Correlation-Based Tuning of a Restricted-Complexity Controller for an Active Suspension System

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Abstract

A correlation-based controller tuning method is proposed for the "Design and optimization of restricted-complexity controllers" benchmark problem. The approach originally proposed for model following is extended to solve the disturbance rejection problem. The idea is to tune the controller parameters such that the closed-loop output be uncorrelated with the disturbance signal. Since perfect decorrelation between the closed-loop output and the disturbance signal is not attainable in the restricted-complexity controller design, the cross correlation between these two signals is minimized iteratively using the stochastic approximation method. Since control specifications can normally be expressed in terms of constraints on the sensitivity functions, a frequency-domain analysis of the criterion is performed. Straightforward implementation of the proposed approach on the active suspension system of the Automatic Control Laboratory of Grenoble (LAG) provides a 2nd-order controller that meets the control specifications very well.

Keywords: restricted-complexity controller; iterative controller tuning; correlation approach; active suspension system;

1 Introduction

The design of restricted-complexity linear controllers has drawn wide attention in the control community. Low-order controllers are usually preferred in industry because controller size may be limited by hardware or/and computational requirements. Moreover, simple controllers are much easier (and cheaper) to implement, to maintain and to understand. There is a wide range of methods for obtaining reduced-order controllers [1] and they can be broadly divided into several categories. In some approaches, a high-order controller is first found, and then an optimization procedure is used to minimize a norm of the error between the full-order and reduced-order controllers. It has be shown that certain information about the plant model and control specification should be considered in the controller reduction procedure [3, 8]. Another approach is to derive a reduced-order model of the plant on the basis of which the controller is designed. However, the controller design step should consider the unmodeled dynamics to ensure robust stability. Other approaches solve an optimal control problem directly for a restricted-order controller [9, 2].

The control parameters of a restricted-complexity controller can also be tuned using data collected on the closed-loop system. In this approach, a control criterion is minimized by a data-driven optimization algorithm. The gradient of the criterion can be computed using additional experiments on the real system, i.e. without using the model of the plant (model-free approaches) [4, 11]. The gradient of the criterion can also be estimated using an approximate model of the plant [12, 5]. In these approaches the model is not explicitly used for the controller design, so the model order has no effect on the controller order.

The correlation approach for iterative controller tuning was originally proposed for the model-following problem [5, 6, 7]. The essential idea of this approach is to modify the control objective so that, instead of minimizing a LQG-like control criterion, one tries to decorrelate the closed-loop output error and an excitation signal. The controller parameters are solutions to a correlation equation involving instrumental variables computed iteratively using the Newton-Raphson algorithm. This method was applied successfully to a magnetic suspension system in [6]. The convergence of the controller parameters to the solution of the correlation equation in the presence of noise and modeling errors was studied in [5]. Since perfect decorrelation is not possible in the context of restricted-order controller design, it is natural to reformulate the design criterion as the two-norm of the cross-correlation function between the closed-loop output error and the reference signal [7]. The frequency-domain analysis of the proposed criterion showed that the algorithm minimizes the difference between the closed-loop transfer function and the reference model weighted by the square of the reference signal spectrum.

In this paper, the correlation approach is adapted to be used for the tuning of a restricted-order controller that rejects the disturbances in prespecified frequency regions. The main advantage is that the controller parameters are not asymptotically affected by noise. The approach is applied to solve the disturbance rejection problem of the benchmark associated with the Special Issue of European Journal of Control. The benchmark problem involves designing the simplest controller able to ensure good disturbance rejection for an active suspension system. The control specifications given in the benchmark are stated in terms of constraints on the sensitivity functions. Although the procedure used in this paper does not accommodate specifications in the frequency domain explicitly, they can be met thanks to the frequency-domain analysis of the criterion [7]. The two-norm of the correlation function is minimized using the extended instrumental variables method.

The paper is organized as follows. Section 2 briefly presents the correlation approach adapted for the regulation problem. The frequency-domain analysis of the algorithm is presented in Section 3. Section 4 describes the application of this approach to the benchmark problem. Concluding remarks are given in Section 5.

2 The Correlation Approach

The correlation approach to controller tuning has already been considered with respect to the model-following problem [5, 6, 7]. The idea is to tune the controller parameters such that the output error between the closed-loop system and the reference model be uncorrelated with the reference signal. This way, the control objective is to make the closed-loop output follow as closely as possible the desired one, and this independently of the output noise characteristics. The controller parameters are solutions of a correlation equation involving instrumental variables. This solution is computed iteratively using the Newton-Raphson algorithm.

In this paper, the correlation approach is applied to the regulation problem. Let the measured output of the plant (see Fig. 1) be described as:

$$y(t) = G(q^{-1})u(t) + F(q^{-1})v_1(t) + v_2(t) \tag{1}$$

where q^{-1} is the delay operator, $u(t)$ the plant input, $v_1(t)$ a measurable disturbance signal, $v_2(t)$ a zero-mean measurement noise independent of $v_1(t)$, $G(q^{-1})$ an LTI SISO discrete-time transfer operator defined as:

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} \tag{2}$$

and $F(q^{-1})$ the model related to the measured disturbance.

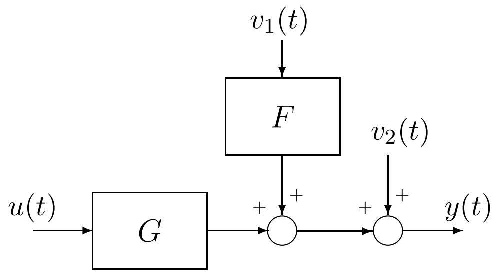

Figure 1: Plant and disturbance models

The system is controlled by the controller $K(q^{-1})$:

$$K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} \tag{3}$$

where

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \cdots + r_{n_R} q^{-n_R} \tag{4}$$
$$S(q^{-1}) = 1 + s_1 q^{-1} + \cdots + s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1}) \tag{5}$$

The controller output can be presented in regression form as:

$$u(t) = -S^*(q^{-1})u(t-1) - R(q^{-1})y(t) = \phi^T(\rho, t)\rho \tag{6}$$

with the regressor vector $\phi(\rho, t)$ and the vector of controller parameters ρ, both of dimension n_ρ, defined as:

$$\phi^T(\rho, t) = [-u(t-1) \cdots - u(t-n_S), -y(t) \cdots - y(t-n_R)] \tag{7}$$
$$\rho^T = [s_1 \cdots s_{n_S}, r_0 \cdots r_{n_R}] \tag{8}$$

The control design objective is to tune the controller parameters such that the effect of the disturbance $v_1(t)$ at the plant output be totally compensated. That is, the output $y(t)$ contains only the measurement noise $v_2(t)$ which is uncorrelated with the disturbance signal $v_1(t)$. Evidently, with a low-order causal controller the perfect decorrelation of $y(t)$ and $v_1(t)$ is not possible. Therefore, it is natural to formulate the design objective as the minimization of some norm of the cross-correlation function between these two signals.

Let the correlation function $f(\rho)$ be defined as follows:

$$f(\rho) = E\{y(\rho, t)\zeta(t)\} \tag{9}$$

where $E\{\cdot\}$ is the mathematical expectation and $\zeta(t)$ a vector of instrumental variables that are correlated with the disturbance signal and independent of the measurement noise. Thus, the tuning objective can be defined as the minimization of the following criterion:

$$J(\rho) = ||f(\rho)||_2^2 = f^T(\rho)f(\rho) \tag{10}$$

where $||\cdot||_2$ represents the two-norm. Hence, the controller parameter vector ρ^* is given by:

$$\rho^* = \arg\min_{\rho} J(\rho) \tag{11}$$

Since this problem cannot be solved analytically, an iterative numerical method is considered. The vector ρ^* is solution of the following gradient equation:

$$J'(\rho) = f^T(\rho)\frac{\partial f(\rho)}{\partial \rho} = 0 \tag{12}$$

This problem can be solved by the Robbins-Monro [10] procedure using the following iterative formula:

$$\rho_{i+1} = \rho_i - \gamma_i \left[Q(\rho_i)\right]^{-1} \left[J'(\rho_i)\right]^T \tag{13}$$

where γ_i is a scalar step size and $Q(\rho_i)$ is a positive definite square matrix. Under the assumption of boundedness of the signals in the loop, with a step size tending to zero appropriately fast, the scheme converges to a local minimum of the criterion as the number of iterations tends to infinity [7].

The gradient of the criterion involves the expectations of signals that are unknown and should be replaced by their estimates from closed-loop sampled data. Let the correlation function be estimated by $\bar{f}(\rho)$:

$$\bar{f}(\rho) = \frac{1}{N}\sum_{t=1}^{N} y(\rho, t)\zeta(t) \tag{14}$$

where N is the number of data. Then, the derivative of the criterion is determined as follows:

$$J'(\rho_i) = \bar{f}^T(\rho_i)\,\frac{1}{N}\sum_{t=1}^{N}\zeta(t)\,\left.\frac{\partial y(\rho,t)}{\partial\rho}\right|_{\rho_i} \tag{15}$$

An accurate value of the gradient cannot be computed because the derivative of $y(\rho, t)$ with respect to ρ is unknown. However, an unbiased model-free estimation of this value can be obtained using two extra closed-loop experiments as is done in the IFT approach [4]. The gradient can also be estimated from an available plant model identified in the open-loop or closed-loop operation using the following expression [6]:

$$\frac{\partial y(\rho,t)}{\partial\rho} \approx \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})S(q^{-1}) + \hat{B}(q^{-1})R(q^{-1})}\phi^T(\rho,t) \tag{16}$$

where $\hat{B}/\hat{A}$ is the identified plant model.

In order to improve the convergence speed, $Q(\rho_i)$ can be chosen as an approximation of the Hessian of the criterion (Gauss-Newton direction). In this case, one has:

$$Q(\rho_i) = \left(\left.\frac{\partial\bar{f}(\rho)}{\partial\rho}\right|_{\rho_i}\right)^T \left.\frac{\partial\bar{f}(\rho)}{\partial\rho}\right|_{\rho_i} + \lambda I \tag{17}$$

where the parameter λ should be chosen to ensure the positive definiteness of the matrix $Q(\rho)$.

3 Frequency-domain analysis

In this section, the frequency characteristics of the achieved closed-loop system are analyzed. The relation between the cross-correlation functions and the spectral density functions helps obtain an asymptotic equivalent of the criterion in the frequency domain.

For the simplicity of the analysis, the following choice of instrumental variables is considered:

$$\zeta^T(t) = [v_1(t+n_z), v_1(t+n_z-1), \ldots, v_1(t), v_1(t-1), \ldots, v_1(t-n_z)] \tag{18}$$

where n_z is a sufficiently large integer number. Thus, the criterion (10) can be presented as:

$$J(\rho) = f^T(\rho)f(\rho) = \sum_{\tau=-n_z}^{n_z} R^2_{yv_1}(\tau) \tag{19}$$

where $R_{yv_1}(\tau)$ is the cross-correlation function between the instrumental variable $\zeta(t)$ and the closed-loop output $y(\rho, t)$ defined as:

$$R_{yv_1}(\tau) = E\{y(\rho, t)v_1(t - \tau)\} \tag{20}$$

The closed-loop output can be expressed as:

$$y(\rho, t) = \mathcal{S}_{yp}(q^{-1}, \rho)(F(q^{-1})v_1(t) + v_2(t)) \tag{21}$$

where $\mathcal{S}_{yp}(q^{-1}, \rho)$ is the output sensitivity function of the closed-loop system defined as follows:

$$\mathcal{S}_{yp}(q^{-1}, \rho) = \frac{1}{1 + K(q^{-1})G(q^{-1})} \tag{22}$$

Then, with the assumption that $v_1(t)$ and $v_2(t)$ are not correlated, one obtains:

$$R_{yv_1}(\tau) = E\{\mathcal{S}_{yp}(q^{-1}, \rho)F(q^{-1})v_1(t)v_1(t - \tau)\} = \sum_{i=0}^{\infty} h(i)R_{v_1v_1}(\tau - i) \tag{23}$$

where $h(t)$ is the impulse response of $\mathcal{S}_{yp}(q^{-1}, \rho)F(q^{-1})$, and $R_{v_1v_1}(\tau)$ the auto-correlation function of $v_1(t)$. On the other hand, $R_{yv_1}(\tau)$ can be expressed as an integral in the frequency domain:

$$R_{yv_1}(\tau) = \int_{-\pi}^{\pi} \mathcal{S}_{yp}(e^{-j\omega}, \rho)F(e^{-j\omega})\Phi_{v_1}(\omega)e^{j\tau\omega}d\omega \tag{24}$$

where $\Phi_{v_1}(\omega)$ is the spectrum of the disturbance signal v_1. Replacing $R_{yv_1}(\tau)$ in the criterion (19) by the expressions from Eqs 23 and 24, one obtains:

$$\begin{aligned} J(\rho) &= \sum_{\tau=-n_z}^{n_z} \left(\sum_{i=0}^{\infty} h(i)R_{v_1v_1}(\tau - i)\right) \\ &\quad \times \left(\int_{-\pi}^{\pi} \mathcal{S}_{yp}(e^{-j\omega}, \rho)F(e^{-j\omega})\Phi_{v_1}(\omega)e^{j\tau\omega}d\omega\right) \end{aligned} \tag{25}$$

$$\begin{aligned} &= \sum_{\tau=-n_z}^{n_z} \int_{-\pi}^{\pi} \sum_{i=0}^{\infty} h(i) e^{ij\omega} R_{v_1 v_1}(\tau - i) e^{j(\tau-i)\omega} \\ &\quad \times \mathcal{S}_{yp}(e^{-j\omega}, \rho) F(e^{-j\omega}) \Phi_{v_1}(\omega) d\omega \qquad (26) \\ &= 2\pi \int_{-\pi}^{\pi} \sum_{i=0}^{\infty} h(i) e^{ij\omega} \left(\frac{1}{2\pi} \sum_{\tau=-n_z}^{n_z} R_{v_1 v_1}(\tau - i) e^{j(\tau-i)\omega} \right) \\ &\quad \times \mathcal{S}_{yp}(e^{-j\omega}, \rho) F(e^{-j\omega}) \Phi_{v_1}(\omega) d\omega \qquad (27) \end{aligned}$$

Finally, when n_z tends to infinity, using the symmetrical property of auto-correlation functions ($R_{v_1 v_1}(\tau) = R_{v_1 v_1}(-\tau)$), one obtains:

$$\lim_{n_z \to \infty} J(\rho) = 2\pi \int_{-\pi}^{\pi} |\mathcal{S}_{yp}(e^{-j\omega}, \rho)|^2 |F(e^{-j\omega})|^2 \Phi_{v_1}^2(\omega) d\omega \qquad (28)$$

Thus, it is clear that the criterion based on the correlation approach is not influenced by the noise signal $v_2(t)$ and that the spectral density of the excitation signal is emphasized with a power of two in the criterion.

If the disturbance signal $v_1(t)$ is white noise with variance 1, and n_z tends to infinity, one has:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} |\mathcal{S}_{yp}(e^{-j\omega}, \rho)|^2 |F(e^{-j\omega})|^2 d\omega \qquad (29)$$

This expression shows that the algorithm tries to minimize the magnitude of the sensitivity function $\mathcal{S}_{yp}$ in the frequency regions where F is large.

4 Application to an Active Suspension System

The benchmark problem aims at designing a reduced-complexity controller for the active suspension system of LAG. The block diagram of the active suspension system is presented in the Figure 2.

The system is excited by the disturbance signal $v_1(t)$ generated by a computer-controlled shaker. The output of the system is the measured voltage corresponding to the residual force $y(t)$. The control input drives the position of a piston via an actuator. The transfer function C/D between the excitation signal of the shaker and the residual force is called the primary path. The disturbance signal $p(t)$ can be measured as the output of the open-loop plant. The secondary path is defined as the transfer function B/A

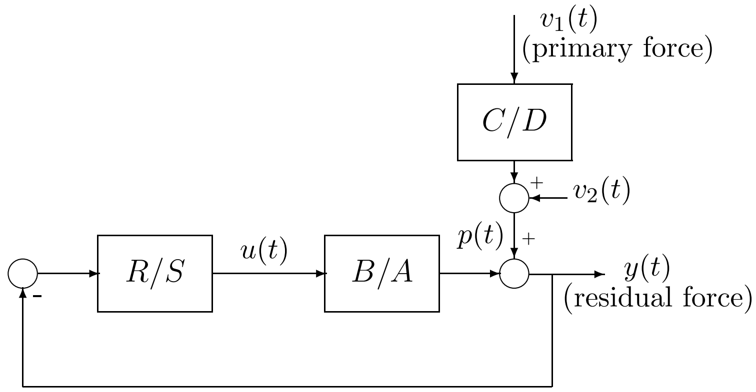

Figure 2: Block diagram of the active suspension system

between the control input and the residual force. The non-parametric model of the primary path shows that there are several vibrational modes, with the first mode at 31.47 Hz and the second mode around 160 Hz being the most important (Fig. 3).

The design objective is to compute a low-order linear discrete-time controller $R(q^{-1})/S(q^{-1})$ which minimizes the residual force around the first and second vibrational modes of the primary path model and tries to distribute the amplification of the disturbances over higher frequencies. The control specifications are expressed as constraints on the output sensitivity function $\mathcal{S}_{yp}$ and input sensitivity function $\mathcal{S}_{up} = K(1+KG)^{-1}$. In addition, the controller gain should be zero at the Nyquist frequency (hence the term $R_{fix}(q^{-1}) = 1 + q^{-1}$ should be incorporated in the controller).

The fixed terms in R and S (i.e., $R = R'R_{fix}$ and $S = S'S_{fix}$) are included in the tuning procedure in the following manner: the augmented plant model is constructed by including the fixed terms R_{fix} and S_{fix} in the plant model B/A (Fig. 4). Then, $u(t)$ in (7) is replaced by the input of the augmented plant $u'(t) = \frac{S_{fix}}{R_{fix}}u(t)$. The estimate of gradient in (16) is calculated by replacing $\hat{B}$, $\hat{A}$, R and S with $\hat{B}R_{fix}$, $\hat{A}S_{fix}$, R' and S', respectively. Finally, R' and S' are computed using the iterative algorithm and later multiplied by the fixed terms to obtain the controller polynomials R and S.

Since the number of real-time experiments is limited, the high-order discrete-time model of the secondary path (available on the benchmark web site) is used to simulate the plant G in the controller tuning procedure. The gradient $J'(\rho_i)$ in (15) and Hessian $Q(\rho_i)$ in (17) of the criterion needed in the optimization are estimated using the same model. However, for the

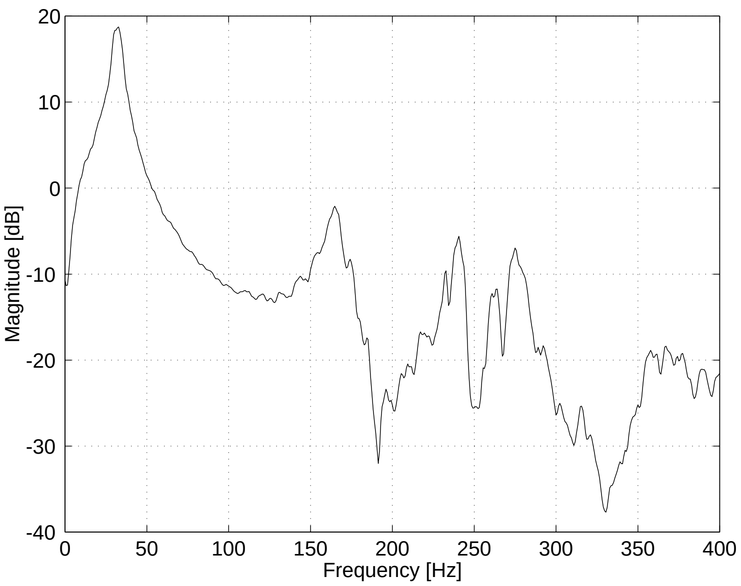

Figure 3: Frequency response of the primary path obtained by spectral analysis

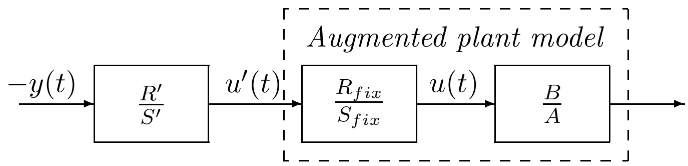

Figure 4: Incorporating fixed terms in the plant model

disturbance signal $p(t) = F(q^{-1})v_1(t) + v_2(t)$, the value measured from an open-loop experiments (also available on the benchmark web site) is used, where $v_1(t)$ is a PRBS generated by a 10-bit shift register with data length $N = 20000$. This way, the model C/D of the primary path is not involved in the simulation.

The following controller structure is adopted:

$$K(q^{-1}) = \frac{(r_0 + r_1 q^{-1})(1 + q^{-1})}{1 + s_1 q^{-1} + s_2 q^{-2}} \tag{30}$$

The 2nd-order controller is chosen as the lowest-order controller that approximately meets the benchmark specifications. All the parameters of the controller are initialized to zero except for $r_0 = 0.0025$. This way, the initial controller $K^0 = r_0(1 + q^{-1})$ stabilizes the closed-loop system.

From Eq. 28 and Fig. 3 it is evident that the algorithm will reduce the sensitivity function $\mathcal{S}_{yp}$ mainly around the first resonant mode. However, in order to accentuate the higher frequencies, the vector of instrumental variables can be filtered by a linear filter $W(q^{-1})$: $\zeta_f(t) = W(q^{-1})\zeta(t)$. In this case Eq. 29 becomes:

$$\rho^* = \arg\min_{\rho} \int_{-\pi}^{\pi} |\mathcal{S}_{yp}(e^{-j\omega}, \rho)|^2 |F(e^{-j\omega})|^2 |W(e^{-j\omega})|^2 d\omega \qquad (31)$$

Hence, the parameters of the linear filter W can be used as design parameters to weigh the sensitivity function $\mathcal{S}_{yp}$. A 3rd-order high-pass Butterworth digital filter with normalized cut-off frequency of 0.25. The length of the instrumental variables vector should be larger then the number of controller parameters to be tuned but much smaller then data length. Here, $n_z = 28$ is chosen.

A local optimum is reached after 8 iterations. In all iterations, the initial step size $\gamma_i = 1$ is used. In the cases where the algorithm provides a controller that makes the closed-loop system unstable (which is readily verified with the discrete-time model), the step size is progressively divided by 2. Figure 5 shows the output $\mathcal{S}_{yp}$ and the input $\mathcal{S}_{up}$ sensitivity function of the closed-loop system before tuning (dash-dot), after 3 iterations (dashed), and after 8 iterations (thick solid line) along with the constraints (thin solid line) provided in the benchmark problem. The resulting controller reduces $\mathcal{S}_{yp}$ considerably around the first and second resonant modes without violating the constraint on the input sensitivity function $\mathcal{S}_{up}$.

The controller obtained in simulation is implemented on the experimental system and Fig. 6 shows the corresponding sensitivity functions. The output sensitivity function estimated by the spectral density method slightly violates the constraints at some frequencies. This can be explained by the fact that the model used in the control design does not describe the experimental system very well around those frequencies. Despite this fact, satisfactory experimental results are obtained using the 2nd-order controller.

5 Conclusions

This paper presents an adaptation of the iterative correlation-based controller tuning scheme for the regulation problem and its application to the

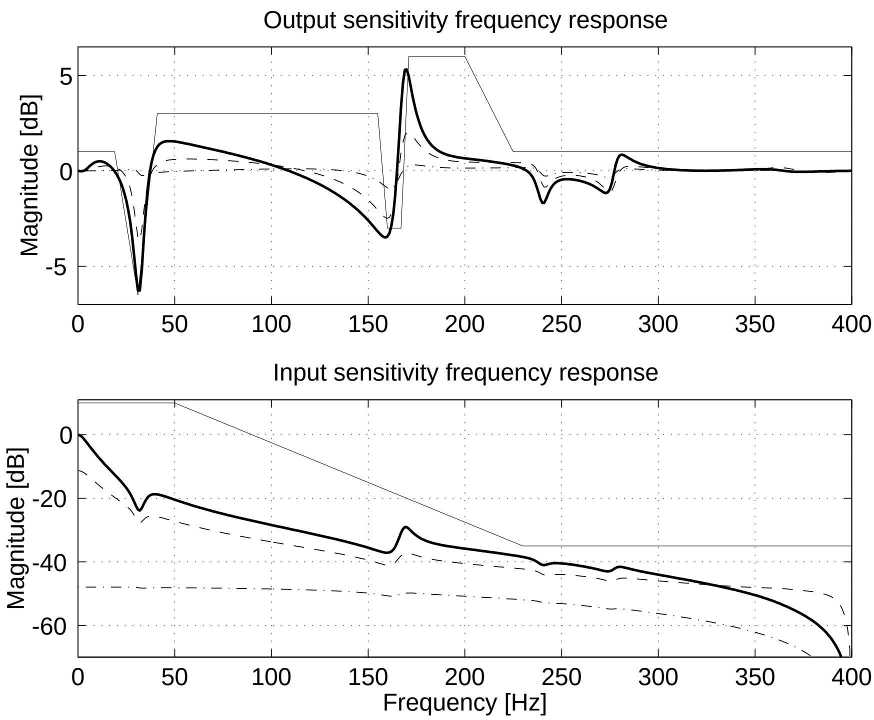

Figure 5: Output and input sensitivity functions of the closed-loop system: before tuning (dash-dot), after 3 iterations (dashed), after 8 iterations (thick solid line) and constraints (thin solid line)

"Design and optimization of restricted-complexity controllers benchmark". With the assumption that the disturbance signal is measurable, it has been shown that reducing the cross-correlation function between the disturbance signal and output of the closed-loop system can be used as an objective for the restricted-complexity controller tuning. This approach can also be used for systems where the disturbance signal is not measurable but there is the possibility of injecting a test signal as an artificial disturbance. Although the proposed algorithm uses data collected in the time domain, the frequency-domain analysis of the criterion shows how the control specifications given in the form of constraints on the sensitivity functions can easily be handled. The resulting restricted-order controller provides satisfactory performance both in simulation and real-time application for the active suspension system of LAG.

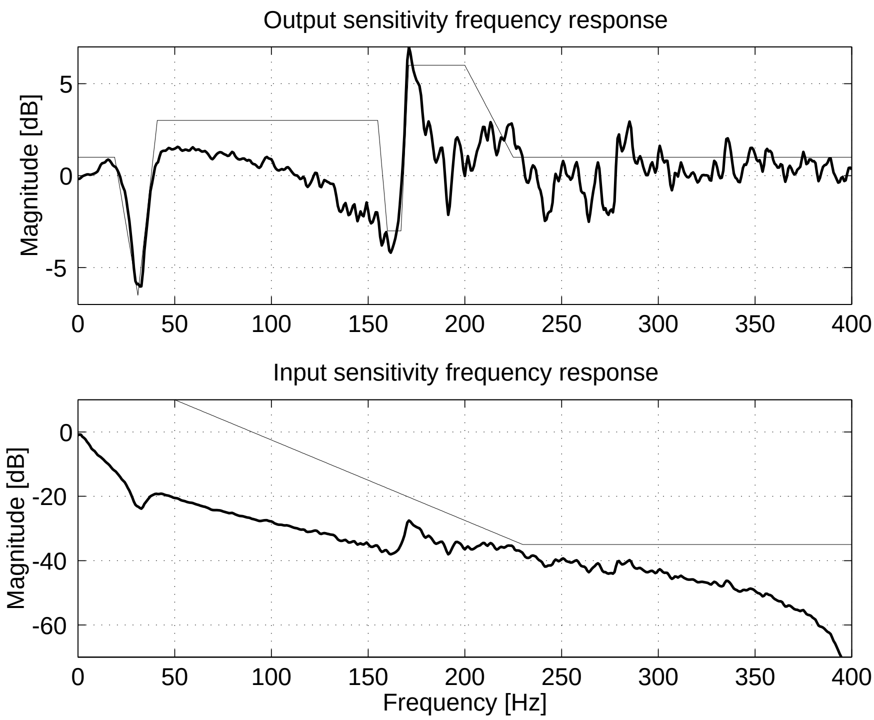

Figure 6: Closed-loop output and input sensitivity function estimates obtained with data collected on the real plant using the final controller (thick line); and constraints (thin line)

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