MODELING AND MOTION PLANNING FOR A CLASS OF WEIGHT HANDLING EQUIPMENTS*

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Abstract

A unified framework for the modeling of a class of weight handling equipment (WHE) is presented. The dynamic equations are obtained using Lagrange multipliers associated to geometric constraints between generalized coordinates. This approach provides a simple way to show differential flatness for all WHEs of the class. The flatness property can then be exploited for motion planning.

1 Introduction

Many different types of weight handling equipment (WHE), and in particular cranes, are used in various industries including construction and naval transport [1]. The aim of their control [2, 6, 7, 8, 9, 11, 13] is to increase productivity and operational security by assisting the human crane operator.

Such devices can be decomposed into a fully actuated, articulated mechanical structure (e.g. a crane with a rotate platform and a boom or a gantry crane with moving bridge) with in general one or two degrees of freedom, and a hoisting system composed of ropes, winches and pulleys.

During operation, a duty cycle of a WHE consists of moving the load from its initial position to its desired final destination in its working space along a trajectory, avoiding obstacles and sway [10, 14]. This requires motion planning for the position of the load.

Our goal is to give a systematic way to obtain dynamic models of a class of WHEs and to show how to find trajectories corresponding to a duty cycle exploiting the flatness property [3, 4, 5] of the dynamic model.

To this aim, the derived model of the class of WHEs involves Lagrange multipliers associated to geometric constraints on the generalized coordinates. This contrasts with choosing a minimal number of coordinates and eliminating the constraints.

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The form of the deduced model shows that each member of the class is differentially flat and the coordinates of the load constitute all or part of the components of a flat output, depending on the number of motors. Thus the solution of the motion planning problem becomes an interpolation problem using sufficiently smooth functions (e.g. polynomials).

The remaining part of the paper is organized as follows. The next section gives three examples of WHEs. Section 3 describes the general model for two and three dimensional WHEs. Flatness of the models is proven in Section 4. Then the solution of the motion planing problem is provided in Section 5. The example of the 3D US Navy crane is studied in details in Section 6 and some simulation results on its real reduced size model are presented in Section 7.

2 Introductory examples

We will present three examples of WHEs and describe some of their common points. This will then lead to a general systematic modeling procedure to be introduced in the next section.

Figures 1 to 3 represent respectively, a 2D overhead crane, a 3D cantilever crane and a 3D US-Navy crane. The following characteristics are noteworthy:

- The load moves in a subspace of either dimension $p = 2$, such as the overhead crane of Figure 1, or $p = 3$ as portrayed in Figures 2 and 3.
- The WHE comprises the following elements:
  - A working load of mass m whose coordinates are x_i, $i = 1 \ldots p$.
  - The hoisting system composed of motors winding ropes and pulleys. The motors are supposed to be torque controlled and each one delivers a torque noted T_j where j numbers the actuator. L_j stand for the different rope lengths.
  - A fully articulated mechanical structure on which are attached the motors winching the ropes. In Figure 1 it is fixed (the rail structure), in Figures 2 and 3 it corresponds to a pole that can rotate under motor actuation.
  - A mobile or main pulley whose coordinates are x_{0i} $i = 1, \ldots, p$
  - A rail constraining the movement of the mobile pulley might (Figures 1 and 2) or might not be present (Figure 3).

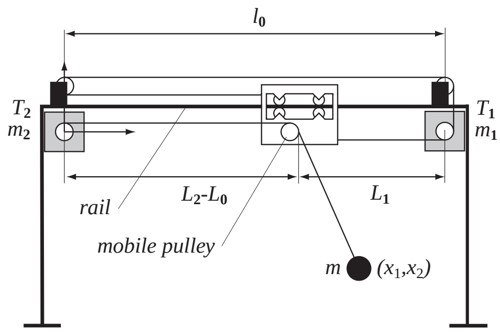

Figure 1: Overhead crane

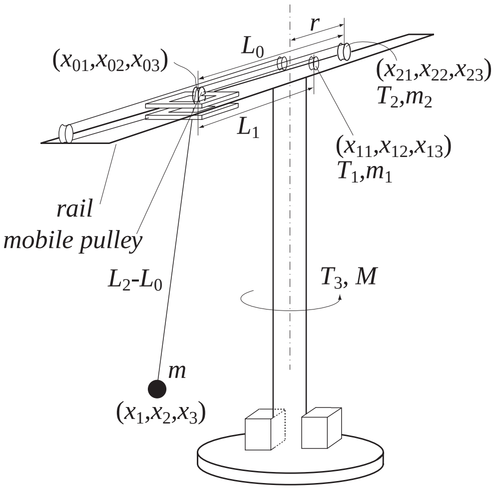

Figure 2: 3D Cantilever crane

3 General formulation for 2D and 3D WHEs

3.1 WHE description

Let p be the dimension of the working space with $p \in \{2,3\}$.

Definition 1 (WHE) *A WHE is constituted by the following elements: i) a rigid articulated actuated mechanical system with $d \in \{0,1\}$ degrees of freedom, ii) motors, iii) ropes, iv) pulleys, v) a load, and enjoys the following topographic properties:*

1. *There are $s+1$ motors fixed on the articulated structure.*
2. *There are as many ropes as motors.*
3. *A motor is linked to a pulley or to the load with a rope.*
4. *s ropes end on a unique pulley, called the mobile pulley. If $s=0$ there is no mobile pulley. Every other pulley is fixed to the structure.*
5. *There is a unique rope going through the mobile pulley and ending on the load.*
6. *Between the load and the mobile pulley there is no other pulley.*

Moreover, the following physical property is assumed. The mobile pulley moves in a manifold of dimension $n \in (p-1,p)$. This manifold is determined thanks to the constraints imposed by the ropes and by possibly restricting the mobile pulley to move along a rail. If $n=p-1$ the manifold is transversal to the gravitational field.

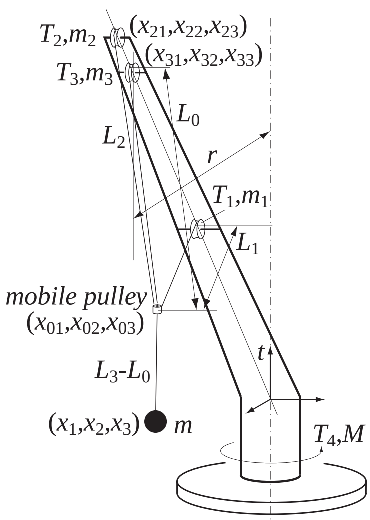

Figure 3: 3D US-Navy crane

Let us enumerate and order the fixed pulleys along each rope starting from the motor winding the rope to the mobile pulley or to the load. This is possible due to the previous definition. Denote by r_i the number of fixed pulleys along the ith rope $(i = 1 \ldots s+1)$.

3.2 WHE modeling

We present here a Lagrangian approach to the WHE modeling. Hence, we start with the choice of generalized coordinates, then express the Lagrangian and the geometric constraints. The model is given in Theorem 1 below.

Consider an inertial base frame such that its pth axis is pointed in the direction opposite to g, the gravity acceleration. We introduce the following coordinates:

1. position of the working load: $(x_1, \ldots, x_p)$,
2. position of the mobile pulley (if it exists): $(x_{01}, \ldots, x_{0p})$,
3. positions of the motors: $(x_{i1}, \ldots, x_{ip})$ for $i = 1 \ldots s+1$,
4. positions of the fixed pulleys: $(w_{ij1}, \ldots, w_{ijp})$ for $i = 1 \ldots s+1$ and $j = 1 \ldots r_i$,
5. rope lengths: L_i for $i = 1 \ldots s+1$,
6. rope length L_0 between the mobile pulley (if it exists) and the motor winching the working load.

The load mass is m and the mobile pulley mass is m_0. To each motor fixed on the structure there is a corresponding equivalent mass m_i, $i = 1 \ldots s+1$. The coordinate L_0 is not associated to any mass. We assume that the rigid body with at most one degree of freedom has an equivalent mass M and its coordinates coincide with the ones of the motor winching the load, namely $(x_{(s+1)1}, \ldots, x_{(s+1)p})$.

The reader can easily check that all fixed pulleys along each rope can be virtually eliminated by placing the corresponding motor at the position of the last pulley with an equivalent mass obtained by adding to its own equivalent mass the sum of the equivalent masses of all the pulleys removed. Each rope length is then reduced by the sum of the constant rope distances between the pulleys removed along that rope. For notational convenience, L_i's stand for these new lengths. Because of space limitations we suppose the following.

Assumptions

(A1) *The mobile pulley is present. Consequently,* $s \geq 1$*.*

(A2) *The angular velocities of the fixed pulleys are small enough to neglect their quadratic effects w.r.t. the structure. We suppose that all the motors are located on the structure along a line determined by the origin of the base frame and by the position of the motor winching the load:* $x_{ji} = \alpha_j x_{(s+1)i}$ *for* $j = 1 \ldots s$ *and* $i = 1 \ldots p$*.*

(A3) *If the mobile pulley moves along a rail, the rail coincides with the above line. Let us introduce a parameter* c *such that* $c = 1$ *if the rail is present and* $c = 0$ *otherwise.*

(A4) *The crane has no redundant actuator or motor:* $s = p - d - c$*.*

(A5) *If* $d = 1$ *the origin of the base frame is on the joint axis of the articulated mechanical structure. The articulated mechanical structure consists of either a rotational joint, to which case the joint axis is colinear with* g*, or a prismatic joint, to which case the joint axis is orthogonal to* g*. This assumption eliminates the variable* $x_{(s+1)p}$*. (The vertical position of the motor winching the load remains constant.)*

p	d	c	s	$d+s+1$
2	0	0	2	3
2	0	1	1	2
3	1	0	2	4
3	1	1	1	3

Table 1: Parameter values compatible with the assumptions

The number of actuators (i.e. the actuator of the articulated structure and the motors winiding the ropes taken together) equals to $s+d+1$. Table 1 gives the possible values of the parameters p, d, c and s compatible with the assumptions.

The Lagrangian reads:

$$\mathcal{L} = \frac{1}{2}\left(m\sum_{i=1}^{p}\dot{x}_i^2 + m_0\sum_{i=1}^{p}\dot{x}_{0i}^2 + M\sum_{i=1}^{p}\dot{x}_{(s+1)i}^2 + m_i\sum_{i=1}^{s+1}\dot{L}_i^2\right) - g(mx_p + m_0x_{0p}) \quad (1)$$

Constraints on the rope lengths are present either due to ropes terminating at the mobile pulley:

$$C_j(x_{01}, \ldots, x_{0p}, x_{(s+1)1}, \ldots, x_{(s+1)p-1}, L_j) = 0 \quad j = 1 \ldots s, \quad (2)$$

or due to the rope terminating at the working load, one for the total length between the mobile pulley and the corresponding motor, and one for the length between the load and the mobile pulley:

$$C_{s+1}(x_{01}, \ldots, x_{0p}, x_{(s+1)1}, \ldots, x_{(s+1)p-1}, L_0) = 0 \quad (3)$$

$$C_{s+2}(x_{01}, \ldots, x_{0p}, x_1, \ldots, x_p, L_0, L_{s+1}) = 0. \quad (4)$$

An additional constraint is imposed by the motion compatible with the degree of freedom of the structure. In view of the above assumptions, the following constraint exists only if $p=3$:

$$C_{s+3}(x_{(s+1)1}, \ldots, x_{(s+1)p-1}) = 0\,. \quad (5)$$

The motion of the mobile pulley along the rail (if it is present) is of the form:

$$C_{s+p+k}(x_{0k}, x_{0p}, x_{(s+1)k}) = 0 \quad k = 1 \ldots p-1. \quad (6)$$

Denote by l the total number of constraints. If (6) is present, $l = s + 2p - 1$ and $l = s + p$ otherwise.

Here, the functions $C_1, \ldots, C_l$ are quadratic functions of all their arguments. Moreover, $C_1, \ldots, C_{s+2}$ contain no product involving L_j, for $j = 0 \ldots s+1$. Their exact form is not needed in the sequel (see Remark 2 below).

In place of obtaining an explicit differential model, we prefer an implicit formulation with additional variables, known as *Lagrange multipliers*.

Theorem 1 *Assume that the constraints are independent in an open subset of the generalized coordinate space. The dynamical model associated to a WHE corresponding to Definition 1 reads:*

$$m\ddot{x}_i = \lambda_{s+2}\frac{\partial C_{s+2}}{\partial x_i} - \delta_{ip}mg \quad i = 1 \ldots p \quad (7)$$

$$m_0\ddot{x}_{0i} = \sum_{j=1}^{l}\lambda_j\frac{\partial C_j}{\partial x_{0i}} - \delta_{ip}m_0g \quad i = 1 \ldots p \quad (8)$$

$$0 = \sum_{j=1}^{l}\lambda_j\frac{\partial C_j}{\partial L_0} \quad (9)$$

$$m_i\ddot{L}_i = \sum_{j=1}^{l}\lambda_j\frac{\partial C_j}{\partial L_i} + T_i \quad i = 1 \ldots s+1 \quad (10)$$

$$M\ddot{x}_{(s+1)i} = \sum_{j=1}^{l}\lambda_j\frac{\partial C_j}{\partial x_{(s+1)i}} + F_i(T_{s+2}) \quad i = 1 \ldots p-1 \quad (11)$$

subject to Constraints (2)–(6), where $\delta_{ip} = 1$ *if* $i = p$ *and* $\delta_{ip} = 0$ *otherwise.* $T_1, \ldots, T_{s+1}$ *are the torques produced by the motors on the structure and* T_{s+2} *the one produced by the structure actuator.* $F_1, \ldots, F_{p-1}$ *are the generalized external forces depending on the torque delivered by the structure actuator.*

Proof: We compute $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = F_q + \tau_q$ where $q = (x_1, \ldots, x_p, x_{01}, \ldots, x_{0p}, L_0, L_1, \ldots, L_{s+1}, x_{(s+1)1}, \ldots, x_{(s+1)(p-1)})^T$, τ_q are the constraint forces. We have

$$F_q = (\underbrace{0, \ldots, 0}_{2p+1}, T_1, \ldots, T_{s+1}, F_1(T_{s+2}), \ldots, F_{(p-1)}(T_{s+2}))^T.$$

Taking total differential of the constraints leads to $\sum_{j=1}^{\dim q}\frac{\partial C_i}{\partial q_j}dq_j = 0$, $i = 1 \ldots l$, expressing that virtual displacements are in $\ker dC$, where dC is the matrix whose entries are $\frac{\partial C_i}{\partial q_j}$. Since the constraint forces compatible with the virtual displacements are workless we have $\sum_{i=1}^{\dim q}\tau_i dq_i = 0$. Therefore $\tau = (\tau_1, \ldots, \tau_{\dim q})$ is a linear combination of the lines of dC:

$$\tau_i = \sum_{j=1}^{l}\lambda_j\frac{\partial C_j}{\partial q_i} \quad i = 1 \ldots \dim q \quad (12)$$

and the theorem is proved. ■

Remark 1 *As announced in the introductory example, the left hand side* $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}}$ *of the model (7)–(11) is independent of the specific topography of the WHE, whereas the right hand side consists of the exterior forces* F_q *plus gravity terms* $\frac{\partial \mathcal{L}}{\partial q}$ *and the terms given by (12) which sum up the topographic specificity.*

Remark 2 *The exact form of the constraints C_j, $j = 1 \dots l$ are:*

$$C_j = \frac{1}{2}\sum_{i=1}^{p}(x_{0i} - \alpha_j x_{(s+1)i})^2 - \frac{L_j^2}{2} = 0 \quad j = 1 \dots s, \tag{13}$$

$$C_{s+1} = \frac{1}{2}\sum_{i=1}^{p-1}(x_{0i} - x_{(s+1)i})^2 - \frac{L_0^2}{2} = 0, \tag{14}$$

$$C_{s+2} = \frac{1}{2}\sum_{i=1}^{p}(x_i - x_{0i})^2 - \frac{(L_{s+1} - L_0)^2}{2} = 0, \tag{15}$$

$$C_{s+3} = \begin{cases} \frac{1}{2}\sum_{i=1}^{p-1} x_{(s+1)i}^2 - r^2 = 0 & \text{for rot. joint} \\ t_1 x_{(s+1)2} - x_{(s+1)1} t_2 = 0 & \text{for prism. joint,} \end{cases} \tag{16}$$

$$C_{s+p+k} = x_{0k} x_{(s+1)p} - x_{(s+1)k} x_{0p} = 0 \quad k = 1 \dots p-1 \tag{17}$$

where $t = (t_1, \dots, t_p)^T$ is the vector of joint axis of the articulated structure and r is the constant distance between the joint axis and the motor winching the load in the case of rotational joint. Note that these formulas are not needed to state and prove our main results.

4 Flatness

Assume that we exclude trajectories in free fall, namely such that $\ddot{x}_p = -g$, and such that $\frac{\partial C_{s+2}}{\partial x_p} \neq 0$.

Theorem 2 *WHEs defined by Definition 1 and satisfying (A1)–(A5) are differentially flat. The flat output, denoted by $\mathbf{x}$ in the sequel, can be chosen as $(x_1, \dots, x_p)$, the coordinates of the load, and $s+d+1-p$ coordinates of the mobile pulley.*

Proof: In view of the assumptions we need to distinguish the four cases of Table 1. We provide the proof for $p = 3$, the simplest cases with $p = 2$ are left to the reader. (Recall that $p = 2$ implies $d = 0$.)

Assume first that $s = 2 = p - 1$ and consider $\mathbf{x} = (x_1, \dots, x_p, x_{0p})$ as a candidate flat output. Combining the pth equation of (7) and (4) and the fact that the C_i's contain no cross-terms involving L_0, L_{s+2} by assumption, one obtains λ_{s+2} as a function of x_p, $\ddot{x}_p$ and x_{0p} since $\frac{\partial C_{s+2}}{\partial x_p} \neq 0$. Next, as long as $\lambda_{s+2} \neq 0$ which is guaranteed by the assumption that $\ddot{x}_p = -g$, the $p-1$ first Equations of (7) express the remaining coordinates $x_{01}, \dots, x_{0(p-1)}$ as functions of x_j, $\ddot{x}_j$, $j = 1 \dots p$, and x_{0p}. Next, we use the $2p+1$ equations (3)-(5), (8) and (9) to express the $2p+1$ variables L_0, L_{s+1}, $x_{(s+1)1}, \dots, x_{(s+1)p-1}$, $\lambda_1, \dots, \lambda_p$ as functions of $x_{01}, \dots, x_{0p}$, $x_1, \dots, x_p$, λ_{s+2} and derivatives up to order 2, which in turn can be expressed as functions of $\mathbf{x}$ and derivatives up to order 4. Now, by (2), one can express $L_1, \dots, L_p$ as functions of the previous ones. By (10), $T_1, \dots, T_p$ are also obtained as functions of the previous ones and derivatives up to order 6, and finally, T_{s+2} and λ_{s+3} are obtained in a similar way by (11), which proves that $\mathbf{x} = (x_1, \dots, x_p, x_{0p})$ is a flat output.

Consider now the case with $s = c = 1$ (i.e. the rail constraints (6) are present) and let $\mathbf{x} = (x_1, \dots, x_p)$ be the candidate flat output. First, we use the $2p$ equations (5)-(6) and (7) to express $2p$ variables $x_{01}, \dots, x_{0p}$, λ_{s+2}, $x_{(s+1)1}, \dots, x_{(s+1)p-1}$ in function of x_j, $\ddot{x}_j$, $j = 1 \dots p$. We proceed using Equations (3), (2), (4) and (9) to express the rope lengths L_0, L_1, L_2 and λ_{s+1} in function of $\mathbf{x}$, $\ddot{\mathbf{x}}$. Next, we use Equation (8) to obtain $\lambda_s, \lambda_{s+p+1}, \lambda_{s+p+2}$ as functions of $\mathbf{x} = (x_1, \dots, x_p)$ and their derivatives up to order 4. Finally, we use Equations (10) and (11) to express $T_1 \dots T_{s+2}$ and λ_{s+3} in function of $\mathbf{x}$ and their derivatives up to order 6 which proves that $\mathbf{x} = (x_1, \dots, x_p)$ is a flat output. ∎

5 Motion Planning

Assume that the position, velocity, acceleration, jerk and all derivatives up to $6th$ order of the flat output (including the position of the load) at the start time t_I are given by $(\mathbf{x}_I, \dot{\mathbf{x}}_I, \ddot{\mathbf{x}}_I, \dots, \mathbf{x}_I^{(5)}, \mathbf{x}_I^{(6)})$ and the desired final configuration of the flat output at time t_F is $(\mathbf{x}_F, \dot{\mathbf{x}}_F, \ddot{\mathbf{x}}_F, \dots, \mathbf{x}_F^{(5)}, \mathbf{x}_F^{(6)})$. We can construct 13 th degree polynomials,

$$x_{ic}(t) = x_{Ii} + (x_{Fi} - x_{Ii}) \sum_{j=1}^{13} a_{ji} \left(\frac{t - t_I}{t_F - t_I} \right)^j \tag{18}$$

where x_{ic} are the reference trajectories of the variables of the flat output $\mathbf{x}$. The coefficients a_{ji} are computed by solving linear equations, whose entries are combinations of the initial and final conditions. Motion planning between two different equilibria can be obtained simply by setting $\mathbf{x_I} = \bar{\mathbf{x}}_\mathbf{I}$, $\dot{\mathbf{x}}_\mathbf{I} = \ddot{\mathbf{x}}_\mathbf{I} = \dots = \mathbf{x_I^{(5)}} = \mathbf{x_I^{(6)}} = \mathbf{0}$ and $\mathbf{x_F} = \bar{\mathbf{x}}_\mathbf{F}$, $\dot{\mathbf{x}}_\mathbf{F} = \ddot{\mathbf{x}}_\mathbf{F} = \dots = \mathbf{x_F^{(5)}} = \mathbf{x_F^{(6)}} = \mathbf{0}$. The input references to be applied that generate the above trajectories are then computed using the flatness property as described in the proof of Theorem 2.

6 Example

Let us illustrate our approach by giving the resulting equations for the US-Navy Crane. The constraints can be easily obtained using Equations (13)-(17) and the notations of Figure 2.

Example: *3D US-Navy Crane.* The parameters are: $p = 3$, $d = 1$, $c = 0$, $s = 2$ and the vector of generalized coordinates is $q = \{x_1, x_2, x_3, x_{31}, x_{32}, x_{01}, x_{02}, x_{03}, L_0, L_1, L_2, L_3\}$.

The kinetic energy reads

$$W_k = \frac{1}{2}\sum_{i=1}^{3}\left(m\dot{x}_i^2 + m_0\dot{x}_{0i}^2\right) + \frac{1}{2}\sum_{i=1}^{2} m_1 \dot{x}_{1i}^2 + \frac{1}{2}m_{L_1}\dot{L}_1^2 + \frac{1}{2}m_{L_2}\dot{L}_2^2 + \frac{1}{2}m_{L_3}\dot{L}_3^2 \tag{19}$$

and potential energy

$$W_p = mgx_3 + m_0 g x_{03}. \tag{20}$$

Define the Lagrangian by $\mathcal{L} = W_k - W_p$. The constraints read:

$$\begin{aligned} \tfrac{1}{2}\left(\textstyle\sum_{i=1}^{3}(x_i - x_{0i})^2 - (L_3 - L_0)^2\right) &= 0 \\ \tfrac{1}{2}\left(\textstyle\sum_{i=1}^{3}(x_{0i} - \alpha_1 x_{3i})^2 - L_1^2\right) &= 0 \\ \tfrac{1}{2}\left(\textstyle\sum_{i=1}^{3}(x_{0i} - \alpha_2 x_{3i})^2 - L_2^2\right) &= 0 \\ \tfrac{1}{2}\left(\textstyle\sum_{i=1}^{3}(x_{0i} - x_{3i})^2 - L_0^2\right) &= 0 \\ \tfrac{1}{2}\left(x_{31}^2 + x_{32}^2 - r^2\right) &= 0 \end{aligned} \tag{21}$$

The model is given by Theorem 1:

$$\begin{aligned}
m\ddot{x}_1 &= \lambda_1(x_1 - x_{01})\\
m\ddot{x}_2 &= \lambda_1(x_2 - x_{02})\\
m\ddot{x}_3 &= \lambda_1(x_3 - x_{03}) - mg\\
m_0\ddot{x}_{01} &= -\lambda_1(x_1 - x_{01}) - \lambda_2(x_{01} - x_{11})\\
&\quad -\lambda_3(x_{01} - \alpha_2 x_{11}) - \lambda_4(x_{01} - \alpha_3 x_{11})\\
m_0\ddot{x}_{02} &= -\lambda_1(x_2 - x_{02}) - \lambda_2(x_{02} - x_{12})\\
&\quad -\lambda_3(x_{02} - \alpha_2 x_{12}) - \lambda_4(x_{02} - \alpha_3 x_{12})\\
m_0\ddot{x}_{03} &= -\lambda_1(x_3 - x_{03}) - \lambda_2(x_{03} - x_{13})\\
&\quad -\lambda_3(x_{03} - \alpha_2 x_{13}) - \lambda_4(x_{03} - \alpha_3 x_{13}) - m_0 g\\
0 &= \lambda_1(L_3 - L_0) - \lambda_4 L_0\\
m_{L_1}\ddot{L}_1 &= -\lambda_2 L_1 + T_{L_1}\\
m_{L_2}\ddot{L}_2 &= -\lambda_3 L_2 + T_{L_2}\\
m_{L_3}\ddot{L}_3 &= -\lambda_1(L_3 - L_0) + T_{L_3}\\
m_1\ddot{x}_{11} &= -\lambda_2(x_{01} - x_{11}) - \alpha_2\lambda_3(x_{01} - \alpha_2 x_{11})\\
&\quad -\alpha_3\lambda_4(x_{01} - \alpha_3 x_{11}) - \lambda_5 x_{11} - T_1 x_{12}\\
m_1\ddot{x}_{12} &= -\lambda_2(x_{02} - x_{12}) - \alpha_2\lambda_3(x_{02} - \alpha_2 x_{12})\\
&\quad -\alpha_3\lambda_4(x_{02} - \alpha_3 x_{12}) - \lambda_5 x_{12} + T_1 x_{11}.
\end{aligned}$$

One can prove, using Theorem 2 that $\mathbf{x} = (x_1, x_2, x_3, x_{03})$ is a flat output (see also [11, 12]).

7 Simulation results

We illustrate the solution of the motion planning problem for the US Navy crane modeled in the previous section. The parameters used are that of a reduced size model (1:80) realized in the authors' laboratory, depicted in Figure 4. The mass of the load is 250g.

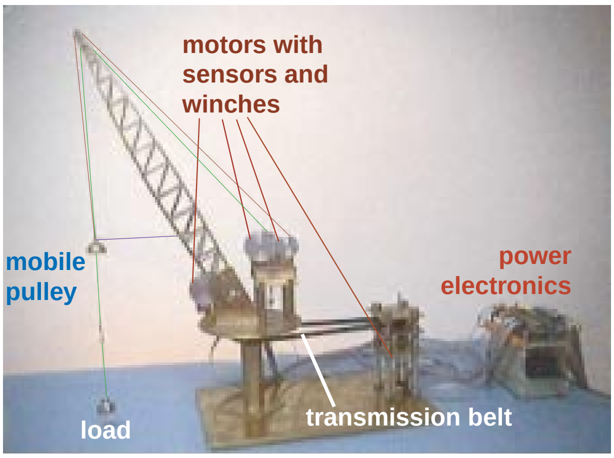

Figure 4: Reduced size model of the US Navy crane

Suppose that we wish to find an idle to idle trajectory for the load implying that the reference trajectory will have no sway. This makes the implementation of a closed-loop control law easy (not presented here), aiming to attenuate and damp the unmodeled perturbations [10].

The trajectory depicted in Figure 5 is a horizontal idle to idle displacement of the load obtained using polynomial interpolation as in Section 5. The corresponding motor torques are given in Figure 6.

The second trajectory is again an idle to idle displacement between the same points but along a parabolic trajectory avoiding an obstacle placed between the initial and final load positions. The trajectory and the generating motor torques are depicted in Figures 7 and 8.

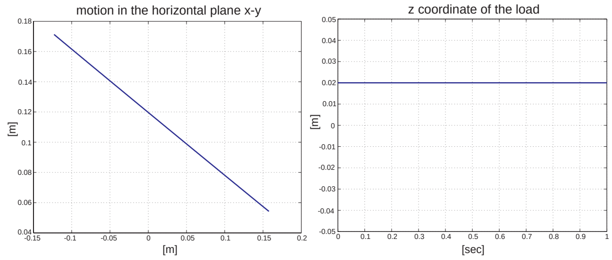

Figure 5: Horizontal displacement of the load

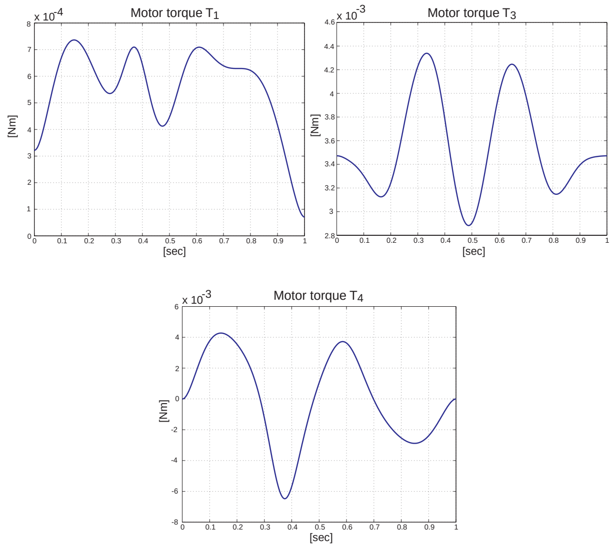

Figure 6: Motor torques generating the horizontal displacement

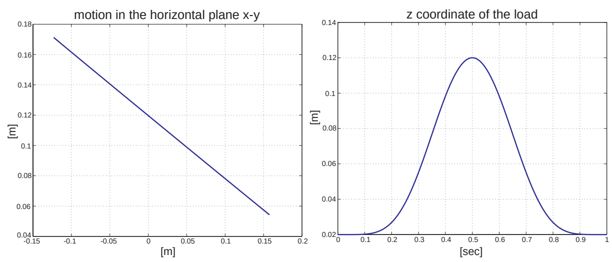

Figure 7: Parabolic displacement of the load

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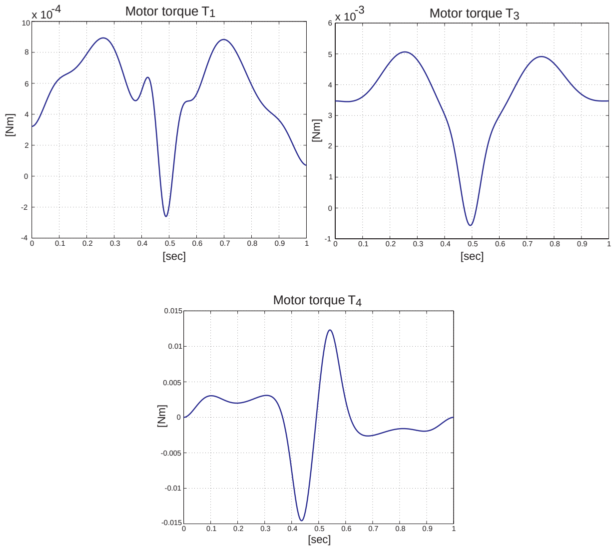

Figure 8: Motor torques generating the parabolic trajectory

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