Spectral Sharpening and the Bradford Transform

Graham D. Finlayson (1), Sabine Süsstrunk (1, 2)

(1) School of Information Systems
The University of East Anglia
Norwich NR4 7TJ

(2) Communication Systems Department
Swiss Federal Institute of Technology (EPFL)
Lausanne, Switzerland

Abstract

The Bradford chromatic adaptation transform, empirically derived by Lam [1], models illumination change. Specifically, it provides a means of mapping XYZs under a reference source to XYZs for a target light such that the corresponding XYZs produce the same perceived colour.

One implication of the Bradford chromatic adaptation transform is that colour correction for illumination takes place not in cone space but rather in a 'narrowed' cone space. The Bradford sensors have their sensitivity more narrowly concentrated than the cones. However, Bradford sensors are not optimally narrow. Indeed, recent work has shown that it is possible to sharpen sensors to a much greater extent than Bradford [2].

The focus of this paper is comparing the perceptual error between actual appearance and predicted appearance of a colour under different illuminants, since it is perceptual error that the Bradford transform minimizes. Lam's original experiments are revisited and perceptual performance of the Bradford transform is compared with that of a new adaptation transform that is based on sharp sensors. Results were found to be similar for the two transforms. In terms of CIELAB error, Bradford performs slightly better. But in terms of the more accurate CIELAB 94 and CMC colour difference formulae, the sharp transform performs equally well: there is no statistically significant difference in performance.

1. Chromatic Adaptation

Adaptation can be considered as a dynamic mechanism of the human visual system to optimize the visual response to a particular viewing condition. Dark and light adaptation are the changes in visual sensitivity when the level of illumination is decreased or increased, respectively. Chromatic adaptation is the ability of the human visual system to discount the colour of the illumination to approximately preserve the appearance of an object. It can be explained as independent sensitivity regulation of the three cone responses. Chromatic adaptation can be observed by examining a white object under different types of illumination, such as daylight and incandescent. Daylight contains far more short-wavelength energy than incandescent, and is "bluer." However, the white object retains its white appearance under both light sources, as long as the viewer is adapted to the light source. [3]

Image capturing systems, such as scanners and digital cameras, do not have the ability to account for illumination, either the viewing illuminant or, for cameras, the capture illuminant. Scanners usually have fluorescent light sources with colour temperatures around 4200 to 4800 degrees Kelvin. Illuminants for digital cameras are less restricted and vary according to the scene, and often within the scene. Images captured with either of these devices are regarded under a wide variety of adapting light sources. Common white-point chromaticities for monitor viewing are D50, D65, and D93. Hardcopy output is usually evaluated under a standard

illuminant of D50 [4]. The profile connection space for colour image data interchange is defined as D50, resulting in each device profile having a chromatic adaptation transform that maps image colours from a source specific illuminant to a D50 illuminant [5]. To faithfully reproduce the appearance of spot and image colours, it follows that all image processing systems need to apply a chromatic adaptation transform that converts the input colours captured under the input illuminant to the corresponding output colours under the output illuminant.

Chromatic adaptation transforms are usually based on corresponding colour data. Specifically, they seek to best model how the same physical (surface) colour appears under two illuminants. The modeling is usually some mathematical minimization of error based on corresponding XYZ.

There are several chromatic adaptation transforms described in the literature, most based on the von Kries model. This model states that the trichromatic responses of corresponding surface measurements under two illuminants are simple scalings apart. For example, if (X, Y, Z) and (X', Y', Z') denote the XYZ tristimuli for an arbitrary surface viewed under two lights, then the von Kries model predicts that (X'=aX, Y'=bY, Z'=cZ). While this model trivially holds for one surface, the same three scalars (a, b, c) must map the XYZ tristimuli for all surfaces. This simple von Kries scaling model is implemented in CIELAB.

While almost all chromatic adaptation transforms adopt the von Kries scaling model in some form, they do so operating on different colour spaces. It is well known that the XYZ colour space is not perceptually relevant in the sense that it is not a colour space that the human visual system appears to use in carrying out colour computation. This said, it may not be surprising that von Kries operating on XYZ poorly describes corresponding colour data.

Based on a single set of colour matching data, the Bradford transform was derived to minimize CIELAB error. That is, the Bradford colour space (the colour coordinates where von Kries coefficients are applied) was found by numerical optimization. The subsequent Bradford transform works better than either von Kries XYZ or von Kries cone based adaptation [6].

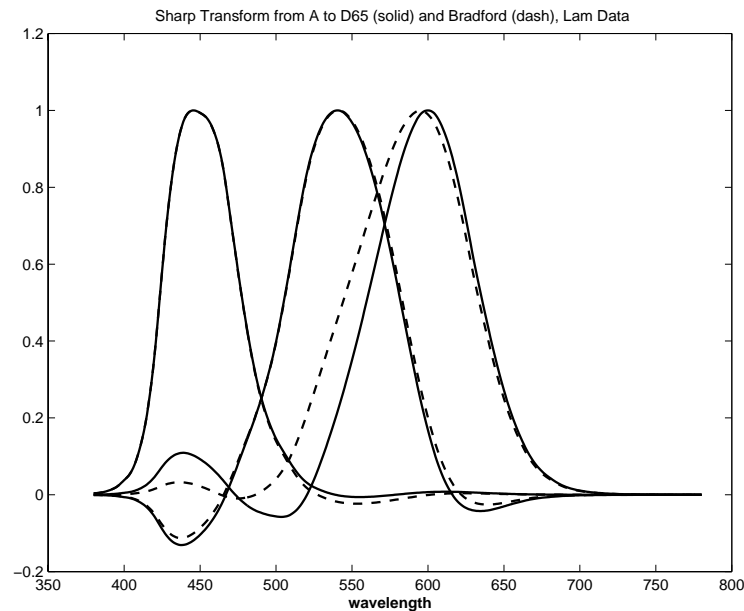Sharp Transform from A to D65 (solid) and Bradford (dash), Lam Data

Figure 1: Normalized white-point preserving sharp transform (solid) from A to D65, derived from Lam's experimental data, compared with the Bradford transform (dash).

Interestingly, the Bradford sensors have narrower support than the cones. In this sense, they might be reasonably described as being 'sharper.' However, they are not optimally sharp. Computational studies [2, 7, 8] have shown that sharper colour channels can be found (see Figure 1). In a mathematical (non-perceptual) sense, the sharper the colour channels, or sensors, the more appropriate the von Kries model becomes. In this paper we compare the perceptual performance of the Bradford and sharp adaptation transforms.

2. Lam's Experiment

In his experiment to derive a chromatic adaptation transform, Lam used 58 dyed wool samples. His main objective when choosing the colours was that the samples represent a reasonable gamut of chromaticities corresponding to ordinary collections of object colours (see Figure 2), and that the samples should have various degrees of colour constancy with regard to change of illuminant from D65 to A.

To evaluate the samples, Lam used a memory matching experiment, where observers are asked to describe the colour appearance of stimuli in relation with a memorized colour ordering system. Lam trained the observers on the Munsell system. Each observer was asked to describe the appearance of the samples in Munsell hue, chroma and value terms. The observers were fully adapted to the illuminant before they began the ordering. He used five observers with each observer repeating the experiment twice, resulting in ten colour descriptions for each surface and for each illuminant, respectively.

Lam converted the average Munsell coordinates of each sample under illuminant D65 and A to CIE 1931 Y, x and y values so that any colour difference formula can be applied to the data. He calculated tristimulus values using the 1931 CIE equivalents of Munsell samples under illuminant C [9]. He corrected for the illuminant change from C to D65 to calculate Munsell equivalent values under D65 by using the Helson et al. [10] chromatic adaptation transform. This correction assumes that the Munsell chips are virtually colour constant when changing illuminants from C to D65. It should be noted that he used the same illuminant to transform the Munsell coordinates of samples estimated under both D65 and A,

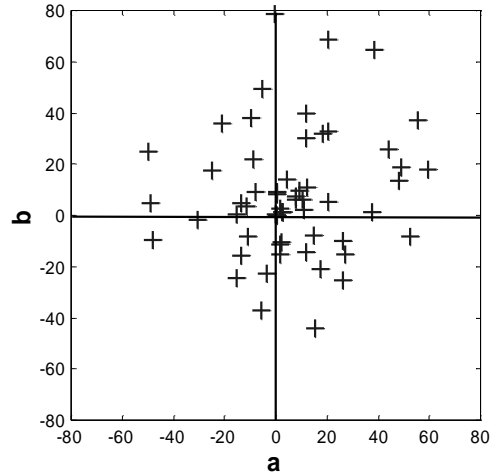justified as he trained the observers on the Munsell coordinate set using D65.

Figure 2: Distribution of Lam's 58 samples in CIELAB space, measured under D65

Lam observed systematic discrepancies between the measured sample values under D65 and those obtained from visual inspection under D65. Newhall et al. [11] found similar effects in their comparisons of successive (i.e. memory) matching with simultaneous colour matching experiments. To calculate the correct corresponding colours under illuminant D65, he therefore added the difference between the measured sample value and the observed sample value under D65 to each observed sample value using additive correction in CIELAB space.

Lam was now in a position to derive a chromatic adaptation transform, i.e. to find a mapping that related his corresponding colour data. In his derivation he adopted the following set of constraints: (1) the transform should maintain achromatic constancy for all neutral samples, (2) it should work with different adapting illuminants, and (3) it should be reversible (i.e. when a particular colour is transformed from A to D65, and back to A again, the tristimulus values before transformation and after transformation back to A should be the same). The Bradford chromatic adaptation transform, called KING1 in his thesis, is based on a modified Nayatani transformation [12] and is as follows:

Step 1: Transformation from X, Y, Z to R, G, B.

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \mathbf{M}_{\text{BFD}} * \begin{bmatrix} X/Y \\ Y/Y \\ Z/Y \end{bmatrix} \qquad (1)$$

where

$$\mathbf{M}_{\text{BFD}} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix}$$

Step 2: Transformation from R, G, B to R', G', B'.

$$R' = R'_w (R/R_w)$$
$$G' = G'_w (G/G_w) \qquad (2)$$
$$B' = B'_w (B/B_w)^p$$

where $p = (B_w / B'_w)^{0.0834}$

Quantities R_w, G_w, B_w and R'_w, G'_w, B'_w are computed from the tristimulus values of the reference and test illuminants, respectively, through equation (1).

Step 3: Transformation from R', G', B' to X', Y', Z'.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{\text{BFD}} \end{bmatrix}^{-1} * \begin{bmatrix} R'Y \\ G'Y \\ B'Y \end{bmatrix} \qquad (3)$$

The RMS CIELAB ΔE error predicting corresponding colours using the Bradford chromatic adaptation transform for Lam's data set is 4.7.

Lam, using his data set, empirically developed a second chromatic adaptation transform, which he called KING2. While the RMS CIELAB error ΔE equal 4.0 of that transform is lower than that of Bradford (KING1), KING2 only works for transformations from illuminant A to D65 and vice versa.

3. Linearized Bradford Transform

In some colour management applications, the non-linear correction in the blue of the Bradford transform is considered negligible and is not encoded [13]. The linear Bradford transform is simplified to:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = [\mathbf{M}_{\text{BFD}}]^{-1} * \mathcal{D} * [\mathbf{M}_{\text{BFD}}] * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad (4)$$

where

$$\mathcal{D} = \begin{bmatrix} R'_w/R_w & 0 & 0 \\ 0 & G'_w/G_w & 0 \\ 0 & 0 & B'_w/B_w \end{bmatrix}$$

Quantities R_w, G_w, B_w and R'_w, G'_w, B'_w are computed from the tristimulus values of the reference and test illuminants by multiplying the corresponding XYZ vectors by $\mathbf{M}_{\text{BFD}}$.

4. Spectral Sharpening

One implication of the Bradford chromatic adaptation transform is that colour correction for illumination takes place not in cone space but rather in a 'narrowed' cone space. The Bradford sensors (the linear combination of XYZs defined in the Bradford transform) have their sensitivity more narrowly concentrated than the cones. Additionally, the long and medium Bradford sensitivities are more de-correlated compared with the cones. However, Bradford sensors are not optimally narrow (see Figure 1). Indeed, recent work has shown that it is possible to sharpen sensors to a much greater extent than Bradford [2]. It has been shown that these 'sharp' sensors are the most appropriate basis for the modeling/computing adaptation of physical quantities (raw XYZs) across illuminants, i.e. for solving the non-perceptual adaptation problems, (treating XYZs as the important units).

Though perceptual data was not used to derive spectrally sharpened sensors, spectral sharpening does appear to be psychophysically relevant. Indeed, sharp sensors have been discovered in many different psychophysical

studies. Foster [14] observed that when field spectral sensitivities of the red and green response of the human eye are determined in the presence of a small background field, the resulting curves are more narrow and de-correlated than the regular cone responses. These sharpened curves tend to peak at wavelengths of 530 nm and 605 nm, respectively.

Poirson and Wandell [15] studied the colour discrimination ability of the visual system when targets are only presented briefly in a complex display. The spectral sensitivities derived from their experimental data peak relatively sharply around 530 and 610 nm.

Thornton [16] derived that the visual response consists of sharp sensors with peak wavelength around 448, 537, and 612 nm by comparing the intersections of the spectral power distributions of matching light sources. He found that light sources designed with these peak wavelengths minimize metamerism.

Brill et al. [7] discussed prime-colour wavelengths of 450, 540, and 605 nm. They proved that monitor primaries based on these wavelengths induce the largest gamut size, and that these monitors are visually very efficient. The colour matching functions derived from these primaries, when linearly related to the CIE 1931 2º colour matching curves, are sharp and de-correlated.

5. The Sharp Adaptation Transform

The sharp adaptation transform used for this experiment is derived from the spectral sharpening algorithms described by Finlayson et al. [2]. The performance of diagonal-matrix transformations that are used in many colour constancy algorithms can be improved if the two data sets are first transformed by a sharpening transform $\mathbf{T}$.

Using Lam's experiment, the prediction of the corresponding colours under D65 should be approximately equal

$$\mathbf{ST} \approx \mathbf{PT}\mathcal{D} \qquad (5)$$

where $\mathbf{S}$ is a 58 x 3 matrix of corresponding colour XYZs under illuminant D65, $\mathbf{P}$ is a 58 x 3 matrix of the measured XYZs under illuminant A and $\mathcal{D}$ is the diagonal matrix formed from the

ratios of the two sharpened white-point vectors RGB_{D65} and RGB_A, derived by multiplying vectors XYZ_{D65} and XYZ_A with $\mathbf{T}$.

The matrix $\mathbf{T}$ is derived from the matrix $\mathbf{M}$ that best maps $\mathbf{P}$ to $\mathbf{S}$ minimizing least-squares error [17].

$$\mathbf{M} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{S} \qquad (6)$$

However, while $\mathbf{M}$ calculated using equation (6) results in the smallest mapping error, it will not fulfill the requirement that particular colours are mapped without error, i.e. preserving achromaticity for neutral colours. Therefore, $\mathbf{M}$ was derived using a white point preserving least-squares regression algorithm [8]. The intent is to map the values in $\mathbf{P}$ to corresponding values in $\mathbf{S}$ so that the RMS error is minimized subject to the constraint that, as an artifact of the minimization, the achromatic scale is correctly mapped. For completeness, the mathematical steps that achieve this are summarized below to allow the interested reader to implement the method. However, it is possible just to assume that such a transform exists and skip over the next two equations.

In order to preserve white:

$$\mathbf{M} = \mathbf{D} + (\mathbf{ZN}) \qquad (7)$$

where $\mathbf{D}$ is the diagonal matrix formed from the ratios of the two white point vectors XYZ_{D65} and XYZ_A respectively. $\mathbf{Z}$ is a 3 x 2 matrix composed of any two vectors orthogonal to the XYZ_A vector. $\mathbf{N}$ is obtained by substituting $\mathbf{Z}$, $\mathbf{N}$ and $\mathbf{D}$ in equation (6) and solving for $\mathbf{N}$.

$$\mathbf{N} = [\mathbf{Z}^T\mathbf{P}^T\mathbf{PZ}]^{-1}[\mathbf{Z}^T\mathbf{P}^T\mathbf{S} - \mathbf{Z}^T\mathbf{P}^T\mathbf{PD}] \qquad (8)$$

The sharpening transform $\mathbf{T}$ can be derived through eigenvector decomposition of the general transform $\mathbf{M}$.

$$\mathbf{M} = \mathbf{UD}_{iagonal}[\mathbf{U}]^{-1} \qquad (9)$$

where $\mathbf{T}$ is equal to $\mathbf{U}$.

It is important to note and easy to prove that the least squares fit between **ST** and **PT** is exactly the diagonal matrix $\mathbf{D}_{iagonal}$ [2].

The predicted corresponding colours under illuminant D65 of Lam's 58 samples, using the sharp transform, are calculated as follows:

$$\mathbf{S} \approx \mathbf{PT}\mathcal{D}[\mathbf{T}]^{-1} \qquad (10)$$

6. Comparison of the Bradford and Sharp Transforms

Applying the resulting sharp transform, derived via data-based sharpening of the corresponding colours of the 58 Lam samples under illuminants A and D65 minimizes the RMS error between corresponding XYZs. It also yields sensors that are visibly sharper than those implied by the Bradford transform (see Figure 1). However, what we are most interested in is to compare the *perceptual error* between actual appearance and predicted appearance of a colour under the different illuminants using both the Bradford and the sharp transform.

The actual and predicted XYZ values were converted to CIELAB space. Three perceptual error prediction methods, ΔE_{Lab}, ΔE_{CIE94}, and CMC(1:1) were applied. Table 1 lists the RMS, mean, minimum and maximum errors for the predictions by the Bradford transform, the linearized Bradford transform, and the sharp transform.

It seems that the Bradford transform performs best, independently of the error metric applied. However, are these small differences actually statistically significant?

A student-t test [18] for matched pairs was used to compare the Bradford, the linearized Bradford and the sharp data sets. The null hypothesis is that the mean of the difference between the Bradford and sharp or linearized Bradford prediction error is equal to zero. The alternative hypothesis is that the mean is not equal to zero, and either one or the other prediction is better. The results are listed in Table 2.

As can be seen from the probability (*p*) values in Table 2, the Bradford transform does perform slightly better than the sharp transform when the colour error metric applied is ΔE. However, the *p* value for both ΔE_{CIE94} and CMC(1:1) is relatively high, meaning that there

is *no statistically significant difference* in using either the Bradford transform or the sharp transform to predict corresponding colours under illuminant D65 for Lam's data set. The *p* values for CIE 94 and CMC colour difference formulae indicate that the performance difference between the Bradford and Sharp adaptation transforms is not significant at the 95% or 99% level.

Table 1: Perceptual errors of predicted colour appearance by the Bradford transform, the linarized Bradford transform, and the sharp transform compared to the actual colour appearance for Lam's data set.

	RMS $\Delta\mathbf{E_{Lab}}$	mean $\Delta\mathbf{E_{Lab}}$	Min $\Delta\mathbf{E_{Lab}}$	Max $\Delta\mathbf{E_{Lab}}$
BFD	**4.73**	4.15	0.43	10.00
Sharp	**5.08**	4.45	0.43	11.67
BFD$_{lin}$	**5.25**	4.44	0.42	11.00

	RMS $\Delta\mathbf{E_{CIE94}}$	mean ΔE_{CIE94}	min ΔE_{CIE94}	max ΔE_{CIE94}
BFD	**3.31**	2.87	0.43	8.46
Sharp	**3.40**	2.93	0.43	8.44
BFD$_{lin}$	**3.54**	3.00	0.41	8.44

	RMS CMC (1:1)	mean CMC (1:1)	min CMC (1:1)	max CMC (1:1)
BFD	**4.09**	3.45	0.58	10.74
Sharp	**4.19**	3.54	0.55	10.68
BFD$_{lin}$	**4.30**	3.57	0.48	10.70

Table 2: Results of the student-t test comparing the significance of the perceptual errors of the Bradford, sharp, and linearized Bradford chromatic adaptation transforms using Lam's data sets.

BFD and Sharp	*t(57)*	*P<=*
ΔE_{Lab}	2.412	0.019
ΔE_{CIE94}	1.011	0.317
CMC(1:1)	1.249	0.217

BFD and BFD$_{lin}$	*t(57)*	*P<=*
ΔE_{Lab}	1.517	0.135
ΔE_{CIE94}	1.462	0.149
CMC(1:1)	0.953	0.345

Comparing the Bradford and the linearized Bradford transform, the null hypothesis cannot be negated for all three colour difference metrics. There is no statistically significant difference using the Bradford transform or the linearized Bradford transform to predict corresponding colours under illuminant D65 for Lam's data set.

7. Conclusions

These results are very interesting. It is well established that ΔE_{CIE94} and CMC are more accurate colour difference formula than CIELAB ΔE. Lam used CIELAB not because he thought it was the most appropriate, but that it was the only standard formula available. He was well aware of its deficiencies and documents these in his thesis. That the Bradford transform is currently predicated using an outdated colour difference metric is unsatisfactory.

More broadly, we believe, the experimental results reported here are significant for a number of other reasons. First, the chromatic adaptation transform in CIECAM97 is based on the Bradford transform. Second, the Bradford transform is being proposed for standardization (CIECAT), yet the performance of the sharp adaptation transform has never been compared. Perhaps one can do better than Bradford? Third, sharp sensors have been discovered in many different psychophysical studies so it seems entirely plausible that sharp sensors are used in colour vision. Yet, to the authors' knowledge, the appearance of the Bradford sensors is unique to Lam's original study. Sharp sensors also have the advantage that they are close to sRGB [19] colour matching curves. So basing adaptation on sharp sensors meshes well with standard colour correction methods used in digital colour cameras.

This is not to say that Bradford may not turn out to be the best transform to use overall but rather that insufficient tests have been carried out to validate its adoption. In any case, the standardization of the Bradford transform is probably premature.

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