COMPUTATIONALLY EFFICIENT QMF FILTER BANKS

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ABSTRACT

This paper introduces a computationally efficient technique for splitting a signal into N equally spaced sub-bands subsampled by 1/N and for near perfectly reconstructing the original signal from the sub-band signals. This technique is based on a multirate approach where some operations are nested to decrease the computation load. Simulation results show that the performances are comparable to that of conventional quadrature mirror filters, but with a very significant reduction in computational complexity.

INTRODUCTION

Conventional quadrature mirror filter (QMF) banks are now widely used, particularly for speech compression [1]. These banks divide the signal into N adjacent sub-bands subsampled by 1/N, and allow the reconstruction of the original signal from the N sub-band signals with negligible distorsion, regardless of the order of the band-pass filters.

In this paper, we expand on earlier work [2-3] to show that conventional QMF banks may be replaced by new filter banks which have nearly the same alias-free reconstruction properties, but with the band-pass filters derived by frequency shifting from a single prototype low-pass filter. We then use a multirate implementation [4] in combination with a nesting technique to develop an implementation which provides performances comparable to that of conventional QMF filter banks, but with a computational complexity which is significantly lower than with QMF banks or earlier pseudo-QMF banks.

PSEUDO QMF FILTER BANK

We consider a signal x(t) which is band-limited to a frequency $f_s/2$ and sampled at rate f_s, with $\omega_s = 2\pi f_s = 2\pi/T_s$. We want to split the sampled input signal x(n) into N sub-band signals $y_k(Nn)$ subsampled at rate f_s/N, by using N equally spaced adjacent band-pass filters with impulse responses $h_k(n)$, and with k=0,...,N-1. The signal splitting must be such that a near perfect approximation $\hat{x}(n)$ of the original signal x(n) may be reconstructed from the subsampled channel signals.

In the following, we derive the N band-pass filters $h_k(n)$ by frequency translation (fig. 1) from a prototype L-tap linear phase low-pass filter h(n) with cut-off frequency at $f_s/4N$, and with

$$h_k(n) = h(n)\cos[2\pi(2k+1)(2n-N)/8N] \quad (1)$$

The z-transforms of the band-pass filters are given as a function of the z-transform H(z) of h(n) by

$$H_k(z) = \frac{W^{N(2k+1)}}{2} H(W^{2(2k+1)}z) + \frac{W^{-N(2k+1)}}{2} H(W^{-2(2k+1)}z) \quad (2)$$

with

$$W = e^{-j2\pi/8N} \quad , \quad j = \sqrt{-1} \quad (3)$$

The subsampled channel signals, with z-transforms $Y_k(z)$ are derived from the input signal x(n) with z-transform X(z), by filtering with the filters $H_k(z)$, and decimating N-1 out of every N consecutive channel samples. Thus, $Y_k(z)$ is given [5] by

$$Y_k(z) = \frac{1}{N}\sum_{u=0}^{N-1} X(W^{8u}z^{1/N})H_k(W^{8u}z^{1/N}) \quad (4)$$

We show now that a near-perfect replica $\hat{x}(n)$ of x(n), with z-transform $\hat{X}(z)$ may be reconstructed from the N channel signals. This is done by inserting N-1 zero-valued samples between successive samples of the channel signals, by filtering the resulting sequences with the filters $h(n)\cos[2\pi(2k+1)(2n+N)/8N]$, and by summing the resulting signals. Hence

$$\hat{X}(z) = \frac{1}{4N}\sum_{k=0}^{N-1}\sum_{u=0}^{N-1} [[W^{N(2k+1)}H(W^{8u+4k+2}z) + W^{-N(2k+1)}H(W^{8u-4k-2}z)][W^{-N(2k+1)}H(W^{4k+2}z) + W^{N(2k+1)}H(W^{-4k-2}z)]]X(W^{8u}z) \quad (5)$$

The aliasing terms in the reconstructed signal are eliminated if the coefficients of $X(W^{8u}z)$ in (5)

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are zero, except for u=0. In our approach, we insure this condition by eliminating the aliasing due to adjacent filters and by designing the prototype low-pass filter H(z) in such a way that non adjacent band-pass filters do not overlap, with

$$H(W^a) = 0 \quad \text{for} \quad |a| > 4 \tag{6}$$

Thus, we have $H(W^a z)H(z) = 0$ for $|a| > 8$, and (6) reduces to

$$\hat{X}(z) = \frac{1}{4N}[G_0(z)X(z) + \sum_{u=1}^{N-1} G_u(z)X(W^{8u}z)] \quad , \tag{7}$$

with

$$G_0(z) = \sum_{k=0}^{N-1} (H^2(W^{4k+2}z) + H^2(W^{-4k-2}z)) \quad , \tag{8}$$

and, for $u \neq 0$,

$$G_u(z) = (-j)^{2N-2u}(-j+(-j)^{-1})[H(-W^{4u+2}z)H(-W^{4u-2}z)] + (-j)^{-2u}(j+j^{-1})[H(W^{4u+2}z)H(W^{4u-2}z)] = 0 \tag{9}$$

Hence the aliasing terms are eliminated and $\hat{X}(z)$ reduces to

$$\hat{X}(z) = G_0(z)X(z)/4N \tag{10}$$

Evaluating $G_0(z)$ on the unit circle yields

$$G_0(e^{j\omega T_s}) = e^{-j2\pi(L-1)\omega/\omega_s} \sum_{k=0}^{N-1} [e^{j\pi(L-1)(2k+1)/2N} H^2(\omega-(2k+1)\omega_s/4N) + e^{-j\pi(L-1)(2k+1)/2N} H^2(\omega+(2k+1)\omega_s/4N)] \tag{11}$$

Assuming the number of taps L of the prototype low-pass filter is odd, with L=2pN+1, the reconstructed signal is a perfect replica of the input signal within a multiplicative constant - 1/4N for p odd and + 1/4N for p even, when

$$\sum_{k=0}^{N-1} [H^2(\omega-(2k+1)\omega_s/4N) + H^2(\omega+(2k+1)\omega_s/4N)] = 1 \tag{12}$$

This condition is realized when $H(\omega) = 1$ in the pass-band, $H(\omega) = 0$ in the stopband and when the usual QMF condition is satisfied in the transition band with in particular a - 3dB response at cut-off frequency.

MULTIRATE IMPLEMENTATION

Since the analysis and reconstruction process are nearly identical, we restrict our discussion to the analysis filter bank. The subsampled channel signals are given by

$$y_k(Nm) = \sum_{n=0}^{L-1} h(n)\cos[2\pi(2k+1)(2n-N)/8N]x(Nm-n) \tag{13}$$

We compute separately the term $y_k^{(1)}(Nm)$ which corresponds to n = L-1 = 2pN and the term $y_k^{(2)}(Nm)$ which corresponds to n = 0,...L-2. In order to compute $y_k^{(2)}(Nm)$, we use the change of index [4]

$$n = 2Nn_1 + n_2 \quad , \quad n_1 = 0,\dots,p-1 \quad n_2 = 0,\dots,2N-1 \tag{14}$$

With (13), this yields

$$y_k^{(2)}(Nm) = \sum_{n_2=0}^{2N-1} a_{n_2}(Nm)\cos[2\pi(2k+1)(2n_2-N)/8N] \tag{15}$$

with

$$a_{n_2}(Nm) = \sum_{n_1=0}^{p-1} h(2Nn_1+n_2)(-1)^{n_1}x(Nm-2Nn_1-n_2) \tag{16}$$

The corrective term $y_k^{(1)}(Nm)$ is given by

$$y_k^{(1)}(Nm) = h(2pN)(-1)^p\cos[2\pi(2k+1)N/8N] \; x(Nm-2pN) \tag{17}$$

Hence, $y_k(Nm)$ becomes

$$y_k(Nm) = \sum_{n_2=0}^{2N-1} b_{n_2}(Nm)\cos[2\pi(2k+1)(2n_2-N)/8N] \tag{18}$$

with

$$b_0(Nm) = a_0(Nm) + h(2pN)(-1)^p x(Nm-2pN)$$

$$b_{n_2}(Nm) = a_{n_2}(Nm) \quad \text{for} \quad n_2 \neq 0 \tag{19}$$

This shows that the N length-L band-pass filters are replaced by 2N filters of length (L-1)/2N, plus the modified cosine transform (18), which may be computed by taking the real part of a DFT, with

$$y_k(Nm) = \mathrm{Re}[W^{-(2k+1)N} \sum_{n_2=0}^{2N-1} b_{n_2}(Nm)W^{2n_2}W^{4kn_2}] \tag{20}$$

We compute (20) with the auxiliary transform [6]

$$g_k(Nm) = \sum_{\substack{n_2=0 \\ n_2\neq N}}^{2N-1} \left(\frac{b_{n_2}(Nm)}{2\sqrt{2}\cos(2\pi n_2/4N)}\right) W^{4kn_2} \tag{21}$$

Finally, $y_k(Nm)$ is derived by

$$y_k(Nm) = \left(\frac{b_N(Nm)}{\sqrt{2}}\right)(\sqrt{2}\cos[(2k+1)\pi/4]) + \mathrm{Re}[\sqrt{2}\,W^{-(2k+1)N}(g_k(Nm)+g_{k+1}(Nm))] \tag{22}$$

The premultiplications of $b_{n_2}(Nm)$ by $1/2\sqrt{2}\cos(2\pi n_2/4N)$ and of $b_N(Nm)$ by $1/\sqrt{2}$ are nested with the filters by premultiplying the coefficients $h(2Nn_1+n_2)(-1)^{n_1}$ of the filters with these factors.

Hence these premultiplications disappear, and the computation of the N band-pass filters reduce to the evaluation of 2N filters of length (L-1)/2N, one DFT of length 2N, plus auxiliary additions and postmultiplications by $\sqrt{2}\cos[(2k+1)\pi/4] = \pm 1$ and $\sqrt{2}\, W^{-(2k+1)N} = \pm (1 \pm j)$. This shows that each set of band-pass filter output samples is computed with 4N auxiliary real additions, plus L real multiplications and L-2N real additions for the filters (including (19)), and the real 2N-point DFT (21). We note that the reduced filters are symmetric for K = 0 and K = N. Therefore, one can save p multiplies in the computation of $b_0(Nm)$ and $b_N(Nm)$.

Furthermore, for N even, the reduced filters for K = N/2 and K = 3N/2 have the same coefficients, and operate on the same set of input samples. Thus, another set of p multiplies can also be saved, and the total number of multiplies for the 2N filters reduces to L-2p for N even. Since the DFT operates on real input sequences, it can be computed either as an N-point complex DFT or as a 2N point DFT on two sets of real input sequences, with 2N auxiliary real additions [7]. In this last case, this yields a total of L-2p real multiplications, 4N+L real additions, and half the number of real operations required for a 2N-point complex DFT.

Applying this approach to a 8-band flat filter bank with a 8 KHz input sampling rate yields a total of 53000 real multiplications per second and 155000 additions per second by using the Rader-Brenner technique for the DFT, and a 49-tap prototype low-pass filter. With a 65-tap prototype low-pass filter, the computational load becomes 67000 real multiplications per second and 171000 real additions per second. This is about half the number of operations required with a conventional implementation of QMF filter banks.

RADIX-4 IMPLEMENTATION

When the proposed technique is implemented in a small signal processor, the use of a conventional FFT algorithm is sometimes undesirable. In this case, one can use the approach discussed above to design a 4-band QMF algorithm which can then be used repeatedly in a radix-4 tree structure similar to the radix-2 tree structure used with the conventional QMF. Each set of output samples of the 4-bank filters is then computed with (3L+9)/4 real multiplications and L+42 real additions, and the algorithm becomes sufficiently simple for easy implementation in a microcomputer with limited memory capacity. This yields a reduction of about 40% of the number of operations when compared to the conventional QMF with comparable filters.

CONCLUDING REMARKS

In this paper, we have considered a particular class of flat frequency response QMF filter banks where the various band-pass filters are derived by frequency shifting from a prototype low-pass filter. We have shown that this filter bank can be implemented very efficiently by using a multirate technique where auxiliary multiplications are nested with the polyphase filters. Simulation results show that the original signal is near perfectly reconstructed from the subsampled channel signals, with a frequency response which is flat within ± 0.2 dB and a rejection of the main aliasing term in excess of 40 dB for a prototype low-pass filters of 65 taps.

REFERENCES

[1] D. Esteban, C. Galand, "Application of Quadrature Mirror Filters to Split Band Voice Coding Schemes", Proc. 1977 Int'l IEEE Conf. on ASSP, pp 191-195.

[2] H.J. Nussbaumer, "Pseudo QMF Filter Bank", IBM Technical Disclosure Bulletin, Vol. 24, No 6, pp 3081-3087, Nov. 1981.

[3] J.H. Rothweiler, "Polyphase Quadrature Filters, A New Subband Coding Technique", Proc. 1983 Int'l IEEE Conf. on ASSP, pp 1280-1283.

[4] M.J. Narasimha, A.M. Peterson, "Design of a 24-Channel Transmultiplexer", IEEE Trans., Acoust., Speech, Signal Processing, Vol. ASSP-27, pp 752-762, Dec. 1979.

[5] R.W. Schafer, L.R. Rabiner, "A Digital Signal Processing Approach to Interpolation", Proc. IEEE, Vol. 61, pp 692-701, June 1973.

[6] C.M. Rader, N.M. Brenner, "A New Principle for Fast Fourier Transformation", IEEE Trans., Acoust., Speech, Signal Procession, Vol. ASSP-24, pp 264-265, 1976.

[7] H.J. Nussbaumer, Fast Fourier Transform and Convolution Algorithms, Springer-Verlag : Berlin, New-York, Heidelberg, 1982

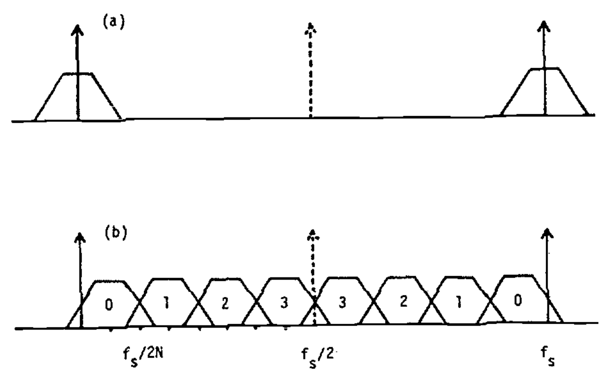

Figure 1 : Module of the transfer function
a) Prototype filter
b) Modulated filters

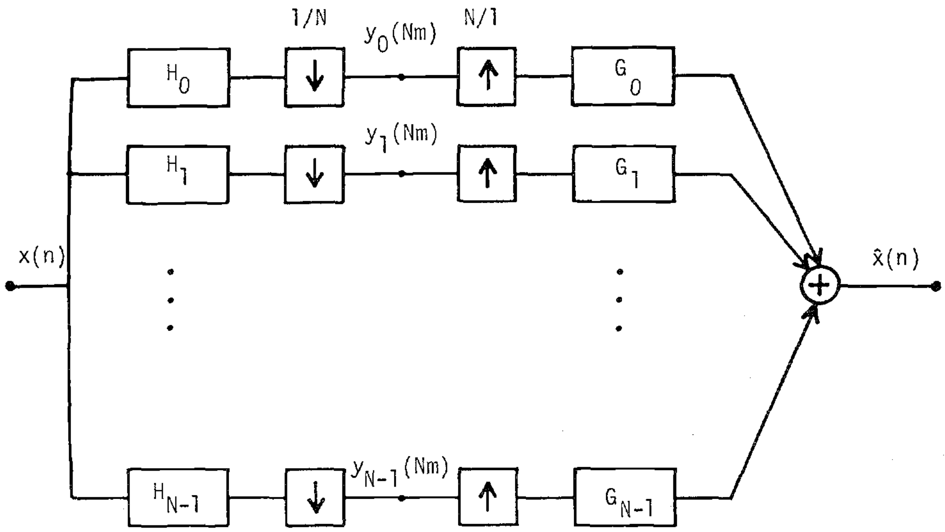

Figure 2 : Analysis and synthesis filter bank for subband coding

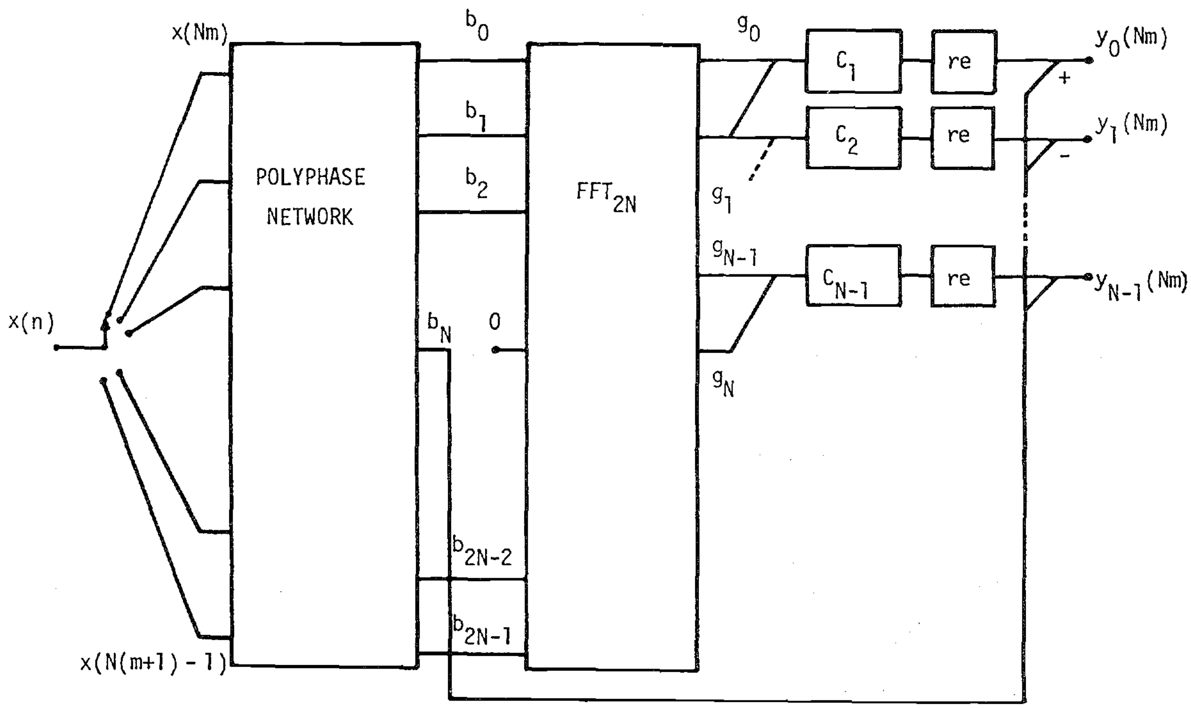

Figure 3 : Efficient implementation of the analysis filter bank with a polyphase network and an FFT