Physical Basis for a Time Series Model of Soil Water Content

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A first-order autoregressive Markovian model AR(1) is formulated on the basis of the hydrologic budget and soil water transport equation. The model predictions compared well with neutron probe measurements of soil moisture content, and the statistical moments were conserved. The applied water events were white noise in structure, and the random shocks generated from the flow dynamics simplifications have a statistical mean of zero and were uncorrelated for all time lags. The derived AR(1) model parameter is used to compute the mean diffusivity of the soil, which is in agreement with reported lab measurements and field estimates obtained from cumulative evaporation measurements made with two large lysimeters.

1. Introduction

Knowledge of soil moisture change in the unsaturated zone near the land surface is an important component in hydrologic and climate studies since nonlinear transport processes such as drainage and runoff are directly linked to precipitation forcing and evaporation output. Stochastic tools, in conjunction with physical descriptions of the hydrologic processes, have gained much attention in the study of geophysical flows. It has been shown by *Mitchell* [1964] and *Gilman et al.*, [1963] that many geophysical variables possess the spectral properties of red noise due to the persistency of meteorological data that can be described by a linear Markov model. Stochastic models have also been utilized to examine climate variability as influenced by land-atmosphere interactions [*Manabe and Delworth*, 1990; *Manabe and Hahn*, 1981]. *Manabe and Delworth* [1990] concluded that land-atmosphere interactions possess features similar to ocean-atmosphere systems where the ocean acts as a long-term integrator of white noise thermal forcing from the atmosphere. In their study, *Manabe and Delworth* [1990] found that the soil acts as an integrator of white noise atmospheric forcing (i.e., precipitation) that supplies a finite memory component to the land-atmosphere system. With the temporal variability of soil moisture described by a simple first-order linear Markovian model, a 50-year integration of a general circulation model was performed by *Manabe and Delworth* [1990] to study the physical mechanisms of the soil variability and its influence on critical climate variables such as surface temperature. Their results indicate that it is not unreasonable to assume that the temporal variability of soil moisture is Markovian and is primarily controlled by precipitation forcing and evaporation losses that can significantly contribute, at low frequencies, to climatic variability. The issues concerning the statistical behavior of soil moisture raised by *Manabe and Delworth*

[1990] in the context of climate change are of great interest in surface and subsurface hydrology as well.

Yevjevich [1963] initiated the physical basis of autoregressive modeling in watershed hydrology [*Salas et al.*, 1980]. A number of physically based Markovian models for runoff have since been proposed and reviewed [*Klemes*, 1973, 1978; *Moss and Bryson*, 1974; *Spolia and Chander*, 1974; *Salas and Smith*, 1981]. *Yu and Brutsaert* [1969a, b] found that an autoregressive Markovian model adequately described evaporation, air temperature, and relative humidity in a study on Lake Ontario. *Yakowitz* [1985] demonstrated that simple Markovian models yield comparable results to nonparametric regression analysis for flood prediction. *Morel-Seytoux* [1988] presented a physical-statistical approach to the study of soil-aquifer-stream interactions and concluded that purely statistical descriptors of hydrologic phenomena may not be adequate when excitations are extremely transient or the media properties are heterogeneous. It should be noted that in most of the simple and more complex models the hydrologic system was driven with a stochastic forcing such as precipitation [*Katz*, 1977; *Kavvas*, 1982a, b], which leads to a stochastic output (e.g., stream flow) that is dependent on the stochasticity of the input [*Salas and Smith*, 1981]. *Ramirez and Bras* [1985] have used a Neyman-Scott cluster model to simulate the precipitation input to a soil-plant model in order to obtain irrigation decisions. *Aboitiz et al.* [1986] derived a state-space model using the hydrologic balance equation in conjunction with an AR(1) time series model for reference evapotranspiration. They successfully used a Kalman filter to generate forecasts of soil moisture depletion and crop evapotranspiration. This approach is sensitive to the error covariance matrix which is difficult to determine since it involves at least the calculation of instrumental variances, spatial averaging variances, calibration variances, errors in the crop coefficients, and some reference evapotranspiration estimates.

In this study we present a simple daily hydrologic balance in which only three governing components at a daily time step interact: applied water, evaporation, and change in moisture content. A field experiment over a uniform and flat bare soil field was designed to insure that only these three hydrologic parameters are interacting where the applied water (irrigation) is known. Under these conditions we study the interaction between water storage and evaporation into the atmosphere, and we show that with simplifications to the Richards's equation and the hydrologic balance the Mark-

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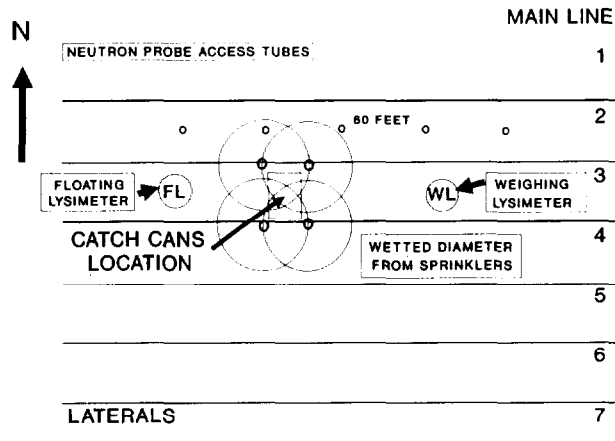[4] Department of Civil Engineering, University of California, Davis.

Fig. 1. Campbell Tract experimental setup displaying the location of the neutron probe access tubes, the weighing lysimeter, the floating lysimeter, and the network of catch cans.

ovian AR(1) process used by *Manabe and Delworth* [1990] can be derived from physical considerations. The derived AR(1) model was compared with daily measured moisture content data obtained from neutron-scattering measurements. We propose that the relationship between water content and diffusivity, hysteresis due to wetting and drying, and assumptions regarding the second stage of evaporation can be modeled with a random stochastic shock for daily time increments that may be small compared to the applied water forcing. It is demonstrated that the AR(1) model transfer parameter is related to soil properties, namely soil water diffusivity.

2. EXPERIMENT

The experiments were carried out from September 4 to December 12, 1990, at the University of California, Davis. The soil is a uniform Yolo clay loam with no layering within the top meter. The site includes a sprinkler system irrigating a surface area that is 150 m by 130 m. The irrigation system consists of six laterals running from east to west, with a main line spacing of 15 m and a lateral spacing of 10 m. The nozzle diameter is 3.175 mm and furnishes a pressure of 345 kPa at the sprinkler head and a wetted diameter of 23 m for wind speeds not exceeding 0.5 m s^{-1}. Under this pressure the gross application rate is estimated to be 0.5 cm h^{-1} [*Rain Bird Sprinkler Manufacturing Corporation*, 1982]. The net application rate, measured in the evenings with wind speeds not exceeding 0.5 m s^{-1}, ranged between 0.439 and 0.458 cm h^{-1} [see *Katul and Parlange*, 1992a, b]. Since application uniformity on the field is critical in this study, a network of cylindrical catch cans (internal diameter, 10.5 cm; height, 17.5 cm) was set to monitor the average uniformity coefficient for each irrigation, as described by *Cuenca* [1989]. The cans were spaced at 3.05 m in the lateral direction and 3.81 m in the main line direction to cover an area of 150 m^2, as shown in Figure 1. A small oil film was sprayed into the cans before each irrigation to reduce evaporation losses from the cans. Uniformity coefficients calculated using Christiansen's formulation [see *Cuenca*, 1989] ranged between 0.79 and 0.88, depending on the mean horizontal wind speed. A sample of the measured spatial variability of applied water within a rectangular area of 150 m^2 bounded at the corners by four sprinklers is shown in Figure 2, with a calculated uniformity coefficient of 0.88. In general, the irrigations supplied an average of 10–20 mm of water and were scheduled in the evenings to maximize net application rates and uniformity coefficients, as discussed by *Katul and Parlange* [1992a, b] and *Parlange and Katul* [1992a, b].

The daily evaporation rate was measured on a 20-min. time step by two large sensitive lysimeters and integrated over each day. The weighing lysimeter is circular in design, 6 m in diameter, and 1 m in depth. The circular design of the lysimeter results in a smaller value for the ratio of the perimeter to the area, which reduces wall-edge effects [*Pruitt and Angus*, 1960; *Pruitt and Lorrence*, 1985]. The weighing
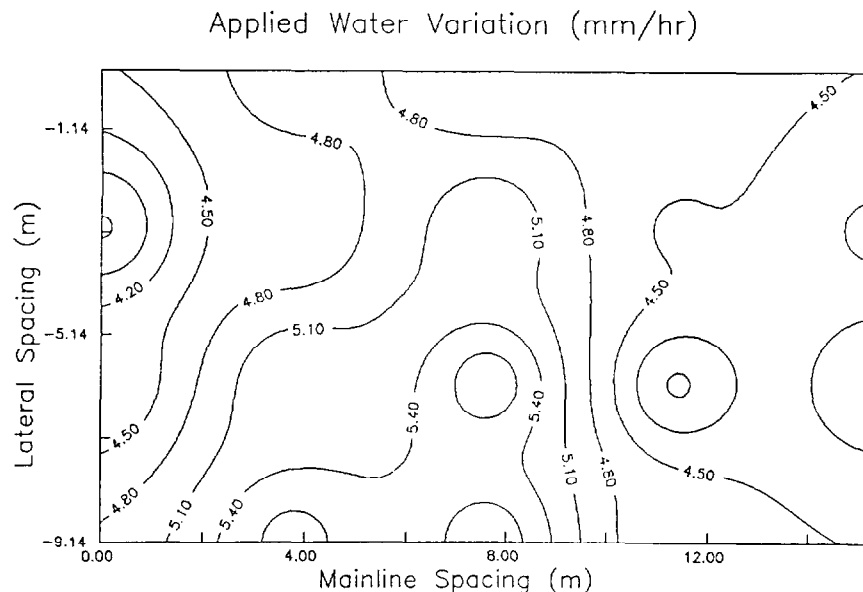

Fig. 2. Spatial variability of the applied water (millimeters per hour) between four sprinkler heads (uniformity coefficient is 0.88), as computed by the network of catch cans.
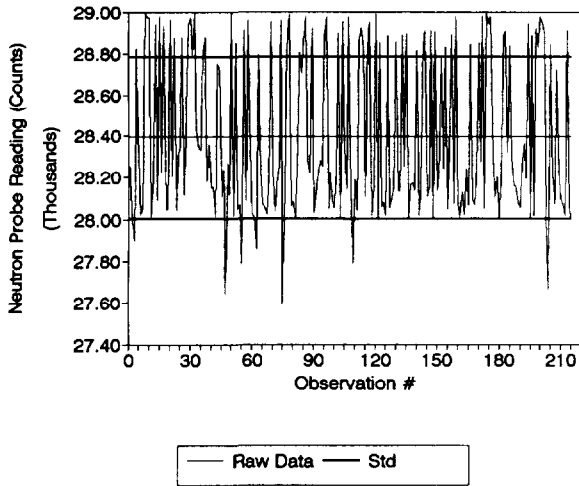
Fig. 3. Variability in consecutive neutron probe readings at 75 cm depth. The standard deviation of the 214 readings around the mean is also shown.
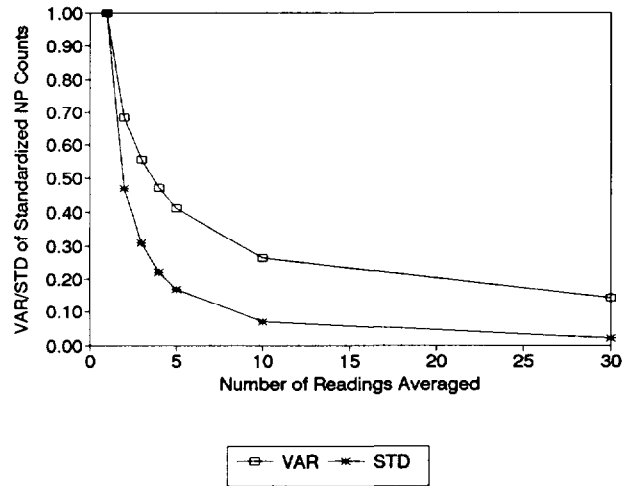

Fig. 4. Reduction in instrumental variance by averaging more readings per depth.

lysimeter accuracy, as reported by *Pruitt and Angus* [1960], is 0.03 mm. The shear stress lysimeter has the same dimensions as the weighing lysimeter and employs water for a floating fluid. The evaporation rate for this lysimeter is determined by measuring hydrostatic pressure changes due to water level fluctuations that are converted to millivolt signals by a pressure transducer. Both lysimeters were calibrated prior to the experiment on August 27, 1990, by applying, incrementally, 20 kg up to 400 kg and reading the corresponding millivolt signal change using a Campbell Scientific CR21X micrologger. Incremental loading and unloading cycles simulating applied water and subsequent evaporation were performed on the lysimeters during the calibration period to insure that no hysteresis effects occurred within the pressure tranducer, the potentiometer, and the lysimeters.

The volumetric moisture content is monitored by a Campbell Nuclear Pacific probe, model 503. Five aluminum access tubes, spaced 20 m from east to west, were drilled using a Soil Conservation Service Madera sampler, as described by *Dickey* [1990], with samples taken every 15 cm to a depth of 1.05 m. The samples from all the drilled tubes were combined to obtain an average field calibration for the neutron probe. The standard error of estimate of the calibration curve was 2.7%, and the coefficient of determination (r^2) was 0.88. Since the neutron gauge detects thermalized neutrons emitted by a radioisotope neutron source that is randomly decaying, variability within readings is unavoidable [*Cuenca*, 1989; *Vauclin et al.*, 1984; *Haverkamp et al.*, 1984]. A study was performed to investigate the instrument variance in which 214 consecutive soundings were recorded at 75 cm depth in the soil with a 32-s count time per sounding. The raw counts versus observation number are displayed in Figure 3. The raw data of Figure 3 were transformed to a standardized series with a mean of zero and a variance of unity. A moving average was performed on the standardized data in order to assess the reduction in instrument variance gained by averaging more sounding at a particular depth. It was concluded from Figure 4 that three readings were sufficient to reduce the variance in the raw data by 70%.

Throughout the course of the experiment, daily neutron probe readings were taken at 15, 30, 45, 60, 75, and 90 cm depth at 0800 PST at the five locations described in Figure 1.

3. MODEL FORMULATION

3.1. *Hydrologic Balance Model*

The local evaporation (E) from a land surface can be estimated from the conservation of mass equation for a soil layer of thickness z_h using

$$E = -\int_0^{z_h} \frac{\partial \theta}{\partial t} \, dz + (P + q_{ri} + q_{si}) - (q_d + q_{ro} + q_{so}) \quad (1)$$

where z is the vertical coordinate (positive into the soil layer), θ is the volumetric moisture content, t is time, P is applied water, q_{ri} is the lateral inflow rate over the soil surface, q_{ro} is the corresponding outflow rate, q_d is the drainage through the lower boundary at $z = z_h$, q_{si} is the lateral subsurface flow into the soil surface, and q_{so} is the corresponding outflow rate [*Brutsaert*, 1982a; *Parlange et al.*, 1989].

For each of the irrigations carried out the application time was short enough that no surface runoff was generated, resulting in $q_{ro} = q_{ri} = 0$. Since the field was flat and the water table depth was in excess of 20 m, it was reasonable to assume that the predominant flow direction was in the vertical, with no horizontal movement of water in the soil, resulting in $q_{so} = q_{si} = 0$. The drainage component can be estimated from Darcy's law:

$$q_d = -k(\nabla \Psi - 1) \quad (2)$$

where k is the hydraulic conductivity of the soil and Ψ is the matric potential. In several field studies [*Nielsen et al.*, 1973; *Brutsaert*, 1982a; *Ahuja et al.*, 1988] it has been observed that during the vertical redistribution of soil water at depths greater than 0.5 m, where the evaporation does not influence the water movement directly, the hydraulic gradient may be taken as minus unity such that $q_d = k$. As z_h is taken deeper, the storage capacity of the control volume increases so that at $z_h > 75$ cm the drainage on a daily basis is small. In this study it was assumed that the daily drainage at 75 cm
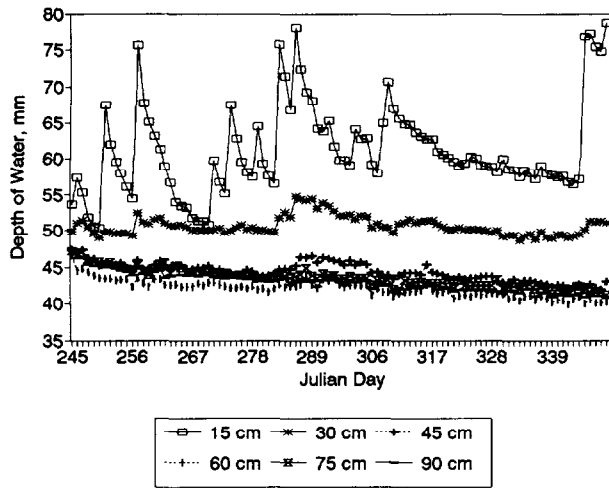
Fig. 5. Average water content variation at each depth. The resulting moisture content at each depth was obtained by averaging the corresponding moisture contents at the five locations.

Fig. 7. Comparison between cumulative evaporation from the weighing lysimeter, floating lysimeter, and neutron probe.

depth was negligible (see Figure 5), and the simplified hydrologic balance is written

$$E = -\int_0^{z_h} \frac{\partial \theta}{\partial t}\, dz + P \qquad (3)$$

Daily evaporation measurements from the two lysimeters were compared with evaporation estimated using (3), where θ was obtained using the neutron probe scattering techniques. The field average water content was obtained by averaging the 15-, 30-, 45-, 60-, and 75-cm neutron probe readings at the five locations and integrating those averages for the 75-cm profile. The average cumulative water (millimeters) in the top 75 cm and the applied water measured by the network of catch cans is presented in Figure 6. The cumulative evaporation was determined from the changes in measured soil moisture (E_{np}) and compared with evaporation measurements from both the weighing lysimeter (E_{wL}) and the floating lysimeter (E_{fL}) (Figure 7). Two linear regression models of the form $E_{wL} = A\,E_{np} + B$ and $E_{fL} =$
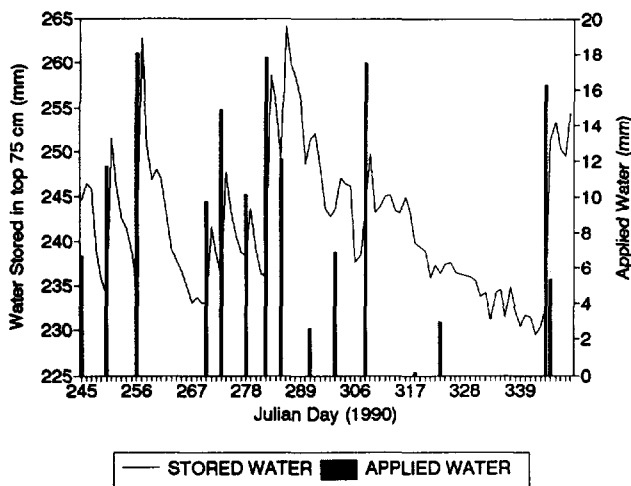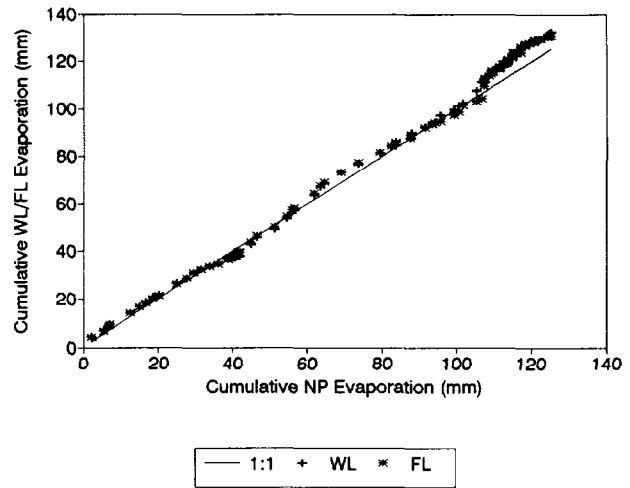
$A\,E_{np} + B$ were used to evaluate the validity of (3), and the results are presented in Table 1. It is clear that the simplified daily hydrologic balance monitored with the neutron probe captures the field scale evaporation measured with the lysimeters.

3.2. Time Series Representation of the Hydrologic Balance Model

From (3) the amount of water stored (W_{t+1}) at time $t + 1$ as a function of the amount of water (W_t) at time t and the atmosphere forcings is

$$W_{t+1} = W_t + P_t - E_t \qquad (4)$$

In the derivation that follows we make some assumptions about the dynamics of the flow and account for these assumptions later by including an integrated random error term in the time series model. One-dimensional, isothermal water transport in homogeneous, isotropic soil is described by the Richards's equation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D\, \frac{\partial \theta}{\partial z}\right] - \frac{\partial k}{\partial z} \qquad (5)$$

where D is soil water diffusivity defined by $D = k\, d\Psi/d\theta$. Gardner [1959] assumed that for the second stage of drying the effect of gravity at the evaporating surface is negligible. Since we assume that the second stage of drying is usually

Fig. 6. Time variation of the applied water events and the average water content of the top 75 cm.

TABLE 1. Comparison Between Lysimeter-Measured Cumulative Evaporation and Neutron Probe Estimations Using the Simple Hydrologic Equation

Parameters	$E_{wl} = A E_{np} + B$	$E_{fl} = A E_{np} + B$
Slope (A), mm mm^{-1}	1.033	1.026
Constant (B), mm	−0.754	−0.384
Standard error of estimate (SEE), mm	1.830	1.670
Coefficient of determination (r^2)	0.9981	0.9984

E_{wl}, E_{fl}, and E_{np} are the cumulative evaporation from the weighing lysimeter, floating lysimeter, and average of five neutron probe access tubes.

attained in less than one day, the second term of (5) can be neglected, and this is treated as a problem of desorption. The initial and boundary conditions for (5) are taken such that $\theta = \theta_i$ for $z > 0$ and $t = 0$, and $\theta = \theta_s$ for $z = 0$ and $t > 0$, where θ_i is the initial moisture content and θ_s is the surface moisture content. Using the Boltzmann similarity variable $B = zt^{-1/2}$ [Bruce and Klute, 1956], assuming infinite initial soil wetting, and neglecting the gravity term, Richards's equation can be simplified to give the evaporation rate as

$$E_t = \frac{1}{2}(De)(t)^{-1/2} \qquad (6)$$

where De is the desorptivity and is a constant determined by the type of soil and given values of θ_i and θ_s [Parlange et al., 1985; Gardner, 1959; Brutsaert, 1982a, b]. Integrating (6) with respect to time yields the cumulative evaporation

$$CE_t = (De)t^{1/2} \qquad (7)$$

Black et al. [1969] proposed a linearized solution for De:

$$De = 2(\theta_i - \theta_s)\left[\frac{D_{avg}}{\pi}\right]^{1/2} \qquad (8)$$

where D_{avg} is a weighted mean diffusivity. If we assume that D_{avg} is constant for a particular soil, and for evaporation $\theta_i \gg \theta_s$, then (7) is rewritten

$$CE_t = 2\theta_i\left[\frac{D_{avg}t}{\pi}\right]^{1/2} \qquad (9)$$

Equation (9) may be written as

$$CE_t = A\theta_i \qquad A = \frac{1}{N}\int_{t=1}^{t=N} 2\left[\frac{D_{avg}t}{\pi}\right]^{1/2} dt \qquad (10)$$

where A is an average soil property and is taken as a constant and N represents the number of days of measurement. All the errors resulting from the assumptions leading to (10) are parameterized as a random shock a_{t+1} at time $t + 1$ so that a_{t+1} represents the integral of all the errors committed due to our assumptions, including the following: neglecting gravity flow at the surface, assuming that the second stage of drying is attained in less than 1 day, taking $\theta_i \gg \theta_s$, simplifying the nonlinear relationship between θ and D_{avg}, and ignoring the hysteresis effects due to wetting and drying on D_{avg}, as discussed by Staple [1976]. The daily evaporation as a function of moisture content at time $t(\theta_t)$ is then given by

$$E_t = A\theta_t + a_{t+1} \qquad (11)$$

Combining (11) with (4) and using $W_t = z_h\theta_t$, the simple time series representation of the hydrologic balance is given by

$$W_{t+1} = W_t - \frac{A}{z_h} W_t - a_{t+1} + P_t \qquad (12)$$

Equation (12) is in the form $W_{t+1} = (\phi 1)W_t + a_{t+1} + P_t$, which is a first-order autoregressive (AR(1)) model if a_{t+1} are independent zero-mean Gaussian and the applied water rates are independent, having a white noise structure in time [Salas et al., 1980] so that when converted to applied water depth, P_t will be similar to a Brownian increment. It should be noted that if the applied water is known, the influence on soil water content is additive, as can be noted from (3). In this study the applied water rates resemble a white noise
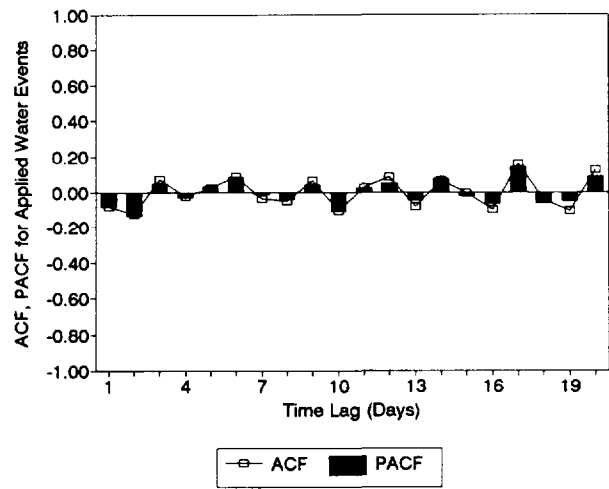

Fig. 8. Autocorrelation and partial autocorrelation functions for the applied water events.

forcing with no correlation or apparent memory structure. This can be seen from Figure 8 which displays the autocorrelation and the partial autocorrelation function for the time series of the applied water rates shown in Figure (6). The applied water depth time series as measured by the catch cans was simply subtracted from the raw data at each instant in time to obtain an outflow-storage relation. The significance of the removal of the applied water depth from the stored water time series will become evident when the mean weighted diffusivity of the soil is computed from the autoregressive parameter; this will be shown in more detail later. It should be noted that the applied water forcings still contribute to the stored water time series since they increase the total amount of water in the soil reservoir, z_h (0–75 cm), and the total daily evaporation rate. The water content time series of the top 75 cm of soil was standardized using

$$N_t = \frac{W_t - \bar{W}}{\sigma} \qquad (13)$$

where N_t is the time series of the standardized water content with the mean equal to 0 and the variance equal to 1, and σ is the standard deviation of the raw water content time series. A plot of the standardized time series (with the applied water depth time series removed) is shown in Figure 9.

The standardization transformation does not affect the absolute value of the coefficient $\phi 1$; however, it is desirable for normalization of subsequent statistical results.

4. RESULTS

Equation (12) demonstrates that the hydrologic balance can be formulated as a first-order autoregressive model, where $\phi 1$ is determined from the available data set. For the standardized water content data the autocorrelation function (ACF) and the partial autocorrelation (PACF) were computed as a function of the lag time (K) and are presented with the 95% confidence band in Figures 10 and 11, respectively. The 95% confidence bands for the ACF and PACF were computed using the Bartlett [1946] equations [see Box and Jenkins, 1970]. The ACF shows roughly a geometric decay that is consistent with an AR(1) process. Moreover, the ACF indicates that the coefficient $\phi 1$ is a positive constant, and no
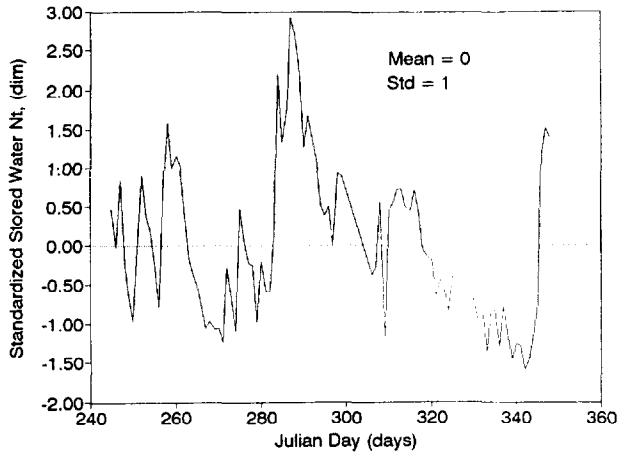
Fig. 9. Time variation of the standardized stored water time series N_t with zero mean and unit variance.

Fig. 11. Partial autocorrelation function for the top 75-cm moisture content time series after applied water events were removed.

periodic trends are apparent. The PACF shows that standardized water content data exhibits a spike at a lag of 1 day, which is consistent with an autoregressive Markovian behavior. The PACF, computed from neutron probe soundings, independently confirmed the finding that the hydrologic balance equation can be represented by an AR(1) model described by (12). Using the method of least squares [*Box and Jenkins*, 1970; *Shumway*, 1988], the parameter $\phi 1$ was determined. Other commonly employed models, including AR(2), AR(3), ARMA(1,1), and ARIMA(1,1,1), were fitted to the standardized time series, and the t test for each model parameter, the variance of the random shocks a_{t+1}, and the sum of the squares of errors (SSE) were computed and are presented in Table 2 for each selected model. It may be noted from Table 2 that the AR(1) model is comparable to the higher-order models (e.g., AR(2) and AR(3)) and that no statistical improvement was obtained by including a moving average term as in ARMA(1,1) or using integrated differencing as in ARIMA(1,1,1). The t test indicated that all the model parameters, except for $\phi 1$, are not statistically different from zero at the 95% level of significance. An Akaike's information criteria (AIC) search was also performed to confirm the results of Table 2 for autoregressive models [see *Shumway*, 1988;
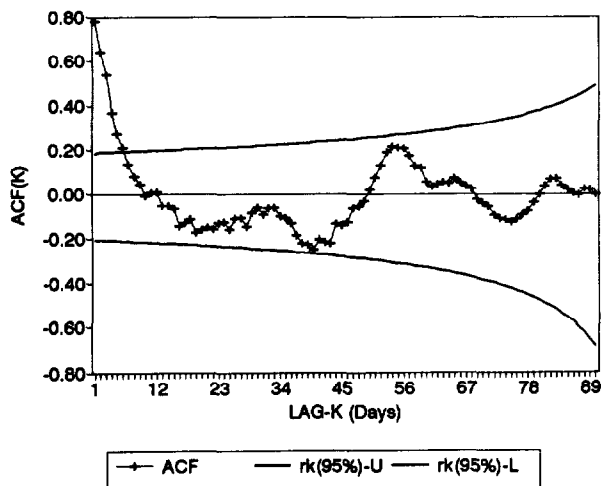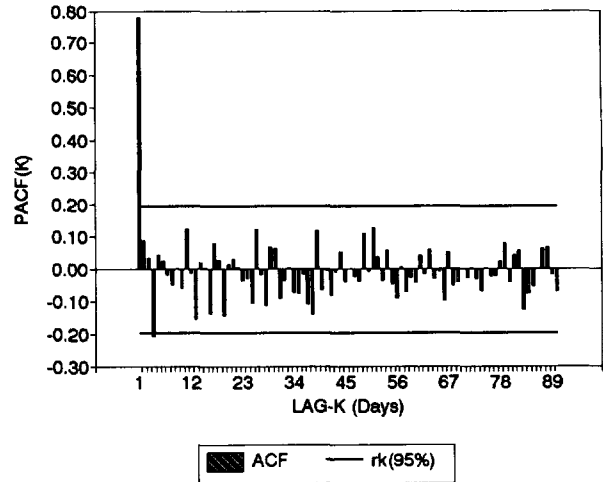
Murphy and Katz, 1985]. The variation of the AIC as a function of autoregressive model order is shown in Figure 12 in which the least AIC value was noted for orders 1 and 4. Therefore the AR(1) appears to be a reasonable choice from a parsimony point of view. The coefficient $\phi 1$ was also computed using the method of moments [*Salas et al.*, 1980; *Yule*, 1927; *Walker*, 1931; *Hipel et al.*, 1977] by solving the Yule-Walker equations to yield $\phi 1 = 0.780$ which is close to 0.8166 as determined by the method of least squares. For the purpose of this study the least squares estimate for $\phi 1$ is used.

The ability of the model to reproduce the neutron probe data set within the 95% confidence band was then investigated. From the standardized neutron probe measurement at time t the model $N_{t+1} = \phi 1 N_t$ was used to forecast the standardized water content variable at $t + 1$, with the results converted to W_{t+1} using (13).

The variance of the forecast error Var (a_t) for any ARMA model was computed using [*Salas et al.*, 1980; *Shumway*, 1988]

$$\text{Var}\,[a_t(L)] = \sigma_a^2 \sum_{j=0}^{j=L-1} \beta_j^2 \qquad (14)$$

Fig. 10. Autocorrelation function for the top 75 cm moisture content time series after applied water events were removed.

TABLE 2. Comparison Between Common Statistical Models

Model	Parameter	Value	t Ratio	SSE	σ_a^2
AR (1)	$\phi 1$	0.8166	13.06	35.63	0.3670
AR (2)	$\phi 1$	0.7523	7.30	35.40	0.3686
	$\phi 2$	0.0818	0.79		
AR (3)	$\phi 1$	0.7486	7.62	33.13	0.3709
	$\phi 2$	0.0336	0.22		
	$\phi 3$	0.0654	0.62		
ARMA (1, 1)	$\phi 1$	0.8539	12.09	35.38	0.3684
	$\theta 1$	0.1048	0.83		
ARIMA (1, 1, 1)	$\phi 1$	0.0953	0.18	37.51	0.3906
	$\theta 1$	0.2837	0.55		

The t ratio is computed for the 95% confidence interval and for the null hypothesis that the parameter is not different from zero. SSE, sum of the square of the error; σ_a^2, variance of the shocks for standardized time series.
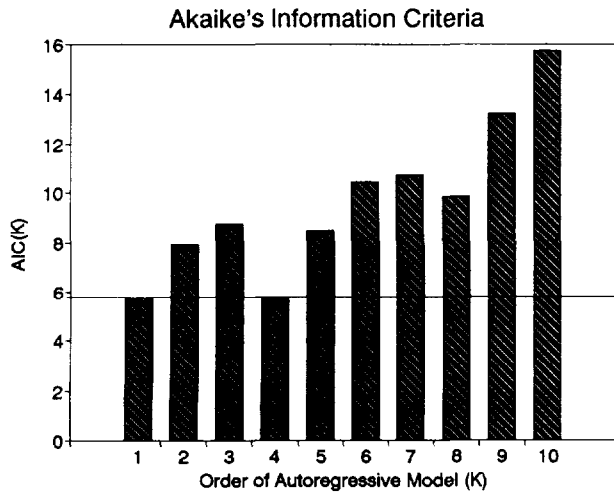
Fig. 12. Variation of the Akaike's information criteria with autoregressive model order.

Fig. 14. The time variation of the random shock a_t converted to millimeters.

where L is the lead time length of the forecasts (equal to unity in this case since the forecasts are for the next day), β_j is a polynomial determined from minimizing the mean square error and is unity for $j = 0$, and σ_a is the standard deviation of the random shocks. Using σ_a, the 95% confidence band was estimated on the basis of equations presented by Salas et al. [1980]. The forecasted water content, the measured water content, and the 95% confidence bands around the forecasts as a function of Julian day are presented in Figure 13. It was assumed during these forecasts that the expected value of a_{t+1} is zero. This assumption was verified by computing a_{t+1} for each day from measured neutron probe data and forecasted values using the AR(1) model. The variation of a_{t+1} as a function of Julian day is presented in Figure 14. A frequency distribution analysis on the shocks was performed, and the results are shown in Figure 15, in which the probability of having the expected value of $a_{t+1} = 0$ is highest. To study the temporal structure of the shocks, the autocorrelation function as well as the partial autocorrelation functions were computed for the a_{t+1} time series, and the results are presented in Figure

16. No apparent structure was observed from Figure 16, and the values of a_t are highly uncorrelated at all lags. The proposed AR(1) model predictions were evaluated for statistical moment preservation, namely, the first, second, and the third moments about the mean, and the results, presented in Table 3, indicate that the simple AR(1) model has preserved the mean (first moment), the variance (second moment), and the skewness coefficient (third moment) of the original data.

In order to estimate how much of the measured water content variation can be explained by the AR(1) model, a linear regression model of the form $W_{NP} = U\,W_{AR(1)} + V$ was fitted between the observed (W_{NP}) and the one time step predicted $W_{AR(1)}$ water contents, as shown in Figure 17. The coefficient of determination (r^2) was 0.79, the standard error of estimate was 3.65 mm, $U = 1.00034$, and $V = -1.07588$ mm. The slope was not statistically different from 1 using the t test with a 95% confidence interval, which indicated a 1:1 behavior between the proposed AR(1) model and the stored water measurements.

Since evaporation is the predominant transport process in this simplified hydrologic balance model, the coefficient ϕ_1 can be used to estimate the mean weighted diffusivity of the
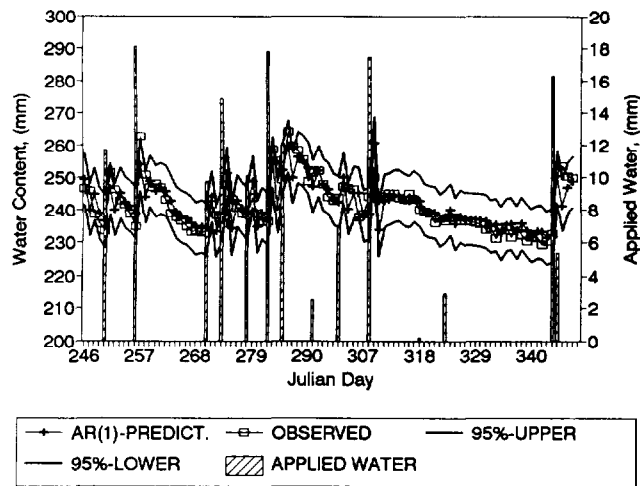

Fig. 13. Variation of AR(1) predictions and neutron probe measurements. The 95% confidence band around the AR(1) predictions as well as the applied water events are shown.
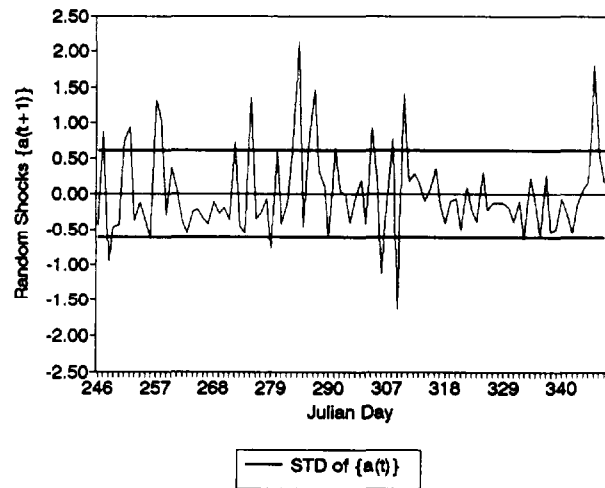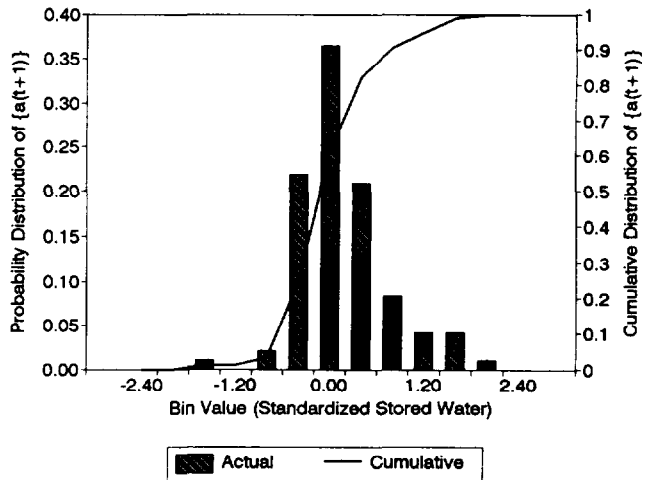

Fig. 15. Probability and cumulative distribution of the random shocks.
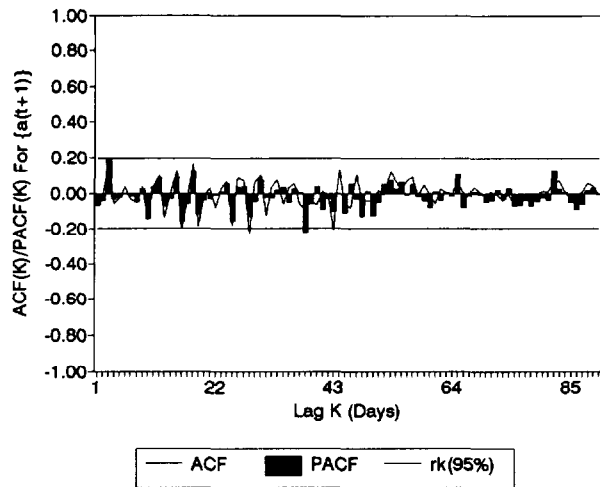
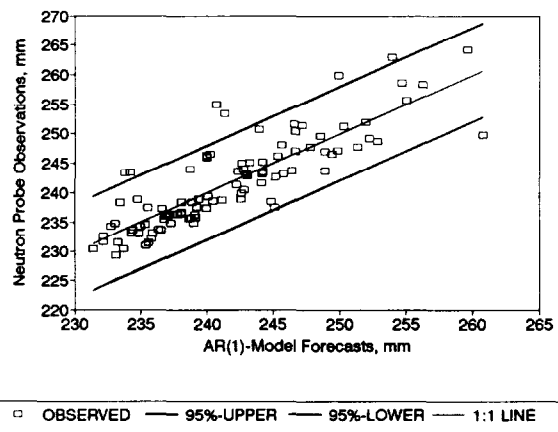Fig. 16. Autocorrelation and partial autocorrelation function for the random shock a_t time series.

Fig. 17. Comparison between AR(1) predictions and neutron probe measurements, with the 95% confidence bands and the 1 : 1 line also shown.

soil, D_{avg}. From (12) the value of $\phi1$ is $(1 - A/z_h) = 0.8166$, so that the average value of A for the 99-day period is 137.55 mm. Equation (10) can be rearranged to give

$$D_{avg} = \frac{\pi A^2}{4} \left[\frac{1}{N} \int_{t=1}^{t=N} t^{1/2} \, dt \right]^{-2} \quad (15)$$

where $N = 99$ days yields $D_{avg} = 334$ mm^2 d^{-1}. This estimate of D_{avg} was compared with an estimate obtained independently using the cumulative lysimeter evaporation and another estimate obtained from published laboratory diffusivity measurements on Yolo clay loam, as discussed by *Lima et al.* [1990]. Using the longest dry down period (Julian days 257–270; 1990) available in the record in which Gardner's assumptions were valid [*Black et al.*, 1969; *Brutsaert*, 1982a], the cumulative evaporation was plotted versus $t^{1/2}$. Due to the expected departure from the $t^{1/2}$ relation [see *Black et al.*, 1969] that may influence the estimate of De, only the first 10 days of this drydown period were plotted in Figure 18. A linear regression line was fitted ($r^2 = 0.994$, standard error of estimate (SEE) $= 0.43$ mm) to the data shown in Figure 18, and the slope was found to be 5.8 mm d$^{-1/2}$. From (8) the calculated regression slope is identical to the average value of the desorptivity De, which is approximately $2\theta[D_{avg}/\pi]^{1/2}$. Using the average moisture content obtained from the neutron probe soundings for this 10-day drydown period (32%), D_{avg} was estimated as 260 mm^2 d^{-1}.

Lima et al. [1990] presented lab results between θ and $D(\theta)$ for Yolo loam soil and for several sodium absorption ratios (SAR). Using the values presented for SAR (0, 0), a regression relation of the form $D(\theta) = D_0 \, 10^{(C_0 \theta)}$ was fitted

to their data set ($r^2 = 0.91$), which gave $D_0 = 1.10$ and $C_0 = 11.5$. Using Crank's equation [*Black et al.*, 1969; *Brutsaert*, 1982a], D_{avg} can be related to θ and $D(\theta)$:

$$D_{avg} = \frac{1.85}{(\theta_i - \theta_s)^{1.85}} \int_{\theta_s}^{\theta_i} (\theta_i - \theta)^{0.85} D(\theta) \, d\theta \quad (16)$$

Using for $D(\theta)$ the functional relation $D(\theta) = 1.1 \times 10^{11.5\theta}$, and for θ_i and θ_s the average moisture content for the 99-day period (33%) and 0, respectively, an approximate value $D_{avg} = 218$ mm^2 d^{-1} was obtained from numerical integration of (16). The three independent methods (autoregressive model, lysimeter data, and lab data) provided rather comparable estimates of the mean weighted field diffusivity.

5. CONCLUSIONS

A simple hydrologic balance model was used to show that the daily variation of moisture content resembles an autoregressive Markovian process under evaporative conditions and white noise forcing. The first-order autoregressive stochastic model was obtained from the hydrologic balance with a constant coefficient $\phi1$ that is dependent on the average diffusivity of the soil. This AR(1) memory structure

TABLE 3. Evaluation of AR (1) Model Predictions by Method of Conservation of Moments About the Mean

Variable	AR (1)	Observed
Mean, mm	241.704	241.45
Standard deviation, mm	6.80	7.50
Skewness coefficient, mm	0.69	0.76

The observed values are measured by the neutron probe scattering technique.
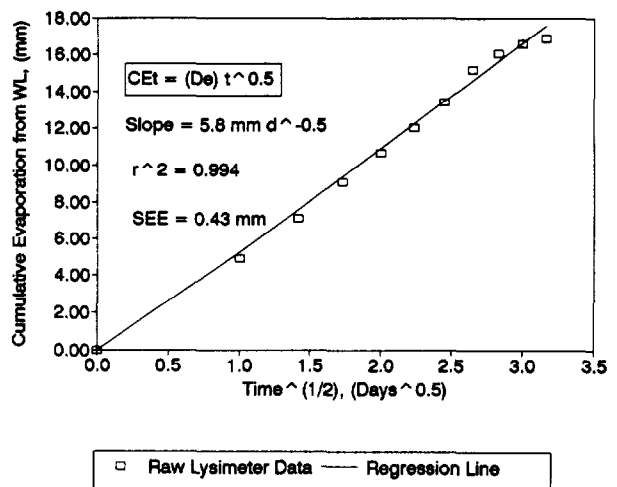

Fig. 18. Cumulative evaporation versus $t^{1/2}$.

indicates that the soil medium can integrate white noise forcings and generate "Brownian" behavior, which is in agreement with the temporal variability of the soil moisture model discussed by *Manabe and Delworth* [1990]. The assumptions leading to the AR(1) form were primarily simplifications to the soil water transport equation while retaining mass conservation as the governing physical mechanism in the soil-atmosphere system. These assumptions may be valid for extended time periods such as daily, monthly, or yearly time increments and would not be expected to hold for shorter time steps (e.g., hourly) where the nonlinearities in the flow dynamics become critical.

The proposed AR(1) model was then evaluated by comparing forecasts of moisture content with neutron probe soundings obtained on a daily time step. The model predictions preserved the mean, the variance, and the skewness coefficient, and a 1:1 correlation with measurements was obtained from linear regression analysis. The random shocks proved to have a mean of zero, and the values were uncorrelated at all lags. The coefficient $\phi 1$ was then used to calculate the average diffusivity of the soil, and the results were comparable to values obtained from lab experiments as well as cumulative evaporation lysimeter measurements at the same site.

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