Data Association in Clutter with An Adaptive Filter

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Abstract – [1] *This paper presents an adaptive filter for tracking targets in clutter. The filter employs a scale factor, which accounts for the target unpredictability at any time as estimated from the available data. The adaptive approach, in which the gain is adapted according to changing target dynamics, is used in conjunction with a widely accepted data association routine called probabilistic data association (PDA) to form the adaptive probabilistic data association filter ($APDAF$). Performance comparison between the PDA and $APDA$ filters is demonstrated through simulations.*

Keywords: Tracking, data association, estimation.

1   Introduction

The performance of a tracking algorithm is mainly governed by the performance of the state estimator used. The Kalman filter is the traditional and the most widely used state estimator in target tracking applications. The filter is the general solution to the recursive linear minimum mean square estimation problem. It will minimize the mean square error between the estimates and the actual target dynamics as long as the target dynamics are accurately modeled.

However, if the tracking is attempted in clutter, i.e., where false returns are present, the standard Kalman filter, on its own will not be sufficient to produce reliable estimations. There are several approaches to be used in collaboration with the Kalman filter to address the false return problem. The PDA approach [1] is one of the widely accepted approaches to be used with the Kalman filter in tracking targets in clutter.

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[1]Part of this work was carried out at Laboratoire d'Automatique of Swiss Federal Institute of Technology, Switzerland.

This paper is organized as follows. The next section explains the motivation behind using an adaptive Kalman filter in place of a standard one where section 2 outlines the PDA approach and how to combine the adaptive filter with it. In section 3 some comparative simulation results are presented to highlight the performance improvement obtained by using an adaptive filter. The paper ends with some concluding remarks.

2   An Adaptive Filter, Why?

The process of state estimation in the Kalman filter comprises two parallel cycles, namely, *i*) estimation of the state and *ii*) estimation of the state covariance, Fig 1. The final estimation of the state is found from the predicted state, innovation and Kalman gain. The Kalman gain is the 'ratio' of the state covariance to the innovation covariance and can be considered as a correction factor on the final estimate. From a frequency domain viewpoint the magnitude of the Kalman gain determines the bandwidth and response speed of the filter. A small gain value produces a substantial noise reduction when the target is not manoeuvring and a large gain value gives a fast response to changes in the target's dynamics, providing the filter with a larger bandwidth to cover maneuvers. Basically, it can be said that the performance of the Kalman filter is determined by the size of the Kalman gain.

While the Kalman gain plays an important role in estimating the target's state, it is independent of the measurements taken. As can be seen from Fig 1 the right hand side frame is not affected by the observations, in fact, given the process and measurement noise covariances as a function of time, the Kalman gain can be computed off line. During the recursive estimation of the state the Kalman gain
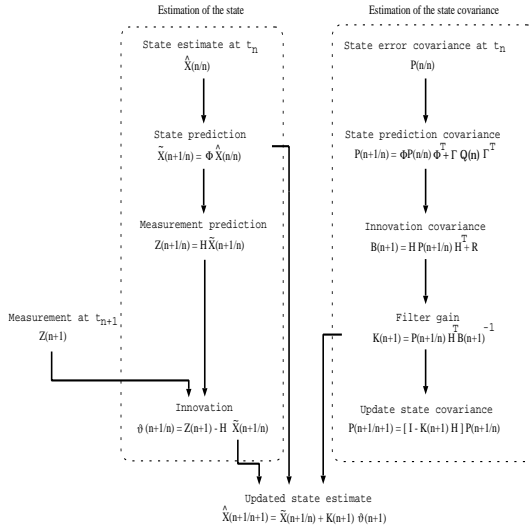
Figure 1: State estimation in the standard Kalman filter

reaches a steady-state value determined by the pre-selected and assumed constant process and measurement noise covariances, Q and R respectively. The use of constant values of the covariances imposes a major restriction on the filter's performance, because if the pre-selected process noise covariance level is not appropriate the correction, by the Kalman gain, on the state prediction will not be suitable and large estimation errors will develop. An intuitive solution to this problem is to adaptively adjust the process noise covariance and thus the Kalman gain, according to estimated changing target dynamics. The simplest method to achieve this is to establish a maneuver detection scheme then modify the process noise covariance following the maneuver detection [2]. The main disadvantage of this approach is the time delay in maneuver detection and changing of the filter. In [3] an adaptive Kalman filtering technique was suggested where the process noise covariance was estimated by means of the difference between the expected prediction error variance and the measurement noise variance. Alternatively, multiple model algorithms can be used, to provide good coverage by using different levels of process noise covariance in each model, at the cost of an increased computational burden and complexity, but even a large number of models does not guarantee a better coverage [4].

The adaptive Kalman filter [5], that will be used in conjunction with the PDA approach, adjusts the gain level of a second order Kalman filter for tracking maneuvering targets. The method introduces a scale factor which represents the current magnitude of process noise covariance, in other words target unpredictability, at time n as estimated from the available data. The aim of the adaptive Kalman filter is to take observations into account while estimating the state covariance (the right hand frame in Fig 1), so that the Kalman gain level is adaptively adjusted in accordance

with the changing target dynamics. With this feature, the filter yields an online gain adjustment without a delay while keeping the computational burden and the complexity at a minimum.

3 Adaptive Rescaling of Process Noise

For the adaptive Kalman filter design, suppose that the target dynamic equation is given by,

$$X(n+1) = \Phi X(n) + \Theta(n)\Gamma W(n) \qquad (1)$$

where $X(n)$ is the target state vector, Φ is the known transition matrix, $\Theta(n)$ is the scale factor that represents the current magnitude of the process noise, Γ is the known disturbance transition matrix and $W(t)$ is the unknown zero-mean Gaussian process noise $\sim N(0, Q)$ independent of previous events. The measurements are in the form of linear combinations of the system state variables, corrupted by uncorrelated noise. Thus, the measurement vector is modelled as

$$Z(n) = HX(n) + V(n) \qquad (2)$$

where H is the measurement matrix and $V(n)$ is white Gaussian measurement noise $\sim N(0, R)$, also independent of previous data. Note that it is assumed that $W(n)$ and $V(n)$ are mutually uncorrelated. At time n, assume that the prior distribution of $X(n)$ for the next transition, using Eqs 1 and 2, is

$$X(n) \sim N(\hat{X}(n), P(n)) \qquad (3)$$

This is based on the previous data and depends on the scale factors $\Theta(0), \Theta(1), ..., \Theta(n-1)$. For the next stage, $\Theta(n) > 0$ is chosen to specify the model for $X(n+1), Z(n+1)$ as viewed through the sensor.

Note that the normality of Eq 3 and any positive specification of $\Theta(n)$ will ensure normality of the posterior distribution $X(n+1) \sim N(\hat{X}(n+1), P(n+1))$ by the standard properties of multivariate normal distributions for a linear/Gaussian model. The calculations lead to the following results.

At $n+1$, before observing $Z(n+1)$ we have

$$X(n+1) \sim N(\tilde{X}(n+1), M(n+1))$$

where

$$\tilde{X}(n+1) = \Phi\hat{X}(n) \qquad (4)$$
$$M(n+1) = \Phi P(n)\Phi^T + \Theta^2(n)\Gamma Q\Gamma^T \qquad (5)$$

Thus, the predictive distribution for the new measurement given by Eq 2 is normal with mean vector

$$\tilde{Z}(n+1) = H\tilde{X}(n+1) = H\Phi\hat{X}(n)$$

covariance matrix

$$S(n+1) = HM(n+1)H^T + R \qquad (6)$$

and the innovation is

$$\nu(n+1) = Z(n+1) - H\tilde{X}(n+1) \qquad (7)$$

Note that, a-priori, the expectation of the sum of squares $|\nu(n+1)|^2 = \nu(n+1)^T\nu(n+1)$ is the sum of the diagonal elements of $S(n+1)$. By Eqs 5 and 6 this expectation contains a term $\propto \Theta^2(n)$. Later, the observed difference $|\nu(n+1)|^2 - \sum_i[S(n+1)]_{ii}$ will be used to construct the new scale factor $\Theta^2(n+1)$.

Given the observation $Z(n+1)$, the posterior distribution of $X(n+1)$ is constructed by standard methods. It is normal with mean vector

$$\hat{X}(n+1) = \tilde{X}(n+1) + K(n+1)\nu(n+1) \qquad (8)$$

where

$$K(n+1) = M(n+1)H^T S^{-1}(n+1) \qquad (9)$$

The new covariance matrix is

$$P(n+1) = \{I - K(n+1)H\}M(n+1) \qquad (10)$$

This completes the transition from one state of information $(\hat{X}(n), P(n))$ to the next $(\hat{X}(n+1), P(n+1))$.

3.1 Construction of $\Theta(n+1)$

By using Eqs 5 and 6,we have

$$S(n+1) = H\Phi P(n)\Phi^T H^T + R + \Theta^2(n)\,H\Gamma Q\Gamma^T H^T \qquad (11)$$

Assume that ψ^2, η^2 and δ^2 are the sums of the diagonal elements of $S(n+1)$, $[H\Phi P(n)\Phi^T H^T + R]$ and $[H\Gamma Q\Gamma^T H^T]$ respectively, then we have;

$$\psi^2(n+1) = \eta^2(n+1) + \Theta^2(n)\delta^2(n+1) \qquad (12)$$

Roughly speaking, if the observed sum of squares $|\nu(n+1)|^2$ is close to $\psi^2(n+1)$, then the filter is reasonably well focused on the target, so $\Theta(n)$ is already set at an appropriate level. If we let a, b, c be fixed constants, where $a \geq 0$, $b \geq 0, c \geq 0$ and $a + b + c = 1$, then the following sequential relationship offers a possible scheme for constructing $\Theta(n+1)$,

$$\begin{aligned}\Theta^2(n+1) &= max\{a\Theta^2(0) + b\Theta^2(n) + \\ & \quad c\,(\frac{|\nu(n+1)|^2 - \eta^2(n+1)}{\delta^2(n+1)}), 0\}\end{aligned} \qquad (13)$$

If $|\nu(n+1)|^2 = \psi^2(n+1)$ then

$$\Theta^2(n+1) = max\{a\Theta^2(0) + (b+c)\,\Theta^2(n), 0\} \qquad (14)$$

The right hand side of Eq 13 is constrained to remain non-negative. The constant c can be regarded as a sensitivity parameter which can be used to adjust the adaptive behavior

of the filter. The initial value of the scale factor was chosen as $\Theta(0) = 1$ so that a gives some weight to the original scale factor. In the simulations a small value of c namely 0.05 was used along with the values $a = 0.8$ and $b = 0.15$.

The idea of dynamically changing the process noise covariance goes back a few decades and various ways of handling the change have been proposed. For instance in [6] a $fudge factor$ was suggested for a similar purpose, however, it has to be noted that Eq 13 is quite different in principle.

4 The Probabilistic Data Association Filter (PDAF)

The $(PDAF)$ is simply a Kalman filter which is used in conjunction with the PDA approach in order to take the measurement origin uncertainty into account. The filter assumes that the target is detected (perceived) and its track has been initialized. At each sampling interval a validation gate is set up. The measurement originating from the target of interest can be among the possible several validated measurements, hence, the track update is done by taking the weighted sum of all observations within the gated region in a probabilistic manner as will be explained later in this section. Measurements outside the validation region are assumed to have originated from false alarms or clutter. The $PDAF$ uses only the latest measurements and the past is summarized approximately by making the following assumption:

$$p[X(n)|Z^{n-1}] = N[X(n); \hat{X}(n), P(n)] \qquad (15)$$

which states that the state is assumed to be normally distributed (Gaussian) according to the latest estimate and covariance matrix. Starting from this point, one cycle of the state estimation is described as follows.

Let $Z(k)$ contain measurements from the elliptical validation region for the track at time n

$$Z(k) = \{z_m(n), \quad m = 1, 2, \ldots, m_k\} \qquad (16)$$

where m_k is the number of validated measurements in that region. Also, the cumulative set of validated measurements up to scan n is denoted by Z^k. Using the nonparametric version of the $PDAF$ [1] the validated tracks are associated to the track. The combined target state estimate is obtained as

$$\hat{x}(n) = \sum_{m=0}^{m_k} \beta_m(n)\hat{x}_m(n) \qquad (17)$$

where $\beta_m(n)$ is the probability that the mth validated measurement is correct and $\hat{x}(n)$ is the updated state condi-

tioned on that event. The conditionally updated states are given by

$$\hat{x}(n) = \hat{x}(n-1) + K_m(n)\nu_m(n) \qquad (18)$$

where $K_m(n)$ is the filter gain and $\nu_m(n)$ is the innovation associated with the mth validated measurement. The association event probabilities $\beta_m(n)$ are given by

$$\beta_m = \frac{e_m}{b + \sum_{j=1}^{m_k} e_j}, \quad m = 1, \ldots, m_k \qquad (19)$$

$$\beta_0 = \frac{b}{b + \sum_{j=1}^{m_k} e_j}, \quad m=0 \qquad (20)$$

where

$$e_m = exp\{\frac{1}{2}\nu_i'(n+1)S^{-1}(n)\nu_i(n+1)\} \qquad (21)$$

$$b = m_k \frac{1 - P_D P_G}{P_D V(n)} \qquad (22)$$

and P_D is the probability of detection of a target originated measurement, V is the volume of the validation gate. Then the updated state is

$$\hat{X}(n+1) = \tilde{X}(n+1) + K(n+1)\nu(n+1) \qquad (23)$$

where

$$\nu_{com}(n+1) = \sum_{i=1}^{m_k} \beta_i(n+1)\nu_i(n+1) \qquad (24)$$

is the combined innovation. One can quite rightly argue that S given in Eq 11 is not the covariance of the combined innovation. In fact covariance of the combined innovation is smaller than S by a certain factor, that is

$$Cov(\nu_{com}(n+1)) = \sum \beta_i^2 S(n+1)$$

However, this is not critical and the difference can be ignored.

The error covariance associated with the updated state estimate is

$$P(n+1) = \beta_0(n+1)M(n+1) + [1 - \beta_0(n+1)] \times$$
$$P^c(n+1) + \tilde{P}(n+1) \qquad (25)$$

where

$$\tilde{P}(n) = K(n+1)[\sum_{i=1}^{m_k} \beta_i(n+1)\nu_i(n+1)\nu_i'(n+1) -$$
$$\nu(n+1)\nu'(n+1)]K'(n+1) \qquad (26)$$

and

$$P^c(n+1) = [I - K(n+1)H(n+1)]M(n+1) \qquad (27)$$

is the covariance of the updated state. Prediction of the state, its covariance, measurement to time $n+1$ and the innovation covariance are calculated as in the standard Kalman filter.

Although, unlike the standard Kalman filter, being a nonlinear filter the $PDAF$ takes the measurements into account in order to address the measurement origin uncertainty issue while estimating the current state covariance, it is still somehow oblivious to the changes in the target dynamics. In other words the target unpredictability/maneuverabilty is still unaccounted for. This is exactly why the standard Kalman filter in the $PDAF$ is replaced with an adaptive one to establish an adaptive $PDAF$.

4.1 Making The PDAF Adaptive

As explained in section 2, the scale factor in the adaptive Kalman filter, which accounts for the target maneuverabilty, is introduced in the predicted state covariance. Also, as mentioned above the covariance of the predicted state is calculated in the same way as it is done in the standard Kalman filter i.e., it is independent of the association event probability calculation procedure (ignoring the fact that the covariance of the combined innovation is smaller than S by a small factor). Thus, since the adaptiveness of the filter is achieved without breaking the general structure of the standard Kalman filter, the replacement of the Kalman filter, in the $PDAF$, by the adaptive Kalman filter will also keep the general structure of the $PDAF$ intact. In other words in the adaptive $PDAF$ the standard $PDAF$ structure, in general, is preserved. Moreover, the assumptions, that are valid for the $PDAF$, are still valid for the $APDAF$. On the other hand by the inclusion of the scale factor, the changed target dynamics are taken into account for the next scan resulting in an algorithm for maneuvering target tracking in clutter. In the adaptive $PDAF$ the equations given in section 3 are still valid except for the predicted state covariance which should be replaced by Eq 5. Also, the combined innovation given by Eq 24 should be used to calculate $\Theta^2(n+1)$ in Eq 13.

5 Simulation Results

The first simulated target motion is generated to perform a straight line motion in two dimensions (i.e. x-y plane) with a sampling interval of 1.0 second and is assumed to last 20 seconds. Measurements are assumed known at the origin of the Cartesian coordinates for the x-y positions of the target with a Gaussian measurement error standard deviation of 100 m used for both axes. The process noise standard deviation is taken as 5 m/s for each axis, whereas 0.9 and 0.002 values are used for the probability of detection and probability of false alarm respectively. The initial values of the state vector are as follows; $[x(0), y(0), \dot{x}(0), \dot{y}(0)] = [1km, 1km, 0.2km/s, 0.2km/s]$.

The simulated target trajectory is given in Fig 2. In the figure crosses indicate the true target positions at each sampling time, the letter d denotes a detection (naturally a detection cannot be obtained from the target of interest at all the sampling instants) and each dot represents a false return.
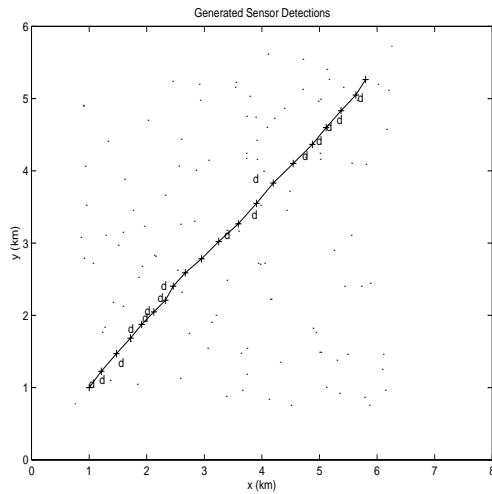

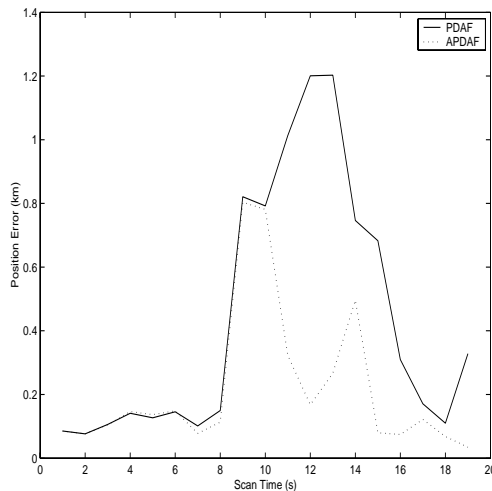Figure 2: Simulated target trajectory (straight line motion)

Figure 3: Estimated position errors

Both the $PDAF$ and $APDAF$ algorithms are configured to used the same process noise covariance as was used whilst creating the simulated target trajectory, i.e., matched filters. Fig 3 depicts the position errors produced by $PDAF$ and $APDAF$ algorithms for the given trajectory. As it can be seen from the figure a clear performance degradation is observed for both algorithms when there is lack of information about the target motion (i.e., no return) between the 9th and the 15th seconds. However, the introduction of the scale factor helps the $APDAF$ keep the position error at a lower level. Moreover, by rescaling the process noise covariance level, the increase in the position error is confined to a short space of time after which the error is pulled back to a reasonable level. This is achieved by introducing a scale factor whose change in value is given in Fig 4. Note that as depicted in Fig 4 the value of the scale factor increases as soon as there is an ambiguity about the target motion.
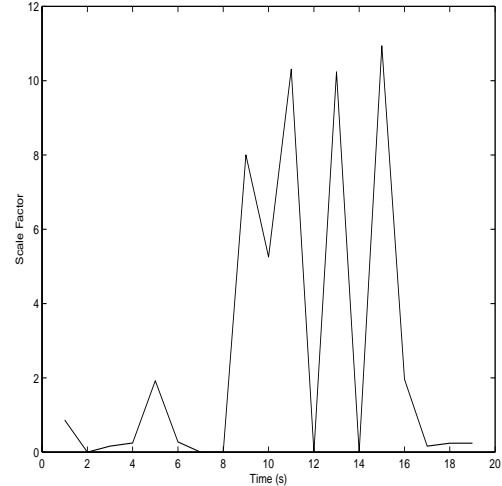

Figure 4: Change in the scale factor

Target manoeuver can also be accounted for by the scale factor. To illustrate this a second target trajectory, where the target performs coordinated turns, i.e., maneuver, has been generated to test the proposed algorithm and also compare its performance with the $PDAF$. Probability of detection and probability of false alarm values as well as the measurement standard errors are kept the same as in the first scenario and As depicted in Fig 5, the target is assumed to start its motion on a straight line and perform a turn at the $9^{t}h$ second. It then moves on a straight line a while longer before it finishes it completes its motion with a mild turn.

For this scenario a higher value of the process noise covariance has been used in the algorithms to account for the manoeuvers. The value of the process noise covariance has been chosen to be some 500 times the value that was used for the target trajectory. Estimated position errors of the algorithms are shown in Fig 6. As expected, the introduction of the scale factor helps the $APDAF$ track the target during manoeuvers better than the $PDAF$.

6   Conclusions

In this paper an adaptive data association filter for tracking targets in clutter is presented. The filter uses an adaptive Kalman filter, in which the level of process noise covariance is adaptively rescaled, in place of standard Kalman filter in the PDA filter. The performance of the proposed algorithm has been demonstrated through simulations and compared with the standard $PDAF$
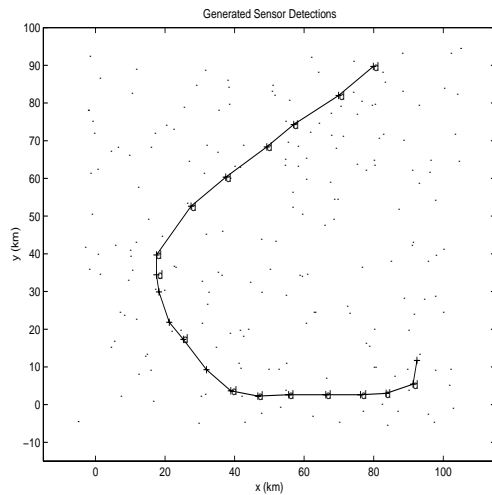
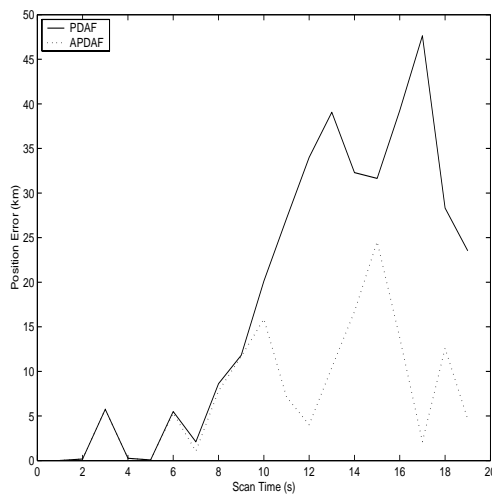Figure 5: Simulated target trajectory (maneouvering)

Figure 6: Estimated position errors for a manoeuvering target

algorithm. As the adaptive $PDAF$ accounts for the target unpredictability/maneuverability as well as the measurement origin uncertainty it outperformed the $PDAF$ in the simulated scenarios.

Acknowledgments:

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