PRECISE & RAPID UNSTEADY PRESSURE TRANSDUCER SIGNAL PROCESSING USING A TRANSFER FUNCTION MODELING TECHNIQUE

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ABSTRACT

The authors have applied a transfer function modeling technique to the calibration of unsteady pressure transducers used for unsteady flow measurements in turbomachinery. This modeling technique has shown to be faster and more precise than previously used curve fitting techniques. In this article, the authors present the theory applied for this identification and give an explicit example of its application and advantages.

INTRODUCTION

Transfer Functions have been used since the beginning of the 19th century to model various systems for stability analysis, prediction, simulation and control purposes. Applied first to mechanical systems, it is now a common modeling technique used in numerous fields such as electronics, chemistry, economics, and neural networks.

Unsteady pressure measurements in turbomachinery are commonly conducted with the use of unsteady pressure transducers. The signals delivered by these piezo-resistive sensors require amplification to meet the range of the data acquisition devices. In order to do so, analog electronic amplifiers are mostly used. Often combined with other signal conditioning features such as filtering (notch, low-pass), the effect of this electronic conditioning on the input signal varies in the frequency domain. In order to reconstruct precisely the true pressure signal from the acquired signal, an accurate description of the transducer's signal amplification chain is necessary. This identification can be conducted by feeding a generated sine sweep signal, with the desired bandwidth and amplitude into the electronic amplification system and measuring the frequency response of the output with a dynamic signal analyzer (Periodic Chirp Response Measurement).

Traditional techniques used to account for the effect of the amplifiers on the source signal consist of extracting the main harmonics (harmonic signal analysis) and getting the corrections corresponding to these discrete frequencies by interpolation on the Periodic Chirp Response results (amplitude and phase versus frequency).

As part of the Brite-Euram "Aeromechanical Design of Turbine Blades" project (ADTurB I), unsteady blade surface measurements were conducted on an axial turbine cascade in the Non-Rotating Annular Test Facility of the "Laboratoire de Thermique Appliquée et de Turbomachines" (LTT) of the Swiss Federal Institute of Technology in Lausanne (EPFL). Two main sources of unsteady pressures in turbomachinery were studied separately and then in combination for different configurations and flow conditions:

(1) The influence of blade motion (Controlled Vibration Measurements).
(2) The influence of upstream generated gusts (Gust Response Measurements)

NOMENCLATURE

DAQ	Data Acquisition
TF	Transfer Function
a,b,c,d,e,f	complex parameters
$\tilde{a},\tilde{b},\tilde{c},\tilde{d},\tilde{e},\tilde{f}$	complex parameters
f	frequency
G(s)	transfer function
g(t)	impulse response
i	index
j	$j=\sqrt{-1}$
$\mathfrak{L}$	Laplace transform
n	number of samples
s	complex variable
u(t)	input time signal
$\boldsymbol{R}$	regression matrix
U	input (frequency domain)
$\boldsymbol{U}$	input Vector
v	degree of TF numerator
w	degree of TF denominator
y(t)	output time signal
Y	output (frequency domain)
$\hat{Y}$	output estimator
$\boldsymbol{Y}$	output vector
$\hat{\boldsymbol{Y}}$	output estimator vector
$\boldsymbol{\varepsilon}$	absolute error vector
$\boldsymbol{\vartheta}$	parameter vector
$\boldsymbol{\vartheta}^*$	optimal parameter vector
ω	pulsation
*	convolution

SYSTEM IDENTIFICATION

Both aerodynamic excitation sources presented in the introduction were tested with a common excitation frequency of 276 [Hz]. Traditional testing of (1) yields in most cases sinusoidal pressure fluctuations requiring only the characterization of the unsteady pressure transducer's amplification chain around a single frequency. On the other hand, the newly installed gust generator consisting of rotating elliptical struts involve four to seven harmonics requiring the same identification for a bandwidth of 250 to 2500 [Hz].

This characterization has been conducted using a 10 [mV] sine sweep input signal generated by a HP35660A dynamic signal analyzer. The frequency response of the system consists of 400 samples from 75 to 3275 [Hz] (every 8 [Hz]) and is represented in figure 1. The effect of the 4 [kHz] low-pass filter is clearly visible and shows the need for a precise identification of the system, especially if one considers the importance of the phase lag introduced over the whole measured bandwidth.

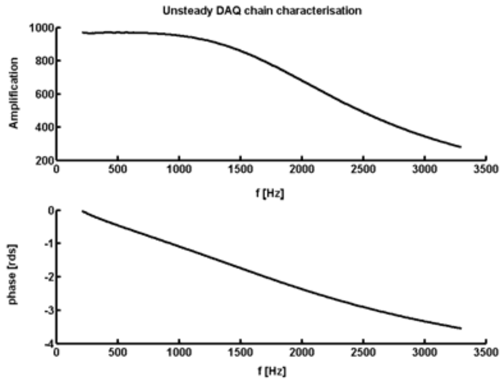

Figure 1 Unsteady DAQ chain characterization

TRANSFER FUNCTION MODELING

Let us consider the unsteady pressure transducer DAQ chain has a single input, *u(t)*, single output, *y(t)*, system. If we suppose small excitations around a nominal frequency (linear system hypothesis) we can write the output *y(t)* as a function of the input signal *u(t)* and of the impulse response *g(t)* as:

$$y(t)=g(t)*u(t)=\int u(t)\cdot g(t-\tau)\,d\tau \qquad (1)$$

Taking the Laplace Transform of (1) will allow a considerable simplification of the mathematical operations:

$$\mathfrak{L}\left[y(t)\right]=\mathfrak{L}\left[u(t)\right]\cdot\mathfrak{L}\left[g(t)\right] \qquad (2)$$

Where $G(s)=\mathfrak{L}\left[g(t)\right]$ is known as the transfer function of the system. (2) becomes:

$$Y(s)=U(s)\cdot G(s) \quad\Rightarrow\quad G(s)=\frac{Y(s)}{U(s)} \qquad (3)$$

G(s) is always a rational fraction in *s* where the degree of the denominator *w* is greater than the degree of the numerator *v* in order to have a causal transfer function. Several combinations of *v* and *w* have been tested starting with low values. The first qualitatively representative combination of *v* and *w* are values of 2 and 3 respectively, giving the following model:

$$G(s)=\frac{\tilde{a}s^2+\tilde{b}s+\tilde{c}}{s^3+\tilde{d}s^2+\tilde{e}s+\tilde{f}} \qquad (4)$$

Where $\tilde{a},\tilde{b},\tilde{c},\tilde{d},\tilde{e}$ and $\tilde{f}$ are the complex parameters of *G(s)* to be determined.

If we consider only harmonic inputs and neglect the transient behavior of the system (i.e.: after the transient response has damped out) then the harmonic response is sufficient for this study. We can therefore replace s by $j\omega$ in (4), where $\omega = 2\pi f$ is a given pulsation and f the corresponding frequency:

$$G(j\omega) = \frac{\tilde{a}(j\omega)^2 + \tilde{b}(j\omega) + \tilde{c}}{(j\omega)^3 + \tilde{d}(j\omega)^2 + \tilde{e}(j\omega) + \tilde{f}} \quad \textbf{(5)}$$

To avoid numerical instability due to large pulsation values ω, (5) can be rewritten as:

$$G(j\omega_i) = \frac{\tilde{a} + \dfrac{\tilde{b}}{j\omega} + \dfrac{\tilde{c}}{(j\omega)^2}}{j\omega + \tilde{d} + \dfrac{\tilde{e}}{j\omega} + \dfrac{\tilde{f}}{(j\omega)^2}} \quad \textbf{(6)}$$

Defining the new complex parameters *a, b, c, d* and f by dividing the numerator and the denominator by $\tilde{d}$ yields the final form of the model to be used:

$$G(j\omega) = \frac{a + \dfrac{b}{j\omega} + \dfrac{c}{(j\omega)^2}}{1 + d \cdot j\omega + \dfrac{e}{j\omega} + \dfrac{f}{(j\omega)^2}} \quad \textbf{(7)}$$

Where:

$$\begin{aligned} a &= \frac{\tilde{a}}{\tilde{d}}; \quad b = \frac{\tilde{b}}{\tilde{d}}; \quad c = \frac{\tilde{c}}{\tilde{d}}; \\ d &= \frac{1}{\tilde{d}}; \quad e = \frac{\tilde{e}}{\tilde{d}}; \quad f = \frac{\tilde{f}}{\tilde{d}} \end{aligned} \quad \textbf{(8)}$$

The parameters appear linearly in the input-output relationship. Only the coefficients vary nonlinearly with respect to the frequency (i.e. it is a linear combination of non-linear functions). Let us proceed with the following regrouping:

Using equation (3), (7) becomes:

$$\begin{aligned} Y(j\omega) = &-d \cdot (j\omega) \cdot Y(j\omega) - \frac{e}{j\omega} \cdot Y(j\omega) \\ &- \frac{f}{(j\omega)^2} \cdot Y(j\omega) + a \cdot U(j\omega) + \\ &\frac{b}{j\omega} \cdot U(j\omega) + \frac{c}{(j\omega)^2} \cdot U(j\omega) \end{aligned} \quad \textbf{(9)}$$

Let us introduce the estimated Y value based on this relationship:

$$\begin{aligned} \hat{Y}(j\omega) = &-d \cdot (j\omega) \cdot Y(j\omega) - \frac{e}{j\omega} \cdot Y(j\omega) \\ &- \frac{f}{(j\omega)^2} \cdot Y(j\omega) + a \cdot U(j\omega) + \\ &\frac{b}{j\omega} \cdot U(j\omega) + \frac{c}{(j\omega)^2} \cdot U(j\omega) \end{aligned} \quad \textbf{(10)}$$

During the identification of the unsteady pressure transducer DAQ chain, we collected measured values for n different frequencies. Therefore we can gather all input and output signals in vectors:

$$\boldsymbol{Y} = \begin{bmatrix} Y(j\omega_1) \\ \vdots \\ Y(j\omega_i) \\ \vdots \\ Y(j\omega_n) \end{bmatrix} \qquad \boldsymbol{U} = \begin{bmatrix} U(j\omega_1) \\ \vdots \\ U(j\omega_i) \\ \vdots \\ U(j\omega_n) \end{bmatrix} \quad \textbf{(11)}$$

Equation (10) can be rewritten for all samples using matrix algebra:

$$\hat{\boldsymbol{Y}} = \boldsymbol{R} \cdot \vartheta \quad \textbf{(12)}$$

Where $\boldsymbol{R}$ is the regression matrix and ϑ the vector of parameters defined as follows:

$$\begin{aligned} \boldsymbol{R} = \Bigg[\boldsymbol{U} \quad \frac{\boldsymbol{U}}{j\omega_i} \quad \frac{\boldsymbol{U}}{(j\omega_i)^2} \quad \cdots \\ \cdots \quad -\boldsymbol{Y} \cdot j\omega_i \quad -\frac{\boldsymbol{Y}}{j\omega_i} \quad -\frac{\boldsymbol{Y}}{(j\omega_i)^2} \Bigg] \end{aligned} \quad \textbf{(13)}$$

$$\vartheta = \begin{bmatrix} a & b & c & d & e & f \end{bmatrix}^T \quad \textbf{(14)}$$

It is clear that there exists a mismatch between the estimated values $\widehat{\boldsymbol{Y}}$ and the measured ones $\boldsymbol{Y}$. A good model (i.e. good parameter vector ϑ) should reduce this mismatch as much as possible. Therefore let us introduce the optimal parameter vector ϑ^* based on a least square criterion and defined as:

$$\vartheta^* = \arg\min_{\vartheta} \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(\vartheta) \right)^T \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(\vartheta) \right) \qquad \textbf{(15)}$$

The optimum is easily computed using (12), see (Kay 1988):

$$\vartheta^* = \left(\boldsymbol{R}^T \boldsymbol{R} \right)^{-1} \boldsymbol{R}^T \boldsymbol{Y} \qquad \textbf{(16)}$$

The estimated values can be computed if $\boldsymbol{R}^T \boldsymbol{R}$ is invertible. The absolute error between the prediction model and the measured values can be written as:

$$\varepsilon = \left| \boldsymbol{Y} - \widehat{\boldsymbol{Y}} \right| \qquad \textbf{(17)}$$

For the example given in figure 1, the above explained technique applied to the model of (7) yields the following results using Matlab® 6.1:

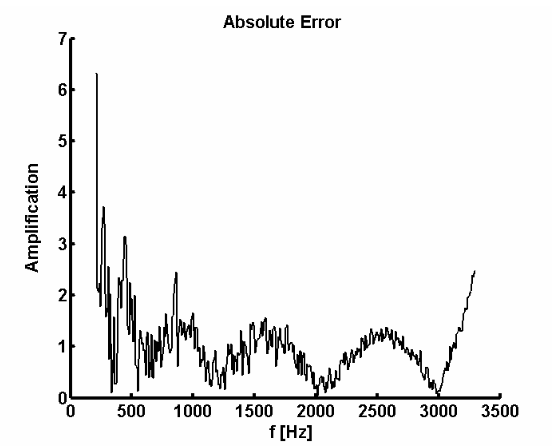

Figure 2 Absolute error of TF model in ‰

This error is based on a nominal amplification of 1000 and represents less than 0.3 % error on the desired bandwidth. Compared to previous methods based on polynomial curve fitting of figure 1 results, this technique and model order turns out to be five times more precise. Furthermore, the measured unsteady pressure signal can be corrected directly by multiplication with the inverse of the transfer function model allowing rapid corrections, especially if the signal's noise isn't filtered through harmonic signal analysis.

A further advantage of this modeling technique is that in similar conditions the same model can be used again and the parameters quickly computed using for example matrix algebra in Matlab®.

Difficulties may arise during the inversion of $\boldsymbol{R}^T \boldsymbol{R}$. In our example this matrix was close to singular (inversion may be inaccurate) but the parameters found produced the results of figure 2. In cases where $\boldsymbol{R}^T \boldsymbol{R}$ can not be inverted, it is possible to introduce a positive definite scaling matrix Σ in equation (15):

$$\vartheta^* = \arg\min_{\vartheta} \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(\vartheta) \right)^T \Sigma \left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}(\vartheta) \right) \qquad \textbf{(17)}$$

(16) becomes:

$$\vartheta^* = \left(\boldsymbol{R}^T \Sigma \boldsymbol{R} \right)^{-1} \boldsymbol{R}^T \Sigma \boldsymbol{Y} \qquad \textbf{(18)}$$

A judicious choice of the eigenvalues of Σ can enable the inversion of $\boldsymbol{R}^T \Sigma \boldsymbol{R}$, allowing the computation of the estimators.

Other difficulties may appear for low frequencies where the phase angles become negative. In this case, instabilities are produced probably due to the violation of the linearity assumption. This problem can be avoided by rejecting these low frequencies data samples.

CONCLUSIONS

The use of transfer functions for the modeling of commonly used unsteady pressure transducer amplification chain has been a success. It has been shown that this technique is faster and more precise than previously used methods and is particularly well suited for on-line corrections on digitized unsteady pressure signals.

REFERENCES

Kay S. M., 1988, *"Modern Spectral Estimation"*, Prentice Hall, ISBN 0-13-598582-X.

Nowinski, M., 1999, *"Experimental Investigation of an Oscillating Airfoil in the Presence of Downstream Generated Gusts"*, Communication du Laboratoire de Thermique Appliquée et de Turbomachines de l'EPFL, No. 30.

Oppenheim, A. V., Schafer, R. W., Buck, J. R., 1999, *"Discrete-Time Signal Processing"*, Prentice-Hall, ISBN 0-13-083443-2.