COMPARISON OF MULTI-CELL MULTI-ANTENNA CDMA AND TDMA SYSTEMS

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Abstract - We evaluate system capacities for multicellular systems with different multiple access strategies. The interference from another cell sharing the same frequency is calculated, and the total interference arising from the frequency reuse pattern is evaluated. A simple analytical model for the inter-cell interference is developed, which is in good agreement with numerical experiments. Irrespective of the multiple access strategy, capacity becomes interference limited. When comparing TDMA and CDMA in the uplink, there is a crossover in capacity. At low SNR CDMA is preferable, at high SNR TDMA performs better.

Keywords - Multi-antenna, CDMA, TDMA, out-of cell interference, spectral efficiency, reuse factor.

I. INTRODUCTION

Fourth generation cellular communication systems (4G) is the next step in the evolution of mobile communications standards. The target is to design a wireless multicellular system with significant increase in data capacities when compared to the 3rd Generation systems. Peak data rates in a wide area network with high mobility are envisioned to be around 100 Mbps, and 1 Gbps in a local area coverage with limited mobility. With limited bandwidth resources this can only be achieved by a substantial increase in spectral efficiency. Supposing a bandwidth of 100 MHz the requirements translate into spectral efficiency of 1 bps/Hz/cell in the first case and 10 bps/Hz/cell in the latter. To reach these spectral efficiencies new technologies need to be used. Interference can be reduced by successive decoding [1], [2], at the price of increasing complexity. However, even in the most optimal case the inter-cell interference can be cancelled only partially. Another way to increase the spectral efficiency is to use multi-antennae (MIMO) technologies [3]. This offers potential for a significant increase in capacity, with reasonable complexity and cost. However, the number of antennas is strictly limited. As a first estimate at most four antennae at the base station and at the mobile terminal have been considered. One of the most important things to decide when starting to construct a cellular radio system is the multiple access strategy. In [4], first steps into analysing the fundamental limits of different multiple access strategies were taken, with the 4G system capacity targets in mind. A comparison between hypothetical DSSS-CDMA and TDMA type uplink MIMO systems with additive white Gaussian noise was carried out. For DSSS-CDMA a frequency reuse factor of one is possible, but with RAKE detection, capacity becomes strongly limited by inter-user interference. In a TDMA system, a higher reuse factor is necessary to mitigate inter-cell interference. Omitting the interference, TDMA system capacity increases monotonically as a function of signal to background noise ratio. In practice, however, there is interference coming from other cells and as a result, TDMA is interference limited, as well. Due to the reuse pattern, the interference factor is much smaller than for DSSS-CDMA. In this work we extend results from [4], by taking into account the inter-cell interference for TDMA type systems with different reuse factors. A simplified analytical model for inter-cell interference is proposed and compared to numerical results.

In section II we give the capacity expression for both CDMA and TDMA systems. The calculation of the inter-cell interference from a distant cell is given in section III and from multiple cells in section IV. The final results are given in section V and some conclusions in section VI.

II. CAPACITY EXPRESSIONS FOR PERFECT POWER EQUALIZATION

Suppose a multi-access multi-antenna channel with N_u users in which all users have the same number of transmit antennae N_t and receive antennae N_r. Assuming single-path channels, the channel model reads

$$\mathbf{y} = \sum_{u=1}^{N_u} \mathbf{H}_u \mathbf{x}_u + \mathbf{z} \ . \tag{1}$$

With i.i.d Rayleigh fading the $N_r \times N_t$ fading matrices $\mathbf{H}_u$ have i.i.d. zero mean complex Gaussian entries h_{ij} with unit variance. We assume perfect power equalization: $E[\mathbf{x}_u] = P_R$, i.e. all users are received with the same average receive power. Power equalization is necessary to reduce the near-far effect of DSSS-CDMA, and to ensure wide area coverage—without equalization, all of the system capacity is consumed at the vicinity of the base station. Here we define the signal-to-noise ratio as SNR $= P_R/(N_0 W)$, where P_R is the received (equalized) power of the useful signal coming from one user, N_0 is two-sided noise power spectral density and W is the given bandwidth. The additive noise is treated as zero-mean complex Gaussian process with variance $N_0 W + \overline{I}_{out}$, where $\overline{I}_{out}$ is the average power of the inter-cell interference that is approximated to be Gaussain. If N is big enough, according to the central limit theorem, it is reasonable to assume that the interference coming from the other users in the cell is also zero-mean Gaussian, with covariance matrix $\mathbf{Q}_I$. Before, we also made a similar assumption for the out-of-cell interference. For a CDMA and TDMA system with a reuse factor of at least three, this approximation makes sense, since the central limit theorem may be applied without making a big error. In the TDMA case with reuse one, however, the out-of-cell interference is far from Gaussian, because of the much stronger contribution of the adjacent cells. To observe this we observe the relative entropy (Kullback-Leibler distance) between two probability density functions $p(x)$ and $q(x)$, defined as $D(p||q) = \int p(x) \log(p(x)/q(x))dx$. In Figure 1, we plot the relative entropy between the real probability density function (pdf) of the out-of-cell interference $p_{I_{out}}(x)$ and the Gaussian pdf $g(x)$, with the same mean and variance, as a function of the reuse factor for different β. We see that for reuse factor 1, the relative entropy distance is significantly bigger than for higher reuse factors.

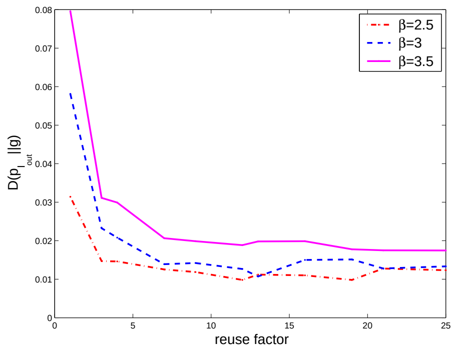

Fig. 1
Kullback-Leibler distance as a function of the reuse factor, for different β.

For simplicity we consider only the open loop case, i.e. with no side information at the transmitter which would allow instantaneous power control or water-filling between the parallel MIMO channels. Introducing $\overline{I} = \overline{I}_{in} + \overline{I}_{out}$ and the coefficient $I_q = \overline{I}/P_R$, we generalize the formulae given in [4] by taking into account the inter-cell interference for TDMA, and write the open loop system capacity in bps/Hz, for both CDMA and TDMA in a general form:

$$C = k_1 \sum_{i=1}^{\min(N_t,N_r)} E\left[\log_2\left(1 + \frac{\text{SNR}\cdot\lambda_i/N_t}{I_q\cdot\text{SNR} + k_2}\right)\right], \quad (2)$$

with λ_i being the squares of singular values of the fading matrix $\mathbf{H}$. This expression holds if the inter-cell interference can be considered as Gaussian. For TDMA this is true when the mobile stations are far enough from the central BS, i.e. with reuse factor larger than one, for CDMA the large number of interferers is enough to make the interference Gaussian for any reuse factor. Hence, the averaging over the interference in front of the logarithm can be moved inside the logarithm using Jensen's inequality. The pre-factor k_1 is related to spreading as discussed in [5], i.e. the proportion of the full frequency and time resources that are used within the cell. The interference factor I_q determines the asymptotic behaviour of the capacity

$$\lim_{\text{SNR}\to\infty} C = k_1 \sum_{i=1}^{\min(N_t,N_r)} E\left[\log_2\left(1 + \frac{\lambda_i}{I_q\cdot N_t}\right)\right]. \quad (3)$$

If I_q is non-zero the capacity is interference limited, otherwise the capacity grows without a limit as a function of SNR. The interference factor depends on the reuse factor q and the interference model used. Finally, k_2 is an inverse processing gain, related to SNR amplification if only part of the bandwidth is used.

For CDMA we have $k_1 = N_u/N_s$, $I_q = (1+f_q(\beta))(N_u-1)/N_s$ and $k_2 = 1$, where N_u is the number of users in a cell and N_s is the spreading factor. $f_q(\beta)$ denotes the ratio between inter-cell and in-cell interference and it depends on the path loss exponent β and the reuse factor q. Since for CDMA we consider only the case $q = 1$, the interference factor simplifies to $I_1 = (1+f_1(\beta))(N_u-1)/N_s$. Note that this expression is valid for CDMA when the number of users is fixed. In the following we will concentrate on the case $N_u = N_s$. Theoretically, as shown in [6], N_u can be higher than N_s, and for a single cell, SISO, random spreading CDMA $N_u \to \infty$ gives the highest throughput $(1/\ln 2)$ that depends neither on the SNR nor on the interference. For a multi-cell MIMO CDMA system, the highest throughput is

$$\lim_{N_u/N_s\to\infty} C = \frac{N_r}{\ln 2(1+f_1(\beta))}. \quad (4)$$

However, in practical systems the highest feasible N_u is $\le N_s$. For TDMA $k_1 = k_2 = 1/q$. The interference factor I_q will be discussed in section III. It will again depend on the path loss exponent β, for any q. If we want to be more precise, we have to add the contribution from the multipath into the interference factor, but this is out of the scope of this paper.

In our simulations we use the path loss exponent $\beta = 3.5$, which corresponds to the propagation in urban microcells. For macro-cells and other wide area networks β is generally between 2.5 and 3. Because the capacity is greatly affected by the path loss our results on spectral efficiency should therefore be quite conservative, reflecting better the achievable peak data rates in microcell environments. According to [7] we take $f_1(3.5) \approx 0.6$ to emulate hard hand-off between cells.

III. Calculating the inter-cell interference

The calculation for the intra-cell and inter-cell interferences for CDMA are well explained in [8], [7]. Here we do the same for the inter-cell interference for TDMA with reuse. For that purpose, we assume perfect power equalization and uniform distribution of users in the cells.

Calculating the interference from a collection of hexagons analytically is hard. To get insight into the interference coming from all cells in the reuse pattern it is good to have an analytical approximation. For that we use a simplified model. The shape of a cell at distance b is approximated by a part of a ring (annulus), as shown in Fig. 2. The dimensions are chosen so that the area of the equivalent cell ϑbd is the same as the area of a hexagonal cell with radius a, $3\sqrt{3}a^2/2$. This gives $\vartheta = 2\tan^{-1}(h/b)$, $h = a\sqrt{3}/2$ and $d = ah/(2\vartheta b)$ (Fig. 2). The interference at the reference base station at $(0,0)$, coming from a single mobile station i that 'belongs' to the cell at distance b, placed at distance r_i from the central line (not from the centre) in that cell is

$$I_{out}^{(i)}(b) = P_{t,i}(r_i)(b + r_i)^{-\beta}, \qquad (5)$$

where $r_i \in [-d/2, d/2]$. As there is a perfect power equalization between the users in the interfering cell we get $P_R = P_{t,i}(r_i)|r_i|^{-\beta}$, with P_R the received power from a mobile station within the interfering cell. Note that this

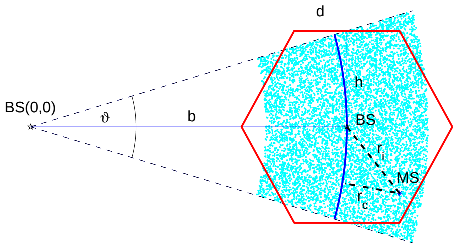

Fig. 2
The ring model.

model gives a lower bound for the inter-cell interference, since in average the user is always closer to the centre line of the ring, than to the centre of the cell.

From (5) we get for the inter-cell interference $I_{out}^{(i)}(b) = P_R\,|1 + b/r_i|^{-\beta}$. Therefore the average ratio of inter-cell interference and the received power is

$$m(b) = E[I_{out}(b)]/P_R \quad = \quad \int_{-d/2}^{d/2} \frac{1}{d}\left|1 + \frac{b}{r}\right|^{-\beta} dr. \qquad (6)$$

Averaging over the positions is a good approximation for reuse factors 3,4,7 and higher. For reuse factor 1, a Gaussian approximation of the inter-cell is unreliable, since in that case fluctuations of the interference coming from the adjacent cells are much bigger.

IV. Interference from multiple cells

Having derived the average interference from a cell at distance b, we compute the total interference from all cells. This depends on the reuse factor. We study hexagonal cells and it can be easily done using the hexagonal coordinate system [9], shown in Fig. 3. The distance between two

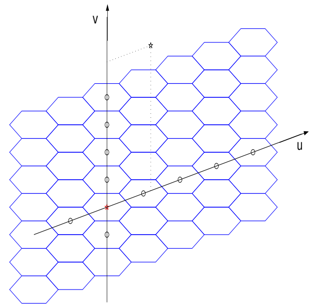

Fig. 3
The hexagonal coordinate system.

points (u_1, v_1) and (u_2, v_2) in this system is

$$2h\sqrt{(u_2 - u_1)^2 + (u_2 - u_1)(v_2 - v_1) + (v_2 - v_1)^2}, \qquad (7)$$

where $h = a\sqrt{3}/2$, and a is the radius of a cell.

The reuse factor $q = i^2 + ij + j^2$, is described by two non-negative integers i and j with $j \le i$, [7], [10]. Hexagonal coordinates of the positions of the base stations using the same frequency as the one at the origin are given by

$$\begin{aligned} k(t, n_k) &= (t \cdot i) \mod q + n_k \cdot q, \; n_k \in \mathbb{N} \\ \ell(t, n_\ell) &= (t \cdot j) \mod q + \frac{n_\ell \cdot q}{\gcd(i,j)}, \; n_l \in \mathbb{N}\,, \end{aligned} \qquad (8)$$

where $t \in \{0, \ldots, \frac{q}{\gcd(i,j)} - 1\}$ and gcd is the greatest common divisor. This parameterisation finds all the cells in the reuse pattern the centres of which belong to the sixth of the plane with polar angles $\in [\pi/6, \pi/2)$. The distances of the centres of these cells from the receiving base station at the origin are thus

$$\frac{b_{t,n_k,n_\ell}}{a\sqrt{3}} = \sqrt{k^2(t,n_k) + k(t,n_k)\ell(t,n_\ell) + \ell^2(t,n_\ell)}\,. \qquad (9)$$

To get this we use the rule explained in [10] for placing the cells in the pattern: start from BS(0,0) and go i cells to the right in the direction of the $u-$axes (which is $\pi/6$) and then j cells up. Repeat the same rule in all 6 directions and for all new cells that are already obtained by this procedure. With a little bit of combinatorics, we get (8).

Knowing the mean $m(b)$ (6) of the interference coming from one cell at distance b, we can compute the total mean interference factor as

$$I_q = 6 \sum_{t=0}^{\frac{q}{\gcd(i,j)}-1} \sum_{\substack{n_k=0 \\ (n_k+t\neq 0)}}^{\infty} \sum_{n_\ell=0}^{\infty} m(b_{t,n_k,n_\ell}) \qquad (10)$$

Here the full plane is recovered by multiplying with 6, and the restriction on the sum over n_k removes the cell at the origin. For practical computation upper limits in sums need not to be infinite. Satisfactory results are obtained for 3 to 4 cell sizes away from the referent BS. In our analysis we consider base stations with 3 sectors. That means the useful power is directed in one of the 3 directions rather than in all directions as is the case with omni-directional antennae. Moreover, the inter-cell interference is also smaller because it comes only from one sector. As a result the total capacity of the cell is increased. We assume perfectly sectored antennae so that there is no interference between sectors. Therefore, to get the interference per sector we divide (10) by 3. From our results we see that the total interference as a function of the reuse factor is reported ia a good approximation.

V. System Capacity Results

Multicellular MIMO system capacity results can be found in Figures 4 and 5. We assume system with $N_u = N_s = 16$. Inter-cell interference is taken into account for different reuse factors according to the analysis above, and fading matrix realizations are averaged over assuming i.i.d. Rayleigh statistics. From Figure 4 we see how both CDMA and TDMA capacities are interference limited. In [4], it was observed that there is always a crossover between the capacity of a CDMA system and that of a TDMA system with reuse and no inter-cell interference, the former giving more capacity at low SNR. This is due to the fact that the interference diminishing CDMA capacity becomes negligible at low SNR, whereas the reuse factor diminishing TDMA capacity does not depend on SNR. Here we observe that the crossover is persistent even when the inter-cell interference is taken into account for TDMA. Also, it should be noted that even with a 4×4 MIMO system, only a spectral efficiency of 7.5 bps/Hz/cell can be reached. Figure 5 shows the dramatic effect of intra-cell interference on TDMA capacity with reuse 3.

Above we have expressed the spectral efficiency as a function of SNR, i.e. the average power used per symbol. In Figures 4 and 5, spectral efficiency is plotted as a function of E_b/N_0. The transmitted energy per bit depends on the capacity, so that $(E_b/N_0)(C) = \mathrm{SNR}/C(\mathrm{SNR})$. Note that there is always a non-zero minimum E_b/N_0 below which no reliable communication is possible. It can be calculated explicitly as [11] $(E_b/N_0)_{\min} = 1/\dot{C}(0)$, where $\dot{C}(0)$ is the first derivative of $\dot{C}(\mathrm{SNR})$ in bps/Hz. Taking the derivative of (2) we get

$$\begin{aligned}\dot{C}(0) &= \frac{k_1}{k_2 N_t \ln 2} \cdot E[\sum_{i=1}^{\min(N_t,N_r)} \lambda_i] \\ &= \frac{k_1}{k_2 \ln 2} \cdot \frac{E[\mathrm{Tr}(\mathbf{H}\mathbf{H}^\dagger)]}{N_t}. \qquad (11)\end{aligned}$$

Since for both CDMA and TDMA $k_2/k_1 = 1$ we get $(E_b/N_0)_{\min} = \ln 2 \cdot N_t / E\left[\mathrm{Tr}(\mathbf{H}\mathbf{H}^\dagger)\right]$. Assuming $N_u = N_s$ we get for both CDMA and TDMA $k_3/k_1 = 1$. Furthermore,

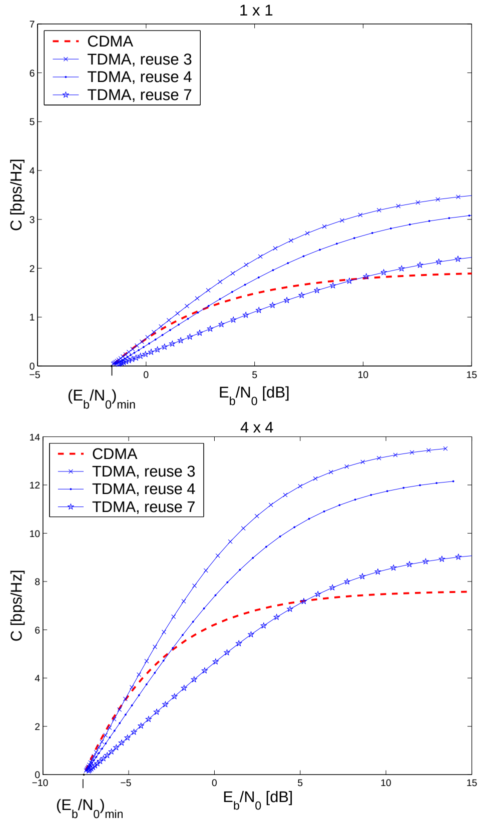

Fig. 4
Comparison between CDMA and TDMA with inter-cell interference and different reuse for a SISO (up) and 4 x 4 MIMO system (down).

if the fading matrix $\mathbf{H}$ has zero-mean entries with unit variance, $E\left[\mathrm{Tr}(\mathbf{H}\mathbf{H}^\dagger)\right] = N_t N_r$, leading to

$$\left(\frac{E_b}{N_0}\right)_{\min} = \frac{\ln 2}{N_r}. \qquad (12)$$

Thus there is a 3dB beamforming gain in $(E_b/N_0)_{\min}$ when doubling the number of receive antennae. This phenomenon is clearly visible in Figures 4 and 5.

It may be of interest to find the bandwidth slope, or the slope of the spectral efficiency at $(E_b/N_0)_{\min}$ in bps/Hz/3 dB, defined in [11] as $\mathcal{S}_0 = \ln 2 \cdot \frac{2(\dot{C}(0))^2}{-\ddot{C}(0)}$. Taking the second derivative of (2) and calculating the expectation values, and using the fact that the fading matrix $\mathbf{H}$ has zero-mean Gaussian entries with variance 1, we get

$$\mathcal{S}_0 = \frac{2 k_1 N_r}{2 I_q + \frac{N_t + N_r}{N_t}}, \qquad (13)$$

which generalizes the result $2N_t N_r/(N_t + N_r)$ of [11] for non- zero interference factor I_q and generic reuse factor q. The bandwidth slope describes the behaviour of the capacity at low E_b/N_0. However, for the TDMA case without

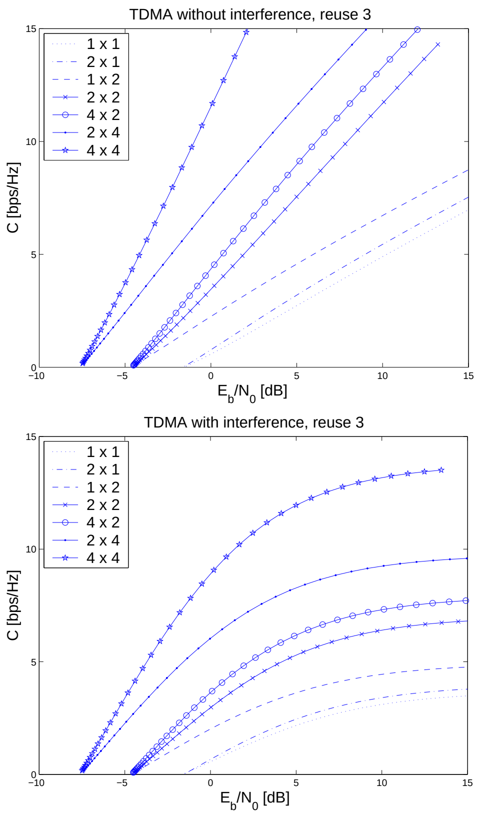

Fig. 5
Comparison of reuse 3 spectral efficiencies per cell for different TDMA MIMO systems without (up) and with (down) inter-cell interference taken into account.

interference it gives an approximate result even for high E_b/N_0: $C = \mathcal{S}_0 \left(E_b/N_0 - (E_b/N_0)_{\min}\right)$. It is easily seen form Figure. 5 that the slope at $(E_b/N_0)_{\min}$ is much smaller for the capacities with interference than for those without, exactly as depicted by (13).

VI. Conclusions

In this work we considered a model for the inter-cell interference in a cellular system with reuse. We found an analytical approximation for the interference from a single cell that doesn't differ much from numerical results and derived a general formula for the aggregate inter-cell interference of a hexagonal reuse pattern. The results show that the capacity of any practical system is interference limited already at rather modest spectral efficiencies, compared to the hypothetical efficiencies promised by single-cell MIMO analysis. The interference is higher in CDMA-based systems than in TDMA, and it is present even if successive cancellation is used. There is a crossover SNR (or E_b/N_0) value, beyond which a TDMA system has higher capacity than a DSSS-CDMA system.

Therefore, in low-SNR regime CDMA has to be used, and in high SNR regime, a regime in which we hope to target high spectral efficiencies, TDMA is recommendable. As we see, even in the TDMA case, in very high SNR regime, the gain in capacity is very small due to the fact that it is interference limited. In figures it can be seen that for high SNR, the weakest system at high SNR is CDMA with RAKE detection. For a fixed SNR as reuse increases, the TDMA spectral efficiency decreases.

When considering prospective 4G system with very wide band, it is inevitable that a multicarrier modulation should be used. The results discussed here extend to multicarriers directly, if spreading in the temporal direction is used. For spreading in the frequency direction, our results can be considered as indicative. The TDMA results can thus be generalized to any orthogonal multiple access strategy. In contrast, the CDMA results show the qualitative behaviour of a multiple access strategy with in-cell interference, such as multicarrier CDMA. In that case we have to model the frequency selectivity appropriately.

References

[1] N. Marina, "On successive decoding without channel state information," in *Proc. IEEE Int. Symp. Telecommunications*, Tehran, Iran, Sep. 2001.

[2] B. Rimoldi and R. Urbanke, A Rate-Splitting Approach to the Gaussian Multiple-Access Channel *IEEE Trans. on Information Theory, Vol. 42, No. 3, March 1996*

[3] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585-595, Nov/Dec 1999, based on AT&T Bell Laboratories, Internal Tech. Memo, June 1995.

[4] S. Gray, O. Tirkkonen, P. Pasanen, "Information theoretic approach to designing a new air interface for 4G," *Wireless Personal Communications*, Sep. 2003.

[5] J. Massey, "Information theory aspects of spread-spectrum communications," in *Proc. IEEE ISSSTA*, 1994.

[6] S. Shamai (Shitz) and S. Verdú, "Spectral efficiency of CDMA with random spreading" *IEEE Trans. on Information Theory*, Vol. 45, no. 2, March 1999.

[7] G. L. Stüber, *Principles of Mobile Communications.* Norwell, MA: Kluwer Academic Publishers, 1996.

[8] A. Viterbi, *CDMA: Principles of Spread Spectrum Communications*, Reading: Addison-Wesley, 1995.

[9] V. H. MacDonald, "The cellular concept," *Bell Syst. Tech. J.*, Vol. 58, January 1979.

[10] J. D. Parsons, *Mobile Radio Propagation Channel. 2nd edition* Chichester, UK: Wiley, 2001

[11] S. Verdú, "Spectral Efficiency in the wideband regime," *IEEE Trans. on Information Theory*, Vol. 48, No.6, June 2002.