ITERATIVE CONTROLLER TUNING BY MINIMIZATION OF A GENERALIZED DECORRELATION CRITERION [1]

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Abstract: A controller tuning method based on the correlation approach is considered. A new, generalized, decorrelation criterion is proposed that allows tuning the controller parameters such that the reference signal be as little correlated as possible with both the input and output closed-loop errors. A frequency-domain analysis of the proposed criterion shows that the discrepancy between the true closed-loop system and the designed one is minimized in terms of the output *and* input sensitivity functions. Furthermore, it is shown that the noise has asymptotically no effect on the controller parameters. The theoretical results are illustrated via a simulation example.

Keywords: Data-driven control, iterative controller tuning, correlation approach

1. INTRODUCTION

Reliable mathematical descriptions of industrial plants are often difficult or impossible to obtain mainly due to the high complexity of the plants and/or the excessive cost of modelling. In these situations, the design of controllers using process information in the form of the experimental data collected under closed-loop operation seems to be a promising alternative to model-based design. Direct adaptive control (Åström and Wittenmark, 1989), iterative feedback tuning (Hjalmarsson, 2002), controller unfalsification (Safonov and Tsao, 1997) and control design based on simultaneous perturbation stochastic approximation (Spall and Cristion, 1998) are but a few examples of such data-driven methods.

In this line of research, a so-called iterative correlation-based tuning method has recently been proposed to address the model-following problem (Karimi *et al.*, 2002*a*; Karimi *et al.*, 2002*c*). The idea behind this approach is to tune the controller parameters to the extent that some external excitation signal be uncorrelated with the closed-loop output error between the true plant and the designed one. This way, the closed-loop output error is not affected by the model mismatch, and the output of the controlled plant tends towards the designed closed-loop output independently of the disturbance characteristics.

The correlation-based tuning approach has been applied to a magnetic suspension system in (Karimi *et al.*, 2002*b*), where the controller parameters are calculated as the solution of a correlation equation involving instrumental variables. Convergence and consistency of the controller parameters in the presence of disturbances and modeling errors has been analyzed in (Karimi *et al.*, 2002*a*). In (Karimi *et al.*, 2002*c*), the design objective is reformulated as the minimization of the 2-norm of the cross-correlation function between the closed-loop output error and the reference signal. Analysis of the proposed criterion in the

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frequency-domain shows that the algorithm, for a special case of instrumental variables, tries to minimize the integral of the difference between the achieved and designed output sensitivity functions weighted by the square of the spectrum of the reference signal. An adaptation of this approach to the regulation problem and its application to a benchmark problem posed for a special issue of European Journal of Control on the design and optimization of restricted-complexity controllers is treated in (Mišković *et al.*, 2002).

The tuning objective proposed in (Karimi *et al.*, 2002*c*) allows the achieved closed-loop system to approach the designed one in terms of the output sensitivity function. However, one could also make demand on the input sensitivity function. In order to handle mixed sensitivity specifications, this paper extends the criterion for controller tuning by adding the 2-norm of the cross-correlation function between the closed-loop input error and the reference signal. This way, the desired closed-loop output can be attained while taking into account some penalty on the control action, i.e. it is possible to make a trade-off between the specifications given in terms of the output sensitivity and those given in terms of the input sensitivity function. Analysis of the proposed generalized criterion in the frequency domain reveals the benefit of incorporating the new term.

The paper is organized as follows. Preliminary material and notations are given in Section 2. Section 3 briefly presents the correlation-based tuning approach. A generalization of the tuning criterion is developed in Section 4. In Section 5, controller tuning using the proposed criterion is illustrated via a numerical example. Finally, some concluding remarks are given in the last section.

2. PRELIMINARIES

Let the output of some unknown true plant be described by the discrete-time model:

$$y(t) = G(q^{-1})u(t) + v(t) \qquad (1)$$

where q^{-1} is the backward-shift operator, $G(q^{-1})$ is a linear time-invariant SISO discrete-time transfer operator, u(t) the input signal to the plant and $v(t)$ a disturbance signal. It is assumed that $v(t)$ is a zero-mean weakly stationary random process.

Consider the closed-loop system depicted in Fig.1, where $K(q^{-1}, \rho)$ is a linear time-invariant transfer function parametrized by the vector $\rho \in \mathcal{R}^{n_\rho}$, and $r(t)$ is an external excitation signal. It is assumed that measurements of $r(t)$ and $y(t)$ are available. The excitation signal $r(t)$ is assumed to be uncorrelated with the disturbance signal $v(t)$.

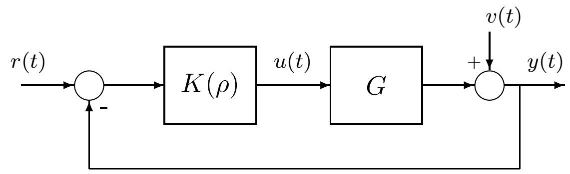

Fig. 1. Controlled plant

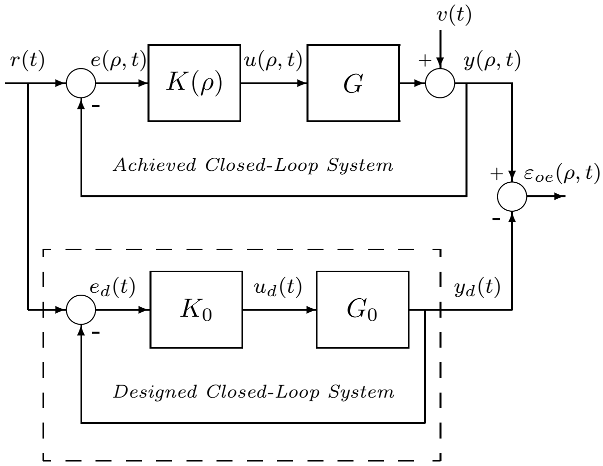

Fig. 2. Closed-loop output error resulting from a comparison of the achieved and designed closed-loop systems

As far as the notations are concerned, the signals collected under closed-loop operation using the controller $K(q^{-1}, \rho)$ will carry the argument ρ. Furthermore, to ease the notation, the backward-shift operator q^{-1} will be omitted in the sequel.

Let us define the following sensitivity functions:

- Output sensitivity function:
$$S(K, G) = (1 + KG)^{-1} \qquad (2)$$
- Input sensitivity function:
$$U(K, G) = K(1 + KG)^{-1} \qquad (3)$$
- Complementary sensitivity function:
$$T(K, G) = KG(1 + KG)^{-1} \qquad (4)$$

3. CORRELATION-BASED TUNING

A block diagram of the model-following problem is represented in Fig. 2. The upper part shows the achieved closed-loop system with the true plant, while the lower part is a realization of the designed closed-loop system that includes the plant model (G_0) and the initial controller (K_0). It is assumed that the initial controller is capable of meeting the specifications of the designed closed-loop system.

The closed-loop output error is defined as:

$$\varepsilon_{oe}(\rho, t) = y(\rho, t) - y_d(t) \qquad (5)$$

where $y(\rho, t)$ is the output of the achieved closed-loop system, and $y_d(t)$ the output of the designed closed-loop system.

Let the initial controller K_0 be applied to the true plant excited by the reference signal $r(t)$. Then, the closed-loop output error contains a contribution due to the difference between G and G_0 (modeling errors) and another contribution stemming from the disturbance $v(t)$. The effect of modeling errors is correlated with the reference signal, whereas that of disturbance is not. Thus, a reasonable way to tune the controller is to make the closed-loop output error $\varepsilon_{oe}(\rho, t)$ uncorrelated with the excitation signal $r(t)$. This way, the improved controller compensates the effect of modeling errors to the extent that the closed-loop output error contains only the filtered disturbance. However, since in practice perfect decorrelation between these two signals cannot be achieved, it is natural to define the tuning objective as the minimization of some norm of the cross-correlation function between $\varepsilon_{oe}(\rho, t)$ and $r(t)$.

Let define the correlation function $f_{oe}(\rho)$:

$$f_{oe}(\rho) = E\{\zeta(t)\varepsilon_{oe}(\rho, t)\} \tag{6}$$

where $E\{\cdot\}$ is the mathematical expectation and $\zeta(t)$ a vector of instrumental variables that are correlated with the reference signal $r(t)$ and independent of the disturbance $v(t)$. Then, the tuning objective can be defined as the minimization of the following criterion:

$$J_{oe}(\rho) = ||f_{oe}(\rho)||_2^2 = f_{oe}^T(\rho) f_{oe}(\rho) \tag{7}$$

where $||\cdot||_2$ represents the 2-norm. The control parameter vector ρ^* is given by:

$$\rho^* = \arg\min_{\rho} J_{oe}(\rho) \tag{8}$$

Since this problem cannot be solved analytically, a numerical method is considered. The vector ρ^* is solution of the following gradient equation:

$$J'_{oe}(\rho) = f_{oe}^T(\rho) \frac{\partial f_{oe}(\rho)}{\partial \rho} = 0 \tag{9}$$

This problem can be solved by the Robbins-Monro procedure using the following iterative formula (Robbins and Monro, 1951):

$$\rho_{i+1} = \rho_i - \gamma_i \left[Q(\rho_i)\right]^{-1} \left[J'_{oe}(\rho_i)\right]^T \tag{10}$$

where γ_i is a scalar step size and $Q(\rho_i)$ a positive definite matrix. Under the assumption of boundedness of the signals in the loop, and with a step size tending to zero appropriately fast, this scheme converges to a local minimum of the criterion as the number of iterations tends to infinity (Karimi *et al.*, 2002*c*).

The gradient of the criterion involves the expectation of signals that are unknown and should be replaced by their estimates from closed-loop data. Let the correlation function be estimated by $\hat{f}_{oe}(\rho)$:

$$\hat{f}_{oe}(\rho) = \frac{1}{N} \sum_{t=1}^{N} \zeta(t)\varepsilon_{oe}(\rho, t) \tag{11}$$

where N is the number of data points. Then, the derivative of the criterion is determined as follows:

$$J'_{oe}(\rho_i) = \hat{f}_{oe}^T(\rho_i) \frac{1}{N} \sum_{t=1}^{N} \zeta(t) \left.\frac{\partial \varepsilon_{oe}(\rho, t)}{\partial \rho}\right|_{\rho_i} \tag{12}$$

An accurate value of this gradient cannot be computed because the derivative of $\varepsilon_{oe}(\rho, t)$ with respect to ρ is unknown. However, an unbiased model-free estimation can be obtained using two extra closed-loop experiments as is done in the IFT approach (Hjalmarsson, 2002). Note that the gradient could also be obtained from a plant model that is identified, for example, using closed-loop data (Karimi *et al.*, 2002*b*).

In order to improve the convergence speed, $Q(\rho_i)$ can be chosen as an approximation of the Hessian of the criterion (Gauss-Newton direction):

$$Q(\rho_i) = \left(\left.\frac{\partial \hat{f}_{oe}(\rho)}{\partial \rho}\right|_{\rho_i} \right)^T \left.\frac{\partial \hat{f}_{oe}(\rho)}{\partial \rho}\right|_{\rho_i} + \lambda I \tag{13}$$

where the parameter λ should be chosen so as to ensure positive definiteness of the matrix $Q(\rho_i)$.

4. GENERALIZING THE CRITERION

In (Karimi *et al.*, 2002*c*), the frequency characteristics of the achieved closed-loop system have been compared with those of the designed closed-loop system for the following choice of instrumental variables:

$$\zeta^T(t) = [r(t + n_z), \ldots, r(t), \ldots, r(t - n_z)] \tag{14}$$

where n_z is a sufficiently large integer number w.r.t. the order of the closed-loop system. On the other hand, the value of n_z should be much smaller than the number of data N in order to have an accurate estimation of the cross-correlation function. Analysis of the criterion of Eq. 7 has shown that the algorithm minimizes the integral of the difference between the achieved and the designed complementarity sensitivity functions weighted by the square of the reference signal spectrum $\Phi_r(\omega)$:

$$\lim_{n_z \to \infty} J_{oe}(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H_{oe}(e^{-j\omega}, \rho)|^2 \Phi_r^2(\omega) \tag{15}$$

where $H_{oe}(\rho) = T(K(\rho), G) - T_0$ with $T_0 = T(K_0, G_0)$ being the designed complementary sensitivity function. If $r(t)$ is white noise with variance 1, and n_z tends to infinity, one has:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} |T(e^{-j\omega}, \rho) - T_0(e^{-j\omega})|^2 d\omega$$
$$= \arg\min_{\rho} \int_{-\pi}^{\pi} |S(e^{-j\omega}, \rho) - S_0(e^{-j\omega})|^2 d\omega \quad (16)$$

where $S_0 = S(K_0, G_0)$ is the designed output sensitivity function. These relations show that both the achieved complementary sensitivity function $T(e^{-j\omega}, \rho)$ and the achieved output sensitivity function $S(e^{-j\omega}, \rho)$ tend to their respective designed functions. Thus, the tuned controller ensures the designed performance for the true plant with respect to tracking and output disturbance rejection.

However, when minimizing the criterion of Eq. 7, the achieved input sensitivity function $U(e^{-j\omega}, \rho)$ does not necessarily approach U_0. At some frequencies, $U(e^{-j\omega}, \rho)$ obtained by controller tuning may grow large, thus affecting robust stability. In addition, the controlled input $u(t)$ may exert a substantial effort on the actuators. To overcome this difficulty, the criterion can be generalized so as to incorporate the new term containing the 2-norm of the cross-correlation function between the closed-loop input error and the reference signal. This way, not only the output but also the input of the achieved closed-loop system will follow respectively the output and the input of the designed closed-loop system independently of the disturbance dynamics. Thus, let us modify the criterion of Eq. 7 as follows:

$$J(\rho) = k_{oe}||f_{oe}(\rho)||_2^2 + k_{ie}||f_{ie}(\rho)||_2^2 \quad (17)$$

where k_{oe} and k_{ie} are positive scalar weighting factors, and $f_{ie}(\rho)$ is the correlation function:

$$f_{ie}(\rho) = E\{\zeta(t)\varepsilon_{ie}(\rho, t)\} \quad (18)$$

The closed-loop input error $\varepsilon_{ie}(\rho, t)$ is given by:

$$\varepsilon_{ie}(\rho, t) = u(\rho, t) - u_d(t) \quad (19)$$

where $u_d(t)$ is the control input of the designed closed-loop system (see Fig. 3).

From Figs. 2 and 3, $\varepsilon_{oe}(\rho, t)$ and $\varepsilon_{ie}(\rho, t)$ can be written as:

$$\varepsilon_{oe}(\rho, t) = (T(\rho) - T_0)\, r(t) + S(\rho)v(t)$$
$$= H_{oe}(\rho)r(t) + S(\rho)v(t) \quad (20)$$

and

$$\varepsilon_{ie}(\rho, t) = H_{ie}(\rho)r(t) - U(\rho)v(t) \quad (21)$$

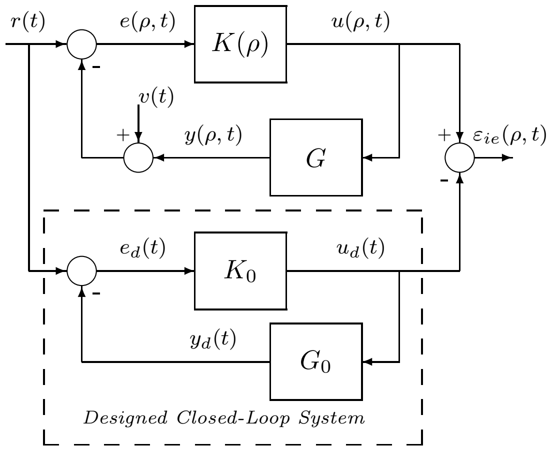

Fig. 3. Closed-loop input error resulting from a comparison of the achieved and designed closed-loop systems

where $H_{ie}(\rho) = U(\rho) - U_0$ with $U_0 = U(K_0, G_0)$ being the designed input sensitivity function.

Considering the vector of instrumental variables given in Eq. 14, and letting n_z tend to infinity, gives asymptotically (after straightforward calculations similar to those in (Karimi *et al.*, 2002*c*)):

$$J(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ k_{oe}|H_{oe}(e^{-j\omega}, \rho)|^2 + k_{ie}|H_{ie}(e^{-j\omega}, \rho)|^2 \right\} \Phi_r^2(\omega) d\omega \quad (22)$$

If $r(t)$ is white noise with variance 1, one has:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} \left\{ k_{oe}|T(e^{-j\omega}, \rho) - T_0(e^{-j\omega})|^2 + k_{ie}|U(e^{-j\omega}, \rho) - U_0(e^{-j\omega})|^2 \right\} d\omega$$
$$= \arg\min_{\rho} \int_{-\pi}^{\pi} \left\{ k_{oe}|S(e^{-j\omega}, \rho) - S_0(e^{-j\omega})|^2 + k_{ie}|U(e^{-j\omega}, \rho) - U_0(e^{-j\omega})|^2 \right\} d\omega \quad (23)$$

This relation shows that there is a trade-off between the minimization of $||S(\rho) - S_0||_2$ and that of $||U(\rho) - U_0||_2$. By minimizing this criterion, the mixed sensitivity specifications are satisfied, and the achieved closed-loop system tries to preserve the robustness properties of the designed one. Furthermore, it is easy to see that the criterion of Eq. 17 is not influenced by the disturbance signal $v(t)$. With regard to this criterion, two extreme cases can be considered: (i) When $(k_{oe}, k_{ie}) = (1, 0)$, Eq. 23 reduces to Eq. 16 and $S(\rho)$ is forced towards S_0; (ii) when $(k_{oe}, k_{ie}) = (0, 1)$, $U(\rho)$ is pushed towards its designed function U_0.

5. SIMULATION EXAMPLE

In this section, the properties of the proposed tuning method are illustrated via an example.

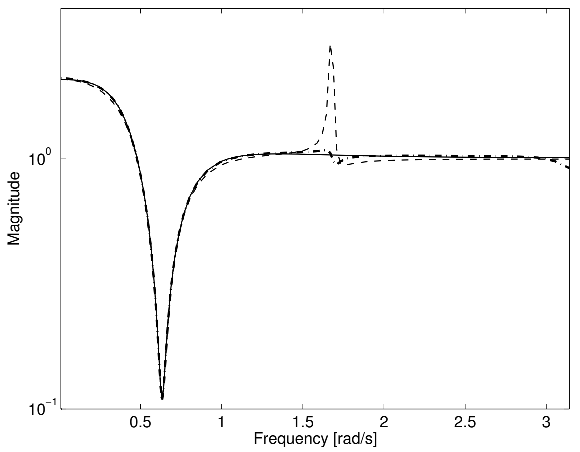

Fig. 4. Output sensitivity functions $S_0(e^{j\omega})$ (solid), $S_{init}(e^{j\omega})$ (dash) and $S(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (1, 0)$

Consider the following 4th-order true plant:

$$G = \frac{0.385q^{-2} + 0.525q^{-3}}{1 - 1.353q^{-1} + 1.55q^{-2} - 1.282q^{-3} + 0.915q^{-4}}$$

The system has two very lightly damped resonant modes and one unstable zero. The following second-order model G_0 has been identified:

$$G_0 = \frac{0.6043q^{-2} - 0.1562q^{-3} - 0.0306q^{-4}}{1 - 1.5822q^{-1} + 0.9629q^{-2}}$$

Let the initial 3rd-order controller K_0 be:

$$K_0 = \frac{-0.1530q^{-1} - 0.038q^{-2}}{1 - 0.8093q^{-1} + 0.2141q^{-2} - 0.012q^{-3}}$$

When K_0 is applied to the true plant G, there is significant deterioration of the performance due to model mismatch (see dashed line in Figs. 4 and 5). To improve the behaviour of the closed-loop system, a 4th-order controller K is to be tuned on the true plant for three different choices of the weighting factors k_{oe} and k_{ie}. The tuning procedure is carried out in 8 iterations, with each iteration being performed using a different realization of the disturbance signal $v(t)$ with a noise-to-signal ratio of 7% in terms of variance. The vector of instrumental variables is chosen as in Eq. 14 with $n_z = 72$, and the reference signal $r(t)$ is a PRBS generated by a 7-bit shift register with data length $N = 2048$. In all iterations, the initial step size $\gamma_i = 0.5$ is used. If the algorithm provides a controller that destabilizes the closed-loop system, the step-size is then divided by 2.

The first choice of weighting factors $(k_{oe}, k_{ie}) = (1, 0)$ corresponds to the minimization of $||H_{oe}||_2$. Fig. 4 shows the output sensitivity functions S_0, $S_{init} = S(K_0, G)$ and S for the designed, initial and final closed-loop systems, respectively. It can be seen that S_0 and S are almost superposed, i.e. the tuning algorithm has succeeded in minimizing H_{oe} to a large extent. However, comparing the corresponding input sensitivity functions U_0, $U_{init} = K_0 S_{init}$ and U shown in Fig. 5, it is easy

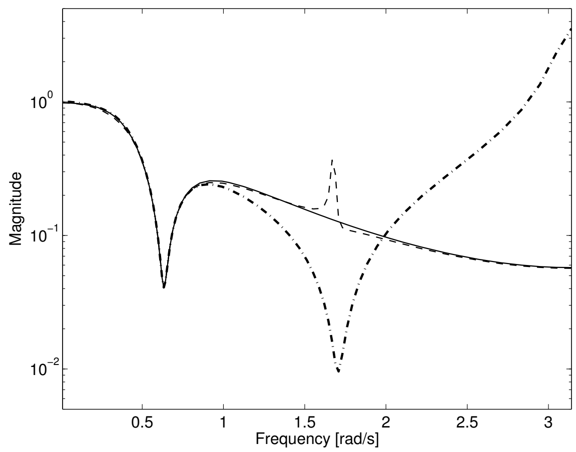

Fig. 5. Input sensitivity functions $U_0(e^{j\omega})$ (solid), $U_{init}(e^{j\omega})$ (dash) and $U(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (1, 0)$

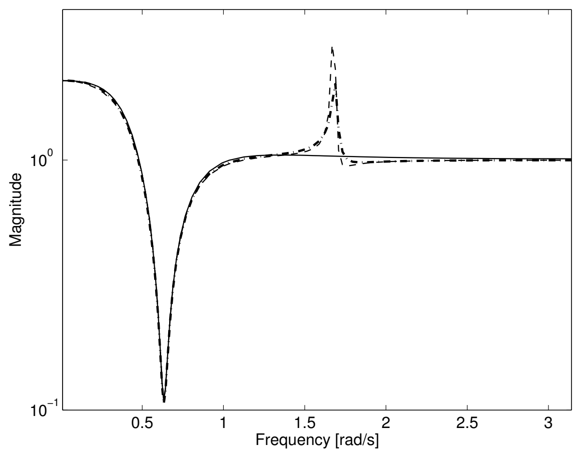

Fig. 6. Output sensitivity functions $S_0(e^{j\omega})$ (solid), $S_{init}(e^{j\omega})$ (dash) and $S(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (0, 1)$

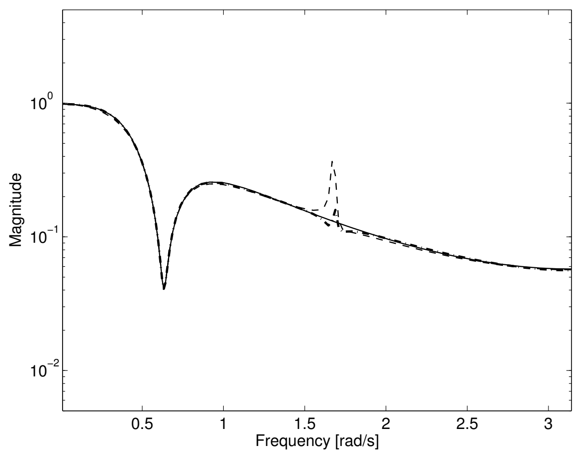

Fig. 7. Input sensitivity functions $U_0(e^{j\omega})$ (solid), $U_{init}(e^{j\omega})$ (dash) and $U(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (0, 1)$

to see that U gets large at high frequencies which can significantly deteriorate the robustness of the closed-loop system.

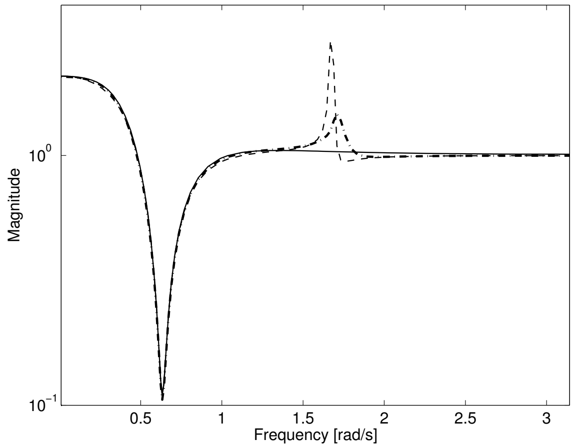

Fig. 8. Output sensitivity functions $S_0(e^{j\omega})$ (solid), $S_{init}(e^{j\omega})$ (dash) and $S(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (0.5, 0.5)$

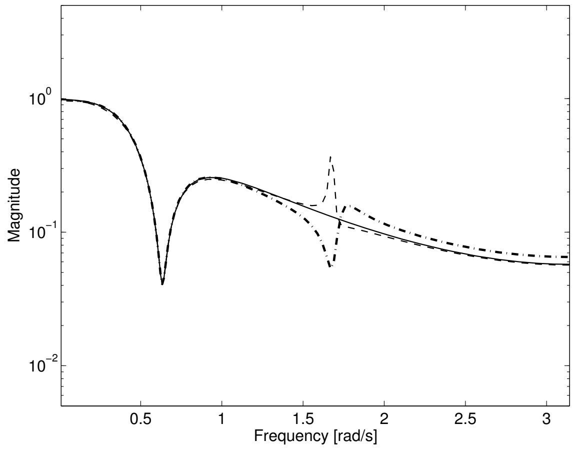

Fig. 9. Input sensitivity functions $U_0(e^{j\omega})$ (solid), $U_{init}(e^{j\omega})$ (dash) and $U(e^{j\omega})$ (dash-dot) for $(k_{oe}, k_{ie}) = (0.5, 0.5)$

For $(k_{oe}, k_{ie}) = (0, 1)$, $||H_{ie}||_2$ is minimized. Figs. 6 and 7 depict the corresponding output sensitivities S_0, S_{init} and S, and input sensitivities U_0, U_{init} and U. A comparison of the curves shows that, though the resulting controller K has not succeeded in reducing the peak of the output sensitivity function S, the final input sensitivity function U is very similar to U_0.

Finally, for the case $(k_{oe}, k_{ie}) = (0.5, 0.5)$, there is a trade-off in minimizing $||H_{oe}||_2$ and $||H_{ie}||_2$. Figs. 8 and 9 shows that the resulting controller K has reduced the peak of the output sensitivity function S and, at the same time, the discrepancy between U_0 and U remains small.

Table 1 gives the performance of the tuning procedure in function of the weighting factors k_{oe} and k_{ie}. These numerical results confirm the qualitative shapes seen in Figs. 4-9. The minima of $||H_{oe}||_2$ and $||H_{ie}||_2$ are achieved for $(k_{oe}, k_{ie}) = (1, 0)$ and $(k_{oe}, k_{ie}) = (0, 1)$, respectively. However, when minimizing only $||H_{oe}||_2$ or $||H_{ie}||_2$, the deviation of the other sensitivity does increase. In contrast, the controller obtained with $(k_{oe}, k_{ie}) = (0.5, 0.5)$ reduces both $||H_{oe}||_2$ and $||H_{ie}||_2$.

Table 1. Results of tuning

	Iteration	$\|\|H_{oe}\|\|_2$	$\|\|H_{ie}\|\|_2$
	1^{st}	0.3002	0.0388
$k_{oe} = 1, k_{ie} = 0$	8^{th}	0.0284	0.4209
$k_{oe} = 0, k_{ie} = 1$	8^{th}	0.1493	0.0091
$k_{oe} = k_{ie} = 0.5$	8^{th}	0.1018	0.0284

6. CONCLUSIONS

An extension of the controller-tuning criterion based on the correlation approach has been proposed. The new criterion is defined as the weighted sum of the 2-norms of the cross-correlation functions between a reference signal and the output and input closed-loop errors. If the assumption of independence between the reference signal and the disturbance holds, the criterion remains asymptotically unaffected by the disturbance characteristics. A frequency-domain analysis of the proposed criterion has shown that, depending on the values of the weighting factors k_{oe} and k_{ie}, there is a trade-off in meeting the designed output and input sensitivities. Simulation results illustrate the features and the applicability of the new tuning approach.

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