A STUDY OF CONVEX CODERS WITH AN APPLICATION TO IMAGE CODING

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ABSTRACT

The concept of convex coding systems is presented together with an application to image coding. A convex coder gives the exact description of a convex set or a group of convex sets in which the input signal is located. The reconstruction is left to the decoder whose role is to pick an estimate in the intersection of these sets. Alternating projections can be used to obtain such estimates which geometrically lie in the intersection of the sets. Convex coding enables us to supplement a criterion of "pleasingness" to the current MSE-oriented criterion. This may be useful for very high compression ratio. An example of convex coding which utilizes the conventional DCT coder as one set is demonstrated and the result shows that the blocking artifacts are efficiently removed.

1. INTRODUCTION

Encoding a signal consists in giving an approximate discrete description. In usual cases, the input signal is already discrete in time and the approximation is due to the quantization in amplitude. In more sophisticated encoding schemes, such as DCT coding, an invertible linear transformation is performed before quantization. However in any case, the encoded signal is interpreted itself as a signal "estimate" of the input signal, either directly or through some linear transformation.

In this paper, we introduce a different approach of coding, that we call **convex coding**. We ask the encoder, not to give the description of an approximate version of the input, but to give the description of a **convex set** which well localizes the input signal. The reconstruction of the input is then performed by the decoder whose role is to pick an estimate in the encoded set. As already hinted in [1], this is in fact what happens in traditional encoding schemes. Hence convex coding is a generalization of coding systems. The goal of convex coding is to design a set so that its elements are a good representation of the input, in the mean squared error (MSE) sense, or according to other characteristics including human perception criteria.

One main contribution of convex coding is the possibility to encode a set of signals gathering several characteristics of different nature. Indeed, the set can be defined as the intersection of several convex sets, each of which corresponds to a desired characteristic and can be defined by a separate encoder. An estimate can then be reconstructed by the decoder thanks to the algorithm of alternating projections onto convex sets (POCS) [2]. This technique has been especially developed in signal recovery and restoration [3, 4], and more recently used in image compression post-processing, especially to remove the blocking artifacts of DCT coding [5, 6]. In the case of post-processing, there is no guarantee that the input image exists in the intersection since one of the sets is only an estimated set. In convex coding systems, all of the sets contain the original signal by necessity, since they were indeed encoded from it.

In this paper, we first present the concept of convex coding. Then, as an illustration, we propose an application using two convex sets among which one is defined by the classical block DCT encoder. The role of the second set is to encode he information of smoothness of the original image across the DCT block boundaries.

2. FORMULATION OF CONVEX CODING

Let us consider an N-dimensional Euclidean space, in which distance is defined by MSE. Assume that an input signal f is a vector in this space, that is, $f \in \mathbf{R}^N$. Traditionally, an encoder describes the input signal with a series of quantization indices. These indices express a product space of quantization bins, which is a subset of $\mathbf{R}^N$. The encoding operation is therefore interpreted as the selection of the subset which contains the input signal. Then, an encoding operation $\mathcal{C}$ can be written as follows.

$$\mathcal{C} : f \longmapsto S_f \in \{ S \subset \mathbf{R}^N \mid f \in S \}$$

while a decoding operation $\mathcal{D}$ is written as follows.

$$\mathcal{D} : S_f \longrightarrow \hat{f} = \mathcal{G}(S_f)$$

where $\mathcal{G}(\cdot)$ denotes a centroid operation and $\hat{f}$ denotes a reproduced signal. Obviously the centroid is an optimal solution in a sense of MSE for uniform distribution. Alternatively, convex coding describes an input signal with a group of sets as follows.

$$\mathcal{C} : f \longmapsto \mathbf{S}_f = (S_{f1}, S_{f2}, \ldots, S_{fz})$$

$$\mathbf{S}_f \in \{ (S_1, S_2, \ldots, S_z), \ S_1, S_2, \ldots, S_z \subset \mathbf{R}^N \mid f \in S_i \}$$

where $1 \leq i \leq z$. This implies that $f \in \bigcap_{i=1}^{z} S_{fi}$. Each S_{fi} is a subset of $\mathbf{R}^N$, which does not necessarily impose a

constraint on every coordinate of f. A decoding operation $\mathcal{D}$ is expressed as follows.

$$\mathcal{D} : \mathbf{S}_f \longrightarrow \hat{f} \in \bigcap_{i=1}^{z} S_{fi}$$

As shown by the above equation, the decoded signal is not unique. Reconstruction can be any vector within $\bigcap_{i=1}^{z} S_{fi}$, which is a set of possible estimates. Hence convex coding is an application of set theoretic estimation [2].

When each set is convex, an element of the intersection can be found by alternating projections. The projections are, in general, iterated. Let $\hat{f}_1$ be an initial estimate for signal f. When the projections onto $S_1, S_2, \ldots, S_z$ are expressed by $P_1, P_2, \ldots, P_z$, we obtain the $(m+1)$-th estimate by

$$\hat{f}^{(m+1)} = P_1 P_2 \ldots P_z \hat{f}^{(m)}.$$

We can start with any (possible) estimate of the signal. Unless our initial reconstruction is already located in the intersection, we obtain a better reconstruction with each projection in the MSE sense. The necessary condition for the convergence is that the intersection is not empty, which should be always true in convex coding.

In the case of scalar quantization, the shape of the encoded set is a hypercube or hyperparallelepiped and the centroid is easy to obtain.

In a convex coding system, an encoder gives a group of sets. Every set may not be a complete set to reconstruct the signal but can be used as a certain constraint. Then we obtain our reconstructed signal in the intersection of the sets at the decoder side. Such intersection can have an arbitrary shape.

In the case of VQ (Vector Quantization), the subsets can be any Voronoi cells. For practical reasons, VQ is usually performed in sub-dimensional signal spaces, such as blocks. One possible "ultimate" coding system is an arbitrarily-dimensional VQ, which may not be implementable. However convex coding can be one approach to the solution, since it offers more flexibility to define arbitrarily shaped subsets of the signal space.

The group of sets may form an overcomplete basis for the space, which means that the coded data can be redundant. The relation between the rate and distortion of the convex coding system is geometrically interpreted as the relation between the price to describe the sets and the size or volume of the intersection. Obviously our target is to obtain the smaller intersection with the simplest description of the sets. Furthermore, we can use other measurement criteria than the MSE.

3. CONVEX CODING WITH A DCT CODER

One possibility of convex coding system is to define a set based on an existing coding scheme such as a standard coding scheme, and to add another set in order to compensate for some "weakness" of the existing scheme. We expect that such a convex set can provide compatibility by appropriate treatment. We propose one approach of convex coding which uses the conventional DCT coding scheme as one set and consider a way to solve a problem of blocking effects.

There are several studies on the reduction of blocking effects such as [7], where block overlapping and low pass filtering across the block boundaries are described; and [5, 6, 8, 9], where several post-processing methods are developed, based on the features of band limitation, bounded variation across the blocks, edge estimation and an image model, respectively. Here we use a convex coding approach. DCT followed by a uniform scalar quantizer amounts to encoding a hypercube in the signal space, since DCT is orthogonal and scalar quantization implies an independent treatment for each coordinate. Let us call this set the DCT set, which is defined by the quantization indices of DCT coefficients. The additional sets should provide some of the information which has been lost in the DCT set. Our motivation is to encode some missing information about the transition between the blocks. Since the DCT basis already defines a complete orthonormal basis of the signal space, any additional data can be redundant. Therefore we do not expect a significant improvement in the rate-distortion. Instead, our target here is to introduce another criterion which aims at reducing the blockiness.

First, we consider an expression of the transitions between the blocks. Assume that we have an original image f and we use $K \times K$ block DCT. Let

$$T_i = [\, t_{-n}, t_{-(n-1)}, \ldots, t_{-1}, t_0, t_1, t_2, \ldots, t_{n-1}\,]_i$$

be a row vector of the image pixels, whose elements $t_{-n}, t_{-(n-1)}, \ldots, t_{-1}$ belong to block b_j while $t_0, t_1, \ldots, t_{n-1}$ belong to block b_{j+1}. Block b_j and block b_{j+1} are adjacent on the image. Suffix i denotes the i-th row within each block b_j and b_{j+1}, that is, $1 \leq i \leq K$. We call this vector an i-th row transition vector between b_j and b_{j+1}. Then, a block boundary between b_j and b_{j+1} can be described by

$$\mathbf{T} = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_K \end{pmatrix}.$$

Next, we define a linear operator U as follows.

$$U = [\, u_{-n}, u_{-(n-1)}, \ldots, u_{-1}, u_0, u_1, u_2, \ldots, u_{n-1}\,]$$

Then an energy parameter B is expressed as

$$B = \left[\sum_{i=1}^{K} |\, U \cdot T_i^t \,|^2 \right]^{\frac{1}{2}}$$

where the superscript t means transposition. Assume that the parameter B for the original image f is given. For an estimate $\hat{f}$, let us call $\hat{T}_i$ the transition vectors. One can easily show that the set of all estimates $\hat{f}$ such that the corresponding transition vector satisfies the following inequality

$$\left[\sum_{i=1}^{K} |\, U \cdot \hat{T}_i^t \,|^2 \right]^{\frac{1}{2}} \leq B \qquad (1)$$

is a convex set which contains the original signal. Let us call this set the "smoothness set". The smoothness set depends on the choice of U. If we choose a Laplacian-like

operator, B represents the lack of smoothness around the boundary. A criterion of smoothness with the Laplacian operator has been utilized in the field of image recovery. In [6], $U = [\,1,-1\,]$ was used to remove the blockiness of DCT coded images. The set was defined by using the upper-bounded energy of the above operator's output. The projection onto this set was also described. The alternating projections were used in a context of post-processing and the upper-bound has not been encoded and therefore had to be guessed. Here we encode the information about the upper-bound as additional information block by block, with a generalized linear operator.

Let $\hat{f}$ be an estimate of f with energy parameter $\hat{B}$. Its projection onto the smoothness set is equal to $\hat{f}$ everywhere except at the pixels of its transition vectors where we have the following transformation

$$\hat{T}_i' = \hat{T}_i + \frac{1}{\|U\|^2}\left(\frac{B}{\hat{B}} - 1\right)\hat{T}_i \mathbf{Q}. \qquad (2)$$

where $\mathbf{Q} = U^t \cdot U$. Note that we simply keep $\hat{T}_i$ in the case where $\hat{B}$ is smaller or equal to B. The derivation of this transformation is omitted due to space limitations. We can derive the above discussion similarly for column vectors of pixels.

It is necessary to encode the energy parameter B of the original image for each block boundary. In this paper we quantize B after normalizing it with the energy parameter $\hat{B}$ obtained from a pure DCT decoded image. Obviously, output levels larger than 1.0 are useless. In this quantization, the output level is not the centroid of each quantization bin but the largest value so that the condition in Eq.(1) is still verified by the original image after quantization of B.

The number of the pixels which are modified by the projection onto the smoothness set depends on the length of U. However, when we project back the modified estimate onto the DCT set, all pixels within some blocks can be modified if the modified estimate is outside of the DCT set.

4. EXPERIMENTAL RESULTS

Experiments on deblocking were performed by computer simulation. We show an example on *Lenna* image (512×512) which was processed with the method. $U = [\,1,2,3,4,-4,-3,-2,-1\,]$ was adopted as our operator to define the smoothness set. $U = [\,1,-1\,]$ was also tested, which was proposed in [6], for comparison. Another set corresponding to the constraint of the pixel intensity range. This set has been used in the literature [4, 6]. We observed that a few iterations were enough for convergence and that the MSE decreased monotonically with the alternating projections as stated by the theory.

Figure 1 is the original *Lenna* (in part). Figure 2 is the image after block DCT encoding at the rate of 0.41 bit/pel and reconstruction according to the conventional method. In Figures 3 and 4, we work at the same bit rate, however 0.32 bit/pel is used for the DCT encoding and 0.9 bit/pel is used for encoding the smoothness set. The reconstructed images are obtained by a DCT reconstruction at first, followed by a single projection onto the smoothness set. The operators $U = [\,1,-1\,]$ and $U = [\,1,2,3,4,-4,-3,-2,-1\,]$

are respectively used in Figures 3 and 4. A comparison of Figures 3 and 4 shows the difference in the effect of additional sets. We see that smooth area which covers several blocks such as cheek is especially improved in Figure 4. Both Figures 2 and 4 are coded at the same rate including the bits for describing the additional set. These two pictures show that our convex coding approach gives a less-blocky reconstruction at the same rate.

5. CONCLUSION

We have presented a concept of convex coding. It is attractive since we can include a supplemental criterion to maintain "pleasingness" even at very low rate and since we have a freedom to pick an estimate as we like. In this sense, convex coding is related with a problem of image recovery and restoration. We also expect that it has a potential for a solution of extra-low rate compression with acceptable quality.

An example of convex coding has been proposed, which utilizes a DCT coding as one set. An additional set with a linear operator was introduced to remove the blocking artifacts. The proposed method is just one application of convex coding and needs further optimization. Other types of convex coding schemes are under study.

References

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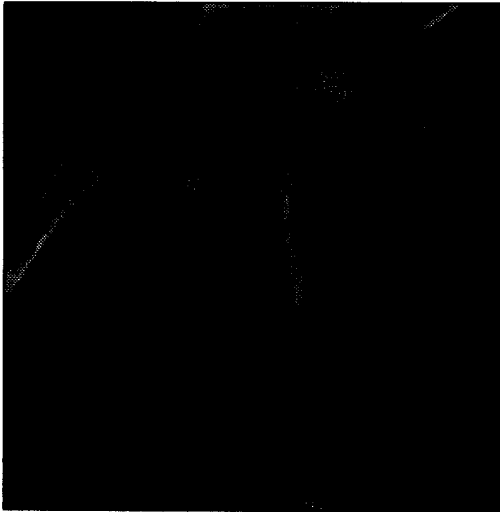[9] R.L.Stevenson, "Reduction of coding artifacts in transform image coding," Proc. of ICASSP, 1993.
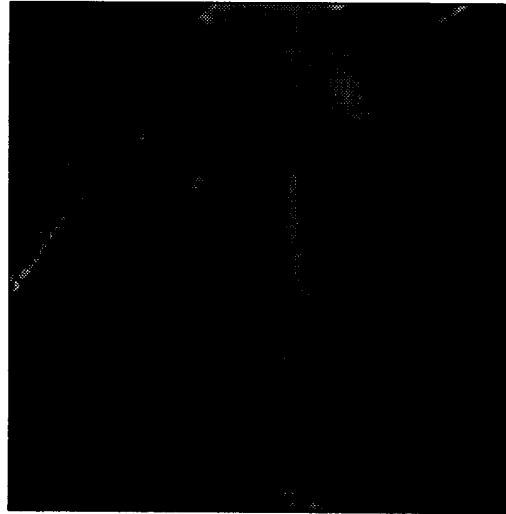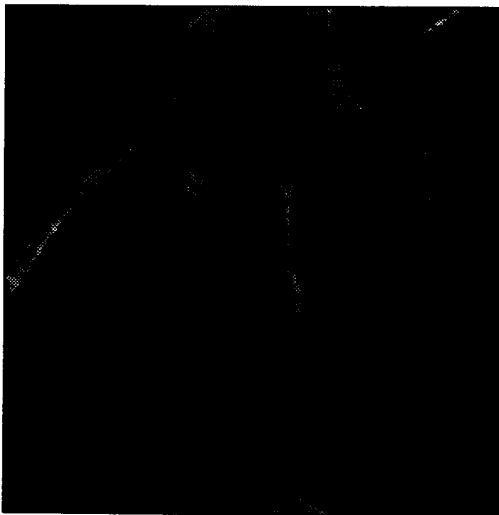
Figure 1: Original *Lenna* (in part).

Figure 2: DCT reconstructed image.

Figure 3: A convex coding approach: result of alternating projections using $U = [1, -1]$.
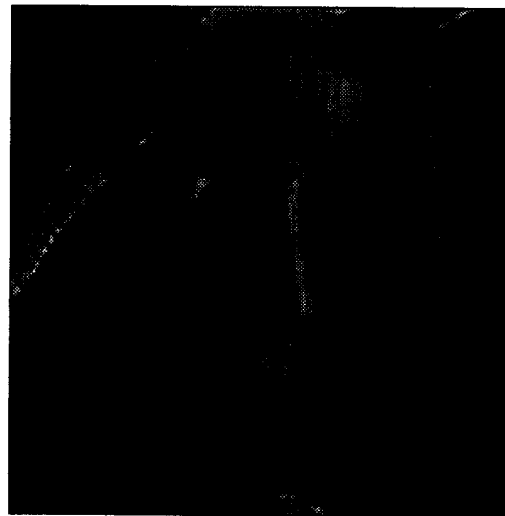

Figure 4: A convex coding approach: result of alternating projections using $U = [\, 1, 2, 3, 4, -4, -3, -2, -1 \,]$.