

Correlation-based Controller Tuning

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In honor of Duncan A. Mellichamp, teacher and humanist

Outline

- Use of data for controller tuning
- Controller tuning based on the correlation approach
- Experimental illustrations
- Conclusions

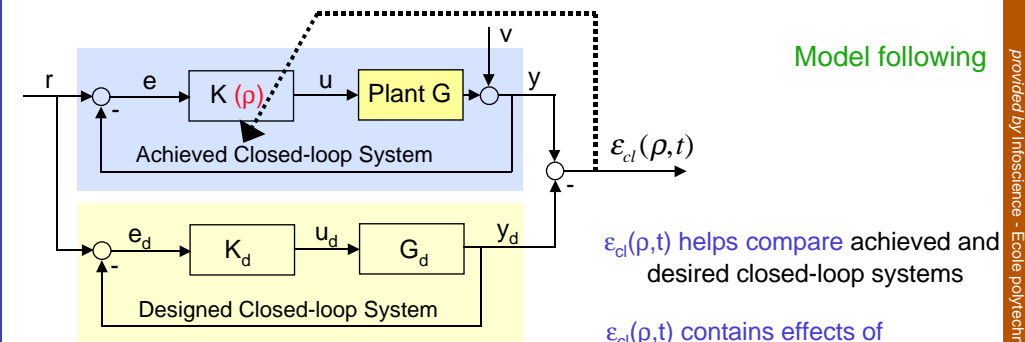
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Use of Data for Controller Tuning

- Indirect model-based approaches
 - Model identification
 - Off-line (e.g. step response)
 - On-line, fast update (indirect adaptive control)
 - Repeated, slower update (identification for control)
 - Controller design
 - Key issue: model validity
- Direct data-driven approaches

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Direct Data-driven Controller Tuning Framework of closed-loop output error (CLOE)



Design objective $\min_{\rho} J = \|\epsilon_{cl}(\rho, t)\|$

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Direct Controller Tuning

Features

- Iterative update

$$\rho_{i+1} = \rho_i - \gamma_i Q^{-1}(\rho_i) J'(\rho_i) \quad \text{Robbins-Monro}$$

ρ : controller parameters

γ_i : step size

$Q(\rho_i)$: positive definite matrix

$J'(\rho_i)$: gradient of criterion

- Difficulties

- Gradient depends on unknown CL plant → gradient estimation
- Presence of noise $v(t)$

- Two data-driven approaches with slow update

- IFT → gradient from closed-loop data (Hjalmarsson et al., 1994)
- CbT → no gradient needed (Karimi et al., 2003)

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Iterative Feedback Tuning

Two Experiments

Evaluation of criterion

$$J(\rho) = E[\varepsilon_{cl}^2(\rho, t)]$$

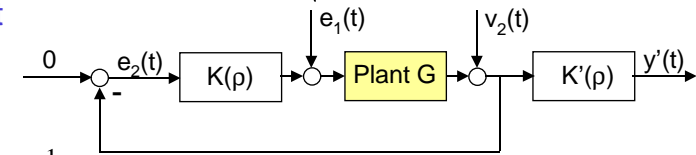
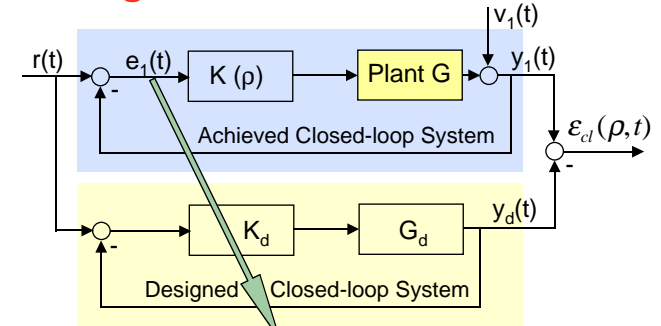
Over N samples:

$$J(\rho) = \frac{1}{N} \sum_{t=1}^N \varepsilon_{cl}^2(\rho, t)$$

Estimation of gradient

$$J'(\rho) = E[\varepsilon_{cl}(\rho, t) y'(\rho, t)]$$

$$y'(\rho, t) = G S(\rho) K'(\rho) e \quad S = \frac{1}{1 + KG}$$



H. Hjalmarsson, Iterative Feedback Tuning: An Overview, Int. J. Adapt. Control Signal Process., pp 373-95 (2002)

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Iterative Feedback Tuning

Properties

- Unbiased model-free estimation of gradient if

- Zero-mean disturbances
- Disturbances in Experiments 1 & 2 are uncorrelated → convergence to (local) minimum

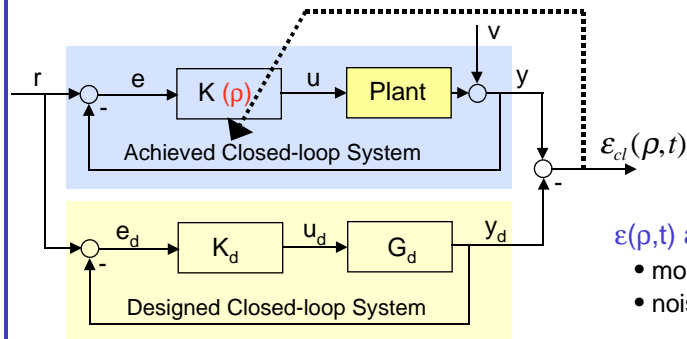
- Features

- Precise local information → bias-free gradient estimation
- Only local information → only gradual changes possible → slow
- Good control → error e small or not sufficiently rich → poor gradient estimation (dual control problem)

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Correlation-based Tuning

Basic Idea



$\varepsilon(\rho, t)$ affected by

- model mismatch, correlated with $r(t)$
- noise, uncorrelated with $r(t)$

Objective: Determine $K(\rho)$ such that $\varepsilon_{cl}(\rho, t)$ is uncorrelated with $r(t)$

→ controller compensates the effect of model mismatch

A. Karimi, L. Miskovic and D. Bonvin, Iterative Correlation-based Controller Tuning with Application to a Magnetic Suspension System, Control Engineering Practice (2003)

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Correlation-based Tuning

Correlation equations

$$f(\rho) = E[\zeta(t)\epsilon_{cl}(\rho, t)] = 0$$

$\zeta(t)$: n_p -dim. vector of instrumental variables

- correlated with $r(t)$, for example, $r(t-n_z) \dots r(t) \dots r(t+n_z)$
- independent of noise $v(t)$

Iterative solution

$$\rho_{i+1} = \rho_i - \gamma_i Q^{-1}(\rho_i) f(\rho_i) \quad \text{Gauss-Newton}$$

γ_i : step size $Q(\rho_i)$: positive definite matrix

$Q(\rho_i) = I$ substitution method

Solution of Correlation Equations

Newton-Raphson Algorithm: $Q(\rho_i) = \left. \frac{\partial f}{\partial \rho} \right|_{\rho_i} = E[\zeta(t)\psi^T(\rho_i, t)]$

$$\psi^T(\rho, t) \equiv \frac{\partial \epsilon_{cl}(\rho, t)}{\partial \rho} \quad \zeta(t) = \hat{\psi}^T(\rho, t) \quad \text{makes } Q \text{ positive definite}$$

Features

- Convergence is not affected by noise
- No need for gradient with the substitution method
- No need for second experiment
- **Existence of a solution?** Perfect decorrelation might require a high-order or non-causal controller → Minimize correlation function

Frequency-domain Interpretation

For $n_z \rightarrow \infty$ and using Parseval theorem

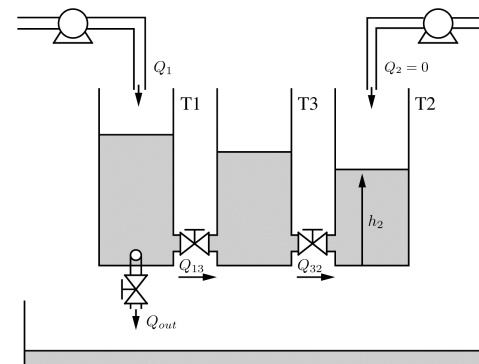
$$\rho^* = \arg \min_{\rho} \int_{-\pi}^{\pi} |T(e^{-j\omega}, \rho) - T_d(e^{-j\omega})|^2 \Phi_r(\omega) d\omega$$

- Difference between achieved and desired closed-loop is minimized
- Noise has no effect on the criterion (Φ_r is the spectrum of $r(t)$)

For IFT (minimization of 2-norm of ϵ_{cl})

$$\rho^* = \arg \min_{\rho} \int_{-\pi}^{\pi} \left[|T(e^{-j\omega}, \rho) - T_d(e^{-j\omega})|^2 \Phi_r(\omega) + |S(e^{-j\omega}, \rho)|^2 \Phi_v(\omega) \right] d\omega$$

Three-tank System Experimental Setup



Manipulated input: Q_1

Measured output: h_2

Mathematical Model

$$A \frac{dh_1}{dt} = Q_1 - Q_{13} - Q_{out}$$

$$A \frac{dh_2}{dt} = Q_{13} - Q_{32}$$

$$A \frac{dh_3}{dt} = Q_2 + Q_{32}$$

$$Q_{13} = a_{13} S \operatorname{sgn}(h_1 - h_3) \sqrt{2g|h_1 - h_3|}$$

$$Q_{32} = a_{32} S \operatorname{sgn}(h_3 - h_2) \sqrt{2g|h_3 - h_2|}$$

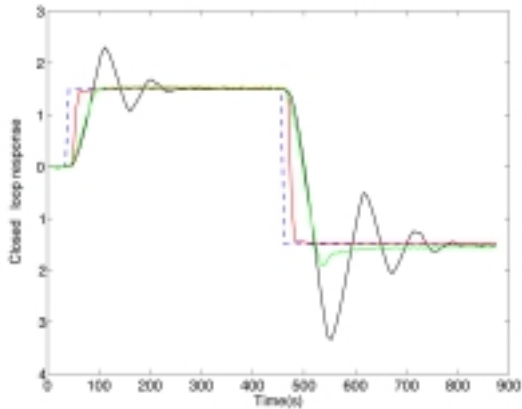
$$Q_{out} = a_{out}(h_1) S \sqrt{2gh_1}$$

A - section of cylinder [m^2]

S - section of connecting pipe [m^2]

a - outflow coefficient (dimensionless)

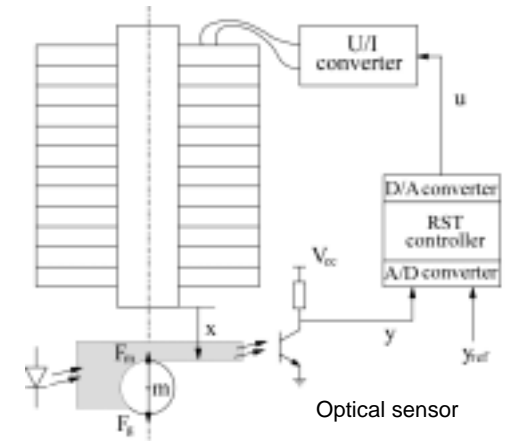
Three-tank System Simulated Closed-loop Response



- ◊ Identification of a linear discrete model
 - ◊ Initial PID-like controller of the form

$$K(q^{-1}) = \frac{s_0 + s_1q^{-1} + s_2q^{-2}}{(1 - q^{-1})(1 + r_1q^{-1})}$$
 designed using pole placement
 - ◊ Sampling period $T_s = 7s$
- Reference signal
— Designed response (linear model)
— Initial CL response (model based)
— CbT after 2 iterations (data based)

Magnetic Suspension System Experimental Setup



Nonlinear, unstable system

Magnetic Suspension System Approximate Model and Initial Controller

- Linearized continuous model with U/I-converter dynamics

$$G(s) = \frac{0.1}{0.017s + 1} \frac{15750}{s^2 - 1238}$$

- Discrete-time model $T_s = 10 \text{ ms}$

$$G_0(q^{-1}) = \frac{10^{-4}(137q^{-1} + 481q^{-2} + 103q^{-3})}{1 - 2.69q^{-1} + 2.19q^{-2} - 0.56q^{-3}}$$

- Initial RST controller

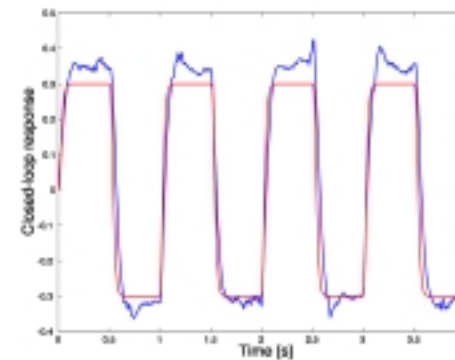
$$R_0(q^{-1}) = 1 + 0.686q^{-1} + .163q^{-2}$$

$$S_0(q^{-1}) = 21.86 - 26.77q^{-1} + 8.15q^{-2}$$

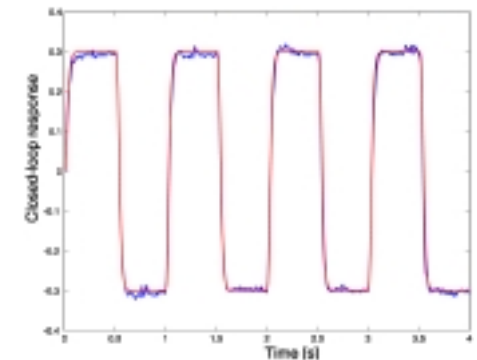
$$T_0(q^{-1}) = 1.83$$

Magnetic Suspension System Correlation-based Tuning

Closed-loop Response



Initial RST controller (model based)

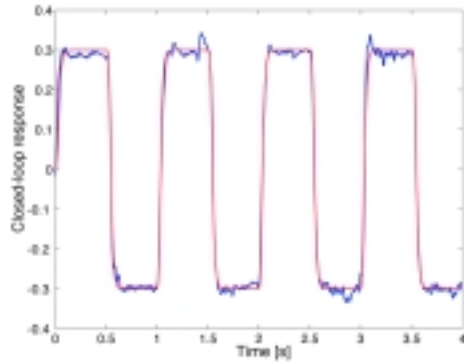


CbT after 6 iterations (data based)

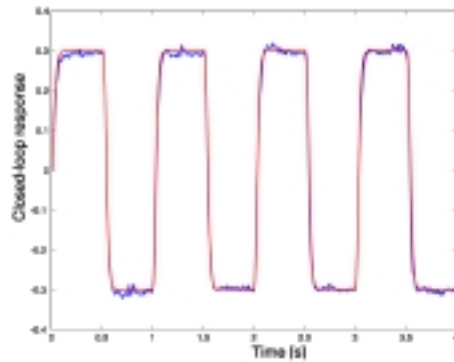
Magnetic Suspension System

IFT vs. CbT

Closed-loop Response



IFT after 24 experiments



CbT after 6 experiments

Conclusions

- Role of the closed-loop output error
 - Allows easy comparison with designed closed loop
 - Expresses the effects of unmodeled dynamics, nonlinearities and noise
- Direct data-driven controller tuning
 - IFT
 - Two experiments per iteration
 - Controller depends on noise
 - CbT
 - A single experiment per iteration
 - Controller independent of noise