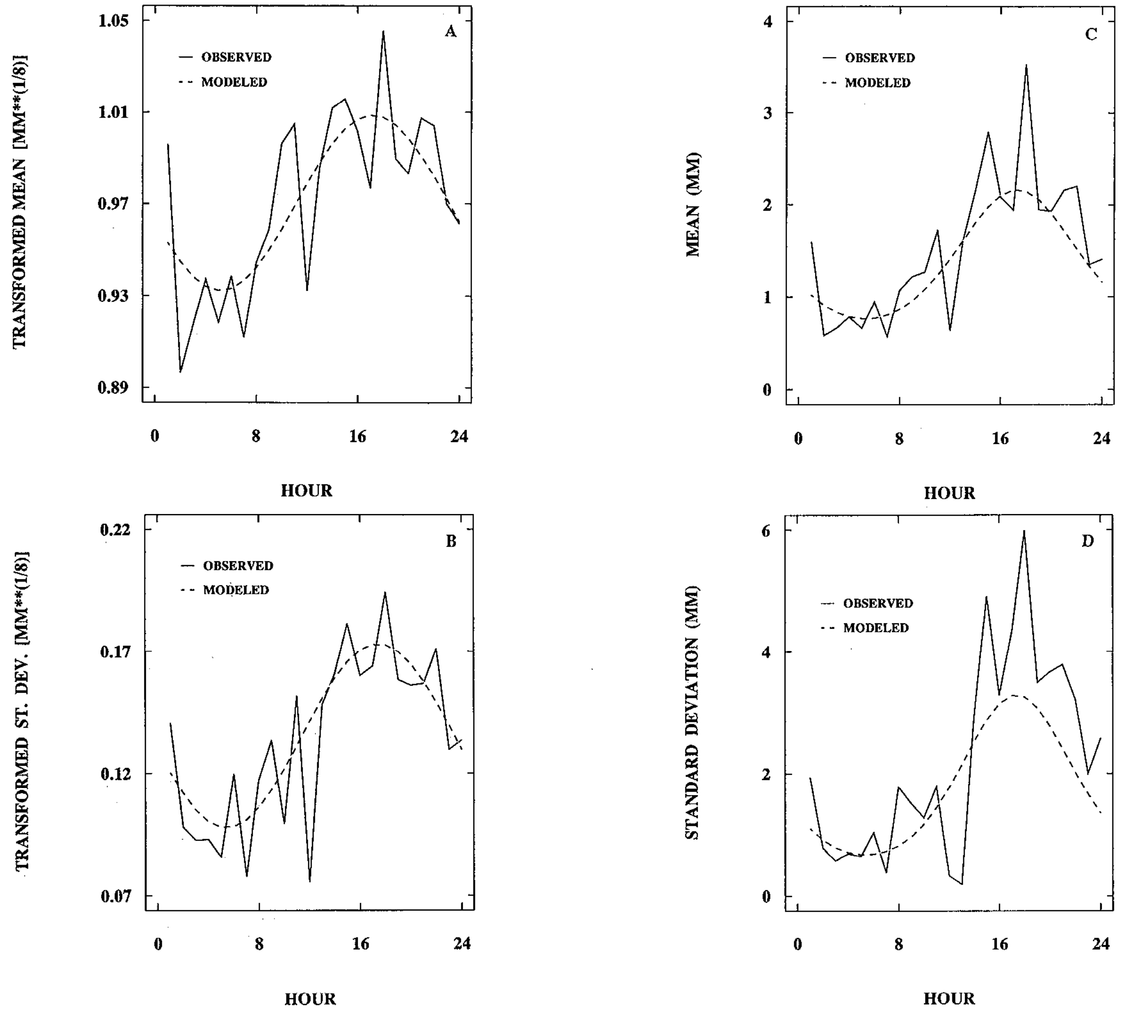


Figure 2. Observed and model statistics for hourly precipitation intensity process in July at Denver. (a) Transformed intensity mean $\mu_1^*(h)$; (b) transformed intensity standard deviation $\sigma_1^*(h)$; (c) mean intensity $\mu_1(h)$; (d) intensity standard deviation $\sigma_1(h)$.

3, $\cdots$. The probability that a specific time period is dry or wet is easily obtained from (11). For instance, the probability of a dry day (i.e., all 24 individual hours being dry) is given by $[1 - \pi_1(1)]$ Pr $\{N_0(1) \geq 24\}$, which reduces to $(1 - \pi_1)P_{00}^{23}$ with no diurnal cycles.

Under the generalizations of a chain-dependent process necessary to adequately model the Denver hourly precipitation data, it is more difficult to analyze the properties of total precipitation. Let the precipitation amount totaled over a time period of H hours starting at the hth hour of the tth day be denoted by

$$S_t(h, H) = X_t(h) + \cdots + X_t(h + H - 1). \quad (12)$$

Of course, the mean of total precipitation is simply the sum of the individual hourly means. That is,

$$E[S_t(h, H)] = \pi_1(h)\mu_1(h) + \cdots + \pi_1(h + H - 1)\mu_1(h + H - 1), \quad (13)$$

which reduces to $H\pi_1\mu_1$ with no diurnal cycles.

Recall [e.g., *Katz and Parlange*, 1993] that for an ordinary chain-dependent process (i.e., with no diurnal cycles and $\phi_1^* = 0$), the variance of total precipitation is approximately

$$\text{Var}\,[S_t(h, H)] \approx H(\pi_1\sigma_1^2 + \pi_1(1 - \pi_1)\mu_1^2[(1 + \rho_1)/(1 - \rho_1)]). \quad (14)$$

This expression can be extended to the case of correlated intensities (i.e., $\phi_1^* \neq 0$) (see Appendix B and section 4), but no simple expression can be obtained with diurnal cycles present. Consequently, a simulation approach is relied on to determine the model distribution of total precipitation (and, in particular, its variance).

Table 3 summarizes the aggregation properties of some of the candidate stochastic models of the hourly precipitation process for July at Denver. In addition to the optimal model (i.e., model 3 in Table 3, with parameter values listed in Tables 1 and 2), models with no diurnal cycles but with intensities either autocorrelated (i.e., model 2) or not autocorrelated (i.e., model 1) are evaluated. The hourly precipitation observations have been totaled over two periods of length 12 hours, morning (A.M.) and afternoon (P.M.), as well over the entire 24 hours of the day.

First the aggregation properties of the hourly occurrence model for Denver are treated, recognizing that whether or not $\phi_1^* \neq 0$ has no bearing (i.e., models 1 and 2 are identical).

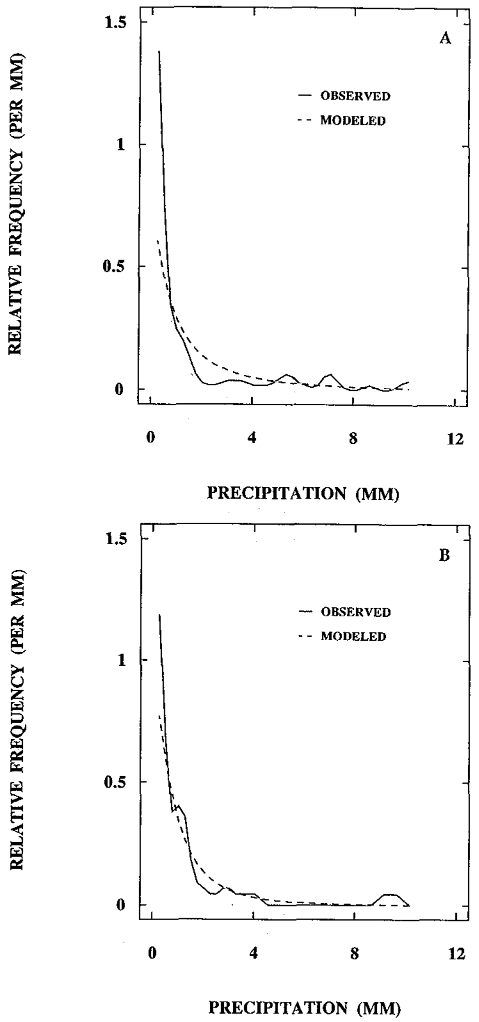


Figure 3. Density function of fitted power transform distribution and empirical distribution for hourly precipitation intensity in July at Denver: (a) 5 P.M.; (b) 11 P.M.

Making use of (11) and its variants, Table 3 includes the model probability of wet periods (i.e., a period during which at least one hour is wet). Without the diurnal cycle in the transition probability $P_{01}(h)$, these probabilities are the same for both A.M. and P.M. and necessarily far off the observed values of about 0.045 for A.M. and 0.273 for P.M. With a diurnal cycle in $P_{01}(h)$, the A.M. and P.M. probabilities are much closer to the observed values, however slightly too high. The model probability of a wet day is again somewhat too high (observed value of about 0.299), with the incorporation of a diurnal cycle not resulting in any improvement.

The role of diurnal cycles in the optimal model for the occurrence process at Denver is further explored by considering the lengths of dry and wet spells. Calculated by using (11), Figure 4a shows how the diurnal cycle in $P_{01}(h)$ produces wide variations in the probability distribution of dry spell lengths depending on the hour h on which the spell begins. For example, the probability that a dry spell lasts at least 15 hours ranges from about 0.67 to about 0.91 depending on whether it starts at 10 A.M. or 10 P.M. Because of a lack of sufficient data to examine the empirical distribution of spell length stratified by hour of the day, simply the overall measure of mean spell length is considered. Figure 4b shows the mean dry spell length for the optimal model, along with the observed values, as a function of the hour on which the spell starts. This curve appears relatively flat compared with those in Figure 4a, because the fact that dry spells typically last one or more days attenuates the diurnal cycle in $P_{01}(h)$. It is evident that the model tends to overestimate the mean dry spell length, with the model means ranging from about 50.0 hours at 1 P.M. to about 59.1 hours at 10 P.M. According to the optimal model, the mean length of wet spells should not have a diurnal cycle because the transition probability $P_{11}(h)$ is constant (see Table 1). The optimal model produces a mean wet spell length of $(1 - P_{11})^{-1} \approx 1.99$ hours, virtually the same as the observed value of 1.97 hours.

Next the aggregation properties of the stochastic models for Denver hourly precipitation (i.e., models 1–3 in Table 3) are examined for the distribution of total precipitation amounts. Obtained by (13), the values in Table 3 include the mean of total precipitation. With the diurnal cycles in both $\pi_1(h)$ and $\mu_1(h)$ being ignored, model 1 produces identical means for both A.M. and P.M. totals, necessarily far off the observed values of about 0.11 mm for A.M. and 1.43 mm for P.M. As the

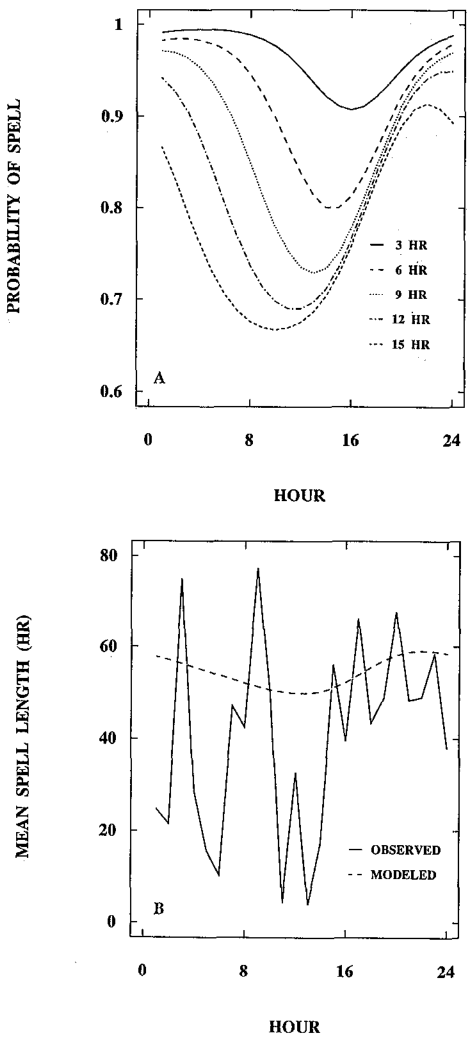


Figure 4. Distribution of dry spells (i.e., runs of consecutive dry hours) in July at Denver as function of hour on which spell starts. (a) Model probability of equaling or exceeding spell of specified length; (b) observed and model mean spell lengths.

intensity autocorrelation has no bearing on the mean, model 2 generates the same mean totals as model 1. The means for the optimal model 3 come much closer to the observed A.M. and P.M. means but are still not identical because of the bias induced by fitting the mean $\mu_1^*(h)$ of the power transformed intensities (equation (6)) (as opposed to the original data). For the daily totals, the means of all three models necessarily come reasonably close to the observed value of about 1.54 mm.

The model standard deviation of total precipitation amounts is not directly calculated because of its complexity, but rather approximated by a Monte Carlo simulation experiment (consisting of 4000 days of hourly precipitation being generated for each of the three models). These simulated standard deviations of total precipitation are also listed in Table 3. With the diurnal cycles in $P_{01}(h)$, $\mu_1^*(h)$, and $\sigma_1^*(h)$ all being ignored, the standard deviations produced by model 1 are far off the observed values of about 0.91 mm for A.M. and 4.80 mm for P.M. For model 2 the effect on the standard deviation of allowing for autocorrelated intensities is swamped by the consequences of leaving out the diurnal cycles. The A.M. and P.M. standard deviations produced by optimal model 3 come much closer to the observed values but still are substantial underestimates. For the daily totals, all three models underestimate the observed standard deviation of about 4.89 mm, with the incorporation of autocorrelated intensities or diurnal cycles each contributing only a slight increase.

Figure 5 shows the simulated distribution of total precipitation for the optimal model as well as the empirical distribution at Denver. It is evident that the model is able to effectively capture the marked difference in A.M. (Figure 5a) and P.M. (Figure 5b) distributions. The shape of the model-simulated distribution of daily totals closely resembles that observed (Figure 5c), despite the underestimation of variance (see Table 3). Because the P.M. period constitutes the dominant part of the diurnal cycle, Figures 5b and 5c are very similar. Again, the discrepancies at the lowest observed precipitation intensity value may be attributable to discretization.

4. Chico Hourly Precipitation

Time series of hourly precipitation at Chico, California, during the month of January for the period 1949–1981 are modeled [*Collander et al.*, 1993]. Because months with any missing observations were excluded, only 23 years remain. Records exist after 1981 but were eliminated because of a switch in the type of rain gauge. Winter precipitation in this region (i.e.,

Table 3. Aggregation Properties of Models for July Hourly Precipitation at Denver

Time Period and Length	Model†	Probability of Wet Period	Mean Total Precipitation, mm	Standard Deviation of Total Precipitation, mm
A.M. (12 hours)	1	0.1949	0.660	2.310
	2	0.1949	0.660	2.394
	3	0.0768	0.138	0.731
	(observed)	(0.0453)	(0.111)	(0.911)
P.M. (12 hours)	1	0.1949	0.660	2.310
	2	0.1949	0.660	2.394
	3	0.3234	1.293	3.418
	(observed)	(0.2734)	(1.431)	(4.799)
Day (24 hours)	1	0.3415	1.319	3.354
	2	0.3415	1.319	3.467
	3	0.3689	1.431	3.499
	(observed)	(0.2987)	(1.542)	(4.890)

†Model 1, no diurnal cycles, $\phi_1^* = 0$; model 2, no diurnal cycles, $\phi_1^* \neq 0$; model 3, optimal (see Tables 1 and 2).

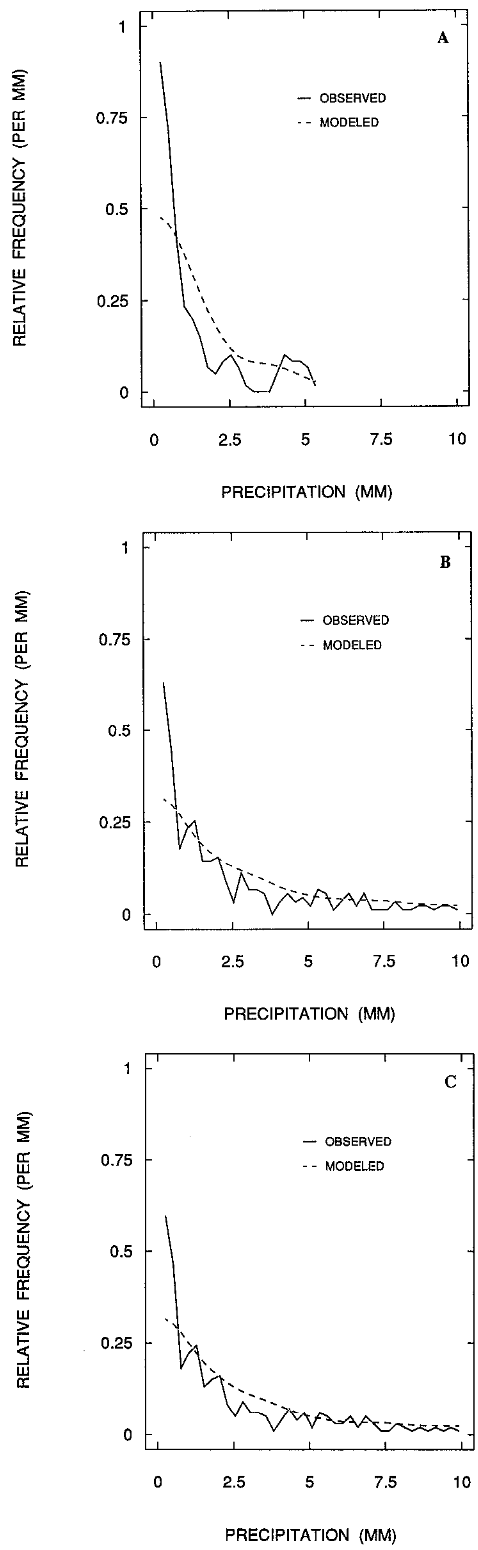


Figure 5. Model simulated and empirical distribution of precipitation amounts in July at Denver totaled over (a) A.M., (b) P.M., and (c) day.

Table 4. Model Identification of Hourly Precipitation Occurrence Process for January at Chico (24 × 31 × 23 = 17,112 Observations)

Markov Chain Order	Log Likelihood Function	Number of Parameters	BIC
1	−2605.1	2	5229.7
2	−2401.2	4	4841.4
3	−2320.7	8	4719.5†
4	−2295.2	16	4746.4

Optimal model: P_{0001} = 0.0144, P_{1001} = 0.1687, P_{1101} = 0.2724, P_{0101} = 0.3896, P_{1011} = 0.6981, P_{0111} = 0.7194, P_{0011} = 0.8207, P_{1111} = 0.8387.
†Minimum.

west of the Sierra Nevada) is strongly related to large-scale atmospheric circulation patterns [*Redmond and Koch*, 1991]. Hence diurnal cycles of substantial magnitude would not be expected [*Wallace*, 1975], and precipitation regimes might be relatively persistent.

4.1. Model Fitting

Diurnal cycles are evidently not present in any of the model parameters (i.e., transition probabilities of the first-order Markov chain and mean and standard deviation of the transformed hourly intensities; recall (9) and (10)). However, the assumption of a first-order Markov chain for the hourly occurrence process does not appear to be tenable. Consequently, higher than first-order Markov chains are also fitted, but still without diurnal cycles in the transition probabilities. Generalizing (2), a kth-order Markov chain ($k \geq 2$) is characterized by transition probabilities

$$P_{i(1)\cdots i(k+1)} = \Pr\{J_t(h+1) = i(k+1) \mid J_t(h) = i(k), \cdots, J_t(h-k+1) = i(1)\}, \quad (15)$$

$i(1), \cdots, i(k+1) = 0, 1$. The transition probabilities in (15) are estimated in a manner completely analogous to the first-order case, and the BIC can again be employed to select the optimal order of Markov chain [*Katz*, 1981]. We note that *Pattison* [1965] fit higher-order Markov chains to hourly precipitation occurrences in California.

Table 4 summarizes the results of applying the BIC to select the order of the Markov chain model of the hourly precipitation occurrence process for January at Chico. The optimal order is $k = 3$, with a fourth-order model being second best. Also listed in Table 4 are the estimated transition probabilities for the third-order model. According to this model, the estimated probability that the next hour is wet ranges from a minimum of about 0.014 to a maximum of 0.839, depending on whether the preceding 3 hours were all dry or all wet. Besides requiring a more complex dependency structure than for Denver in July (i.e., third order versus first order), the hour-to-hour persistence of precipitation occurrence is much greater, with a first-order autocorrelation coefficient of about 0.79.

Table 5 summarizes the results of applying the BIC to determine whether the hourly intensity process for January at Chico needs to allow for autocorrelation (i.e., whether $\phi_1^* \neq 0$). The likelihood function involved is a special case of (A6) in Appendix A. As a value for the power transform parameter, $p = 1/8$ is selected by Hinkley's index of symmetry (the same as for Denver in July), and the model that allows the intensities to be autocorrelated is identified as superior. The estimated

Table 5. Model Identification of Hourly Precipitation Intensity Process for January at Chico (1885 Wet Hours)

$\phi_1^* \neq 0$?	Log Likelihood Function, ln $(\text{mm}^{1/8})^2$	Number of Parameters	BIC
no	3912.9	2	−7810.6
yes	4186.0	3	−8349.3†

Optimal model: μ_1^* = 1.0024 mm$^{1/8}$, σ_1^* = 0.1253 mm$^{1/8}$, ϕ_1^* = 0.5484.

first-order autocorrelation coefficient for the transformed intensity process is about 0.548, somewhat greater than for Denver (about 0.384). Finally, Figure 6 shows that the positively skewed shape of the observed hourly intensity distribution is captured well by the fitted power transform distribution.

4.2. Aggregation Properties

Even with a higher-order Markov chain, it is straightforward to derive aggregation properties for the hourly occurrence process. For a third-order Markov chain, the probability of a dry day is

$$\Pr\{J_t(h) = 0, h = 1, 2, \cdots, 24\} = (1 - \pi_1)P_{00}P_{000}P_{0000}^{21}, \quad (16)$$

where the lower-order probabilities in (16) (i.e., π_1, P_{00}, P_{000}) can be derived from the given third-order transition probabilities, $P_{i(1)\cdots i(4)}$, in (15) [e.g., *Lloyd*, 1974].

Not having diurnal cycles in the parameters removes one obstacle to analyzing the aggregation properties of total precipitation amounts. In particular, with a first-order Markov chain, the approximate effect on the variance of total precipitation of introducing autocorrelation among the hourly transformed intensities (7) can be seen. Generalizing (14),

$$\text{Var}\,[S_t(h, H)] \approx H\{\pi_1\sigma_1^2[(1 + P_{11}\phi_1)/(1 - P_{11}\phi_1)] + \pi_1(1 - \pi_1)\mu_1^2[(1 + \rho_1)/(1 - \rho_1)]\}, \quad (17)$$

where the parameters of the Markov chain are related by $P_{11} = \pi_1 + (1 - \pi_1)\rho_1$ (see Appendix B for an outline of the

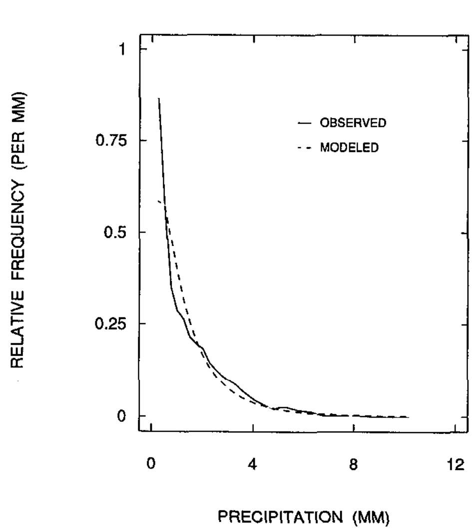


Figure 6. Density function of fitted power transform distribution and empirical distribution for hourly precipitation in January at Chico.

derivation of (17)). Here ϕ_1 denotes the first-order autocorrelation coefficient between the original, untransformed hourly intensities, and it is evident that $\phi_1 > 0$ serves to increase the variance of total precipitation relative to the situation in which $\phi_1 = 0$.

If the hourly intensities were not autocorrelated, then an analytical expression for the variance of total precipitation can be obtained for higher than first-order Markov chains. The basic idea is to employ a state space representation of a higher-order Markov chain as a first-order chain with vector states and to apply the more general expression for the variance of the sum of a chain-dependent process with a Markov chain having more than two states [*Katz*, 1977b]. This expression involves the inverse of a matrix and is not reproduced here [see *Klugman and Klugman*, 1981]. Instead, a simulation approach is again relied on to determine the distribution (and variance) of total precipitation for the Chico model.

Table 6 summarizes the aggregation properties of some of the candidate stochastic models for the hourly precipitation process for January at Chico. Because none of the candidate models involve diurnal cycles, only daily total precipitation is considered. Using (16), Table 6 includes the probability of a wet day as determined by the optimal third-order Markov chain (see Table 4), as well as for a simpler first-order Markov chain. This model probability is only slightly too high, whereas the first-order model produces much too high a value (compared with the observed value of about 0.321).

The mean daily total precipitation is necessarily approximately correct no matter what the form of model (observed value of about 4.04 mm). The corresponding model standard deviation is determined by the same type of simulation approach as for Denver (i.e., 4000 days of hourly precipitation amounts are generated for each model). Both the incorporation of autocorrelated intensities and the change from first- to third-order Markov chain lessen the tendency to underestimate the observed standard deviation of about 9.62 mm. Finally, Figure 7 compares the simulated and observed distributions of daily total precipitation, with the shapes of the two distributions being similar in spite of the underestimation of variance.

5. Discussion

Various ways in which chain-dependent processes may be extended to more adequately model time series of hourly precipitation amounts have been demonstrated. These extensions include autocorrelated intensities, diurnal cycles in the model parameters, and higher-order Markov chains for the occurrence process. A flexible modeling approach has been advocated. It is appropriate for fitting precipitation totaled over timescales different than hourly, as well as consistent with conventional models for daily precipitation. In spite of the generalizations required, many theoretical properties of the models can be determined by either analytical formulas, recursive numerical methods, or simulations. Furthermore, the form of such models is convenient for generating synthetic precipitation time series, needed as inputs in many hydrologic applications.

Table 6. Aggregation Properties of Models for January Hourly Precipitation at Chico

Type of Model: Order	Type of Model: $\phi_1^* \neq 0$?	Probability of Wet Day	Mean Daily Total Precipitation, mm	Standard Deviation of Daily Total Precipitation, mm
1	no	0.4849	4.029	6.738
1	yes	0.4849	4.029	7.400
3	no	0.3697	4.029	7.925
3	yes	0.3697	4.029	8.244
(observed)		(0.3212)	(4.042)	(9.619)

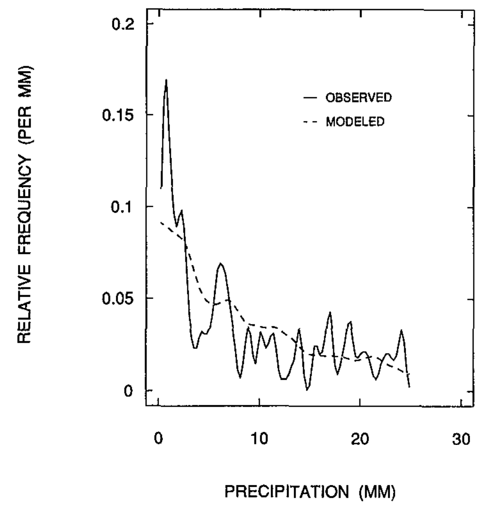


Figure 7. Model simulated and empirical distribution of daily total precipitation in January at Chico.

Attention has been devoted to the performance of these stochastic models when hourly precipitation is aggregated (e.g., to daily totals), being comparable in some respects and superior in others to conceptual models. *Islam et al.* [1990] and *Rodríguez-Iturbe et al.* [1987, 1988] established that virtually all conceptual models tend to underestimate the variance of daily total precipitation and are unable to reproduce the relative frequency of wet days unless they are "tuned" by using the appropriate daily sample statistics. Similar deficiencies have been encountered here for chain-dependent models. Nevertheless, the chain-dependent process approach is clearly superior in its ability to reproduce the marked diurnal cycles characteristic of precipitation in certain regions and seasons. The conceptual models employed have generally ignored these cycles even when they are well known to exist (e.g., for Denver in late spring or summer), in part because their incorporation would make already difficult parameter estimation problems more complex, if not infeasible. As illustrated here, explicit treatment of diurnal cycles is required in order to adequately reflect statistics such as the length of dry or wet spells. In summary, while conceptual models should still play an important role in the physical understanding of the precipitation process, for practical purposes chain-dependent processes do have some clear advantages.

Future work would deal with an examination of ways in which chain-dependent processes could be further extended to produce improved aggregation properties. For instance, the underestimation of the variance of daily total precipitation might be attributable to failing to produce a high enough degree of persistence in occurrences on a daily timescale. In this regard, it might be feasible to fit higher than first-order Markov chain models to the hourly occurrence process for

Denver while still retaining any diurnal cycles in the transition probabilities. Another approach could involve fitting stochastic models for hourly precipitation conditional on large-scale atmospheric circulation patterns. When applied to daily precipitation, this approach has been shown by *Katz and Parlange* [1993] to yield a superior overall model, at least in terms of model variance for monthly total precipitation.

Appendix A: Likelihood Functions for Model Identification

This appendix outlines the general procedure for estimating parameters and obtaining likelihood functions for the various forms of models fit to the time series of hourly precipitation amounts.

A1. Occurrence Process

A1.1. Fitting the logistic model. The maximum likelihood estimator of $P_{ij}(h)$ is

$$\hat{P}_{ij}(h) = n_{ij}(h)/n_{i\cdot}(h), \tag{A1}$$

where $n_{ij}(h)$ denotes the number of times in the sample of observations of precipitation occurrences that a transition from state i on the hth hour of the day to state j on the $(h+1)$th hour occurs and $n_{i\cdot}(h) = n_{i0}(h) + n_{i1}(h)$. To fit the logistic model, these estimates $\hat{P}_{i1}(h)$, $h = 1, 2, \cdots, 24$, are substituted into the left-hand side of (9). The model is fitted by weighted least squares, with the weights being given by [see *Neter et al.*, 1983, pp. 361–367; *McCullagh and Nelder*, 1983]

$$w_i(h) = n_{i\cdot}(h)P_{i1}(h)[1 - P_{i1}(h)]. \tag{A2}$$

Again the estimates $\hat{P}_{i1}(h)$ determined by (A1) are substituted into (A2). Once the estimates of the parameters A_i, B_i, and C_i have been obtained, the smoothed values of the transition probabilities $P_{i1}(h)$ can be determined through the inverse of the logistic transformation (i.e., left-hand side of (9)). That is,

$$\text{if } x = \ln[p/(1-p)], \text{ then } p = e^x/(1+e^x). \tag{A3}$$

A1.2. Likelihood function. The logarithm of the likelihood function, say $L(\{J_t(h)\})$, for a first-order Markov chain with diurnal cycles in the transition probabilities can be expressed as

$$\ln L(\{J_t(h)\}) = \sum_{i,j,h} n_{ij}(h) \ln P_{ij}(h). \tag{A4}$$

The maximized log likelihood function is obtained by substituting the smoothed estimates of the transition probabilities $P_{ij}(h)$, as described in part A1.1, into (A4).

A2. Intensity Process

A2.1. Fitting the cosine wave. First the mean hourly transformed intensities $\mu_1^*(h)$, $h = 1, 2, \cdots, 24$, are estimated by the corresponding sample means. To fit the cosine wave, these estimates are then substituted into the left-hand side of (10). Because these sample means are based on unequal sample sizes, the model is fitted by weighted least squares, with the weights being given by

$$w_1^*(h) = n_{1\cdot}(h). \tag{A5}$$

The cosine wave for the standard deviations of the hourly transformed intensities $\sigma_1^*(h)$, $h = 1, 2, \cdots, 24$, is fit in an analogous manner, using the same weights (A5). The only complication is that the sample standard deviations are computed about the cosine wave for the mean hourly transformed intensities, not about the individual hourly sample means (i.e., the estimate of $\sigma_1^*(h)$ is a function of the estimate of $\mu_1^*(h)$). Finally, the estimate of the first-order autocorrelation coefficient for the hourly transformed intensities ϕ_1^* is based on the cosine waves for both the hourly means and standard deviations (i.e., the estimate of ϕ_1^* is a function of the estimates of $\mu_1^*(h)$ and $\sigma_1^*(h)$).

A2.2. Likelihood function. The logarithm of the likelihood function, say $L(\{X_t^*(h)\})$, for the hourly transformed intensity process, with diurnal cycles in the means and standard deviations and with first-order autocorrelation, can be expressed as

$$\ln L(\{X_t^*(h)\}) = -(1/2) \sum_1^{24} n_{01}(h-1) \ln\{[\sigma_1^*(h)]^2\} - (1/2) \sum_1^{24} n_{11}(h-1) \ln\{[\sigma_1^*(h)]^2[1 - (\phi_1^*)^2]\}. \tag{A6}$$

Here the convention is adopted that $n_{ij}(0) = n_{ij}(24)$. The maximized log likelihood function is obtained by substituting the smoothed estimates (i.e., cosine wave) of the standard deviations $\sigma_1^*(h)$, as described in part A2.1, along with the estimate of the first-order autocorrelation coefficient ϕ_1^* into (A6). Strictly speaking, the joint likelihood of all the parameters is not necessarily maximized because of the sequential manner in which the optimization is performed.

Appendix B: Variance Derivation

The approximate expression (17) for the variance of total precipitation $S_t(h, H)$ is now derived. No diurnal cycles in any of the parameters are permitted, with the occurrence process being a first-order Markov chain. The hourly intensities within a wet spell are assumed to constitute an AR(1) process with parameter ϕ_1 (for convenience, the autocorrelation is introduced into the original intensities rather than into the power-transformed process as in (7)). By conditioning on the states of the occurrence process (i.e., $J_t(h), J_t(h+1), \cdots, J_t(h+l)$), it is straightforward to show that the autocovariance function for the time series of hourly precipitation amounts can be expressed as

$$\text{Cov}[X_t(h), X_t(h+l)] = \pi_1\sigma_1^2(P_{11}\phi_1)^l + \pi_1(1-\pi_1)\mu_1^2\rho_1^l \qquad l = 1, 2, \cdots. \tag{B1}$$

Substitution of (B1) into the general asymptotic expression for the variance of a sum of a stationary process [e.g., *Brockwell and Davis*, 1991, p. 219] yields (17).

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