1-1-2009

A fuzzy hierarchical decision model and its application in networking datacenters and in infrastructure acquisitions and design

Michael Khader
Walden University

Walden University

COLLEGE OF MANAGEMENT AND TECHNOLOGY

This is to certify that the doctoral dissertation by

Michael Khader

has been found to be complete and satisfactory in all respects,
and that any and all revisions required by
the review committee have been made.

Review Committee
Dr. Anthony Lolas Committee Chairperson,
Applied Management and Decision Sciences Faculty

Dr. Raghu Korrapati, Committee Member,
Applied Management and Decision Sciences Faculty

Dr.Teresa Lao, External Committee Member,
Applied Management and Decision Sciences Faculty

Chief Academic Officer

Denise DeZolt, Ph.D.

Walden University
2009

ABSTRACT

A Fuzzy Hierarchical Decision Model and Its Application in Networking Datacenters and in Infrastructure Acquisitions and Design

by

Michael Khader

M.S. Computer Science, Stevens Institute of Technology, 1991
B.S. Electrical Engineering, Polytechnic University of NY, 1983
B.S. Biomedical Engineering, Cairo University, 1980

Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
Applied Management and Decision Sciences
Information System Management

Walden University
November 2008

ABSTRACT

According to several studies, an inordinate number of major business decisions to acquire, design, plan, and implement networking infrastructures fail. A networking infrastructure is a collaborative group of telecommunications systems providing services needed for a firm's operations and business growth. The analytical hierarchy process (AHP) is a well established decision-making process used to analyze decisions related to networking infrastructures. AHP is concerned with decomposing complex decisions into a set of factors and solutions. However, AHP has difficulties in handling uncertainty in decision information. This study addressed the research question of solutions to AHP deficiencies. The solutions were accomplished through the development of a model capable of handling decisions with incomplete information and uncertain decision operating environment. This model is based on AHP framework and fuzzy sets theory. Fuzzy sets are sets whose memberships are gradual. A member of a fuzzy set may have a strong, weak, or a moderate membership. The methodology for this study was based primarily on the analytical research design method, which is neither quantitative nor qualitative, but based on mathematical concepts, proofs, and logic. The model's constructs were verified by a simulated practical case study based on current literature and the input of networking experts. To further verify the research objectives, the investigator developed, tested, and validated a software platform. The results showed tangible improvements in analyzing complex networking infrastructure decisions. The ability of this model to analyze decisions with incomplete information and uncertain economic outlook can be employed in the socially important areas such as renewable energy, forest management, and environmental studies to achieve large savings.

A Fuzzy Hierarchical Decision Model and Its Application in Networking Datacenters and in Infrastructure Acquisitions and Design

by

Michael Khader

M.S. Computer Science, Stevens Institute of Technology, 1991
B.S. Electrical Engineering, Polytechnic University of NY, 1983
B.S. (equivalent), Biomedical Engineering, Cairo University, 1978

Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
Applied Management and Decision Sciences
Information System Management

Walden University
November 2008

DEDICATION

To Ramy and Maha, thanks for your understanding, support and encouragement to stay with this project to the end. To my daughters, Amanda, Nadia, and Daniela, sorry for the times I could not spend with you. I love you all. I promise you I will make up the lost time.

ACKNOWLEDGMENTS

My deepest and most sincere appreciation is extended to Dr. Anthony Lolas, the committee chair, for his ongoing encouragement and support from the start of this endeavourer till this crucial point. His continuous and professional guidance and patience continue to provide me with courage and motivation to get to the end of the road of this undertaken. Thanks to Dr. Raghu Korrapati who provided me with a roadmap and helped me capture the vision of Walden University through the course work I have completed with him. His diligence and conference calls during the execution of his classes helped make this distance learning experience an enjoyable one. I really appreciate him taking time from his busy schedule to serve on this committee. Dr. Teresa Lao introduced me to the concept of qualitative research during a residency in Houston. It was enlightening and provided me with an appreciation of how careful a researcher needs to be when selecting a research methodology. Her early decision to accept to serve made me feel I was getting closer to achieving my goal. Her effort is and will always be greatly appreciated.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION TO THE STUDY

Introduction

Making the wrong decision sometimes sounds right (Elisberg, 2007). For many, when asked whether certain decisions that resulted in failures were correct, the answer, according to Elisberg, was “Yes, based on what we knew at the time.” He disagreed with this answer. He agreed with Nutt (2002 & 2005) and argued that more should have been done to achieve successful outcomes that could have avoided these failed or challenged decisions. According to Nutt, about half of all decisions fail. Nutt’s definition of failure is that the decision does not achieve the desired outcomes. He compiled a database of 400 actual decisions made by top managers in private, public, and nonprofit organizations across the United States, Canada, and Europe during a 20 year period. His research included a wide variety of decisions, from purchasing equipment, to renovating space, to deciding which products or services to sell. Nutt found a failure rate of approximately 50%. He contended that failure rates would be higher if it were possible to study a random selection of decisions. He found that failure is four times more likely when decision makers embraced the first idea they came across without taking the time to analyze uncertain information. Decisions related to networking infrastructures and services were among those that Nutt studied. They too failed at a rate of 50% or higher.

Vertical Markets (2007), a marketing research organization that specializes in studying the telemarketing sector, indicated that the expenditure on networking infrastructures and services was over $200 billion. According to the research of Elisberg and Nutt, valuable resources that could have been redirected to areas to improve the organization's health and market position were wasted because of many failed decisions. This dissertation research analyzed managerial and technical processes and challenges in the networking areas and attempted to derive solutions to them. It treated networking acquisitions and designs as uncertain multicriteria decision problems. To improve the decision making process, a fuzzy hierarchical decision model was developed to enable managers to analyze complex and uncertain parameters.

Chapter 1 presents background information and a historical perspective of the dynamic nature of the telecommunications industry and its environment. Issues and challenges faced by managers when making decisions related to designing and implementing networking infrastructure are presented. Uncertainty and vague information are addressed as significant reasons for a decision not to realize its ultimate goal. The major topics presented in this chapter are: (a) Statement of the problem and an attempted solution, (b) Significance of the problem, Theoretical framework, and (d) Research questions and objectives.

Further, the limitations, assumptions, delimitations, and research methodology are discussed. The next section provides background information and a historical perspective.

Background

Information networks have become an essential strategic component of today's enterprise. As stated earlier, Vertical Markets (2007) reported that total business expenditure on telecommunications infrastructures and services reached $204 billion in 2006 and is expected to climb to $250 billion in 2009. Stallings (2006) and Tanenbaum

(2003) agree that information networks have been gaining in complexity and dimensions because of the deregulation decision to break up AT&T in 1984. The deregulation of the telecommunications sector reached its peak in the mid 1990s and thus opened the market for competition. This resulted in innovations that brought many successive advances to the field of telecommunications. Kuo and Chen (2007) argued that the acquisition of information networks, equipment, and services are multicriteria complex decision problems. An organization embarked on acquiring networking infrastructure should be aware of many issues related to the vendors' ability to deliver. Criteria related to the vendor's ability to deliver products and services at a competitive cost, and to maintain, enhance, and retrofit these products over an extended timeframe are a few of the factors that may affect the decision making process in the acquisition of telecommunications and networking infrastructures.

Prior to 1984, regulated telephone companies provided all U.S. telecommunications products and services (Schoening, 2004). Businesses had largely acclimated to the 1984 changes and had established internal telecommunications departments. These internal organizations were responsible for providing telecommunications services to their companies usually through the work from internal employees. This type of arrangement was considered insourcing. Today, outsourcing is commonplace. For many businesses, outsourcing has become the preferred way of handling internal functions. Schoening (2004) concurs with Kuo and Chen (2007 that the acquisition of telecommunications services and infrastructures has become a complex undertaking that involves many factors which require the consideration of many alternatives. For instance, consider the issues that face a multinational firm that wants to take advantage of the benefits of internet protocol (IP) telephony. The firm may decide to eliminate current networking installations that span continents and countries, or it may leverage its investment in current installations and perhaps find a way to upgrade the existing networking infrastructure. Depending on the direction the firm takes, different sets of issues emerge.

In the case of new installations, some of the issues are

1. Advanced technologies.
2. Internet protocol standards.
3. Ability to deploy internationally.
4. Network management.
5. Information security.
6. Scalability to accommodate future growth.
7. Cost.

In the case of upgrading existing infrastructure, a different set of issues will have to be considered.

1. Compatibility with existing infrastructures.
2. Exposing the firm's internal networks to vendors may present a breach of security that could lead to the exposure of trade secrets.
3. Cost to a lesser extent.

There are issues common to approaches for either new installations or upgrading current infrastructure (Schoening , 2004). Questions related to dealing with foreign governments, tariffs, and providers' portfolios need to be answered. Service and management contracts as well as the provider's ability to survive in a very competitive market are pertinent. The problem is compounded if the firm cannot find a single provider and is forced to use multiple vendors. Partnership agreements, project management, and the ability of vendors to interact legally, logistically, and ethically in a highly competitive international environment are some of the complications that add other dimensions to the decision making process. Based on these issues and questions, Schoening argued that multicriteria decisions to acquire networking infrastructures are fuzzy and complex where the boundaries among factors and alternatives are blurred.

Decision support systems are among the tools that assist managers and decision makers in deciding which path is appropriate when dealing with problems of this nature. Multicriteria analysis is often a prerequisite to successfully arrive at a decision that may yield the best outcome (Triantaphyllou, 2001). In an attempt to deal with complex multicriteria decision problems, Saaty (1980, 1994, 2001) developed the analytic hierarchy process (AHP). The AHP is a method that formulates and analyzes decisions by decomposing a complex multicriteria decision problem into a hierarchy of irreducible criteria and a set of alternatives. AHP uses numerical ratings from pairwise comparisons to establish a priority or weight for each criterion. AHP has been used in numerous applications such as planning (Poh & Ang, 1999), setting priorities (Stan & Duarte, 2003), choosing the best policy alternatives (Byun, 2002), and ensuring system stability (Fahmy, 2001). Other researchers used AHP to tackle problems in areas such as software selection (Lai, Wong & Cheung, 2002), operating system choices (Nagi, 2003), and telecommunications vendor assessments (Tam & Tummala, 2001). AHP allows decision makers to make qualitative decisions using the judgment of experts in a relatively quantitative process. It also enables systematic decision making by expressing the interaction and hierarchy of factors, thus reducing the risk of a rough estimation.

According to Saaty (1980, 1994, 2001), AHP involves a three-step process: decomposition, comparative judgment, and synthesis. In the first step, a hierarchical structure is established to present the problem, which is labeled as problem formulation. The next step is to compare factors at the same level in the hierarchy in pairs, and compare their contributions to the decision objective. A comparison matrix is developed by comparing pairs of criteria or alternatives. A crisp scale that ranges from 1 to 9, where 1 represents equally important and 9 represents extremely more important, is used to express the evaluator's preferences. The final step is to synthesis priorities to calculate a composite weight for each alternative, based on the preferences derived from the comparison matrix. The expected outcome of this weighting process is the selection of the best, or highest scoring, solution among multiple alternatives. In the case of acquiring networking infrastructures and services, the outcome would result in the selection of a solution that scores most favorably on the weighting scale. Additional AHP details are presented in chapter 2.

Problem Statement

Making a wrong procurement decision or designing an improper solution for networking infrastructures can lead to catastrophic outcomes such as significant financial losses and misplaced resources. The lack of a multicriteria decision models designed specifically to treat these types of decisions in an environment of uncertainty makes it difficult for managers to arrive at conclusions with a high degree of confidence. Nutt's research (2002, 2005) indicated that such failures in about half the business decisions could have led to major problems. Incorrect business decisions in private, public, and nonprofit organizations may cost large sums of money that can be directed elsewhere for better results. Similarly, decisions to acquire networking infrastructures may cause an organization to lose its competitive advantage when the desired outcome is not achieved.

Despite the attractiveness of AHP and the potential of its use in analyzing decisions to procure networking infrastructures and services, decision analysts have voiced concerns over a major deficiency of the classic AHP. Peng, Chen, and Qi, (2006), Pohekar and Ramachandran (2004), and Mikhailov (2003) agree that a main problem with AHP is its difficulty in handling uncertainty in the decision process. The crisp scale that it uses appears inefficient and incapable of capturing uncertainty. The causes of uncertainty may be due to incomplete or vague data about a particular factor in the solution exploration analysis. Since some of the evaluation criteria are subjective and qualitative in nature, it is very difficult for a decision maker to provide exact pairwise comparison judgments (Arslan & Khist, 2007; Efedigil, Onuit, & Kongar, 2007; Mikhailov & Tsvetinove, 2004). These authors agree that, under many conditions, crisp data used in AHP are inadequate to model real life situations because human judgments, including preferences, are often vague and cannot be assigned an exact numerical value. Mikhailov and Tsvetinove outlined other difficulties with AHP related to consistency and rank reversal. They considered these issues less critical than AHP's difficulty in handling vague and uncertain decisions.

Given AHP's deficiencies, it appears inappropriate to use AHP to model decisions for developing solutions for networking infrastructures. This is primarily because of uncertainty and complex networking architecture (Cheng et al., 2007; Kuo & Chen, 2005). Overcoming AHP's difficulties requires models that address the inherent multicriteria and fuzziness of the decision process in acquiring networking infrastructures and services. Developing such a model that overcomes these deficiencies is the problem that needs to be explored in this research.

Definition of Terms

This section presents the terms used in this research. The following are these definitions:

Access layer: Grouping of computers and servers that networking devices interface with. Typically this layer includes demilitarized zones (DMZs)(see below), firewalls, switches and hubs.

AHP: A process that decomposes a complex multicriteria decision problem into a number of irreducible factors (criteria), sub factors (criteria), and alternative solutions that can be relatively weighted. Its main contribution is to quantify qualitative factors and alternatives (Saaty, 1980, 1996, 2001).

AHP criterion: A factor related to the main objective of a decision being analyzed. Each factor receives a weight describing its importance with respect to the objective of the decision. When normalized, the total weights for all factors add to 100. For example, one is about to purchase a house, then there are a set of factors may need to be considered. Price, square footage, geographical location, and quality of schooling may be the factors (criteria) in the purchasing decision.

AHP weighting: Each criterion gets a numerical number indicating its importance to the decision. In our above example: price may get a weight of 20, square footage is assigned 25, geographical location may be weighted at 20, and schooling gets a weight of 35. Note that the weights add to 100.

AHP pairwise comparison matrix: A table that includes entries describing the decision analyst opinion (judgment) to which criterion is more (less) important than another in terms of their importance to achieving the goal of the decision under analysis.

AHP scale: This scale ranges from 1 to 9 and 1/9 to 1. It represents crisp numerical presentation of linguistic judgments in the pairwise comparison matrices for relative importance of criterion or alternative. The interval [1, 9] is for the category more important and the interval [1/9, 1] is for the category least important.

Alternatives: Different choices of solutions or actions available to the decision maker. In this study, the alternatives are assumed to be finite, ranging from two to ten.

Conflict among criteria: Different criteria represent different measures and dimensions. Thus they may conflict with each other. For instance, cost may conflict with profit. In this research, no such conflicts are assumed unless explicitly stated (Triantaphyllou, 2001).

Core layer: Grouping of switches that provide a backbone (high-speed line) between data centers.

Demilitarized Zone: One or more servers, routers and switches that act as a buffer between the external users and the internal network. This small network usually prevents unauthorized access to the network.

Distribution layer: Grouping of switches or routers that communicate between different access layers and the core layer.

Decision Weight: Most multicriteria decision methods require that criteria be assigned weights of importance relative to achieving a main objective. Usually, these weights are required to be normalized to add up to one. However, other normalization scales can be used.

Decision: A decision matrix A is an (m x n) matrix in which element a_{ij} indicates the preference of alternative A_i when it is evaluated in terms of decision criteria Cj (for i = 1, 2, 3, …, m and j = 1, 2, 3, …, n).

Electronic Connectivity (Econnectivity): A general term used when a company used data centers to do business through internet or intranet.

Electronic Service (Eservice): A general term for the services a data center can provide via the internet or intranet.

Fuzzy Sets: extensions of classical set theory used in fuzzy logic. Contrary to classical set theory, which permits membership in binary form, fuzzy sets allow for gradual membership. The degree of belonging to the fuzzy set ranges within the interval [0,1] (Zadeh, 1965; Zimmerman, 1968).

Fuzzy linguistic variable: A variable described in linguistic terms. For example one may describe a room temperature to be too hot, hot, warm, cold, or too cold.

Fuzzy membership function: A mathematical function that maps a linguistic variable into a membership value in a fuzzy set. The function transforms the linguistic definition into a value within the interval [0, 1] inclusive of 0,0, 0.1, 0.2,…, 0.5, 0.6,…, 0.9, 1.0. A high values indicate a strong membership of an element in a fuzzy set. Low values indicate weak membership of an element in a fuzzy set.

Incommensurable Units: Different criteria may be associated with different units of measures. For instance, in the case of purchasing a house, the criteria cost and square footage may be measured in terms of thousands of dollars and square feet respectively. It is this nature of having to consider different units in a comparison which makes multicriteria problems intrinsically difficult to solve (Triantaph, 2001).

Load Balancer: A device that distributes traffic onto different links to prevent congestion on networks.

Servers: Computers that are used to either store data or provide services for the company, e.g., e-mail services.

Switch: A device that allows computers to connect to a network and to access services. Each computer gets a communication link to transfer data to and from the network.

Router: A device that allows networks to expand by acting as a central location for computers or other networking devices to connect into. However, routers allow communication between different networks. They are more expensive than switches and usually have a lower number of access ports.

Purpose of the Study

The purpose of this research was to develop a new decision model that overcomes AHP difficulties in handling uncertainty and vagueness related to designing networking data centers and networking infrastructures. It is intended to enable managers with a better process to analyze vague and uncertain data in the decision making process.

The model used AHP as a framework because of AHP's apparent reputation to structure multicriteria decision problems. It overcomes AHP's deficiencies through the use of the theory of fuzzy sets. This model was then used to analyze a practical multicriteria example, data center design.

According to Dey and Sakara (2000), model development research that uses existing frameworks needs to provide continuity of the research methods. They reasoned that research that builds on previous work needs to provide continuity in methodology to gain acceptance and avoid misunderstanding.

Saaty's (1980, 1996, 2001) work in developing the AHP and Triantaphyllou's (2001) comparative study of multicriteria decision making systems used the analytical research design method. This method depends on mathematical concepts, proofs, and formulation (Buckley, Buckley, & Chaing, 1976, Martin, 2004, Moole, 2005). The model developed in this research used the analytical research design method since AHP was developed using the same method.

According to Buckley, Buckley, and Chaing (1976); Martin (2004); and Moole (2005), the analytical research method does not require data collection. Consequently, no surveys, formal interviews, or other instruments of this type were used in this research. This study did not use the quantitative or typical qualitative research design methods.

Significance of the Problem

Decisions to select appropriate vendors of networking infrastructures and the development of networking solutions are of great importance. There are two generally accepted primary reasons: one is the significance of the financial stake and the other is that decision makers operate in a difficult and uncertain telecommunications environment (Bello, 2003; Schoening, 2003).

Experts generally agree that no best way exists to evaluate and select suppliers, and thus organizations use a variety of approaches. Bello and Schoening agree that the overall objective of the decision maker is to reduce risk and maximize value. Some experts suggest that many large acquisition decisions involving millions of dollars do not adhere to a formal process. They are based on spreadsheets with massive amounts of unstructured information. A possible outcome of not adhering to formal decision structures is missing business targets and objectives (Byun, 2002; Stam & Duarte, 2003; Stallings, 2006; Tam & Tummala, 2001). This study filled a gap in multicriteria decision making research. It provided improvements in using uncertain and fuzzy information.

Nature of the Study

This research was analytical in nature. The rationale for using this methodology was that the underlying framework for the study is also analytical as provided in the AHP research. A main feature of the proposed model is its ability to handle uncertainty in the decision making process. This was accomplished through the development of algebraic and algorithmic fuzzy operations for use with a fuzzy scale for the new model. This part of the research was to overcome one of AHP's limitations manifested by its crisp scale.

Further, this research was grounded in decision sciences with a focus on both the breadth and depth of analyzing AHP-structured complex, multicriteria decision problems. Additionally, synthesis and contrasts of AHP methodology, structure, scales, judgment matrices, and weights were presented with a focus on introducing the concept of fuzzifying the decision process. Techniques to derive range maxima and range minima, a feature of fuzzy sets, were also developed.

To further validate the model, a software tool was developed. This work dealt with developing algorithms to implement the fuzzy hierarchical model and its mathematical and logical operations. The software platform enabled the decision maker to develop a fuzzy model of the networking infrastructure problem through the use of

1. Fuzzy criteria structures.
2. Fuzzy alternative structures.
3. Sensitivity analysis.
4. Fuzzy judgment matrices.
5. Fuzzy weighting and ranking.
6. Degree of uncertainty.
7. Decision maker's pessimistic and optimistic attitudes.

A simulated practical case study that emphasized the design and architecture of datacenters and their complexities was the basis for input data used to validate the developed model. The investigator devised the case study based on the work of

Arregoces (2004), Khader and Barnes (2000), Dennis and Fitzgerald (2005), Stallings (2006), and Tannenbaum (2003). Further, two experts (not representing their organizations) in the networking fields were consulted to provide feedback related to the relevant criteria and issues pertinent to networking and datacenters design. The experts represented two important industrial perspectives. One expert is a chief global networking architect in a leading telecommunication manufacturing company. The other is a vice president of networking operations in a major United States bank, a procurer of telecommunication equipment and services. The case study effort emphasized four phases:

1. Design
2. Criteria formulation
3. Fuzzy modeling
4. Sensitivity analysis

The design phase treated the development of an illustrative fuzzy multicriteria decision problem in the field of datacenter design. The design emphasized the challenges a decision maker might face with alternate datacenter solutions and designs. The formulation phase focused on selecting the factors that influence the main decision outcome. In this phase, fuzzy judgment matrices for criteria and alternatives were developed. The fuzzy modeling phase included fuzzy weighting of attributes, ranking of alternatives. The sensitivity analysis dealt with repeating fuzzy modeling with variation of the degree of certainty. The results were analyzed to compare the risk factors. The importance of the sensitivity analysis stems from the fact that a decision support system is not the final arbiter in making decisions (Arregoces, 2006). Sensitivity analysis, through the use of different degrees of uncertainties, provides the decision maker with additional insights that should assist in making sound judgments.

The abovementioned research tasks were intended to fill a gap in decision analysis research. To that end, the study addressed a new decision model. This model formulates complex decisions made under difficult economic operating conditions and with incomplete input information. The study filled a gap in current research through the use of fuzzy set operations in conjunction with the AHP framework to produce a model that may have the potential to standardize networking infrastructure decision making processes. Furthermore, the extensive review of the literature revealed that lack of decision modeling research that deals with dynamic and uncertain telecommunication industry. The model developed in this study addressed such issues.

Research Objectives

The objective of this research was to develop a new decision model that overcomes AHP difficulties in handling uncertainty and vagueness related to designing networking data centers and networking infrastructures. The research activities and the capabilities of the model were focused on achieving the research objectives in overcoming AHP's difficulties in handling decisions under uncertainty and risk.

The developed model was able to provide

1. Consideration of monetary and nonmonitory attributes.
2. Quantification of qualitative factors and thus making it easier to rank factors of network architectures and designs.

3. Treatment of uncertain subjective judgments.
4. Formulation of a decision making fuzzy hierarchical model based on AHP and the fuzzy theory to deal with uncertainty and vagueness.
5. Development of software tools to implement a fuzzy hierarchical decision-making modeling to verify models components and constructs.
6. Application to a simulated practical case study.

Research Questions

This research aimed at answering the following four research questions to achieve the above-mentioned objectives:

1. Does the model provide improvements in handling uncertainty compared to AHP?
2. In providing maximum benefit and acceptance, is the model consistent with underlying heuristic framework (Russel & Norvig, 2003; Moole, 2005)?
3. Does the developed model take into account the decision maker's pessimistic and optimistic attitudes?
4. Does the newly developed model improve the multicriteria decision process?

Theoretical Framework

This research was primarily framed around fuzzy set theory. The fuzzy set theory has been used to tackle ill-defined and complex problems due to incomplete and imprecise information that characterizes real-world systems. Zadeh (1965), the original author of the fuzzy theory, stated that "as the complexity of a system increases, human ability to make precise yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance become mutually exclusive" (p. 28). This was identified as the principle of incompatibility. This follows that modeling complex or ill-defined systems may not be achieved precisely. Fuzzy set theory is not intended to replace the theory of probability, but rather to provide solutions to problems that lack mathematical rigor inherent to probability theory. Essentially, fuzzy set theory is an extension of classical set theory.

Classical and fuzzy sets are different in the way they treat the idea of membership. Membership is defined as whether an object belongs to a set or not. In classical set theory, a set is a collection of objects having a general property, for example, a set of clients. In classical logic, an element is, therefore, either a member or not a member of a set (Ross, 1996). The boundaries of these concepts are very rigid or crisp, and there is no room for grey or in between states. There are no intermediate grades of membership between full and non-membership. This deterministic yes-or-no response approach, or dichotomous approach, is currently a widespread practice in system modeling, reasoning processes, and computing. A major problem with the classical set approach is that it fails to convey information effectively. Specifically, the states between full and non-membership are ignored, yet they may be very important. Meanwhile, many real-world systems are very complex and ill defined to be well understood and modeled precisely using the classical set theory.

The essence of fuzziness, in contrast to classical set theory, is that the transition from a membership to non-membership state of an element of a set is gradual rather than abrupt. Thus, fuzzy set theory allows a generalization of the classical set concept to model complex or ill-defined systems within a range. The main concepts associated with fuzzy set theory, as applied to decision systems, are membership functions, linguistic variables, natural language computation, linguistic approximation, fuzzy set arithmetic operations, set operations and fuzzy weighted averages (Schmucker, 1984; Zimmerman, 1968). More details about these concepts are provided in the next sections.

Linguistic Variables

Research in cognitive psychology suggests that individuals base their thinking on conceptual patterns and mental images rather than on quantities or numbers, according to Ross (1996) and Zimmerman (1986). The concept of linguistic variables lies at the core of fuzzy set theory, since the basics of fuzzy set theory is the manipulation of linguistic expressions instead of numbers. Although natural language is imprecise, it conveys valuable information and, despite the vagueness inherent in natural language, humans can understand each other quite well. The values assumed by linguistic variables are words. A linguistic variable differs from a numerical variable in that its values are not numbers but words or sentences in a natural or artificial language. Since words in general are less precise than numbers, the concept of linguistic variables serves the purpose of providing a mean to approximate the characterization of phenomena that are too complex or too ill defined to be amenable to description in conventional quantitative terms. Examples of linguistic variables are expressions such as, need for job, qualification of bidders, and market conditions. These linguistic variables may assume different values, such as very high, high, moderate, low and very low, which are fuzzy sets (membership functions) and represent the perception of the decision maker on the magnitude of any risk factor.

Membership Functions

A crisp set can be considered as a container and the elements belong to this set as the objects contained in it. In this sense, an object will be either in the container or not in the container. On the other hand, a fuzzy set has varying degrees of membership. The degrees of membership of an element are expressed by a membership function. A membership function is a function that maps a universe of objects, X, onto the unit interval [0, 1]. The universe of objects represents the elements of the set and the interval corresponds to the set of grades. The grades of membership in fuzzy sets may fall anywhere in the interval [0, 1]. A degree of 0 (zero) means that an element is not a member of the set at all. A degree of 1 (one) represents full membership. Membership functions in fuzzy set theory are used to represent uncertainty. In contrast to crisp sets that have only one membership function, fuzzy sets have a large number of membership functions.

The inputs to the decision support system are the assessments of the different judgments of relative importance of factors and alternatives specific to a project in linguistic terms (high, medium, low). The system checks the knowledge base and databases and performs natural language computations and produces the risk impact for each group of risk factors as well as the overall risk (combination of partial risk impacts) and the corresponding likelihood in linguistic terms. For example, the overall risk impact can be low with high likelihood. The system can also provide recommendations on the most appropriate risk response strategies from the results of risk analysis. The decision maker can then make her or his judgment and take appropriate measures to mitigate project risks and thus improve the likelihood of project success.

Scope, Limitations, Delimitations

The research method used in this research is analytical. Limitations in analytical studies are usually due to interpretations, logical errors, and semantics. To minimize such limitations, substantiating claims were based on being thorough in developing formulae and adhering to well established mathematical formulations and proofs (Moole, 2005). A software tool was developed to further validate the model constructs. Although the developed model can be used for other types of fuzzy and uncertain decisions, its application in this research was limited to the selection of a datacenter solution.

The data used to validate the model was simulated. It was intended to exercise the boundary conditions and the model behavior under different sensitivity analysis scenarios. The input to the model was mainly based on the literature, the investigator's years of expertise, and experts in the field of networking and datacenter design. It is important to note, as it was stated earlier, no surveys, interviews or instruments of this type were used in this study.

The following activities were within the scope of the research:

1. Develop a fuzzy multicriteria model to analyze decisions to acquire and design networking infrastructures and services.
2. Develop the fuzzy logical and algebraic operations required for development of the model.

3. Develop the algorithms for the model to accept evaluation criteria, and alternatives.
4. Perform ranking and generate reports.
5. Apply the model to a simulated practical case study with numerical fuzzy data.

The study did not discuss any comparison with proprietary non-published models. Any comparison was limited to those available in the literature. Publication of the results in peer reviewed journals and conferences should assist in the research acceptance.

Assumptions

A decision maker is a rational person. Rational persons are defined as individuals who try to minimize their regret, minimize losses, or maximize profit (Simon, as cited in Triantaphyllou, 2001). In this dissertation research, a decision maker is assumed to be a rational person.

The model developed in this research is for a single decision maker. However, experts can contribute to defining decision criteria, but consensus must be reached on the final criteria before they are entered into the model. A consensus can be reach with the help of instruments such as brainstorming, qualitative questionnaires, or geometric means (Saaty, 2001).

Social Change

This study can lead to a standardization of multicriteria decision processes intended for networking infrastructure acquisitions. According to many experts in the networking field (Arregoces, 2006; Khader & Barnes, 2000; Stallings, 2003; Tanenbaum, 2003), standardization usually leads to savings in product development. It is a better method of operation compared to proprietary activities. Stallings and Tanenbaum argue that standardization of networking component development can lead to huge savings as well as to opening the market for increased competition. In their opinion, a competitive market leads to technological advances and lower pricing of networking products.

Further, this study used the AHP as a framework. AHP was used in the past to analyze forest management, water resource management, and renewable energy planning studies (Anada & Herath, 2007; Liebowitz, 2005; Pohekar & Ramachandran, 2004; Wang 2005). These studies are related to issues of social impacts. Since the model was developed to overcome AHP's difficulties, its use can provide improved analysis outcomes in similar socially important areas such as outsourcing decisions, poverty reduction projects, and public capital development projects.

Summary

One of the business areas that has an inordinate number of failed decisions is in the acquisition of networking infrastructures (Nutt, 2002, 2005). Considering the magnitude of multibillion dollar expenditures in the area of networking platforms and services (vertical markets, 2007), failed decisions may lead to significant repercussions. The major loss of resources and misdirected funds may lead to unfavorable positions in a competitive and dynamic market (Schoening, 2004). According to Nutt, uncertain and vague data are among the factors leading to failed decisions. This study proposed a model to assist managers in analyzing complex, uncertain, and vague data about complex decision problems. The intent was to minimize risk to appropriately align resources.

Presented in chapter 1 was the high rate of decision failures in business and its impact on the bottom line. The significance of failed decisions in terms of lost funds and misaligned resources was dealt with. A major focus of the chapter was on the complexity of the multicriteria decision-making process. The challenges of acquiring networking infrastructures were woven into a complex fuzzy decision-making problem that will serve as a model to highlight the proposed method to improve the decision making process.

AHP's deficiencies were explored to provide the framework for a solution to overcome these deficiencies through the development of a model that takes into account uncertainty, complexity, risk and the decision maker's equivocation. The proposed fuzzy hierarchical model in the decision making process is designed to provide a rationale to deal with the trends and complexity of the emerging environment in which the telecommunications industry operates.

The focus in chapter 2 was on reviewing the pertinent literature to provide a background in identifying the core factors involved with problem identification and analysis. The literature review addressed the changes that the telecommunications and information systems industries have experienced during the past decade. Current literature in multicriteria research to solve complex decision problems in an environment of uncertainty is also a theme in chapter 2.

Addressed in chapter 3 was the research methodology. The focus was on the analytical research method, which was the primary research methodology used in this study. The advantages and disadvantages of this method were highlighted in this chapter with the objective to overcome the deficiencies through the proposed fuzzy hierarchical model. Further, outlined in chapter were the detailed steps to verify the applicability of the proposed model. The verification process mainly relied on numerical examples and a simulated practical situation based on the literature and the opinions of two industry experts. A software platform was used as well. The intent was to use these techniques collectively to answer the research questions and achieve the objectives of the study. The simulated case study illustrated how the proposed model may be used in practical settings and applications.

The emphasis in chapter 4 was on the results of this research. Decision making frameworks, multicriteria analysis, and the underlying framework used to develop the fuzzy hierarchical decision model under considerations were among the topics covered in chapter 4. Further, the steps used to synthesize, formulate, derive, and develop the research model were presented. Additionally, contrasting the research model with AHP was among the presentations in chapter 4. Also presented were the development of the model algorithm and the software aids used to verify and validate the model constructs. Finally, Covered in this chapter was a simulated practical situation to elucidate the applicability of the fuzzy hierarchical decision modeling support system. A sensitivity analysis was conducted to examine the model behavior under different degrees of uncertainty. The sensitivity analysis also took into account the pessimistic and optimistic attitudes of the experts used in this research. The conclusion of this research, and a discussion of the answers to the research questions that embedded a contrast between the

AHP decision modeling techniques and the fuzzy hierarchical model were part of the discussion in chapter 5. The discussion in chapter 5 encompassed implication of the research and recommendations for future studies.

CHAPTER 2: LITERATURE REVIEW

Overview

This chapter presents a review of the literature concerning the research questions and objectives stated in chapter 1. The review revolved around four main themes: (a) networking infrastructure acquisition issues and challenges, (b) multicriteria decision making models that focus on quantifying qualitative factors, (c) decision-making under uncertainty, and (d) a linkage between current literature and the dissertation research. These major themes encompassed discussions of the analytical hierarchy process and its deficiencies in dealing with uncertainties. This chapter also presents reviews of literature related to fuzzy theory and its applications as well as deterministic and nondeterministic decision models.

Requirements for Acquiring Networking Infrastructures

The process of acquiring networking infrastructures requires technology management skills and purchasing process skills. Technology skills are needed to identify and select the right products and services for a specific business environment. Purchasing process skills are needed to obtain the product at the best life-cycle cost. Schoening (2004) defined four requirements that must be incorporated into the acquisition planning process to ensure that a selected product meets the objectives of a business environment. A summary of these requirements were: (a) the selected product is backed by a viable business that will be around during the product life cycle and will continue to improve the product, (b) the product has a track record through its entire life cycle, (c) quality maintenance support will be available during the product life cycle, and (d) the total life cycle cost of the selected product at least matches or performs better than other acceptable products (p. 416).

There are some crucial issues to successfully plan and implement a networking infrastructure according to Schoeing (2004). Some of these pertinent issues are the translation of business requirements into technical specifications and the development of selection criteria. The selection of a specific vendor's product or services and implementing the systems are just as important. Lichtenthaler (2007) suggested additional important issues including acquisition payment options, leasing and early lease termination, and management and consulting service contracts. Long distance contracts and outsourcing versus insourcing as well as common carrier services are also relevant areas of concern. The quality of services and the achievement of the specifications and requirements are further challenges for the decision maker.

Granat and Wiercbicki (2004) dealt with a multicriteria analysis in telecommunications from a technical perspective. They primarily treated the support for strategic networking management, planning, and design. They also dealt with routing problems and regulations. However, they did not focus on the acquisition issues and their importance in the planning stages.

Hui and Foo (1998) presented a concept for standardizing internet telephony systems. They outlined the internet telephony environment and the importance of the TCP/IP (transport control protocol/internet protocol) for its operation. They dealt with dynamic IP address resolution and H.323 standards for real-time voice communications as well as an interoperability model. Just like Granat and Wiercbicki, the research of Hui and Foo did not treat the acquisition as part of the network planning stages.

There are several mathematical models that may be used to minimize the risks inherent in telecommunications operations from an operator perspective. Agrell and Lindbroth (2004) developed a model to reduce the risk in a telecommunications supply chain induced by uncertain demands, outsourcing and unclear interfaces as well as heterogeneous business logic. Although the mathematical model is convincing, it did not consider the acquisition point of view. Further, the model ignored the requirements of the standard decision making process. Rather, it focused on the operational aspect of a telecommunication supply chain.

The common thread in the above literature is the lack of treatment of acquiring networking infrastructures in the decision making process. The dynamic global nature of the telecommunications environment and its associated issues, mainly outsourcing and the inability of many vendors to continue over an extended time frame, may contribute greatly to decisions involving infrastructure acquisitions. The global economy apparently has impacted the telecommunications industry as well as many other types of business entities. The uncertainty and the radical changes in the global environment have made the decision making process much more complex.

The service sector is typical of those in this category. The dynamic nature of this type of environment resulted in the emergence of new players and the disappearance of many others. According to Agrell and Lindbroth (2004), a new level of uncertainty was reached, and it is even more in 2008. Therefore, a multicriteria decision making model is needed to deal with such an environment and its complexities. The following sections present a preview of the concept of multicriteria decision processes and some of their types.

The theoretical underpinning of this study is grounded in three areas: (1) theories of multiattribute decision sciences, (2) fuzzy sets theory, and (3) mathematical theories of matrices. Multiattribute theories are concerned with decomposing complex decision problems into irreducible factors and sets of actionable solutions. Fuzzy sets theory focuses on dealing with imprecise information and modeling complex systems that are ill-defined. The mathematical techniques for manipulating matrices are relevant to both multiattribute analysis and manipulation of fuzzy sets. Detailed analysis of the theoretical underpinning is provided in the remaining of this chapter.

Multiattribute analysis in decision-making processes focuses on the theories and techniques that aim at quantifying qualitative data when one considers complex decision problems. To complicate matters, they must transform a problem with components that have incompatible measurement units to a problem with unified relative or absolute measurement scales. As Triantaphyllou (2001) eloquently presented it, the essence of the problem is to provide the decision maker with the ability to compare apples and oranges. In general, these methods decompose the problem and transform it into sets of delineated clusters of factors that are easier to analyze and gain concurrence. Lang and Merino (1996) used the term irreducible to refer to decomposed factors. Multiattribute analysis has been used in evaluating a wide range of multicriteria projects. Some of these projects ranged in size and complexity from the small and simple, such as selecting a convenient store location (Kuo, Chi, & Kao, 1999), to the more complex, such as the selection of a large scale semiconductor equipment manufacturer (Chan & Chan, 2004), and even to such abnormal projects similar to the semantic-based facial expression recognition system using analytical hierarchy processes (Cheng et al., 2007).

Although the literature is replete with research articles related to the way people make decisions (prescriptive theories) and the way people ought to make decisions (normative theories), the development of the perfect real-life decision making method remains an elusive goal. According to Triaphyllou (2001), multiattribute analysis techniques are steps in the direction of developing models for decision making that approximate perfection, if there is such a thing. Multiattribute decision making methods concentrate on problems with discrete decision spaces. In these problems, the set of decision alternatives has been predetermined.

Multi-attribute methods may be diverse in their structures, methods of assessment and scales. In general, many of them have certain aspects in common. Chen and Hwang (1991) define the terms alternatives, attributes, and weights as follows:

1. Alternatives: They represent a set of different choices of actions available to the decision maker. The primary assumption is that this set is finite ranging from a few to tens and maybe hundreds. The focus is to screen the alternatives, prioritize them, and eventually the decision maker will rank them through the use of one or more methods.
2. Attributes: They describe where each multicriteria decision problem is associated with attributes, also referred to as factors or criteria. These factors

represent a different dimension from which the decision maker views each alternative. In cases where these factors (criteria) are large, more than dozens, they are arranged in a structural (hierarchical) manner. When a criterion is major, it may encompass several sub-criteria. This lends credence to the need for a hierarchical arrangement.

3. Conflicts among criteria: A situation that may surface since different criteria may represent different dimensions. Also different criteria may be associated with different units of measure. For example, in the case of buying a used car, the criteria cost and mileage may be measured in terms of dollars and thousands of miles, respectively. It is this endemic nature of multiattribute analysis that makes problems of this type hard to solve as the weighting for each criterion may be different to each buyer.
4. Decision weights: Most multiattribute analysis methods require that the criteria be assigned weights that are usually normalized. The weighting of the criteria depends on the method used. The performance of the criteria is usually presented in a matrix format. A typical decision matrix is normally established according to the following: A is an (m x n) arrangement in which a_{ij} indicates the performance of alternative A_i when it is evaluated in terms of decision criteria C_j (for i = 1, 2, 3, …, m, and j = 1, 2, 3, …, n).

Classification of Multiattribute Analysis Methods

There exists more than one way to classify multiattribute decision making methods, primarily according to the type of data they use, or according to the number of decision makers involved in the process. According to the data types, there are deterministic, stochastic, or fuzzy (Chen & Hwang, 1991). Multiobjective and probabilistic models may also fit this classification. On the other hand, if the classification is according to the number of decision makers, we may have single decision maker multiattribute methods, or group decision making methods.

This dissertation research concentrated on single decision maker methods. Cheng and Hwang (1991) classified the single decision maker methods according to the type of information as shown in Figure1. Figure 1 presents the taxonomy of multicriteria decision-making at the root.

The Analytical Hierarchy Process

The analytical hierarchy process (AHP) decomposes a complex multi-attribute decision making problem into a system of hierarchies. This system of hierarchies uses a pairwise comparison technique aimed at eliciting numerical evaluations of qualitative phenomena from experts and decision makers. This section presents an examination of the method used in AHP to process the a_{ij} values after they have been determined. The entries a_{ij} in the m x n matrix represent the relative value of alternative Ai when it is considered in terms of criterion C_j. In AHP the sum $\sum_{i=1}^{n} a_{ij}$ is equal to 1.

According to AHP, the best alternative results from the maximization of values. As presented by Saaty (2001), this is indicated by the following relationship provided by the two siblings: no information available and some information exists with respect to the attributes. Under the no information available sibling, Saaty identified three techniques in analyzing decision attributes that he labeled as dominance, maximin, and maximax. Under the existence of some information's sibling, there are three types standard, cardinal and ordinal. Under each one of these types figure 1 presents the modeling technique used to describe the multiattribute decision making process.

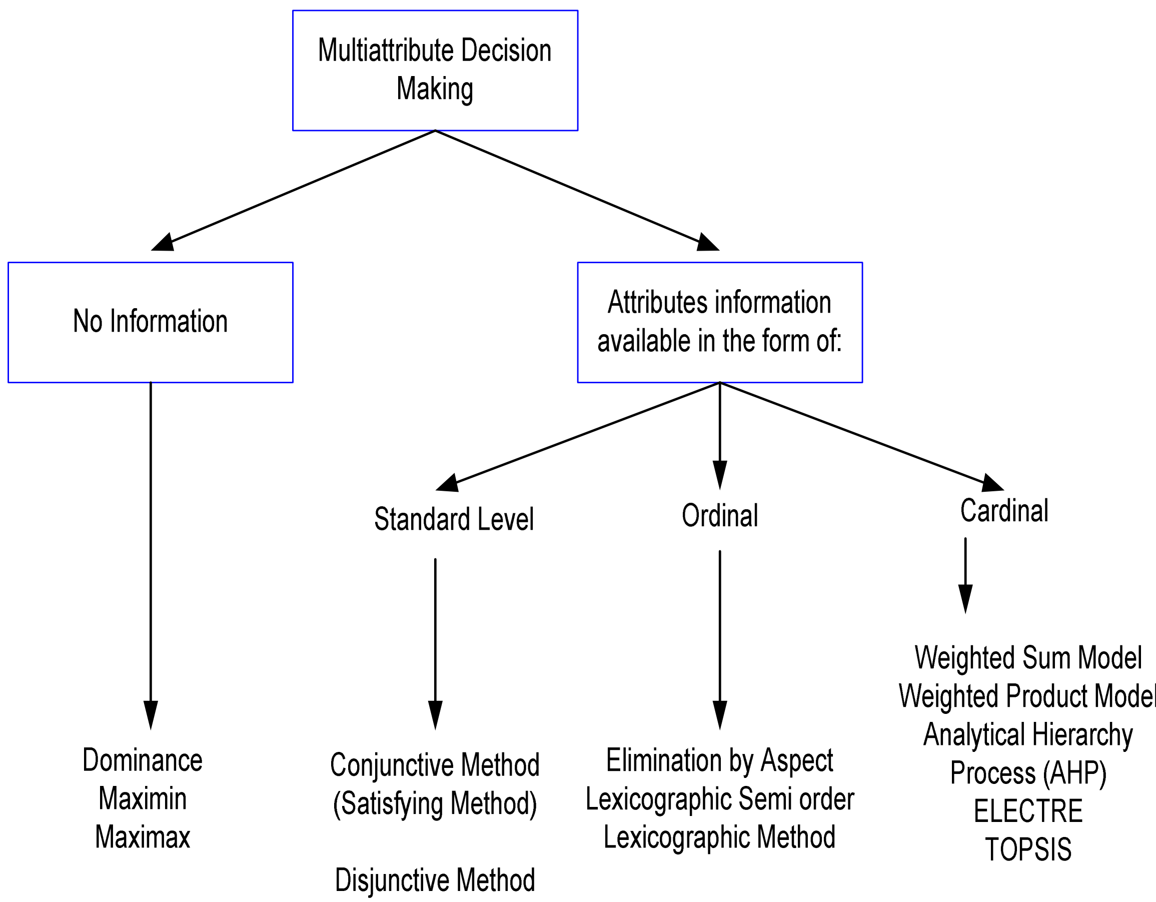


Figure 1. Taxonomy of multicriteria decision making methods, adapted from Chen and Hwang's (1991) description.

AHP-score = $\max_i \sum a_{ij} w_j, \quad for \quad i = 1,2,3,...,m, j = 1,2,3,...n.$

In the case of minimization the following relationship indicates the best alternative:

AHP-score = $\min_i \sum a_{ij} w_j, \quad for \quad i = 1,2,3,...,m, j = 1,2,3,...n.$

where *a and w* represent the weight of the relative importance of a criterion and an alternative respectively. Stated differently, in the maximization case, the decision maker is looking for the alternative with the most benefit or profit. In the minimization case, the goal is to determine the alternative with the least cost. Although the relationships appear to be similar to the weighted sum method, it does not have the restriction of expressing all the criteria in terms of the same unit.

Numerical Example

To make this notion clearer, consider the following numerical example that treats four criteria and three alternatives:

	Criteria			
	F1	F2	F3	F4
	0.20	0.15	0.40	0.25
Alternatives				
A1	25/65	20/55	15/65	30/65
A2	10/65	30/55	20/65	30/65
A3	30/65	5/55	30/65	5/65

The Factor F1 has 0.20 in terms of its relative relevance to the principle goal being decided; F2 has 0.15, F3 has 0.40, and F4 has 0.25. The columns in the decision matrix have been normalized to add up to 1, as Saaty (1980) requires. The ratios represent the comparison in pairs of the contribution of each of the alternatives (A1, A2, and A3) toward each of the factors (F1, F2, F3, and F4).

The AHP scores for each alternative can be derived:

$A1_{AHP\text{-}score} = (25/65) \times 0.20 + (20/55) \times 0.15 + (15/65) \times 0.40 + (30/65) \times 0.25 = 0.34.$

Similarly,

$A2_{AHP\text{-}score} = 0.35$

$A3_{AHP\text{-}score} = 0.31$

Thus, in applying the maximum case in which the decision maker chooses the alternative with the most benefit, the best alternative is A2 (because it has the highest AHP-score: 0.35). Also the alternative can be ranked (A2 > A1 > A3). This example raises an interesting question related to a choice of an alternative A2 just because it weights .01 more than that of alternative A1. Since the resultant scores are so close between the two options, it poses a question that the decision maker must consider seriously before making the final selection. The process may need to be repeated to elicit weights that produce results with greater differentiating gaps between the alternatives.

One problem with this ranking is that it does not show any risk that an alternative may carry. One can go as far as saying that this ranking appears misleading. Not too many experts can claim that they are absolutely certain of a preference judgment they render. This is the drawback of crisp scales.

Scaling Interval [9 – 1/9] AHP

Saaty (1980) used a discrete pairwise scale ranging from a lower bound of 1 and an upper bound of 9 with 2 as a stepping increment. In other words, when two criteria C1 and C2 are compared in terms of their relative importance to each other, the way a decision maker sees them, the values of the pairwise comparison can take [9: C1 is extremely important relative to C2, 7: C1 is very important relative to C2, …, and so on, 1/9: C1 is of little importance relative to C2]. Table 1 presents the AHP relative judgment scale in terms of more important pairwise comparison and Table 2 presents the AHP relative scale in terms of less important pairwise comparison.

Table 1.

Weighting Scale of Pairwise Comparison: More Important.

Relative importance of a factor compared with any other factor	Scale
Equally important	1
Moderately more important	3
Strongly more important	5
Very strongly more important	7
Extremely more important	9
Intermediate judgments	2,4,6,8

Consider any two factors (attributes) Fx and Fy. If Fy is strongly more important than Fx, then the relative importance of Fy as compared to Fx is, according to Table 1, equal to 5. Conversely, the relative importance of Fx as compared to Fy is the reciprocal of 5, that is, 1/5. This suggests another table, Table 2, depicting the less important pairwise comparison scale.

Table 2.

Weighting Scale of Pairwise Comparison: Less Important.

Relative Importance of a factor compared with any other factor	Scale
Equally important	1
Moderately less important	1/3
Strongly less important	1/5
Very strongly less important	1/7
Extremely less important	1/9
Intermediate judgment levels	1/2,1/4,1/6,1/8

Saaty's (1980) rationale for using this number of points on the relative discrete judgment scale is that people, according to psychological theories, are unable to make choices from an infinite set of available selections. For example, people cannot make a distinction between two very close values of importance, say 5.00 and 5.02. Psychological experiments also have shown an intriguing fact that most individuals cannot simultaneously compare more than 7, give or take 2 (Miller, as cited in Saaty, 1980). This is the rationale for using a judgment scale with 1 as a lower bound and 9 as an upper bound, and a unit difference between successive scale values. If we call the scale between [9 – 1/9] scale 1, it is not unreasonable to present an alternative scale, say scale 2, which has the values on the subinterval [9, 1] evenly distributed with the intervals [1, 1/9] as the reciprocals. These considerations lead to the scale {9, 9/2, 9/3, 9/4, 9/5, 9/6, 9/7, 9/8, 1, 8/9, 7/9, 6/9, 5/9, 4/9, 3/9, 2/9, 1/9}. It is still possible to follow Saaty's recommendation of limiting the multi-attribute comparisons to yield two sets of two 5 values (1, 3, 5, 7, 9) for more important and (1/9, 1/7, 1/5, 1/3, 1) for the less important comparisons with intermediate values of (2, 4, 6, 8) and (1/8, 1/6, 1/4, 1/2) for refinement, if needed.

AHP Hierarchy

The AHP method is best presented in a hierarchical structure of criteria and alternatives. At the top of the structure is the goal of the multi-attribute analysis as depicted in Figure 2. The alternatives are at the bottom level of the structure. Between the goal and the alternatives lie the criteria and sub-criteria. A structure containing three levels would be built as follows:

Level 1: The objective of the analysis

Level 2: The attributes considered in achieving the objective

Level 3: The alternatives

The three levels of hierarchy shown in Figure 2 indicate that the criteria affecting the choice of the best alternative are arranged in level 2. Whereas Level 3 shows the various alternatives. At this level (3), the alternatives are evaluated for their contribution with respect to each criterion. C1, C2, C3…, and Cn denote the criteria; A1, A2, A3…, and Am denote the alternatives. As the number of levels in the hierarchy increases, so does the level of complexity of the analysis and the number of contradictions (Traintaphyllou, 2001). Figure 3 indicates a hierarchy with four levels where each criterion has two sub-criteria. The multi-attribute analysis of this structure will follow the following steps:

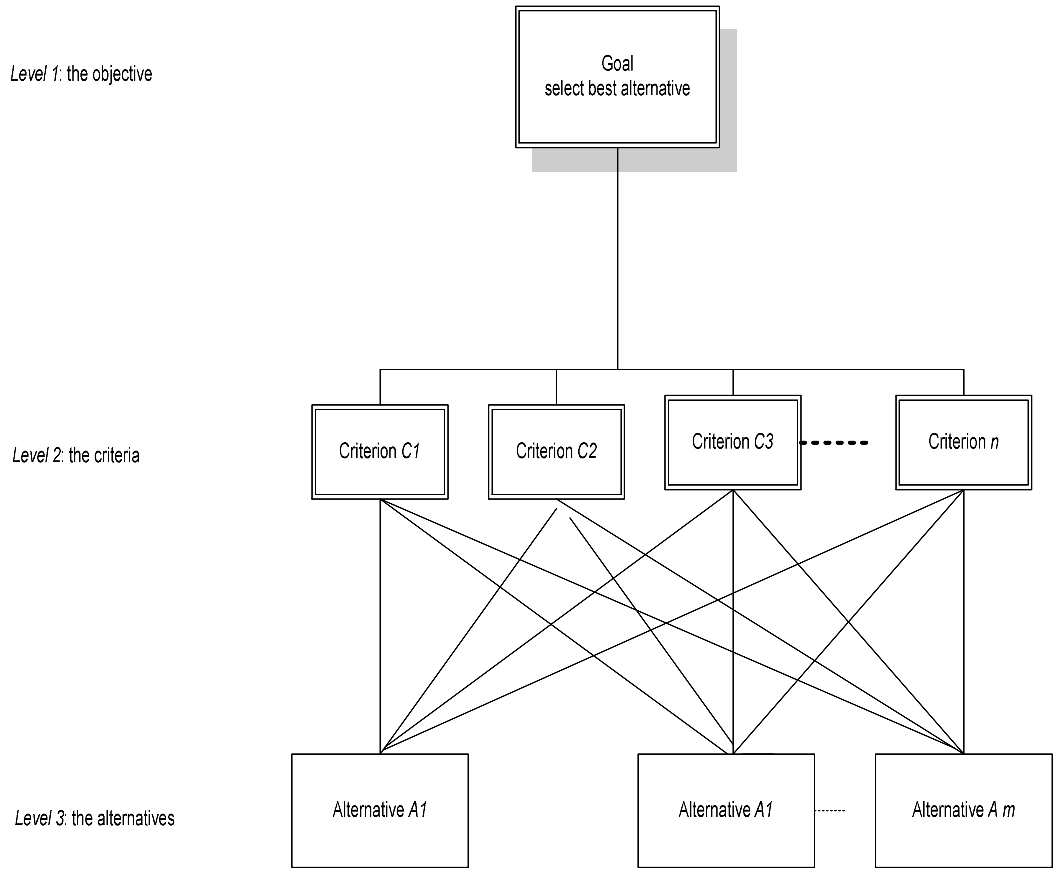


Figure 2. Three-level AHP hierarchy with n criteria and m alternatives.

Step 1: Weight each criterion's relative importance in achieving the goal.

Step 2: Weight each sub-criterion's relative importance contribution to the criterion to which it belongs.

Step 3. Weight each alternative's contribution to the sub-criterion and ultimately to each criterion.

Select the best alternative based on steps 1, 2, and 3.

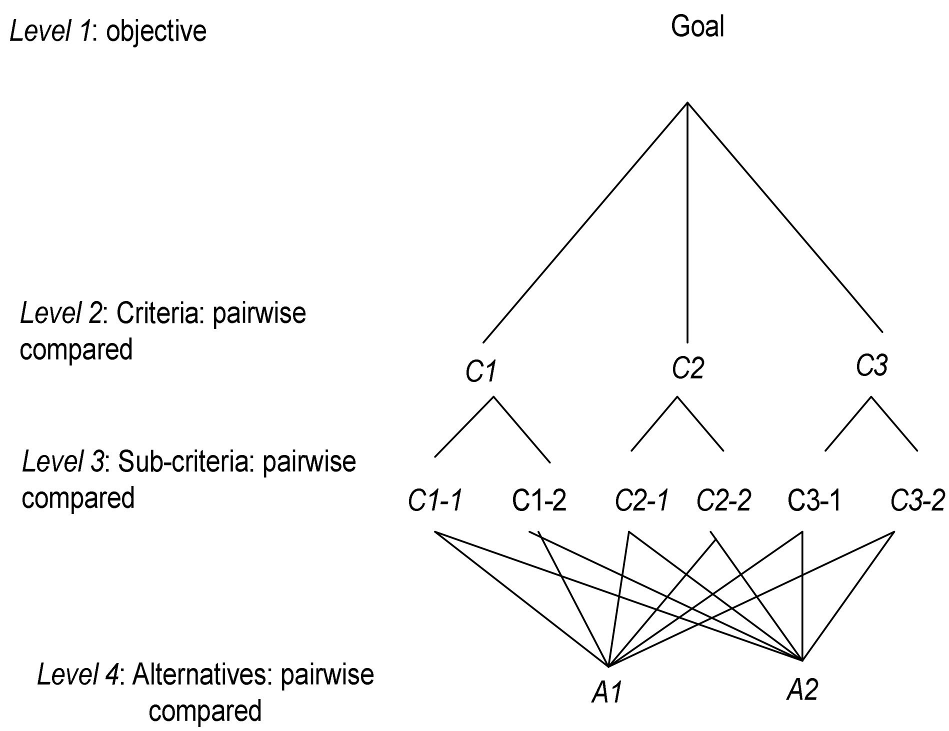


Figure 3. AHP hierarchy with sub criteria.

AHP Judgment Matrix

To perform the analysis of comparison scaling and weighting, Saaty (1980) used what he refers to as the judgment matrix. For each criterion, sub criterion, and alternative, a judgment matrix is created. The entries in these matrices indicate the values of relative importance of the elements compared to each other (criteria, sub-criteria, or alternatives), using the scales given in Table 1 or 2.

To prepare such a matrix, first put 1s in the diagonal table for which the rows match the columns, since each criterion or alternative is equally important to itself. Then the decision makers, the experts, or whoever is given the task of rating the criteria, sub criteria, and the alternatives, populates the rest of the matrix with the judgment values using the pairwise scale. The following numerical example of selecting corporate relocation site from among four alternative sites illustrates this idea. In this example, four criteria are considered, C1: saving due to relocation, C2: recreational facility, C3: schooling, and C4: housing. The criteria judgment matrix in Table 3 takes the general form of:

$$M = \begin{bmatrix} w_1 / w_1 & . & ..w_1 / w_n \\ .. & .. & ..... \\ w_n / w_1 & .. & ..w_n / w_n \end{bmatrix}$$

where M is the pairwise comparison matrix. For instance, w1/w1 is the ratio resulting from comparing C1's contribution to itself which should be 1, w2/w2 is the ratio of comparing C2's contribution to itself. It also should be 1. On the other hand ratio w2/ w1 resulting from comparing C2's contribution to C1's contribution is 3 if C1's contribution is three times as important as that of C2. The reciprocal of that is true, we can say w2/w1 = 1/3 which means C1 is three times less important than C2. The relative ratio scale derived from the pairwise comparison reciprocal matrix M is derived by solving:

$$\sum_{j=1}^{n} a_{ij} w_j = \lambda_{\max} w_i$$

with $a_{ji} = 1/a_{ij}$ or $a_{ji}a_{ij} = 1$, also $0 < a_{ij}$, thus M is known as positive matrix whose solution, known as the principle right eigenvector, is normalized as follows. When $a_{ij}a_{jk} = a_{ik}$, the matrix M = (a_{ij}) is said to be consistent and its principal eigenvalue is equal to n. Normalization is obtained by adding each column in the matrix M and dividing each weight in the column by the sum. The numerical example in Table 3 illustrates this idea.

$$\sum_{i=1}^{n} w_i = 1$$

Table3.

Matrix of Paired Comparison of Criteria.

	C1	C2	C3	C4
C1	1	1/3	1/9	1/9
C2	3	1	1/5	1/5
C3	9	5	1	1/2
C4	9	5	2	1

Then the alternatives need to be compared to each other in terms of their contribution to each of the criterion. This will result in 4 additional judgment matrices, one for each site compared to all others in terms of monitory, recreation, schooling, and housing.

The final selection follows:

A1 (AHP-score) = 4x(0.25) + 10x(0.25) + 36x(0.05) + 50x(0.33) = 21.8

A2 (AHP-score) = 4x(0.25) + 10x(0.25) + 36x(0.14) + 50x(0.08) = 12.5

A3 (AHP-score) = 4x(0.25) + 10x(0.25) + 36x(0.53) + 50x(.41) = 43.1

A4 (AHP-score) = 4x(0.25) + 10x(0.25) + 36x(0.28) + 50x(0.18) = 22.6

The values within parentheses are from the matrix of Table 4. The values outside the parentheses are the priority weights multiplied by 100. Site 3 is clearly the winner.

Table 4.

Normalizing the Criteria Judgment Matrix.

	C1	C2	C3	C4	C1	C2	C3	C4	Row Total	Average /Total/4
C1	1	0.33	0.11	0.11	0.05	0.03	0.06	0.17	0.17	0.04
C2	3	1	0.2	0.2	0.13	0.09	0.06	0.11	0.39	0.10
C3	9	5	1	0.5	0.4	0.44	0.30	0.28	1.43	0.36
C4	9	5	2	1	0.41	0.44	0.61	0.55	2.01	050
Total	22	11.3	3.31	1.8					4.0	1.00

Table 5.

Normalized Weighting Alternative Matrix with Respect to Criterion C3: Schooling.

	A1	A2	A3	A4	A1	A2	A3	A4	Row Total	Average Total/4

A1	1	0.20	0.11	0.20	0.05	0.02	0.06	0.06	0.19	0.05
A2	5	1	0.2	0.33	0.25	0.11	0.11	0.09	0.56	0.14
A3	9	5	1	0.2	0.45	0.54	0.55	0.57	2.12	0.53
A4	5	3	0.5	1	0.25	0.32	0.28	0.28	1.13	0.28
Total	20	9.20	1.8	3.53	1.00	1.00	1.00	1.00	4.0	1.00

AHP Consistency Ratio

Inconsistency has the potential to appear during the pairwise comparison of relative importance of criteria and alternatives. The more criteria and alternatives, the greater the chance of this situation occurring. Consider this straight forward example: someone indicates that they prefer A over B, B over C, and C over D. After more reflection, he or she tells you that D is preferred over A. To deal with this issue, a consistency index, CI, is calculated from the judgment matrix and it follows the following equality:

$$CI = (\lambda_{max} - n)/(n-1), \quad \lambda_{max} \geq n.$$

Where CI is the consistency index, λ_{max} is the maximum eigenvalue and n is the number of criteria. To calculate the consistency ratio for the criteria judgment matrix for our example the following procedures are followed where the matrix P and the vector Q are taken from Table 4.

$$P \qquad X \quad Q \quad = \quad R$$

$$\begin{vmatrix} 1 & 0.33 & 0.11 & 0.11 \\ 3 & 1 & 0.2 & 0.2 \\ 9 & 5 & 1 & 0.5 \\ 9 & 5 & 2 & 1 \end{vmatrix} \text{X} \begin{vmatrix} 0.04 \\ 0.10 \\ 0.36 \\ 0.50 \end{vmatrix} = \begin{vmatrix} 0.17 \\ 0.39 \\ 1.47 \\ 2.08 \end{vmatrix}$$

Matrix Q is a column matrix of the respective priority weights of the pairwise comparison matrix P. For the pairwise comparison matrix and the priority weights Q, the value of value R. was computed as follows:

1 x 0.04 + 0.33 x 0.1 + 0.11 x 0.36 + 0.11 x 0.50 = 0.17

The rest of the values in the vector R follow matrix multiplication.

The next step is to divide each element of R by the corresponding element in Q and average the results.

R/Q

0.17/0.04	=	4.25
0.39/0.10	=	3.90
1.47/0.36	=	4.08
2.08/0.50	=	4.16
Total		16.39
Average		4.10

The average is a characteristic of eigenvalue. We have been referring to it as λ. The consistency index (CI), for a square matrix of order N (in this example N = 4) is then

$$CI = (\lambda - N) / (N-1) = (4.10 - 4) / (4 -1) = 0.03$$

For the denominator of the CR, we use the random index approximations as given by (Saaty, 2001). Similar procedures are followed to calculate the consistency ratio for the alternative judgment matrices.

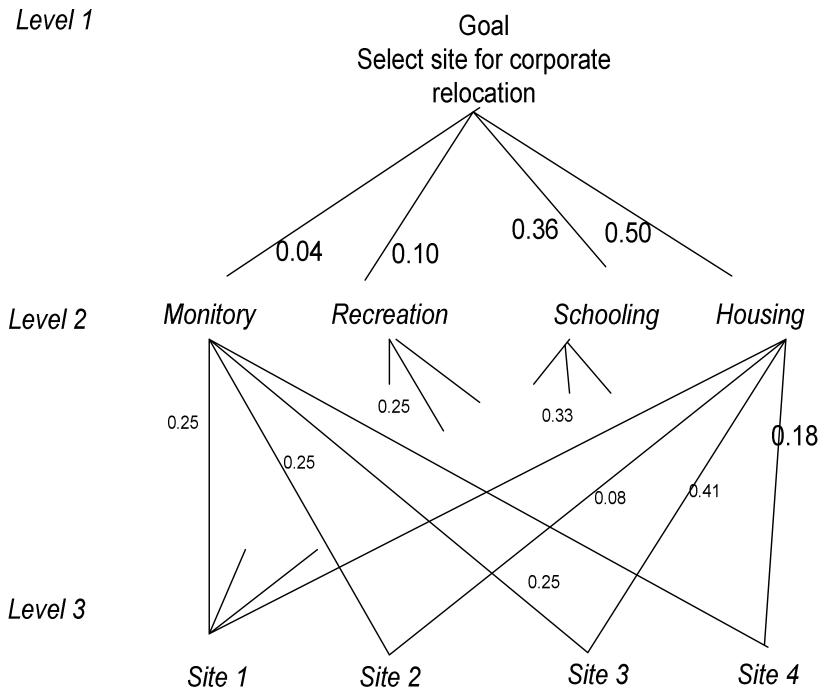


Figure 4. AHP hierarchy for a site selection example.

A consistency ratio CR is derived from dividing CI by a random average consistency index, RCI. RCI is derived from a sample of 500 of judgment scale measurement (9, 8, 7,…,1/2, 1/3,…, 1/9). According to Saaty, if CI is less than 10%, the decision maker can proceed with the analysis. If CI is greater than 10%, more refinement of the judgment matrix is needed. More details and examples of consistency ration calculation is given in chapter 4.

AHP Application: Semiconductor Supplier Selection (Chan & Chan, 2004)

Globalization and outsourcing in the past few years have elevated the supplier selection decision to a level of importance that is considered critical. Especially for the manufacturers of advanced semiconductor assembly equipment, supplier selection is a critical decision because it is a multicriterion decision problem and it can involve the expenditure of millions of dollars. Chan and Chan (2004) propose using the AHP to handle this problem. The pressure of the fierce competition in today's global economy has forced many organizations to outsource many parts of their operations. The field of semiconductor manufacturing involves large sums of capital investment and large scale operations. Chan and Chan contend that their case study will make the selection process systematic while providing some of the risk analysis needed for large scale industrial projects. Other reasons for using the analytical hierarchy process is because of its ability to structure the problem and its intangible attributes, its ability to structure the problems in a hierarchical manner to gain insights into the decision making process, and its ability to monitor consistency with which a decision maker makes a judgment.

In their research methodology, they categorized their techniques into 3 subcategories:

1. Background review of recent business environment of semiconductor equipment manufacturing industry, the background of the company being studied (its products, practices, and competitors).

2. Design of questionnaire, to interview the company's purchasing personnel and to further analyze the data. From the result of their analysis, the research direction is established and the foundation of the model is built on the research findings.
3. Development of a supplier selection model that includes the establishment and use of supplier selection criteria, the construction of the AHP model, design of an evaluation questionnaire, interviews with respondents, and synthesis of the model.

The main categories of the data they collected included:

1. Cost factor, measured on the basis of the total cost, supplier willingness and ability to share cost data and unit price.
2. Delivery factor, measured on the basis of the ability and willingness to expedite orders, speed by which a supplier can deliver, time needed to deliver prototypes, ability to meet due dates, and supplier location.
3. Flexibility factor, measured on the basis of the ability and willingness to change order volumes and change the mix of order items.
4. Innovations factor, measured on the basis of the technological capability of the supplier, willingness to share technological information, and ability to design new products or make changes to existing products.
5. Quality factor, measured in the form of the ability to process durable and reliable inputs that conforms to the buying firm's specifications. The quality factor was established as a primary concern in the supplier selection process.

6. Services factor, measured in the form of the attitude of the supplier in handling complaints and sharing of logistic information.

The following diagram was created to clarify and illustrate the factors addressed above that are associated with the model. It is based on the analytical hierarchy process in which pertinent criteria are measured with respect to their levels of importance to each other. The level of importance measurement is obtained from the data collected through interviews and questionnaires of purchasing personnel and people in key and strategic position within the buying firm.

The goal that Chan and Chan set for their study, which was the development of an AHP-based model for supplier selection for semiconductor manufacturing equipment, was achieved. They developed a model based on the AHP process. They used a case-study research method based on interviews with stakeholders and extensive analysis in the form of ranking of criteria, categorization and sub categorization of data.

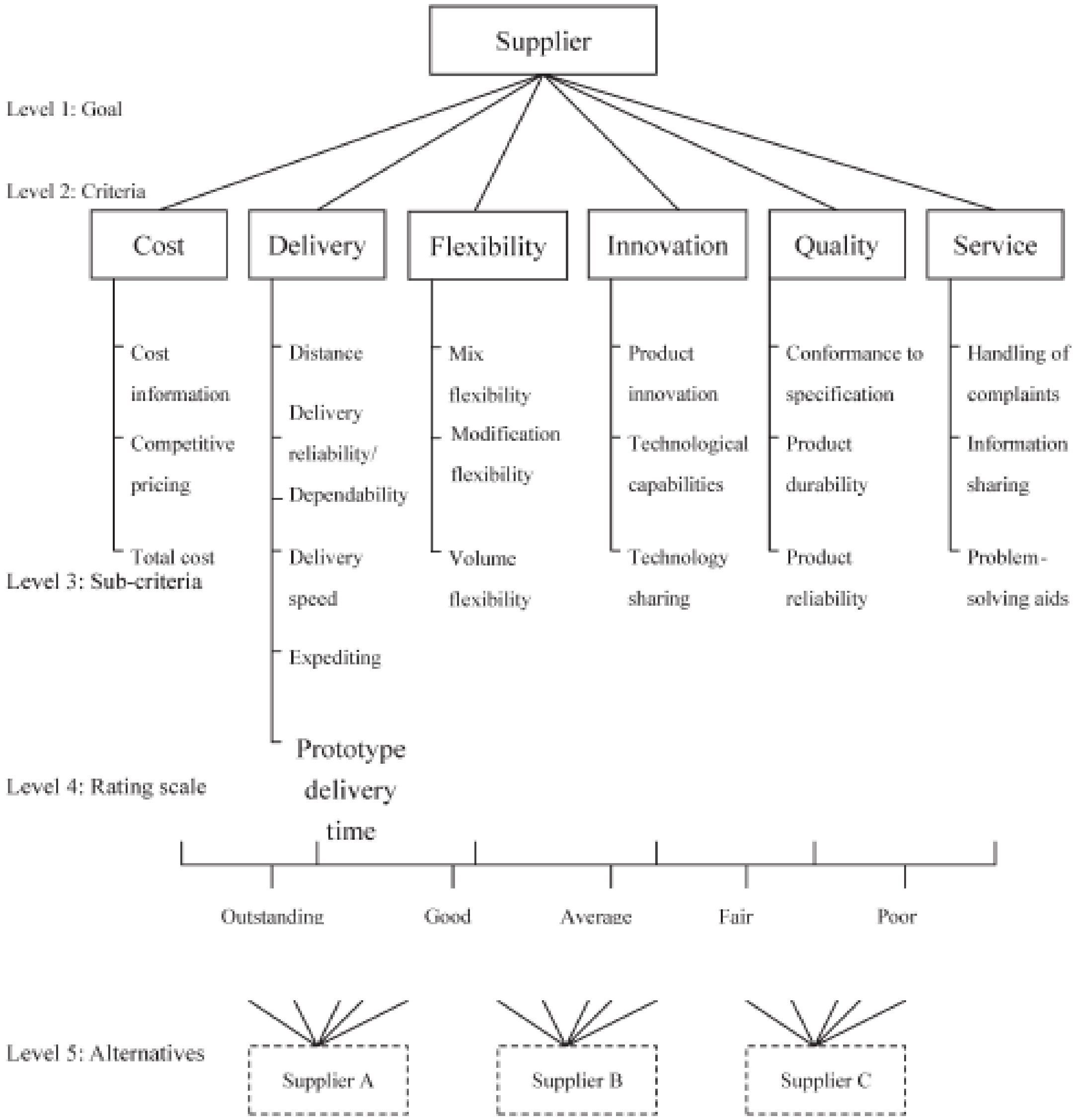


Figure 5. AHP decision structure – semiconductor manufacturer supplier selection.

They validated their constructs through getting the supplier to implement the model. This in itself is an achievement that worked toward the validation of their hypothesis. According to Chan and Chan, it is usually difficult to implement changes to the decision making processes in large organizations. However, they should have added measures to quantify and gain an insight to the level of satisfaction of both the suppliers and the vendors with the model.

This application, like others of this type, still has deficiencies related to the consistency ratio, crisp scale assumptions, and forcing the decision makers to provide absolute subjective judgment. These factors collectively make the classical AHP insufficient to capture uncertainty in the market, vague information, and decision maker's equivocation. To provide the decision analyst with the tools necessary to analyze complex decisions with imprecise input information solutions the AHP deficiencies are needed. This provides the motivation for further studies in this area of decision sciences. Specifically when major decisions fail, large financial losses and negative social impacts through misalignment of resources are the outcomes. Improving decision processes under uncertainty when only imprecise information is available provides for improved risk management. Improved risk management results in higher degree of confidence in major and complex decisions.

Benefits and Drawbacks of AHP

The AHP provides the same benefits as do multicriteria decision modeling (MCDM) in terms of focusing the decision maker's attention on developing a formal structure to capture all the important factors likely to differentiate a good choice of an option from a poor one. Multiattribute comparisons are generally found to be readily accepted in practice as a means of establishing information about the relative importance of criteria and the relative performance of options. The fact that the multiattribute comparison matrix provides some redundant information about relative values allows some cross checking to be done. Arguably, the resulting weights or scores may be more stable and consistent than if they were based on a narrower set of judgments. AHP also fits comfortably with circumstances where judgments, rather than measurements of performance, are the predominant form of input information. AHP usefulness stems from its ability to translate practical human judgments into crisp numbers. Nonetheless, despite these attractions, decision analysts have voiced a number of concerns about the AHP. The primary concerns are:

1. The crisp AHP scale has the potential to be internally inconsistent. A may score 3 in relation to B and similarly B may score 5 relative to C. Thus, based on the AHP scale, this means that a consistent ranking of A relative to C requires a score of 15, which is out of range when a bounded interval [1 – 9] is used.
2. Weights are elicited for criteria before measurement scales for criteria have been set. Thus the decision maker is induced to make statements about the relative importance of items without knowing what, in fact, is being compared.
3. Introducing new options can change the relative ranking of some of the original options. This rank reversal phenomenon is alarming and arises from a failure to consistently relate scales of performance to their associated weights.

Saaty (2001) rejects these concerns and contends that it is natural in a business environment for these situations to arise. Even if we accept Saaty's defense with respect to these concerns, a main disadvantage that is of a major concern is the apparent difficulties of AHP to handle uncertain decisions. Crisp scale can be inefficient and may lead to wrong decisions with unforeseen consequences. The causes of uncertainty may be due to incomplete or vague information about a particular factor or a supplier. Since some of the supplier evaluation criteria are subjective and qualitative, it is very difficult for a decision maker to provide exact pairwise comparison judgments (Mikhailov & Tsvetinove, 2004). The authors argue that, under many conditions, crisp data used in AHP are inadequate to model real life situations because human judgments that included preferences are often vague and cannot be assigned an exact numerical value.

A more realistic approach may be to use linguistic assessments instead of numerical values. In other words, the ratings and weights of the criteria in the problem are assessed by means of linguistic variables (Bellman & Zadeh, 1970; Chen, 2000; Delgado et al., 1992; Herrera et al., 1996; Herrera and Herrera-Viedma, 2000). Ross (1996) also deals with the relationship between linguistic expressions and fuzzy mathematics.

This research further extended the concept of AHP to develop a methodology for solving networking decision problems in a fuzzy environment. It considered the fuzziness and uncertainty in the decision data and the decision making processes. Linguistic variables were used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. The focus was to convert the classical AHP decision matrix into a fuzzy matrix. Furthermore, the developed model embedded the experts' pessimistic and optimistic attitudes into the decision analysis. A weighted-normalized fuzzy decision matrix was then constructed once the decision makers' fuzzy ratings were pooled. The following sections present some of the concepts that motivated this research.

Uncertainty

Uncertainty is a term used in subtly different ways in a number of fields, including economics, finance, statistics, insurance, psychology, and engineering. It applies to measurements that range from those already made or those yet to be identified (Gil-Aluja, 2004). Economic life in all its varied aspects is submerged within this context. Many decisions to be taken within this field are frequently getting more complex because of the consequence of uncertainty in the outcome of future events. Increasingly, research into techniques for the treatment of problems within the sphere of uncertainty becomes more necessary. Treating formal, exact, or even probable data is convenient because there is the sense of knowing with a degree of confidence where we are proposing to go. Gil-Aluja explains that treating uncertain data, accepting certain economic criteria without being sure of oneself, relying on the will to comprehend, almost constitutes an undertaking with irrationality.

Theory of Probability and Uncertainty

The theory of what is fuzzy and its valuation with its many variations is the mathematical tool to deal with uncertainty, while the theory of probabilities is the theory used relative to chance. Uncertainty and chance do not correspond to the same level of information. Uncertainty is not known to possess laws; probability, on the other hand, does. This leads to the conclusion that uncertainty is deficiently structured and it is subjectively explained. The concept of probability, on the other hand, is linked to chance which in itself is a measurement based on repeated observations in time and/or space. Thus, probability constitutes an evaluation that, if desired, can be as objective as possible.

According to Gil-Aluja (2004), the classification of models intended to solve problems can fall into one of the following categories ranging from the uncertain to the known:

1 Nondeterministic with unknown situations.

2 Nondeterministic with known possible situations but the assignment of an objective scale of value to them is not known.

3 Nondeterministic with situations and events that can be evaluated but not measured.

4 Nondeterministic with known situations and with measurable probability events.

5 Deterministic model in which the situations are known and a hypothesis can be considered that the event of a specific situation is known.

From an optimum point of view, one should build a model based on category 5 in which all parameters of the decision are predetermined. The cost in this case may inhibit such action and force researchers to stop at category 3. In this case the model deals with the most general of theories that are capable of describing an uncertain environment, namely the theory of fuzzy logic.

Fuzzy Logic

Dr. Lotfy Zadeh, in 1965, proposed a theory called fuzzy sets. According to Zadeh's definition, a fuzzy set is a class of elements or objects that lack definite boundaries between them. The fuzzy logic is useful to define objects which are characterized by vagueness and uncertainty. Fuzzy logic is a multivalued theory where intermediate values are expressed in a range, such as high, moderate, or low, instead of yes or no, true or false as in the classical crisp logic theory. The fuzzy sets are defined by the membership functions. The fuzzy sets represent the grade of any element x of space X that have partial membership in A (where A is a fuzzy set). The degree to which an element belongs to a set is defined by the value between 0 and 1.

An element x really belongs to A if $\mu(A(x) = 1$, and clearly not if $\mu(A(x) = 0$. As the value of $\mu(A(x)$ moves toward 1, the degree of membership of an element x increases in a fuzzy set A. Therefore, if $\mu(A(x) = 0.5$, then we can say x somewhat belongs to A. On the other hand, if $\mu(A(x) = 0.8$, then we can say x has a strong membership in A.

Fuzzy Numbers and Linguistic Variables

In this section, some basic definitions of fuzzy sets, fuzzy numbers and linguistic variables are reviewed from Buckley (1985), Kaufmann and Gupta (1991), Negi (1989), and Zadeh (1975). The basic definitions and notations below will be used throughout this research unless otherwise stated.

Definition 1: A fuzzy set A in a universe of discourse X is characterized by a membership function $\mu(A(x)$ which associates with each element x in X a real number in the interval [0,1]. The function value $\mu(A(x)$ is termed the grade of membership of x in A.

Definition 2: A fuzzy set A in the universe of discourse X is convex iff (if and only if)

$$\mu_A(\alpha x_1 + \alpha x_2) \geq \min((\mu_A(x_1), \mu_A(x_2))$$

for all x_1, x_2 in X and all $\alpha \in [0,1]$, where min denotes the minimum operator.

Definition 3: The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set A in the universe of discourse X is called normalized when the height of A is equal to 1.

Definition 4: A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. Figure 6 depicts a fuzzy number n in the universe of discourse X that conforms to this definition.

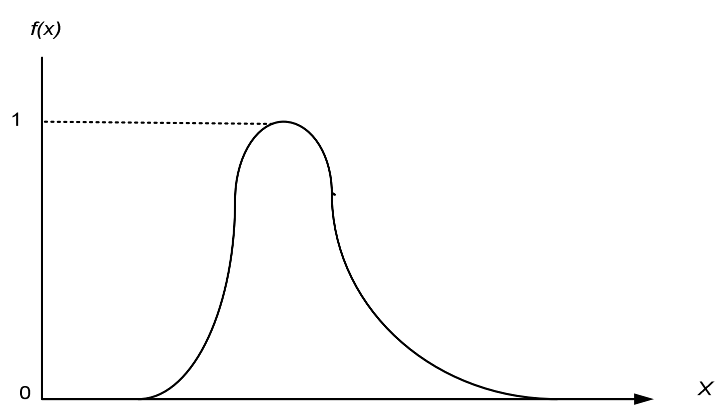


Figure 6. Fuzzy number n where f(x), also known as μ(x) = membership function.

Definition 5: The α-cut of fuzzy number A is defined as $A^{\alpha} = \{ x_i : \mu_A(x_i) \geq \alpha, x_i \in X \}$ where α is within the range [0,1]. The symbol A^{α} represents a non-empty bounded interval contained in X, which can be denoted by $A^{\alpha} = [A_l^{\alpha}, A_u^{\alpha}]$ where A_l^{α} and A_u^{α} are the lower and upper bounds of the closed interval, respectively (Kaufmann and Gupta, 1991; Zimmermann, 1991). For a fuzzy number A, if $A_l^{\alpha} \geq 0$ and $A_u^{\alpha} \leq 1$ for all $\alpha \in [0,1]$, then A is called a standardized (normalized) positive fuzzy number.

Definition 6: A positive trapezoidal fuzzy number (PTFN) A can be defined as (n_1, n_2, n_3, n_4) as shown in Figure 7. The membership function, $\mu_{\tilde{n}}(x)$ is defined as (Kaufmann and Gupta, 1991)

$$\mu_A(x) = \begin{cases} 0, & x \leq n1, \\ \dfrac{x - n1}{n2 - n1}. & n1 \leq x \leq n2, \\ 1, & n2 \leq x \leq n3, \\ \dfrac{x - n4}{n3 - n4}, & n3 \leq x \leq n4, \\ 0, & x \geq n4. \end{cases}$$

For a trapezoidal fuzzy number $A = (n_1, n_2, n_3, n_4)$, if $n_2 = n_3$, then is called a triangular fuzzy number. A non-fuzzy (crisp) number r can be expressed as (r, r, r, r). By the extension principle, as expressed by Dubois and Prade (1980), the fuzzy sum and fuzzy subtraction of any two trapezoidal fuzzy numbers are also trapezoidal fuzzy numbers; but the multiplication of any two trapezoidal fuzzy numbers is only an approximate trapezoidal fuzzy number. Given any two positive trapezoidal fuzzy numbers, A = (n_1, n_2, n_3, n_4), B = (m_1, m_2, m_3, m_4), and a positive real number r, some main operations of fuzzy numbers A and B can be expressed as follows:

1. Addition: A + B = ($n_1 + m_1$, $n_2 + m_2$, $n_3 + m_3$, $n_4 + m_4$)
2. Subtraction: A - B = ($n_1 - m_1$, $n_2 - m_2$, $n_3 - m_3$, $n_4 - m_4$)
3. Multiplication by a scalar: A x r = (rn1, rrn2, rn3 ,rn4), where r is a scalar value.

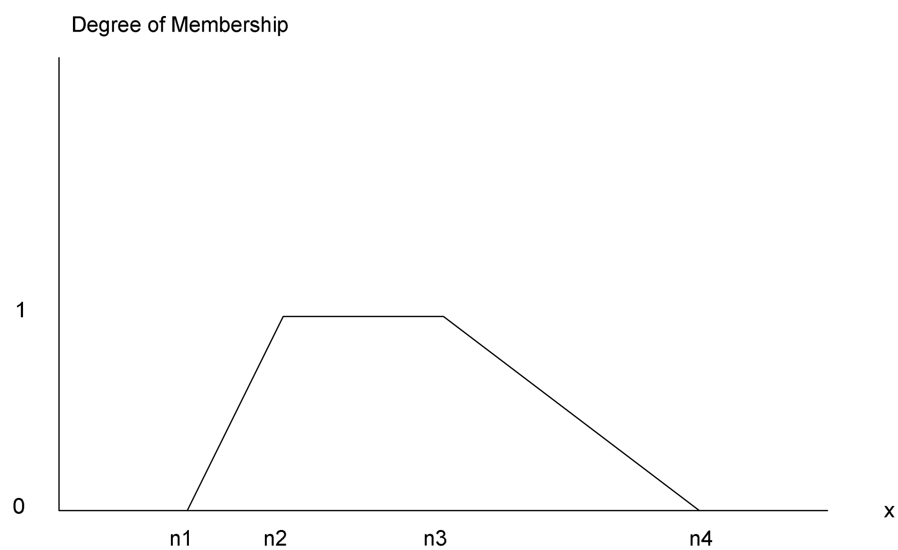


Figure 7. Trapezoidal fuzzy number A.

Definition 7: A matrix D is called a fuzzy matrix if at least one element is a fuzzy number (Buckley, 1985).

Definition 8: A linguistic variable is a variable whose values are expressed in linguistic terms (Zimmermann, 1991). The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions (Zimmermann, 1991). For example, weight is a linguistic variable whose values are very low, low, medium, high, very high, etc. Fuzzy numbers can also represent these linguistic values. Let A = (n_1, n_2, n_3, n_4) and B = (m_1, m_2, m_3, m_4) be two trapezoidal fuzzy numbers, then the distance between them can be calculated by using the vertex method (Chen, 2000) as

$$d(A,B)=\sqrt{[(n1-m1)^2+(n2-m2)^2+(n3-m3)^2+(n4-m4)^2)]/4}$$

Let A = (n_1, n_2, n_3) and B = (m_1, m_2, m_3) be two triangular fuzzy numbers, then the distance between the two fuzzy numbers A and B is given by

$$d(A,B)=\sqrt{[(n1-m1)^2+(n2-m2)^2+(n3-m3)^2]/3}$$

Note that a triangular fuzzy number is a special case of trapezoidal fuzzy numbers. (More details about the operations and ranking of triangular fuzzy numbers will be discussed later in the upcoming sections of this chapter). The vertex method is an effective method to calculate the distance between two trapezoidal fuzzy numbers. According to the vertex method, two trapezoidal fuzzy numbers A and B are identical if and only if d(A,B) = 0. Let A, B and P be three trapezoidal fuzzy numbers. Fuzzy number A is closer to fuzzy number B than the other fuzzy number P if and only if (iff) d(A,B) < d(B,P).

Triangle Fuzzy Numbers

Dubois and Prade (1980) defined a triangle fuzzy number (TFN) as a special class of fuzzy number whose membership defined by three real numbers, expresses as (l, m, u) with the following properties:

$$\mu A(x) = \begin{cases} x - l / m - l, & l \le x \le m, \\ u - x / u - m, & m \le x \le u, \\ 0, & otherwise. \end{cases}$$

Where m is the most possible value of a fuzzy number A, also known as the modal (Tang, and Beynon, 2007), l and u are the lower and upper bound, respectively. If the element falls before or beyond them, it will have no membership to the set. Note that $\mu A(x) = 0$, if $x < l$ and $x > u$. This is shown in Figure 8, $x < l$ and $x > u$ will have no membership in the fuzzy number A = (l, m, u).

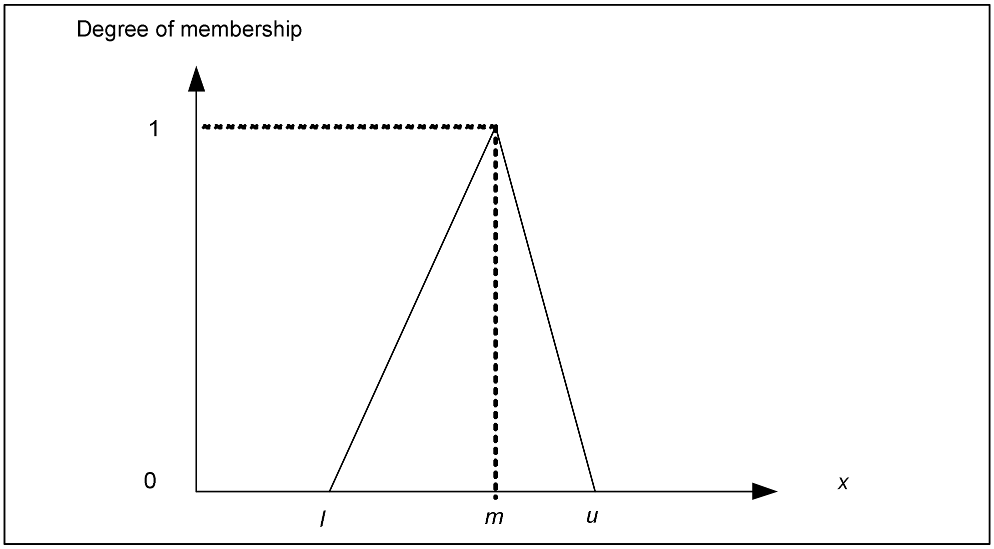


Figure 8. Graphical representation of triangular fuzzy number.

Operations on Triangular Fuzzy Numbers

Here are some of the fuzzy arithmetic operations on triangular fuzzy numbers. Let A and B be two triangular fuzzy numbers where A = (l_a, m_a, u_a) and B = = (l_b , m_b , u_b), where l, u are the lower and upper bounds of each of the triangular fuzzy number and m represents the middle value.

Addition: A + B = ($l_a + l_b$, $m_{a+} m_b$, u_a+ ,u_b).

Subtraction: A - B = ($l_a - l_b$, $m_a - m_b$, $u_a - u_b$).

Multiplication: A.B = ($l_a. l_b$, $m_{a.} m_b$, $u_a u_b$):

Scalar multiplication: $\forall k \quad greater \quad than \quad 0, k \in R, kA = (kla, kma, kua)$

This mathematical formulation reads: for every scalar value k greater than 0 and k belong to the set of real numbers R. If k is multiplied by a fuzzy number A = (la, ma, ua), then the result is a new fuzzy number that is equal to (kla, kma, kua).

Division:

$$\frac{A}{B} = \left\langle \frac{la}{ub}, \frac{ma}{mb}, \frac{ua}{lb} \right\rangle$$

Inverse:

$$A^{-1} = \left\langle \frac{1}{ua}, \frac{1}{ma}, \frac{1}{la} \right\rangle$$

Natural Logarithm: ln(A) = (ln(l), ln(m), ln(u))

Exponential: exp(A) = (exp(l), exp(m), exp(u))

A Triangular Number Example

Suppose, for example, that you are driving along a highway where the speed limit is 55 miles per hour (mph). You try to hold your speed at exactly 55 mph, but your car lacks cruise control, so your speed varies from moment to moment. If you plot your speed over a period of several minutes and then plot the result in Cartesian coordinates, you will get a function that looks like the diagram shown below in Figure 8. This curve represents a fuzzy number A = (50, 55, 60). If $x < 50$ and $x > 60$, then we can say x has no membership in the fuzzy number A, x's membership in the fuzzy set A = 0. This means that the speed is out of the range [50, 60]. If we take a membership value at 0.6, then speed is within the fuzzy number with a range of 53 to 57. If the membership = 1, then the vehicle speed is exactly at 55 mph. The membership concept can also be interpreted as a degree of fuzziness. Higher level of membership means a lower degree of fuzziness. A membership of 1 leads to an expression of the fuzzy number in the form of (55, 55, 55) in which l = m = u = 55 mph.

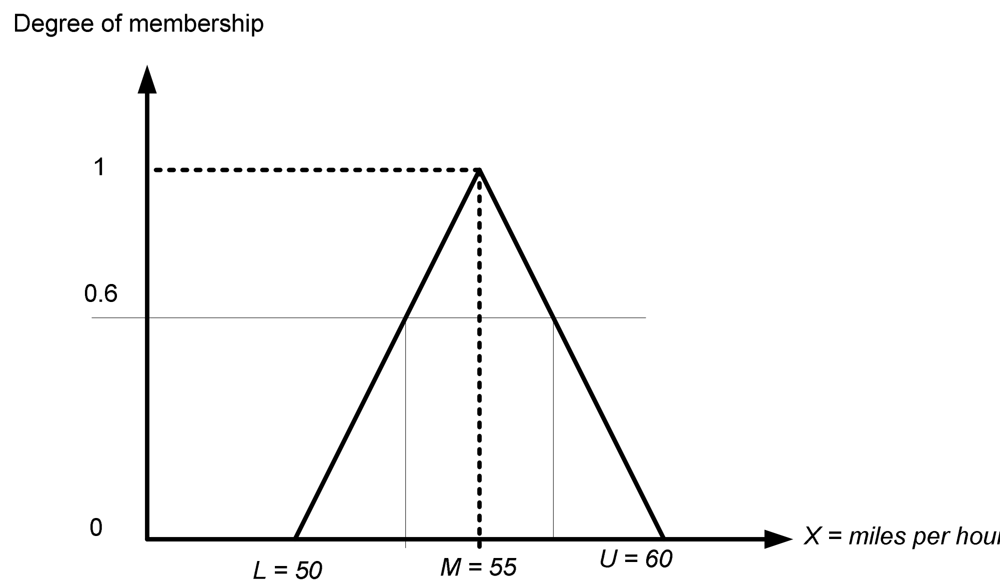


Figure 9. Triangular Fuzzy Number (TFN); A = (50, 55, 60).

Ranking of Triangular Fuzzy Numbers

Zhu, Jing, and Chang (1999) defined a ranking method of fuzzy numbers. Their procedure is as follows. Let μ_i (x) denote the membership function for the fuzzy numbers A_i. Next, the following relation is defined:

$$e_{ij} = \max_{x>y}\{\min(u_i(x), u_j(y))\},$$
$$\text{for all} \quad i, j = 1,2,3..., m.$$

Then A i dominates, or outranks, A j, written A i > A j iff (if and only if) $e_{ij} = 1$ and eij < Q where Q is some fixed positive fraction less than 1. Values such as 0.7, 0.8, or 0.9 might be appropriate for Q and value of Q should be set by the analyst and possibly be varied for sensitivity analysis (Triantaphllou, 2001).

Example

As an example of the previous discussion, suppose that the importance of two fuzzy alternatives ALT1 and ALT2 are represented by the two fuzzy triangular numbers $A_1 = ($ 0.2, 0.4, 0.6) and $A_2 = (0.4, 0.7, 0.9)$, respectively. Next it can be observed from Figure 10 that $e_{21} = 1$ and $e_{12} = 0.4 < Q$, where $Q = 0.9$ as shown in Figure 10. Therefore, $A_2 >$ A_1 and thus the best fuzzy alternative is ALT2.

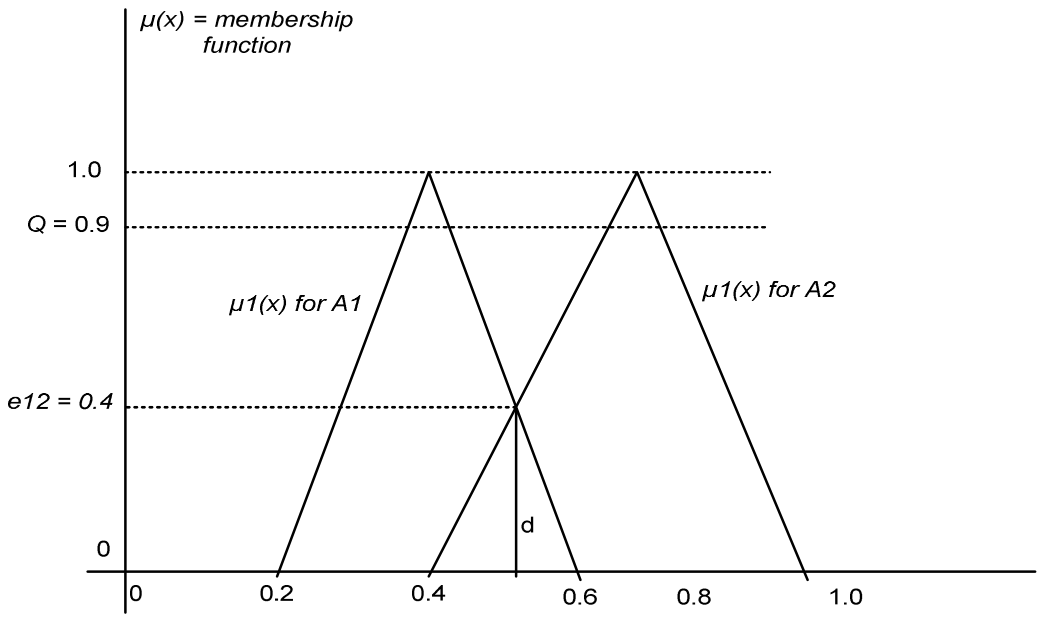


Figure 10. Membership function for two triangular fuzzy numbers A1 and A2.

Fuzzification

Fuzzification is the process of making a crisp quantity fuzzy. This is done by recognizing that many quantities that are considered crisp and deterministic are actually not deterministic at all (Ross, 1996). They carry considerable uncertainty. If the form of uncertainty happens to arise because of impression, ambiguity, or vagueness, then the variable is probably fuzzy and can be presented by a membership function. There are many ways to assign membership values or functions to fuzzy variables. According to

Duboise and Prade (1980), the assignment process can be intuitive or based on some algorithmic or logical operations. Ross (1996) lists some of these assignment methods as follows:

1. Inference
2. Rank ordering
3. Angular fuzzy sets
4. Neural networks
5. Genetic algorithms
6. Inductive reasoning
7. Soft partitioning
7. Meta rules
8. Fuzzy statistics

Sequential Elimination

A common limitation of AHP, whether fuzzy or crisp, is the potential for criteria to grow in volume and diversity to a level that is difficult to manage. Just consider a decision problem analyzed using AHP to make pairwise comparisons of all the criteria and the options. The benefit of this is that humans are quite good at making such pairwise comparisons (Saaty, 2001). However, they are not particularly good at ranking a long list based on some arbitrary criteria. The downside of using pairwise comparisons is the potentially very large number of such comparisons which need to be made. Applying

AHP to a decision involving n options and m criteria would require $\binom{n}{2} \times m + \binom{m}{2}$ multi-attribute comparisons.

Example

Applying this combinatory formulation, a complex decision might have, say, 8 criteria and 6 options, which would necessitate:

$$Numberofpairwi\sec omparisons = \binom{6}{2} x8 + \binom{8}{2}$$
$$= \frac{6!}{2(6-2)!} x8 + \frac{8!}{2(9-2)!} = \frac{5760}{48} + \frac{40320}{1440} = 120 + 28 = 148$$

Where 6! reads as 6 factorial, 6! = 6(6-1)(6-2)(6-3)(6-4)(6-5)(1). By definition, 0! (zero factorial) = 1. This can be said to be true because of the convention that the product of no numbers at all is one. In essence, in performing no multiplication at all is equivalent to multiplying by one.

Dealing with a possible 148 comparisons would be quite a task to remain focused for any individual! But the argument in favor of AHP is that while making 148 separate pairwise comparisons would be time consuming, it would still be quite feasible. This is more than can be said for many of the other potential methods. The sequential elimination method, if introduced to the analysis in the early stages, can result in a reduction in the number of criteria and alternatives because they failed to meet certain thresholds.

Lang and Merino (1996) used a matrix, shown in Figure10, to help present the sequential elimination method. The attributes, both monitory and non-monitory, are identified by subscripts ranging from 1 to M, with j representing any particular attribute. The presentation of the alternatives ranges from 1 to, i representing any particular alternative. Figure 10 depicts this matrix presentation where A_{ij} is the j^{th} attribute of the i^{th} alternative. If any criterion, monitory or non-monitory, is the same for alternatives or has very little bearing on any of them, it can be culled for lack of relevancy. Any criterion that fails to satisfy specified standards or thresholds is also removed, as those that are dominated on all counts by others. The decision maker can employ this sequential method of elimination in the early phases of the analysis to weed out alternatives that crowd the domain needed for, say AHP or fuzzy AHP analysis. This approach of integrating the sequential elimination with other analysis theory should, at least, lead to two benefits: a manageable decision problem and more accurate analysis due to the reduction in the number of pair compared criteria.

Alternative Attributes	1 2 3 …..	i	… ………………….N
1 2 3			
J		A_{ij}	

Figure 11. A matrix representation of multi-attribute sequential elimination method.

Fuzzy AHP

Laarhoven and Pedrycz (1983) proposed the first studies that applied fuzzy logic principle to AHP. Buckley (1985) initiates trapezoidal fuzzy numbers to express the decision maker's evaluation on alternatives with respect to each criterion while Laarhoven and Pedrycz were using triangular fuzzy numbers. Chang (1996) introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for a pairwise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pairwise comparisons

Zhu, Jing, and Chang (1999) provided the basic theory of the triangular fuzzy number and improved the formulation of comparing the triangular fuzzy number's size. On this basis, a practical example on petroleum prospecting was introduced. Leung and Cao (2000) proposed a fuzzy consistency definition with consideration of a tolerance deviation. Essentially, the fuzzy ratios of relative importance, allowing certain tolerance deviation, were formulated as constraints on the membership values of the local priorities.

Enea and Piazza (2004) focused on the constraints that have to be considered within fuzzy AHP. They used constrained fuzzy AHP in project selection. Kahraman, Cebeci, and Ulukan (2004) used the fuzzy AHP for comparing catering firms in Turkey. The means of the triangular fuzzy numbers produced by the customers and experts for each comparison were successfully used in the pairwise comparison matrices. Tang and Beynon (2007) used the fuzzy AHP method for the application and development of a capital investment study. They tried to select the type of fleet car to be adopted by a car rental company. Tolga, Demircan, and Kahraman (2005) used fuzzy replacement analysis and the analytic hierarchy process in the selection of the operating system. The economic part of the decision process had been developed by Fuzzy Replacement Analysis.

Non-economic factors and financial figures had been combined by using a fuzzy AHP approach. Chan and Kumar (2005) proposed a model for providing a framework for an organization to select the global supplier by considering risk factors. They used the fuzzy extended analytic hierarchy process in the selection of a global supplier in their current business scenario. Verma and Pulman (1998) examined the differences between managers' ratings of the perceived importance of different supplier attributes and their actual choice of suppliers in an experimental setting. They used two methods: a Likert scale set of questions and a discrete choice analysis (DCA) experiment. Ghodsypour and Obrien (1998) proposed an integration of an analytical hierarchy process and linear programming to consider the tangible and intangible factors in choosing the best supplier and placing the optimum order in a maxima format for the value of acquisition.

Parakash (2005) introduced an approach of combining scoring methods with fuzzy expert systems to perform land suitability analysis for agricultural crops. Bevilacqua and Petroni, as cited in Parakash (2005), developed a system for supplier selection using fuzzy logic. Kahraman, Ruan, and Ibrahim, (2003) used fuzzy AHP to select the best supplier firm providing the most satisfaction. Chan and Kumar (2006) developed a fuzzy model for global supplier selection that considered factors such as: overall cost of the product, quality of the product, service performance of the supplier, supplier's profile, and risk factors.

Current Research and the Research Model

All the above research assumed a single degree of fuzziness across the pairwise comparison matrix for factors, sub factors, and alternatives. Using a single degree of fuzziness has the inherent inability to capture the differences in the confidence levels when criteria and alternatives are judged. A single expert may be sure of how criterion A fared when compared to criterion B, but may not be as confident when criterion C is examined against criterion B. Furthermore, none of this research appears to have dealt with the experts' pessimistic and optimistic attitudes toward the economic outlook or the issues related to the decision problem under consideration.

The fuzzy hierarchical model overcame these limitations through assignments of different levels of ά – cuts to each individual fuzzy judgment. The mathematical representation of each pairwise comparison judgment was in the form (l, m, u, ά – cut). For applications when the numbers of criteria and alternatives are large, the model provided the user with the option to set the same ά – cut across all judgments. In either case, whether the ά – cut was assigned individually to subjective judgments or across the board, it provided a useful tool for sensitivity analysis. The decision maker will have the option to examine different what if scenarios and the effect of the changes in the degree of fuzziness on the outcome.   The use of the ά – cut is to vary the membership function from 0 to 1, where 0 denotes most fuzzy and 1 denotes absolute crispness. The scale for the ά – cut is 0, 0.1, 0.2,…, 0.9, 1.0, with 0.1 increments. Using a value of 1 for the ά – cut lets the model revert to the classical AHP method. To a embed experts, attitude within the analysis of decision problems, a delta function was introduced and applied to the defuzzified pairwise comparison matrix to adjust the modal value of the fuzzy judgment to the left (pessimistic) or to the write (optimistic). More on alpha-cut and delta-function is discussed in chapter 4.

Summary

The literature review component of this research project provided an important element to deal with issues of relevancy, applicability, and significance. The literature review in this chapter attempted to assess pertinent literature related to the problem at hand which is overcoming AHP deficiencies in analyzing uncertain decisions and the developed solution to overcome these deficiencies.The focus was on four related themes: (a) the challenges in networking architecture and design (Bello, 2003; Schoeing, 2003); (b) multicriteria decision modeling (Saaty, 1980, 1996, & 2001; Triantaphyllou, 2001); (c) the theory of fuzzy sets, mathematical concepts related to AHP; and (c) the theory of fuzzy sets was discussed. Additionally, a linkage to current research (Chan, Kumar 2006 Parakash, 2005; Tang, Beynon (2007); and Tolga, Demircan, Kahraman, 2005) was provided.

The discussion in this chapter focused on a review of the literature related to the turbulent telecommunications industry and the difficulties of the environment in which it operates. The acquisition of telecommunications services and infrastructures as a complex multicriteria decision problem was presented. A focuses of the review was on the issues and challenges related to networking infrastructure acquisitions. The review dealt with the classical multicriteria decision making (MCDM) models and the drawbacks of such methods. Some of the drawbacks that have been indicated include the deficiencies in its scales and its inability to capture uncertainties in the economic and business environments.

Addressed in this chapter was the current research to improve upon the classical multicriteria methodology. The fuzzy sets, specifically their operations and rankings, were reviewed with illustrative examples. The review was concluded with a linkage between existing literature and the proposed research. Further, a review of the sequential elimination method was given. This method is useful in reducing the number of criteria and alternatives. Thus, it simplifies handling of complex multicriteria decision problems with a large number of attributes.

The focus in the next chapter (chapter 3) is on the research methodology. The analytical research design method is used in this research. This methodology was used to develop AHP, the underlying framework for the model developed in this dissertation. To provide continuity and minimize unintended deviations, the AHP's research methodology is extended to the research in this dissertation. Provided in this chapter is the rationale for using this research method compared to others. The research methodology is grounded in mathematical concepts, proofs and formulation. These components form the main effort to develop the model. The tasks to achieve the research objectives and to answer the research questions are outlined. Further, a simulated case study to verify the applicability of the developed model in a practical setting is introduced.

CHAPTER 3: RESEARCH METHOD

Overview

This chapter discusses the research design method that was used to develop the fuzzy hierarchical model. It presents different strategies to select a research method and previews these methods. Justification for selecting the research method is also discussed. The advantages and disadvantages of the research method used and how to mitigate the disadvantages are discussed. Further, an outline of the steps to develop a practical application to verify the model constructs is put forward.

The aim of this dissertation was to improve the analytical hierarchy decision modeling (AHP). To achieve this objective, the following tasks were performed: (a) Enhanced AHP in an attempt to overcome its deficiency in handling decisions under uncertain and vague conditions, (b) Devised fuzzy modeling synthesis based on AHP framework, (c) Developed the required mathematical fuzzy set operations and concepts, (d) Developed the required fuzzy matrices as they relate to the fuzzy hierarchical model, (e) Developed a software tool to expedite verification of model operations, and (f) Applied the model, using the software application to a simulated practical case study for further validation. These tasks are embedded in the general framework outlined below:

1. Analysis of fuzzy mathematical concepts.
2. Derivation of the mathematical hierarchy of the proposed solution.
3. Fuzzification of crisp judgments.
4. Defuzzifcation of fuzziness to crisp weights.
5. Consistency testing.

6. Ranking of alternatives.

According to Buckley, Buckley, and Chiang (1976), Martin (2004), and Moole (2005), there are multiple methods of conducting scientific research. Moole stated that "suitable research methods depend on the subject being researched" (p. 51). The research problem was identified by reviewing prior research. The analytical research method is used in this research. It should be noted that the analytical research method and the analytical hierarchy process are not related. The analytical research method is a research design methodology while the analytical hierarchy process is a multicriteria decision modeling framework.

Research Design

The research problem and the research design methodology discussed in this dissertation were identified through extensive review of the current literature (Saaty, 1980, 1996, 2001; Tryantaphyllou, 2001) as it related to the framework of AHP. The works of Arslan and Khist (2007); Efedigil and Kongar, (2007); and Mikhailov and Tsvetinove (2004) were studied with a focus on AHP's deficiencies in handling decisions under uncertainty. Other literature was reviewed as well as described in chapters 1 and 2. Cheng et al. (2007); Isiklar (2007); and Kuo and Chen (2007) argued for the need to apply decision modeling under uncertainty to networking problems. The analytical research design method was the predominant methodology used in the reviewed studies. These studies were based on mathematical concepts, derivation, and formulation based on proven mathematical techniques and proofs. The analytical method appears suited for this research because of the need for an analysis of fuzzy set theoretical concepts and the

AHP axioms without reference to empirical data (Moole, 2005). Adherence to proven mathematical formulation ensures correctness of the mathematical concepts advanced in this research project. Other methods (quantitative, qualitative, and experimental) are not suited for this problem because of its defined mathematical nature (Martin, as cited in Moole, 2005). The use of deductive logic on both the fuzzy set theory and the framework of AHP was the predominant analysis method. The general theory of fuzzy sets, the constructs of AHP, and operations on fuzzy matrices were dealt with as they apply to the fuzzy hierarchical model. The study compared and contrasted the fuzzy hierarchical model and the classical AHP to elicit their relative strengths and weaknesses.

The quantitative and qualitative research design methods depend mainly on data collections and the use of instruments such as interviews and surveys. On the other hand, the analytical research design method does not involve data collection. Moole (2005) reiterated this notion when he stated "unlike other methods, such as quantitative and qualitative methods, which consist mainly of data collection and interviews" (p. 52). The analytical method uses step by step derivation of new formulae from proven fuzzy set theory and AHP framework and constructs. The derivation of the new formulae adhered to the techniques used in the AHP framework to provide consistency and completeness.

Justification for Using the Analytical Research Method

Following the logic of Buckley, Buckley, and Chiang (1976); Martin (2004); and Moole (2005), the research problem was derived from a deductive syllogistic work whereby the researcher used internal logic to perform mathematical analysis of the subject under study. The mathematical analysis presented in chapter 2 is based on the mathematical modeling of multicriteria decision making problems. This research project was a logical extension of the reviewed work that primarily used the same research method. Martin and Moole described a framework for selecting the methodology. The analytical methodology was applied in the reviewed literature and was applied to this dissertation research project as well. According to Martin, selecting a research method strategy as presented in Moole (2005, p. 53) is a function of the subject under study. The strategy encompassed one of four approaches: opinion, empirical, archival, and analytical. Each research method involves domains, and techniques. The techniques can be formal or informal. The following is an outline of these research methods, their domains, and the associated techniques:

1. Opinion Domain.
    a. Individual.
        i. Formal techniques (survey).
        ii. Informal technique (interview).
    b. Group.
        i. Formal techniques (Delphi).
        ii. Informal technique (brainstorming).
2. Empirical Domain.
    a. Case.
        i. Observation technique (formal and informal, observation).
    b. Field.
        i. Formal techniques (time and motion study, observation).

c. Laboratory.

i. Formal techniques (simulation).

ii. Informal techniques (observation).

3. Archival Domain.

a. Primary.

i. Content analysis technique (scanning).

b. Secondary.

i. Sampling techniques (scanning).

c. Physical.

i. Erosion/accretion techniques (observation).

4. Analytical Domain.

a. Internal logic.

i. Mathematical modeling (formal).

ii. Philosophical argument (informal).

The analytical research method which is based on internal logic of the authors has several advantages and disadvantages

Advantages

1. Analytical research does not need additional data and also it is not limited by existing data, according to Moole (2005).
2. It provides a broad scope for imagination and creativity.

3. It is best suited for operational and logic research techniques, according to Buckley and Chiang (1976).

Disadvantages

1. Difficult to criticize and can be abused to mislead.
2. Subject to logical errors, according to Martin as cited in Moole (2005).

The abuse factor is rare according to Perkins (2006). Perkins presented an example of a scientist who spent an inordinate amount of time to develop a mathematical model that exaggerated the return on investment. This model was used to secure loans from the international monitory funds (IMF). Perkins explained that the limited time frame deprived managers from a thorough review. The oversight was quickly discovered and corrected.

To overcome the disadvantages of the analytical methods, the investigator developed a practical application to test and verify the model in this study. To avoid logical errors, numerical examples were used. Further test of the logic was conducted by comparing and contrasting the model to the classical AHP that used crisp subjective judgments. This was accomplished by having the developed model revert to the classical AHP operation mode.

Descriptive research includes surveys and fact-finding enquiries of different kind. The major purpose of descriptive research is description of the state of affairs as it exists at present. In social science and business research the term *Ex post facto* is often used for descriptive research studies (Kothari, 1990). Descriptive research usually relies on quantitative and qualitative research techniques. The main characteristic of the descriptive research method is that the researcher has no control over the variables; he can only report what has happened or what is happening. According to Kothari, most *ex post facto* research projects are used for descriptive studies in which the researcher seeks to measure such items as, for example, frequency of shopping, preference of people, or similar data. *Ex post facto* research studies also include attempts by researchers to discover causes even when they cannot control the variables. In analytical research, on the other hand, the researcher has to use facts or information already available, and analyze these to make a critical evaluation of the material, according to Kothari. This study is of the analytical research type where facts and information relevant to decision making under uncertainty are used to evaluate a current situation in networking infrastructure decisions. Deficiencies in the existing situation are identified and solutions to such difficulties are proposed.

With this logic in mind, pure quantitative or qualitative methods would be inappropriate for this study. One primary reason is that the framework (AHP) which the study used as basis for the proposed solutions was developed with a pure analytical research design method. To provide continuity, as stated earlier in this chapter, the analytical method must be used. In this study the analytical research method was used to develop a proposal for a solution to a current situation in multicriteria decision making.

Validating analytical techniques often uses simulated data, according to Kothari (1990). In this study two approaches were used one was applying the proposed solution to a simulated case study. Integrated with this approach a software platform was used to further verify the components of the proposed solution. A key question in research is the reliability the research method and results. After all to a large extent that is what a researcher is looking for - independent, objective results and analysis that reflects reality (Asia Market Research, 2008). The reliability of the analytical research method is derived from the adherence to well established and proven mathematical techniques (Moole, 2005). In this study mathematical analysis of matrices, fuzzy sets theory, and multicriteria decision sciences are strictly enforced to formulate the proposed solution. Simulated case study and collaboration with networking experts to develop the case study and review the results of solution applications are the basis for ensuring the reliability of the research results. The following sections describe these validation approaches.

Experimental Phase: Scenario Development

Once the algorithms were developed and the model was created, it was important to validate the results. Validation was carried out through the creation of a series of scenarios that included variations of the degree of fuzziness across all alternatives and criteria. Also, the degree of fuzziness variation relative to the individual criterion and alternative was performed.

Sensitivity analysis was performed to examine the impact of changing the degree of fuzziness on risk factors. It was used also to experiment with the ability of the model to sift between input data and return tangible and easily understood results, Using these results, the decision maker or the manager can make a quantifiable choice.

Practical Situation

A practical case study (application) was developed through the review of relevant literature related to networking infrastructure acquisitions and design. Two networking specialists assisted in developing a simulated case study relevant to the issues address by the study. The simulated case study focused on the practical aspects of the proposed solution. A case study focuses on either the case or on an issue the case illustrates in-depth. According to Creswell (1998),

> Case study is the study of a bounded system with the focus being either on the case or an issue that is illustrated by the case. A case study provides an in depth study of this system, based on diverse array of data collection materials, and the researcher situates this system or case within its larger context or setting. (p. 251)

The application of the model to a problem in the networking field was to verify its constructs individually and collectively in solving problems in practical settings. Two experts in networking and datacenter design collaborated in defining design and evaluation criteria for a simulated application that represented a substantial business decision. This decision dealt with handling the increased demands for on-line service in the banking industry. Model application encompassed a number of steps:

1. Identify relevant practical case related to networking acquisitions from literature.
2. Define acquisition criteria.
3. Refine acquisition criteria.
4. Develop fuzzy pairwise comparison matrices for each criterion.
5. Derive fuzzy consistency ration.
6. Derive fuzzy weights for each criterion.
7. Identify alternative solutions.

8. Develop pairwise matrices for each alternative as it relates to each criterion.
9. Derive consistency for each alternative.
10. Derive weights for each alternative.
11. Defuzzify.
12. Rank alternatives.
13. Perform sensitivity analysis.

The investigator developed a software platform to add credence to the simulation of data used to validate the solution. The software application focused on testing the mathematical formulation used to develop the solution. An emphasis in the software application was on using standardized techniques for user interfacing and graphical representation of results reporting

Summary

There were four research domains available to researchers to use. The method selected to conduct a research objective depends on the subject under investigation, according Buckley, Buckley, and Chiang (1976), Martin (2004), and Moole (2005). The analytical research method is grounded in the internal logic of the investigator. It is based on mathematical concepts, formulations, and derivations. The research problem was identified through extensive review of relevant literature that used the analytical method for research design. The research is a logical extension of the reviewed current literature. For this reason, the analytical research design was best suited for this project.

This chapter presented the research method used in this dissertation. It presented the type of available research methods and previewed strategies for selecting the appropriate research methodology. The effort and steps needed to achieve the research objectives and answer research questions were also discussed. A practical application to verify the constructs of the model was outlined.

Validation procedures were described in this chapter. Validation relied mainly on a simulated case study pertinent to a major decision in the area of networking infrastructures planning and design. A software platform was developed to implement the solution formulation into a tool that a decision analyst can use. A decision analyst should be able to use the software application to enter decision information and vies results in graphical form. A decision analyst can also use the software to perform sensitivity analysis

Chapter 4 is often dedicated to presenting the results of the research. It is usually includes data collection methods, data analysis and the result of a quantitative, qualitative, or some times a mixed method of analysis. However, this presentation did not follow the Walden often used format. Instead, it followed the format used in Moole (2005). It focused on the formulation of the model. It laid the groundwork for the framework, the detailed tasks undertaken to develop the model, and the mathematical derivation that the investigator carried out. Presented in this chapter were the software algorithm and a simulated practical application to verify the model’s constructs. The simulated data and developed software were used to validate the research and answer the research questions. The results of applying the model to a practical application with simulated data and the software were presented.

CHAPTER 4: RESULTS

This chapter focused on developing a fuzzy hierarchical model intended to handle decisions under uncertainty with special emphasis on decision criteria for data centers and networking design. The steps and techniques taken to accomplish this task encompassed five categories: (1) development of the mathematical formulations of the new fuzzy hierarchical model including fuzzy pair comparison matrices and fuzzy weighting, (2) development of software algorithm and tools to implement the developed model, (3) development of a current literature and networking experts' based simulated practical model application, (4) sensitivity analysis to provide the decision maker with insights needed to gain a better understanding of the decision problem, and (5) comparison and contrast of the fuzzy hierarchical model with the classical AHP decision modeling. Before proceeding further, it will be helpful to preview multicriteria decision making framework.

Decision Making Framework

Planning and decision making processes are executed in four major phases: "intelligence, design, model formulation, and choice or decision" (Sharifi, 2003, p. 15), as shown in Figure 12.

1. Intelligence: The description of the system under consideration and understanding of the system's behavior.
2. Designing and planning of a decision model: This phase integrates the following two components: (1) assessment of current situation, and (2) decision objective formulation.

3. Model formulation: This is a critical phase because using the wrong model can result in catastrophic outcomes that may achieve no ultimate value for the organization.
4. Deciding or choosing an alternative (solution): This phase encompasses the following tasks:
    i. Generation of alternate solutions to the problem on hand.
    ii. Assessment of the impact of each solution on the decision objective.
    iii. Evaluation of each of the alternatives with respect to achieving the desired goal.
    iv. Explaining and visualizing the decision.

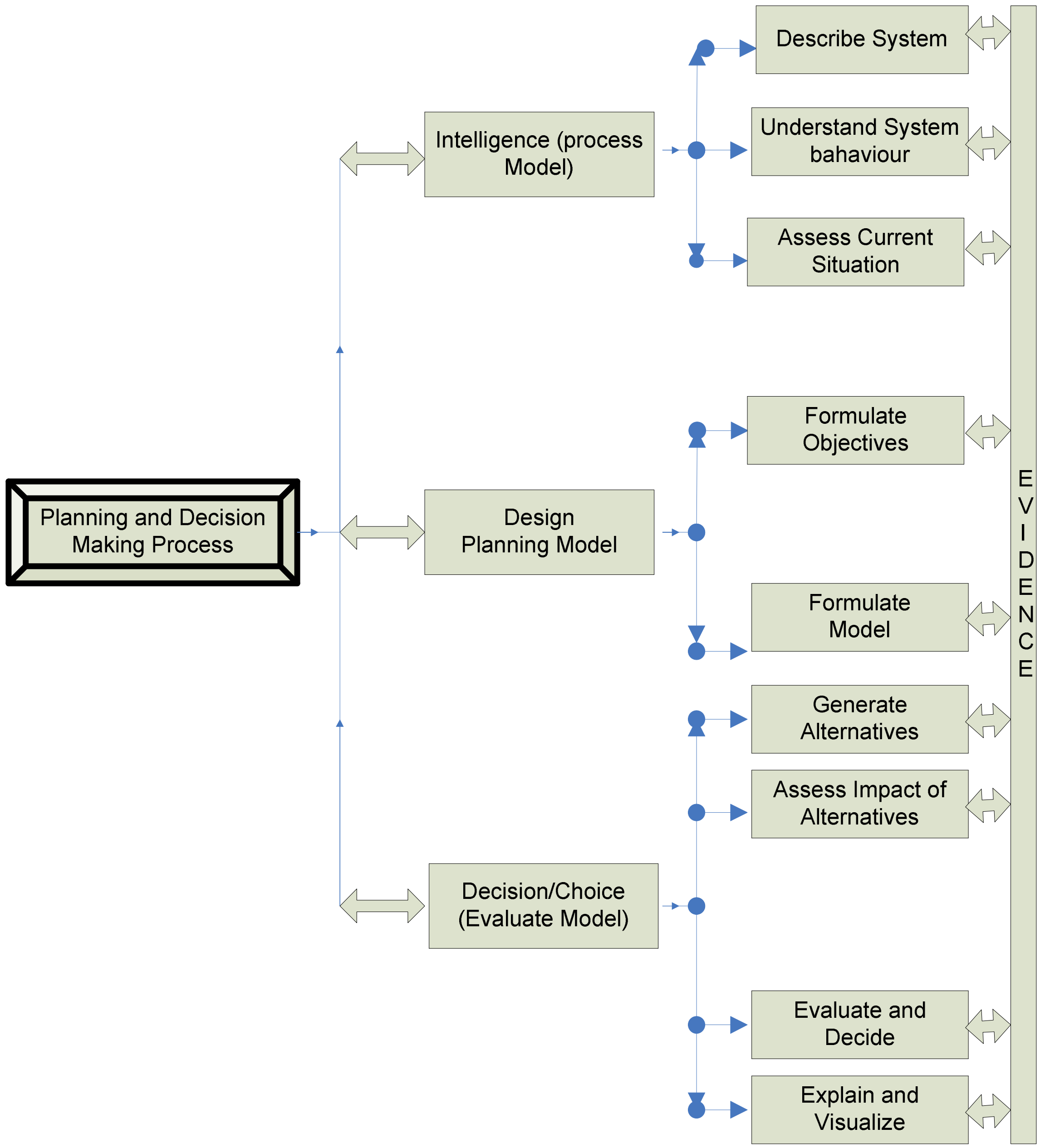


Figure 12. Framework for planning and decision making process.

Fuzzy Hierarchical Model Structural Design

The design of the research model encompassed the formulation of the mathematical fuzzy computations based on fuzzy set theory, the development of fuzzy scale to capture uncertainty, and the use of a decision hierarchy consistent with the underlying framework, which is the classical AHP. The structure is mainly concerned with the mathematical representation of the pairwise comparison matrices of fuzzy judgments. These fuzzy matrices are the result of pairwise evaluation of each criterion against all other criteria with regard to achieving the main goal which is decomposing a complex decision under uncertain conditions. Furthermore, a fuzzy evaluation of each alternative decision solution with respect to each criterion is also carried out. The following are the primary tasks that were conducted to achieve the structure in question:

1. Fuzzy Weight Derivation. This research task was concerned with derivation of fuzzy weight from the fuzzy PCM, development of mathematical techniques for ranking fuzzy sets (weights).
2. Alpha-cut and derivation of judgment intervals. This formulation step is concerned with integrating degrees of uncertainties into with the analysis to reflect the economic and business environment. This helps obtaining an optimum judgment with a reasonable degree of confidence.
3. Delta function analysis. Some decision makers and experts may have a highly uncertain and pessimistic views and some others may have somewhat certain and optimistic attitudes. The delta function analysis and α-cut embed these types of attitudes into the decision making process.

4. Alternative Solutions Analysis. The logical steps after completion of criteria evaluation, ranking, and fuzzy PCM derivation is the alternative solutions analysis. This analysis is similar to criteria evaluation. Each alternative (A1) is evaluated against all others in terms of its contribution to say criteria C1 , then same alternative (A1) is evaluated against all others in terms of their contribution to C2. This process is repeated for A1 until all criteria are exhausted, say Cn was reached. The process repeats again for A2, and C1, C2, C3,…, Cn; A3, C1, C2, C3,…,Cn;…; and Am, C1, C2, C3, Cn. This process produces n sets of m by m matrices. The entries into these matrices are fuzzy judgment in the forms of triplets: lower bound, upper bound, and the most possible value of a fuzzy judgment.
5. Software Algorithm Development. Processing fuzzy pairwise evaluation can be a daunting task for a decision maker, a software tool to implement the construct of the model was necessary to alleviate this burden. The FHM software algorithms are consistent with the FHM axioms, concepts, fuzzy formulation, criteria evaluation, evaluation of alternate solutions, and ranking. In summary the software was a reflection of the developed model and provided consistent outcomes. The software was developed in MS studio 2005. This platform is rich in features and languages appropriate for this intricate task.
6. Practical Situation. It was necessary to apply the model to a substantial practical decision problem related to an area of significance that posses the financial magnitude and the depth in technical complexity.

Principles and axioms of the Fuzzy Hierarchical Model

The fuzzy hierarchical model formulations are based on the following principles and axioms:

1. Decomposition: Structuring a complex problem into different clusters at various hierarchies. The intent is to reduce the complexity of a difficult decision problem into a set of manageable tasks.
2. Pairwise comparison: Creating Pairwise Comparison Matrices (PCMs) for all the criteria, sub-criteria, and alternative solutions under evaluation to derive the weights or the preference judgment in terms of how important a criterion or alternative solution when compared with all others composing the decision problem.
3. Hierarchical composition: Aggregating these local comparisons over the hierarchy to arrive at the final evaluation.

The following five axioms constitute the theory of the fuzzy hierarchical model:

1. Reciprocal axiom: If the pairwise comparison between two elements A and B with respect to an element C is Pc(xab), then the comparison between B and C must be 1/Pc(xab).
2. Homogeneity axiom: Elements clustered and arranged under a hierarchy must be homogeneous, i.e. they must be comparable with an order of magnitude. It means that elements within a cluster should preferably be compared within the fuzzy scale: (1, 1, 1) to (8, 9, 11) or other variation of the scale depending on the degree of uncertainty. For example a scale of (1,1,3), (1,3,4), (3,4,5),…,

(8, 9,10). Where the scale values are triangle fuzzy numbers whose intervals vary depending on the level of uncertainty related to a given judgment.

3. Fuzzy matrix axiom: Entries into the pairwise comparison matrices are in the forms of fuzzy triplets and an alpha-cut. The fuzzy triplets represent the fuzzy judgments and the alpha-cut entry reflects the degree of uncertainty.
4. Independence of judgment axiom: Judgment at one level of a hierarchy should be independent of the elements under it. One should carefully consider this axiom while making decisions, as human tendency forces one to look at the elements under the hierarchy during evaluation (Prakash, 2003). The requirement of a judgment being adequately represented or incorporated into the decision hierarchy must be met. This guarantees results that match expectations.
5. Consistency axiom: Preference fuzzy judgments need to be consistent. This means that if a decision maker prefers A over B and B over C, must also prefers A over C.

Additional operations needed for model development

1. Given a triangular fuzzy number t = (l, m, u), then the reciprocal value of t is given by 1/t = (1/u, 1/m. 1/l).
2. The power of a triangular fuzzy number t is given by $t^n = (l, m, u)^n = (l^n, m^n, u^n)$. Given n experts rendering n (greater than 2) independent judgment with regards to a preference of criterion C_i over C_j, the aggregate judgment of the experts is given by the geometric mean relation as follows:

$$aij = \sqrt[n]{\prod_{k=1}^{n} B_{ijk}} \text{ ,}$$

where a_{ij} is the aggregate preference of the n experts and B_{ijk} is the fuzzy preference of the k^{th} expert. According to Saaty and Vargas (2001), this holds true given the following conditions:

a. No dictator: No single individual preferences determine the group order.

b. Decisiveness: The aggregation procedures must produce a group order.

c. Unanimity: If all individuals prefer alternative A over B, then the aggregation must produce a group order indicating that the group prefers A over B.

Fuzzy Hierarchical Decision Model

Figure 13 depicts the overall flow diagram of the developed fuzzy model. It encompasses a number of major steps:

1. Assessment of the decision problem on hands which includes development of actionable objectives, development of criteria that characterize the problem, and attempts to define alternate solutions. Also embedded in this stage is the construction of an overall hierarchy for the fuzzy hierarchical decision model. The result of this construction is a number of sub-criteria associated with each of criterion in the hierarchy.

2. Application of the sequential elimination method, described in chapter 2, to reduce the number of criteria and alternatives to a bounded number such that the decision problem remains manageable. The technique used is that if two criteria contribute equally or similarly to the main goal of the decision, then these two criteria are removed from the analysis. A similar approach is taken when comparing alternatives' performance with respect to each criterion.
3. Once the process of redundant elimination is completed, then experts are sought for their judging the criteria, sub-criteria and alternatives. If a single decision maker, then this process is straight forward. If a group of experts are involved in making the decision, then the geometric mean is used to solicit an aggregated judgment (comparison) for each C_{ij} and A_{ij} with respect to all others. C represents criteria and A represent alternatives. At this stage in the model, the solicited judgments are still crisp, just as in the Saaty's model. The geometric mean relies on multiplying the crisp comparison values up to n for n experts then taking the nth root of the result of each judgment. The resulting crisp judgment is given by the equality $M_{ij} = \sqrt[n]{\prod_{k=1}^{n} B_{ijk,}}$, where i,j = 1,2,…n; and k = 2, 3,4,…m. B_{ijk} is preference judgment for expert k.
4. Fuzzification of the aggregated judgments is the process of converting a crisp preference into a fuzzy preference taking into account uncertainty, vagueness, and sometimes lack of information of the entities being considered. A mapping function is used to convert the crisp value into a fuzzy entity. In the model

developed in this research, triangular fuzzy numbers were used since they embed the crisp judgment as the most probable. They also included lower (left) bounds, and upper (right) bounds. The interval to the right and to the left represent the pessimistic and optimistic attitudes of the decision makers. More details are presented later on fuzzification and the fuzzy scale in the sections dealing with mathematical derivations.

5. Construction of a fuzzy pairwise comparison matrix is at the core of the model. A number of matrices are constructed: An n by n fuzzy matrix for a decision problem with n criteria, and n matrices; one for each alternate solution performance with respect to each criterion. The size of each of these matrices depends on the number of alternatives. For 3 alternatives, the model will construct n 3 by 3 pairwise alternatives' evaliation matrices with respect to each criterion. The matrix below depicts a preference judgment matrix.

$$A = \begin{pmatrix} W_1 / W_1 & W_2 / W_1 \mathrm{L} & W_N / W_1 \\ W_1 / W_2 & W_2 / W_2 & W_N / W_2 \\ W_1 / W_3 & W_2 / W_3 & W_N / W_3 \end{pmatrix}$$

6. Normalization is the process of obtaining fuzzy eigenvector priority weight for each of the criteria and alternatives. This is done be fuzzy addition on all rows of the comparison matrices and derivation of weights relative to the total for each criterion and alternative. The equation below gives the process of fuzzy normalization:

$$T = (w_1 / \sum w_i, \quad w_2 / \sum w_i, \quad w_3 / \sum w_i, \ldots, w_n / \sum w_i)$$

where T is the normalized eigenvector. From the normalized eigenvector, the priorities or importance of the attributes under analysis are extracted. W denotes the weighted preferences in fuzzy judgment forms.

7. Consistency testing is the process through which the matrices generated are assured to be consistent. A consistency ratio of 10% is acceptable. In some cases, because of the possible overlapping of the fuzzy number, the consistency ratio may be over the recommended 10%. As long as the reason for such inconsistency is understood, there is little problem with proceeding with analysis. However, if this overlapping condition does not exist and the consistency ratio is still higher than the recommended 10 %, then the process of soliciting experts for their judgments must restart all over until an acceptable consistency ratio is reached. A consistency test is performed for both alternatives and criteria.
8. Defuzzification is the process of presenting the ranking of criteria and alternatives to the decision makers in a form familiar to them.
9. Fuzzification is a mapping of fuzzy sets to crisp values. There are three techniques that the model is capable of using depending on the user's preference.
   i. Delta function and alpha-cut
   ii. Delphi
   iii. Centroid

These techniques are discussed in the appropriate sections in this chapter.

10. Presentation of the ranking of the alternate solution to the stakeholder or the decision makers calumniates the major steps of the model.
11. Finally, if the decision maker wishes to perform sensitivity analysis with respect to different alpha-cuts which represents the degree of certainties, the model provides this capability. It also provided the capability to view the fuzzy representation of the ranking to gain an insight as to the level of risk related each solution. Also, pessimistic and optimistic scenarios can be performed to arm the decision maker with most of the tools that may be needed to arrive at an informed choice.

In summary, the steps of the fuzzy hierarchical model are:

1. Acquisition of crisp pairwise evaluation matrix
2. Acquisition of normal (crisp) pairwise comparison matrix
3. Fuzzifying the crisp PCM to fuzzy PCM
4. Fuzzy analysis for performance rating
5. Performing fuzzy consistency tests
6. Weight multiplication from hierarchy
7. Alpha-cut analysis for embedding uncertainty of decision maker confidence.
8. Defuzzification using delta function for embedding attitude of decision maker.
9. Normalizing the effect table
10. Performing overall weighting of normalized fuzzy matrix
11. Performing overall ranking over the entire hierarchy.
12. Performing sensitivity analysis.

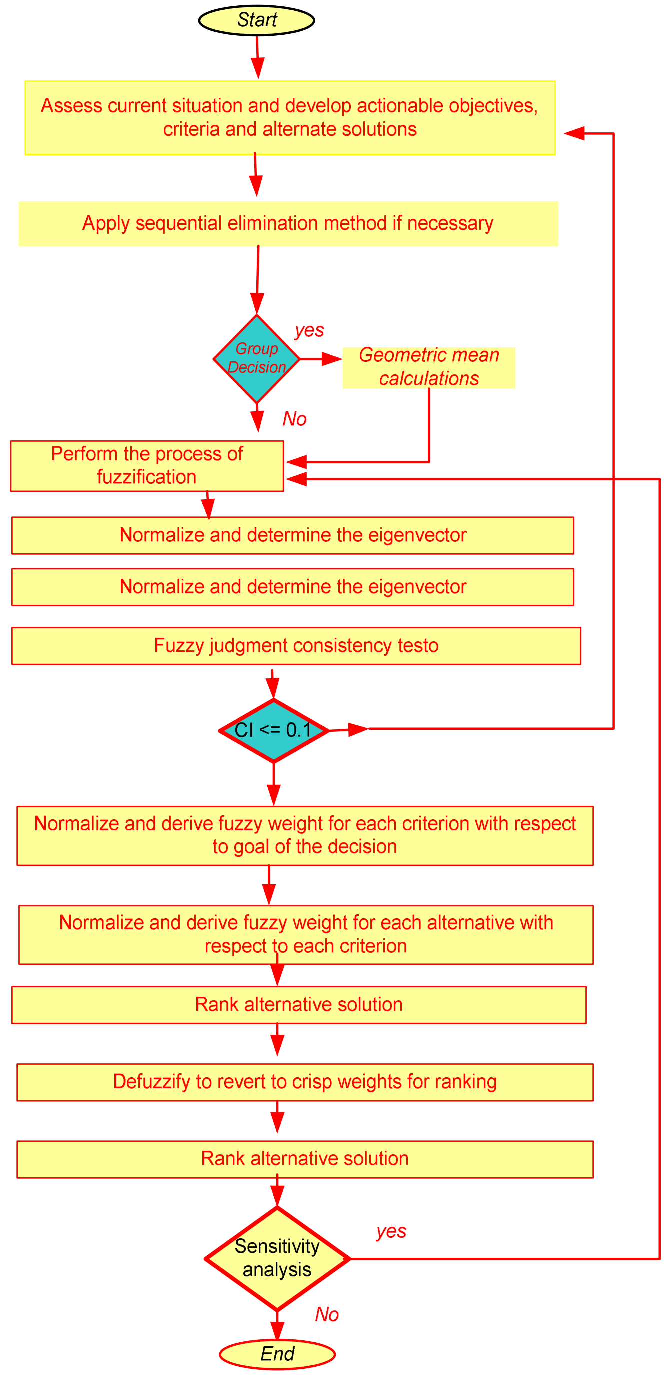


Figure 13._Flow diagram of fuzzy hierarchical decision model.

The fuzzy hierarchical modeling (FHM) method was developed due to the imprecision in assessing the relative importance of attributes and the performance ratings of alternatives with respect to attributes. According to Chan and Kumar (2006), imprecision may arise from a variety of reasons, including unquantifiable information, incomplete information, unobtainable information and partial ignorance. Conventional MCDM methods cannot effectively handle problems with such imprecise information. To resolve this difficulty, fuzzy set theory has been used and is adopted herein. Fuzzy set theory attempts to select, prioritize or rank a finite number of courses of action by evaluating a group of predetermined criteria. Solving this problem thus requires constructing an evaluation procedure to rate and rank, in order of preference, the set of alternatives.

The AHP of Saaty uses the pairwise comparison matrix to evaluate the ambiguity in multicriteria decision marking problems. Let $C_1, C_2, \dots, C_n$ denote the set of criteria, while a_{ij} represents a quantified judgment on a pair of criteria C_i, C_j. The relative importance of two elements is rated using a scale with the values 1, 3, 5, 7, and 9, where 1 refers to equally important, 3 denotes slightly more important, 5 equals strongly more important, 7 represents demonstrably more important and 9 denotes absolutely more important. An n-by-n matrix A is developed as follows:

$$A=[a_{ij}]=\begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & & & \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{pmatrix} \end{array}$$

where $a_{ii} = 1$ and $a_{ji} = 1/a_{ij}$, i, j = 1, 2, … , n.

In the fuzzy hierarchical model, instead of crisp judgments, a fuzzy triplet and an alpha-cut are used for each a_{ij}. The fuzzy triplet represents the preference judgment in a fuzzy interval form. The alpha-cut is the uncertainty index related to each judgment. The preference judgments continued to be solicited in the crisp forms form experts. The model performs the fuzzification process to deal with criteria measurement and determine the fuzzy consensus problem in judgments. Different α-cuts are then converted. Relative weights of the elements of each level are calculated as follows:

Fuzzification

It is a process through which a crisp value is mapped to a fuzzy set through a mapping function. If the crisp number represents a subjective judgment on preference to which criterion is more important than another criterion, then fuzification permits a range of uncertainty when making this judgment. There is an interval to the right of the crisp judgment and an interval to the left of the crisp judgment. Depending on business and economic conditions, and whether a decision’s maxima or minima category is being considered, the left and right intervals may vary to reflect an expert's optimistic or pessimistic attitude.

A triangular fuzzy number (TFN) is denoted simply as (l, m, u). The parameters, l, m and u, respectively, denote the smallest possible value, the most promising value and the largest possible value that a fuzzy interval may describe a fuzzy event. The triangular fuzzy numbers u_{ij} are established as follows:

$$u_{ij} = (l_{ij,} m_{ij}, u_{ij},) \qquad l_{ij} \le m_{ij} \le u_{ij}\,, \qquad and$$
$$l_{ij}, m_{ij}, u_{ij} \in [1/11,1] \cup [1,11]$$

where [1/11, 1] and [1,11] are the ranges of less important and more important linguistic variables.

Since each number in the pairwise comparison matrix represents the subjective opinion of decision makers and is an ambiguous concept, fuzzy numbers work best to consolidate fragmented expert opinions. To calculate the geometric mean in a group of decision makers, we used the multiplicative method. All crisp values with regard to a preference judgment of n experts are multiplied. The nth square root is then taken as follow:

$$m_{ij} = \sqrt[n]{\prod_{k=1}^{n} B_{ijk,}} \qquad \text{Equation 1}$$

$$l_{ij} = \min(B_{ijk}),$$

$$u_{ij} = \max(B_{ijk,})$$

where B_{ijk} represents a judgment of expert k for the relative importance of two criteria C_i, C_j.

Table 5 represents a fuzzy scale used to map the crisp experts' judgments to uncertain and vague judgments with the reliance on the triangular fuzzy numbers. Figure 14 depicts the overlapping characteristics of the fuzzy scale derived from crisp experts' preference judgments. This is natural since with vagueness and uncertainty delineation between pessimistic and optimistic scenario may appear difficult to attain. As far as the participating experts concerns, their preference judgments are solicited in a crisp form. The Fuzzy hierarchical model performs the fuzzification of the judgments to deal with uncertainties arising from ill-defined problems, vagueness, and incomplete information.

Furthermore, Equation 1 intended for solicitation of n experts independently with reference to the solution of a single uncertain decision problem. The multiplicative mathematical technique is used on the n crisp judgments of the n experts. Then the nth square root is taken of the multiplicative result. The assumption here is that these experts work independently in providing their subjective judgment.

Combining the technique of fuzzification, the fuzzy scale, and the multiplicative method, we are now ready to develop a fuzzy pairwise comparison matrix to express the preference judgment in fuzzy formats instead of the crisp values used in the classical Saaty's AHP. Table 6 represents a fuzzy scale with a fuzzy interval spread = 4. Table 7 represents a tighter fuzzy interval scale with a spread = 2. An argument can be made to use other fuzzy scales. It is even recommended to use more than one fuzzy scale in solving the same complex decision problem especially when sensitivity analysis is perform to provide what is called what-if-scenarios and to take into account the pessimistic and optimistic attitude of decision makers.

Table 6.

Fuzzification of the AHP Crisp Scale with a Spread = 4.

Crisp PCM value	Linguistic Definition	Fuzzy PCM Value	Crisp PCM Value	Fuzzy PCM Value
1	Equal importance	(1,1,1), if diagonal (1,1,3), otherwise	1/1	(1/1,1/1,1/1), if diagonal (1/3,1,1), otherwise
3	Weak importance	(1,3,5)	1/3	(1/5,1/3,1/1)
5	Demonstrated importance over the other	(3,5,7)	1/5	(1/7,1/5,1/3)
7	Strong importance	(5,7,9)	1/7	(1/9,1/7,1/5)
9	Absolute importance	(7,9,11)	1/9	(1/11,1/9,1/7)

Table 7.

Fuzzy Scale with Lower Degree of Fuzziness – Spread = 2.

Crisp PCM value	Fuzzy PCM Value	Crisp PCM Value	Fuzzy PCM Value
1	(1,1,1), if diagonal (1,1,2), otherwise	1/1	(1/1,1/1,1/1), if diagonal (1/2,1,1), otherwise
2	(1,2,3)	1/2	(1/3,1/2,1/1)
3	(2,3,4)	1/3	(1/4,1/3,1/2)
5	(4,5,6)	1/5	(1/6,1/5,1/4)
7	(6,7,8)	1/7	(1/8,1/7,1/6)
9	(8,9,10)	1/9	(1/10,1/9,1/8)

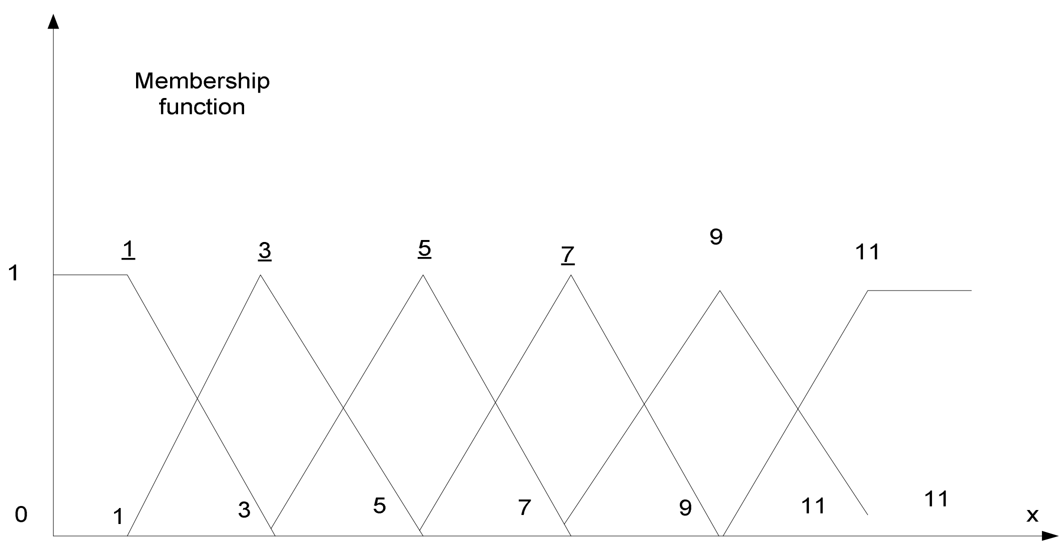


Figure 14. Overlapping characteristics of symmetric fuzzy scale.

Substituting the fuzzy preference judgment and the geometric mean derivation of crisp judgments the following fuzzy pairwise comparison matrix is derived. The diagonal fuzzy judgments are expressed as (1,1,1). The fuzzy interval (1,1,1) is used when a criteria or an alternative is compared to itself (equally important). In the non-diagonal case, the fuzzy interval (1, 1, 3) is used instead.

$$\tilde{A} = [\tilde{a}_{ij}] = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_n \end{array} & \begin{pmatrix} 1,1,1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1,1,1 & \cdots & \tilde{a}_{2n} \\ \vdots & & & \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \cdots & 1,1,1 \end{pmatrix} \end{array}$$

Equation 2

where $\tilde{a}_{12}$ denotes a triangular fuzzy number for the relative importance of two criteria C_1 and C_2. Generalizing, then $\tilde{a}_{ij}$ represents the fuzzy triplet judgment of criterion C_i compared to criterion C_j. The following matrix is an expanded representation of Equation 2. Each entry is expressed as a fuzzy triplet and an associated α-cut. This is the format that the decision maker uses to input the preference fuzzy judgment after the fuzzification process is completed. α-cut = 0 is used in the diagonal fuzzy entries because a degree of certainty does not have any meaning with a fuzzy triplet of (1,1,1) since its interval between the lower and upper bounds is 0.

$$\tilde{A} = [((l_{ij}, m_{ij}, u_{ij}, \alpha_{ij} - cut))] = \begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_n \end{array} & \begin{pmatrix} 1,1,1,0 & (l_{12}, m_{12}, u_{12}, \alpha_{12} - cut) \cdots & (l_{1n}, m_{1n}, u_{1n}, \alpha_{1n} - cut) \\ 1/(l_{12}, m_{12}, u_{12}, \alpha_{12} - cut) & 1,1,1,0 \cdots & (l_{2n}, m_{2n}, u_{2n}, \alpha_{2n} - cut) \\ \vdots & & \\ 1/\tilde{a}_{1n} & 1/(l_{1n}, m_{1n}, u_{1n}, \alpha_{1n} - cut) \cdots & 1,1,1,0 \end{pmatrix} \end{array}$$

An important feature of the above matrix is the ability to associate a degree of uncertainty with each fuzzy triplet judgment. What is unique about this approach is that: (1) each expert may have a different degree of uncertainty for each comparison rendered. Furthermore, a group of experts' preference judgment geometric mean may have a different degree of confidence. This type of modeling mirrors real world situations when dealing with uncertainty.

In the model developed in this research, the user is permitted to use a global α-cut and judgment by judgment α-cut. The main advantage of this approach is to capture general economic uncertainties, and also being able to capture individual decision makers' (experts) degree of confidence in their preference pairwise comparison judgment with respect to criterion C_i and $C_{j..}$ In later sections when the practical model application is discussed, further information about the global and localized degree of confidence will be presented.

<u>*Defuzzification*</u>

The model dealt with three methods to attain defuzzification that is required for ranking, contrasting, and weighting of criteria and alternatives in a form that is familiar to managers or decision makers. These three methods play different roles according to the stage at which the model is operating. These techniques are the delta function, Delphi, and centroid. The delta function requires α-cut as part of the calculations. As shown in Equations 4 and 7, the α-cut method expresses fuzzy perception. Owing to its ability to explicitly display the preference and risk tolerance of decision makers, risk may be understood according to the economic and business climates.

Defuzzification: δ-function and α-cut

Notably, α-cut can be viewed as a stable or fluctuating condition. The range of uncertainty is the greatest when $\alpha = 0$. Meanwhile, the decision-making environment moves to stabilizes when α is increased, while simultaneously, the variance for decision-making decreases. Additionally, α can be any number between 0 and 1, and the analysis is carried out with one of value for α from the following 10 numbers, 0.1, 0.2, … , 1 for uncertainty emulation. Besides, while $\alpha = 0$ represents the upper-bound u_{ij} and lower-bound l_{ij} of triangular fuzzy numbers, and while, $\alpha = 1$ represents the geometric mean m_{ij} in triangular fuzzy numbers. On the other hand, δ can be viewed as the degree of a decision maker's pessimism. When δ is 0, the decision maker is more optimistic and, thus, the expert consensus is upper-bound u_{ij} of the triangular fuzzy number. Conversely, when $\delta = 1$, the decision maker is pessimistic, and the number ranges from 0 to 1; however, five numbers 0.1, 0.3, 0.5, 0.7, and 0.9, may be used to emulate the state of mind of decision makers:

$$(a_{ij})^{\delta} = [\delta . l_{ij}^{\alpha} + (1-\delta) . u_{ij}^{\alpha}], \qquad 0 \le \delta \le 1, 0 \le \alpha \le 1, \qquad \text{Equation 3}$$

where:

$l_{ij}^{\alpha} = (m_{ij-} - l_{ij}).\alpha + l_{ij}$, represents the left-end value of α -cut for a_{ij} Equation 4

$u_{ij}^{\alpha} = u_{ij-}(u_{ij} - m_{ij}).\alpha$, represents the right-end value of α -cut for a_{ij} Equation 5

Optimistic Attitude Example of Defuzzification with α-cut and δ-function

Consider a fuzzy pairwise comparison a_{ij}, expressed in fuzzy triangular number with the following parameters:

Lower bound $l_{ij} = 2$, upper bound $u_{ij} = 6$, crisp value (middle) = 4,

$\alpha = 0.5$, and $\delta = 0.2$.

Applying equations 4 and 5,

$l_{ij}(0.5) = (4 - 2)0.5 + 2 = 3.0$

$u_{ij}(0.5) = 6 - (6 - 4)0.5 = 5.0$

The above two values indicate a triangular fuzzy number (fuzzy pairwise comparison) of (3, 4, 5).

The next step is to convert this fuzzy number to a crisp judgment. Applying Equation 3 we obtain:

$a_{ij\,(0.5)}^{0.2} = [0.2 \,.\, 3 + (1 - 0.2).5.0] = 0.6 + 4 = 4.6$

The original central value of fuzzy symmetric judgment was 4.0. However, with α = 0.5 and λ = 0.2, the defuzzified value is 4.6. This indicates an optimistic decision maker. This illustrates that lower values for δ moves the defuzzified judgment to the right while higher values move the judgment to the left (pessimistic attitude when maxima analysis is being considered).

Pessimistic Attitude Example of Defuzzification with α-cut and δ-function

Repeat the above numerical example with same $\alpha = 0.5$ but with $\delta = 0.9$. l_{ij} (0.5) and u_{ij} (0.5) will be as above. Now Equation 3 is applied to obtain $a_{ij\,(0.5)}^{09}$ as follows: $a_{ij\,(0.5)}^{0.2}$ = [0.9 . 3 + (1 -0.9).5.0] = 2.7 + 0.5 = 3.2. It can be seen from this result for the defuzzified value that the decision maker has a pessimistic attitude. This is evidence from the original middle value (equals 4) now moved to the right by 0.8, almost one whole unit when δ assumed higher value (0.9).

Defuzzification: Weighting Using Delphi

The Delphi method uses a techniques of the averaging of the fuzzy triplet of a triangular fuzzy number $t_{ij} = (l_{ij}, m_{ij}, u_{ij})$ by giving the central value of the fuzzy number more weight than the lower and upper bound (Gil-Aluja, 2004). For example, if we double the weight of the middle value of the fuzzy triplet m_{ij}, then the resulting crisp weight is as follows:

$a_{ij} = l_{ij} + 2.\, m_{ij,} + u_{ij} / 4$

Defuzzification: Weighting Using Center of Gravity

The center of gravity method, also known as centroid, integrates over the fuzzy triplet limits from lower bound to upper bound. Then the integration result is divided over the fuzzy interval. This integral is given by the following equation:

$$a_{ij} = \frac{\int_l^m F(x)xdx}{\int_l^m F(x)dx} + \frac{\int_l^m F(x))xdx}{\int_l^m F(x)dx}$$

For a triangular fuzzy number, the above integral yields the following average:

$a_{ij} = l_{ij} + m_{ij}, + u_{ij} / 3$

When the process of defuzzification is completed, a crisp pairwise comparison matrix is constructed. It is expressed by Equation 6, taking into account α-cut and δ. δ -function with α-cut pairwise comparison matrix is given by:

$$(A^{\alpha})^{\delta} = [(a_{ij})^{\delta} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C2 & & C_n \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ \mathrm{M} \\ C_n \end{array} & \begin{bmatrix} 1 & (a_{12}^{\alpha})^{\delta} \ldots & \ldots\ldots\ldots\ldots (a_{1n}^{\alpha})^{\delta} \\ (a_{21}^{\alpha})^{\delta} & 1 \quad \ldots & \ldots\ldots\ldots\ldots (a_{2n}^{\alpha})^{\delta} \\ \mathrm{M} & & \mathrm{M} \\ (a_{n1}^{\alpha})^{\delta} & (a_{n2}^{\alpha})^{\delta} \ldots\ldots\ldots\ldots\ldots\ldots & 1. \end{bmatrix} \end{array} \qquad \text{Equation 6}$$

where the entries in the matrix are single values. Similar matrices are derived with Delphi and centroid defuzzification.

Calculation of eigenvalue and eigenvector

If $\bar{\lambda}$ is assumed to be the eigenvalue of the single pairwise comparison matrix $(A^{\alpha})^{\lambda}$, then:

$$(A^{\alpha})^{\lambda} \cdot W = \bar{\lambda}_{\max} \cdot W, \qquad \text{Equation 7}$$

$$[(A^{\alpha})^{\lambda} - \bar{\lambda}_{\max}] \cdot W = 0,$$

Equation 8

where w denotes the eigenvector of $(A^{\alpha})^{\lambda}$, $0 \leqslant \lambda \leqslant 1$, $0 \leqslant \alpha \leqslant 1$. Comparing Equations 1 and 7, the traditional AHP only uses a specific figure geometric mean to represent the expert opinions for the pairwise comparison matrix. However, the triangular fuzzy numbers are used to present the fuzzy opinions and expert consensus. Meanwhile, both approaches use the eigenvector method for weight calculation.

Consistency test

The essential idea of the AHP is that a matrix A of rank n is only consistent if it has one positive eigenvalue $n = \lambda_{max}$ while all other eigenvalues are zero. Further, Saaty developed the consistency index (CI) to measure the deviation from a consistent matrix:

$$CI=(\lambda_{max}-n)/(n-1) \qquad \text{Equation 9}$$

The consistency ratio (CR) is introduced to aid the decision on revising the matrix or not. It is defined as the ratio of the CI to the so-called random index (RI) which is a CI of randomly generated matrices:

$$CR=CI/RI \qquad \text{Equation 10}$$

for n = 3 the required consistency ratio (CR^{Goal}) should be less than 0.05, for n = 4 it should be less than 0.08 and for $n \geqslant 5$ it should be less than 0.10 to get a sufficient consistent matrix. Otherwise the matrix should be revised (Saaty, 1994, 2001).

Consistency ratio algorithm and procedures

The role of the defuzzification step is to present the ranking outcomes in a form familiar to decision makers. This is to assure them that the results are within the framework, theories and methodologies of multicriteria analysis. The consistency ratio is one of the measures to provide this sought after assurance. In this section, a numerical example is presented to further illustrate this important criterion of multicriteria analysis.

The method described in this section, accompanied with a numerical example, illustrates an algorithm to approximate the consistency of a set of pairwise comparisons. As previously stated, it defines a consistency ration (CR) as fraction for which the numerator is a consistency index (CI) and the denominator is a random index. Thus,

$$CR = CI/RI$$

To get the consistency index (CI) of a set of paired comparisons, the first step is to compute the product of two matrices, referred to as P and Q in what follows. This product is matrix R. Matrix P is square matrix (n by n). It has the same number of rows and columns. Let us choose n to equal 4. The numerical example of P x Q = R is shown below.

$$\begin{array}{ccccc} P & X & Q & = & R \\ \begin{vmatrix} 1 & 0.33 & 0.11 & 0.11 \\ 3 & 1 & 0.2 & 0.2 \\ 9 & 5 & 1 & 0.5 \\ 9 & 5 & 2 & 1 \end{vmatrix} & X & \begin{vmatrix} 0.04 \\ 0.10 \\ 0.36 \\ 0.50 \end{vmatrix} & = & \begin{vmatrix} 0.17 \\ 0.39 \\ 1.47 \\ 2.08 \end{vmatrix} \end{array}$$

Matrix Q is a column matrix of the respective priority weights of the pairwise comparison matrix P. For the pairwise comparison matrix and the priority weights Q, the value of value R. was computed as follows:

$1 \times 0.04 + 0.33 \times 0.1 + 0.11 \times 0.36 + 0.11 \times 0.50 = 0.17$

The rest of the values in the vector R follow matrix multiplication.

The next step is to divide each element of R by the corresponding element in Q and average the results.

R/Q

0.17/0.04	=	4.25
0.39/0.10	=	3.90
1.47/0.36	=	4.08
2.08/0.50	=	4.16
Total		16.39
Average		4.10

The average is a characteristic of eigenvalue. We have been referring to it as λ. The consistency index (CI), for a square matrix of order N (in this example N = 4) is then

$$CI = (\lambda - N) / (N-1) = (4.10 - 4) / (4 - 1) = 0.03$$

For the denominator of the CR, we use the random index approximations as given by (Saaty, 2001, p. 165)

N:	1	2	3	4	5	6	7	8	9	10
CR:	0	0	.52	.82	1.11	1.25	1.35	1.40	1.45	1.49

These were based on a large number of simulations, for which the pairing of paired comparisons were done randomly. For our example, N equals 4 and RI equals 0.90. The consistency ratio is therefore

$$CR = CI/RI = 0.03/0.90 = 0.03,$$

which is lower than the recommended 10%. However, it should be noted that because the fuzzy nature of the model and the left (pessimistic) leaning of some decision and the right (optimistic) leaning of others, we may obtain values greater than 0.10. That is still acceptable as long as the source of such discrepancy can be identified.

Compute the overall hierarchy weight

Consider the fuzzy PCM matrix A we derived from the crisp judgment of a single decision maker or the geometric mean of a group of experts. After deffuzification and consistency testing as we have seen in early section, the next step is to normalize the fuzzy PCM in order to derive the eigenvector priority weights. This process involves fuzzy addition of all the rows in the matrix shown below. The result is obtaining an average for each of the criteria.

Deriving overall hierarchical weights

Consider the fuzzy PCM matrix A which is given as the following:

$$A = \begin{pmatrix} (a_{11l}, a_{11m}, a_{11u}) \quad \dots\dots\dots\dots\dots\dots\dots\dots (a1_{1nl}, a_{1nm}, a_{1nu}) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ (a_{m1l,} a_{m1m}, a_{m1u}) \dots\dots\dots\dots\dots\dots\dots\dots (a_{mnl}, a_{mnm}, a_{mnu}) \end{pmatrix}$$

the fuzzy analysis is applied to obtain the fuzzy performance matrix. To obtain the fuzzy decision performance matrix X and the fuzzy weight W using the fuzzy analysis, the AHP weighting method, and the operations defined for triangular fuzzy sets, then

$$x_i ..or.. w_j = \sum_{j=1}^{k} a_j \Big/ \sum_{i=1}^{k} \sum_{j=1}^{k} a_{ij}$$

where i = 1, 2, 3,……p, and j = 1, 2, 3,……q, or k = q, depending upon the elements under operation, whether it is an alternative or criteria (the number of rows and columns in the PCM)

$$X_i = \begin{pmatrix} (x_{11l}, x_{11m}, x_{11u}) \\ (x_{21l}, x_{21m}, x_{21u}) \\ \dots\dots\dots\dots \quad \dots\dots\dots\dots \quad \dots\dots \\ \\ \dots\dots\dots\dots \quad \dots\dots\dots\dots \quad \dots \\ (x_{ijl}, x_{ijm}, x_{iju}) \end{pmatrix}$$

where j = the number of classes in the sub criteria (lowest level) and the number of criteria in the other upper levels

$$W_j = [(w_{1l}, w_{1m}, w_{1u}) \; w_{2l}, w_{2m}, w_{2u}) \dots\dots\dots\dots (w_{nl}, w_{nm}, w_{nu})]$$

where n is the number of criteria under the hierarchy. A fuzzy weighted performance matrix P can thus be obtained by multiplying the weight from the weight vector with the decision matrix.

$$P = X_i * W = \begin{pmatrix} (w_{1l}x_{11l}\ , w_{1m}x_{11m}\, , w_{1u}x_{11u}) \\ (w_{2l}x_{21l}\, , w_{2m}x_{21m}\, , w_{2u}x_{21u}) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (w_{il}x_{ijl,} w_{im}x_{ijm}\, , w_{iu}x_{iju}) \end{pmatrix}$$

The above overall weighting vector already takes into account the computations of alpha-cut and delta function. The two indices reflect the certainty of the expert's preference judgment as well as well as the pessimistic and optimistic attitudes of the decision maker.

Algorithmic and Procedural Operations of the Model

Rationale 1

Revisiting the issue raised in chapter 2 regarding the voluminous numbers of preference judgments, a human will have to process if computers are not available. Applying multicriteria analysis to a decision involving n options and m criteria would require $\binom{n}{2}$ × m+$\binom{m}{2}$ multi-attribute comparisons. Applying this combinatory formulation, a complex decision might have, say, 8 criteria and 6 options, would necessitate:

$$NumberofPairwise\,Comparisons = \binom{6}{2} x8 + \binom{8}{2}$$

$$= \frac{6!}{2(6-2)!} x8 + \frac{8!}{2(9-2)!} = \frac{5760}{48} + \frac{40320}{1440} = 120 + 28 = 148$$

where 6! reads as 6 factorial, 6! = 6(6-1)(6-2)(6-3)(6-4)(6-5)(1). By definition, 0! (zero factorial) = 1. Dealing with a possible 148 comparisons would be quite a task to remain focused for any individual. The issue of the large number of comparison is among the reasons that necessitated a need for a software application to implement the fuzzy hierarchical model.

Rationale 2

To focus the research in this dissertation toward practical uses, it was necessary to develop decision applications in areas with levels of complexities that are worthy of multicriteria decision modeling. It was apparent from the start of this dissertation research that application of the developed model to areas with even a moderate level of complexity would require a software application form of the model to prove valuable. As will be shown later in this chapter, a substantial networking and data center design application was the catalyst for validating this fuzzy decision model. Dealing with the uncertain and complex nature of decisions in this technological field may involve a large number of attributes. Dealing with such attributes and developing complex solutions in this very advanced field may not be easily handled without the aid of software.

Rationale 3

The investigator believed that the dissemination of research results would be easier if the recipients of the research can have a software application that accompanied it. This was true in the case of the experts who provided inputs as to the priorities of what should be included in the model to be industrially applicable. Furthermore, the software proved valuable in highlighting the advantages that the model provided over the classical AHP decision modeling methodology.

The software application dealt with three main modules: (1) graphical user's definition construction of criteria and alternative multicriteria matrices, (2) processing engine, and (3) graphical reporting function. The graphical user's interface afforded the decision maker a friendly interface to define the multicriteria and the proposed alternative solutions to a multicriteria decision problem in fuzzy formats. Additionally, this module permitted the decision maker to introduce what is perceived as degree of uncertainties associated with each fuzzy preference judgment.

The processing engine was concerned with implementing the mathematical computations of the relevant constructs necessary to carry out the analysis of the decision. The main interwoven services the processing engine provided are:

1. Definition of criteria and alternative and fuzzy pairwise comparison insertions according to decision maker's requirements.
2. Defuzzification and fuzzy weighting of preference judgments.
3. Assurance of symmetry of matrices and all other necessary axioms of the fuzzy hierarchical model.
4. Fuzzy weighting of preference judgment which involved eigenvector priority weighting.
5. Normalization of preference judgment to fit into scales familiar to decision makers and managers.
6. Ranking of attributes of the decision under solution.

7. Ranking of alternate solutions and tracking of uncertainty and decision makers pessimistic and optimistic attitudes.

The graphical reporting module focused on presenting the ranking of alternate solutions in fuzzy and crisp forms that the decision makers can easily accept. It also presented the risk each solution carried in the form of uncertainty intervals. Thus, the final decision is still left to the decision maker. It should be noted that the three software application modules were not stand alone agents. Instead, they performed in concert to arrive at the final results. Also, sensitivity analysis was carried out by having the decision maker repeat the analysis while varying the degree of uncertainty (alpha-cuts) as well as the pessimistic and optimistic attitudes either by varying the delta-function or by changing the fuzzy judgment triplets either to the left or to the right from the central value of each preference.

In the next sections, these three models are presented in software algorithmic formats. Additionally, the time and space complexity of each module was also given. Furthermore, an overall discussion of the time and space complexity was presented. The algorithmic operations were presented in the standard formats of pseudo code. The actual code of the software is available upon request. The operations of the fuzzy modeling software are illustrated later in this chapter in the section dealing with a simulated practical application. The software was developed in Microsoft Visual Studio 2005. It is rich in object design features and languages. Also, it allowed for many types of graphical user interfaces. The entire applications was developed using object design schema.

User Interface

A primary concern is that the software does not know in advance the size of the table (pairwise comparison matrix). This matrix is a three dimension cube with each entry requiring four cells, namely, three for the fuzzy triplet and one for the degree of uncertainty, the alpha-cut.

Criteria PCM Definition and Construction

```
    FUZZ_PCM(C_i, …, C_n)
    BEGIN
    IF CRITERION = SELECT
    I  = 1;  J = 1
    FOR I IN 1 TO  N
      FOR J IN 1 TO N
              CRITERIA (I) = CRITERION (NAME)
              ALLOCATE (CRITERION, FUZZY_PCM[I][J][1] = 0)
              ALLOCATE (CRITERION, FUZZY_PCM[I][J][2] = 0)
              ALLOCATE (CRITERION, FUZZY_PCM[I][J][3] = 0)
              ALLOCATE (CRITERION, FUZZY_PCM[I][J][4] = 0)
     WHILE D_MAKER_INPUT NOT EQUAL NULL
    SAVE N
    FORM OBJECT (FUZZY_PCM(CRITERIA))
END
```

User Input Time and Space Complexity Analysis

Assuming a cost of 1 for each operation of allocating a cell and initializing it with zero, the worst case of an operation of O(4N x 4N) = O(16N^2). Actually, the worst case is not too bad. The reason is that N is bounded and N does reach infinity. In fact, N should not exceed 10.

Alternatives Fuzzy PCM Definition and Construction

```
FUZZY_PCM(A_i,… A_M)
BEGIN
WHILE CRITERION IN 1 TO N
    FOR ALTERNATIVE IN 1 TO M
        DO
            ALTERNATIVE (I) = ALTERNATIVE(NAME, C(I))
                ALLOCATE (ALTERNATIVE, FUZZY_PCM[I][J][1] = 0)
                ALLOCATE (ALTERNATIVE, FUZZY_PCM[I][J][2] = 0)
                ALLOCATE (ALTERNATIVE, FUZZY_PCM[I][J][3] = 0)
                 ALLOCATE (ALTERNATIVE FUZZY_PCM[I][J][4] = 0)
            FORM OBJECT FUZZY_PCM(ALTERNATIVE)
        WHILE D_MAKER_INPUT NOT EQUAL NULL
       SAVE M
END
```

Alternative Fuzzy Time and Space Complexity Analysis

The order of operation is similar to that of the criteria cube insertion. Keeping the cost of a single operation at 1, worst case number of operations is determined by the number of criteria N and the number of alternative M. Thus, the time and space complexity is given by $O(4M \times 4M \times N) = O(16M^2 \times N)$. Usually, however not guaranteed, M is less N. What should be noted is that all alternatives are compared to others with respect to each individual criterion. The goal here is to derive a construct and define the space necessary for an N data cube for alternative comparison. One fuzzy alternative PCM is constructed for each of the N criteria. It is still manageable because of the requirement that N and M be bounded.

Fuzzy Preference Judgment Entry

Although the model was designed for a single decision maker's use, group preference judgments can be processed. The procedures are to determine the geometric mean of the experts' judgments. Then the fuzzy triplets and associated degree of uncertainty (alpha –cut) are entered the same as single decision maker's interfacing.

Processing Engine: Criteria PCM

```
CRITRIA_PCM(FUZZY_PREFERNCES)
BEGIN
FOR I IN 1 TO N
    FOR J IN 1 TO N
            FUZZY_PCM_CRITERIA[I][J][1]  = C_IJ(L)
                    FUZZY_PCM_CRITERIA[J][I][1]  = 1/C_IJ(U)
```

FUZZY_PCM_CRITERIA[I][J][1] = C_{IJ}(M)

FUZZY_PCM_CRITERIA[I][J][1] = C_{IJ}(U)

FUZZY_PCM_CRITERIA[J][I][1] = 1/C_{IJ}(L)

FUZZY_PCM_CRITERIA[I][J][1] = C_{IJ}(ALPHA-CUT)

FUZZY_PCM_CRITERIA[I][J][1] = C_{IJ}(ALPHA-CUT)

END

where A_{IJ}(L), A_{IJ}(M), and A_{IJ}(U) are the fuzzy triplet for each of the pair comparison of criterion C_i against C_j. A_{IJ}(ALPHA-CUT) is the degree of uncertainty associated with each of the preference judgment. The alpha-cuts can be all the same for all judgments or they can vary according the decision maker's equivocation with respect to each of the pairwise comparison of each to criteria.

Criteria PCM Time and Space Complexity Analysis

Similar to the criteria data cube construction and allocation, given N criteria and 4 cells for each comparison, the worst case order of operation is $O(4N \times 4N) = 16N^2$). The Figure below illustrates the date cube and the cell values.

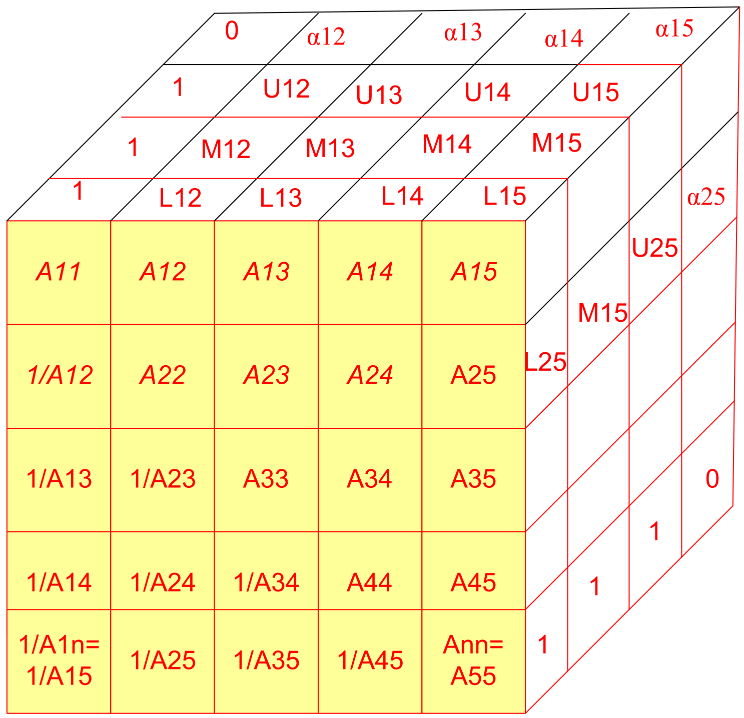


Figure 15. Illustrative data cube for fuzzy PCM.

Fuzzy Alternative PCM Construction

ALTERNATIVE_PCM(FUZZY_PREFERENCES)

BEGIN

FOR CRITERION IN 1 TO N

FOR I IN 1 TO M

FOR J IN 1 TO M

FUZZY_PCM_ALTERNATIVE[I][J][1] = $A_{IJ}(L)$

FUZZY_PCM_ALTERNATIVE[J][I][1] = $1/A_{IJ}(U)$

FUZZY_PCM_ALTERNATIVE[I][J][1] = $A_{IJ}(M)$

```
            FUZZY_PCM_ALTERNATIVE[I][J][1]  = A_IJ(U)
                    FUZZY_PCM_ALTERNATIVE[J][I][1]  = 1/A_IJ(L)
            FUZZY_PCM_ALTERNATIVE[I][J][1]  = A_IJ(ALPHA-CUT)
            FUZZY_PCM_ALTERNATIVE[J][I][1]  = A_IJ(ALPHA-CUT)
            CREATE_OBJECT FUZZY_PCM(ALTERNATIVE [I])
        REPEAT UNTIL CRITERION = N
    END FOR
    SAVE M
END
```

Fuzzy Alternative Time and Space Algorithm Complexity Analysis

The analysis in the fuzzification of an alternative case is similar to that of the fuzzification of criteria fuzzy pairwise matrix. However, the cost is compounded by N criterion. Recall that each single alternative must be compared to each of the other alternatives with respect to its performance with respect to each of the N criteria. The order of complexity is given as: $O(N \times 4M \times 4M) = O(N \times 16M^2) = O(16 \times N \times M^2)$

Proof:

Given M alternatives, and assigning the cost of each operation a value of one, then for M alternatives, four fuzzy operations are needed for each evaluation. Therefore, 4M x 4 M operations are required. Taking into account that the pairwise comparisons have to be repeated N iterations for the performance for each alternative with respect to each criterion, then for N criteria, the time and space order of complexity is given by:

O(16 x N x M^2). Given that N and M are bounded, the complexity of this module is bounded.

<u>*Criteria Defuzzification*</u>

```
BEGIN
ALPHA-CUT ANALYSIS (CRITERION IN 1 TO N)
FOR I IN 1 TO N
   FOR J IN 1 TO N
      FOR K 1 TO N
           CRITERIA_PCM_FUZZY[I][J][1] = CIJ_L(ALPHA) = (CIJ_M –CIJ_L)
                       x ALPHA + CIJ_L
       CRITERIA_PCM_FUZZY[I][J][3] = CIJ_U(ALPHA) = CIJ_U (CIJ_U –
                       CIJ_M) x ALPHA
      REPEAT UNTIL I = N AND J = N
     END FOR
IF DEFUZZIFY_TYPE = DELTA
     FOR I = 1 TO N
      FOR J = 1 TO N
               PCM_CRISP[I][I] = DELTA x CIJ_L(ALPHA) – (1 –
                       CIJ_U(ALPHA)
          END FOR
     END FOR
END DEFUZZIFY_TYPE(DELTA)
```

```
IF DEFUZZIFY_TYPE = DELPHI
    FOR I IN 1 TO N
        FOR J IN 1 TO N
            PCM_CRISP[I][I] = CIJ_L(ALPHA) + 2 x CIJ_M + CIJ_U(ALPHA) / 4
        END FOR
    END FOR
END DEFUZZIFY_TYPE (DELPHI)
IF DEFUZZIFY_TYPE = CENTROID
    FOR I IN 1 TO N
        FOR J IN 1 TO N
            PCM_CRISP[I][I] = CIJ_L(ALPHA) + CIJ_M + CIJ_U(ALPHA) / 3
            J = J + 1
        END FOR
    I = I + 1
    END FOR
END DEFUZZIFY_TYPE (CENTROID)
```

Defuzzification Time and Space Algorithm Complexity

Again, following the processing of matrices with N by N dimension and assigning 1 to the cost of operation, the order of operations is given by $O(3N \times 3N + 1(NxN) = 9N^2 + N^2) = O(10N^2)$

Proof:

One element of the criteria fuzzy matrix was not accessed during ALPHA analysis, thus the number of operations was reduced by one, from 4 to 3. The cells that were accessed were lower bound, upper bound, and alpha. Therefore, the alpha analysis yielded an order of operation $O(9N^2)$. The remaining three methods of defuzzification yielded one of three choices, thus, during run time, only one N by N matrix is traversed. Thus, the additional penalty is N^2. Thus, the order of time and space complexity is given by $O(10N^2)$.

Alternative Defuzzification

```
BEGIN
FOR CRITERION IN 1 TO N
ALPHA-CUT ANALYSIS (ALTERNATIVE  IN 1 TO M)
FOR I IN 1 TO M
   FOR J IN 1 TO M
      FOR K 1 TO 4
            ALTERNATIVE_PCM_FUZZY[I][J][1] = AIJ_L(ALPH) =
            (AIJ_M –IJ_L) x ALPHA + AIJ_L
         ALTERNATIVE_PCM_FUZZY[I][J][3] =
               AIJ_U(ALPHA) = AIJ_U (CIJ_U – AIJ_M) x ALPHA
      REPEAT UNTIL J = M
      END FOR
     REPEAT UNTIL I = M
     END FOR
```

```
END ALPHA-CUT ANALYSIS (ALTERNATIOV IN 1 TO M)


IF DEFUZZIFY_TYPE = DELTA
     FOR I = 1 TO N
      FOR J = 1 TO N
               ALTERNATIVE_PCM_CRISP[I][J] =
               DELTA x AIJ_L(ALPHA) – (1 – AIJ_U(ALPHA)
          END FOR
     END FOR
END DEFUZZIFY_TYPE(DELTA)
IF DEFUZZYFY_TYPE = DELPHI
   FOR I IN 1 TO M
      FOR J IN 1 TO M
           ALTERNATIVE_PCM_CRISP[I][I] =
           AIJ_L(ALPHA) + 2 x AIJ_M + AIJ_U(ALPHA) / 4
         END FOR
    END FOR
END DEFUZZIFY_TYPE (DELPHI)
IF DEFUZZIFY_TYPE = CENTROID
FOR CRITERION IN 1 TO N
   FOR I IN 1 TO M
      FOR J IN 1 TO M
```

```
ALTERNATIVE_PCM_CRISP[I][J] =
AIJ_L(ALPHA) + AIJ_M + CIJ_U(ALPHA) / 3
END FOR
END FOR
END DEFUZZIFY_TYPE (CENTROID)
```

Defuzzification Time and Space Algorithm Complexity

Again following processing of matrices with N by N dimension and assigning 1 to the cost of operation, the order of operations is given by O(3M x 3M + (MxM) = $9M^2$ + M^2) = O($10M^2$). This gets repeated N criteria times. Then the order of operation is given by O($10NM^2$).

Fuzzy Criteria Normalization

```
FUZZY_ROW_TOTAL (FUZZY_TRIPLET)
BEGIN
FOR ROW IN 1 TO N
    FOR CLUMN IN 1 TO N
        (Add lower bounds for row i)
            CRITERIA_FUZZY_PCM[ROW][N+1][1] +=
            CRITERIA_FUZZY_PCM[ROW][COLUMN][1]
            (Add middle value for row I)
            CRITERIA_FUZZY_PCM[ROW][N+1][2] +=
            CRITERIA_FUZZY_PCM[ROW][COLUMN][2]
            (Add middle upper bound for row I)
```

```
                CRITERIA_FUZZY_PCM[ROW][N+1][3] +=
                CRITERIA_FUZZY_PCM[ROW][COLUMN][3]
                COLUMN = COLUMN +1
            END FOR
            ROW = ROW + 1
END FOR
END
FUZZY_COUMN_TOTAL(FUZZY_TRIPLET)
BEGIN
FOR COLUMN IN 1 TO N
    FOR ROW IN 1 TO N
        (Add lower bounds for column i)
                CRITERIA_FUZZY_PCM[N+1][COLUMN][1] +=
                CRITERIA_FUZZY_PCM[ROW][COLUMN][1]
                (Add middle value for column I)
                CRITERIA_FUZZY_PCM[N+1][COLUMN][2] +=
                CRITERIA_FUZZY_PCM[ROW][COLUMN][2]
                (Add middle upper bound for column I)
                CRITERIA_FUZZY_PCM[ROW][N+1][3] +=
                CRITERIA_FUZZY_PCM[ROW][COLUMN][3]
                ROW = ROW +1
            END FOR
```

COLUMN = COLUMN + 1

END FOR

END

Normalization Time and Space Complexity

Given by O (3N x 3(N + 1) = O (9(N^2+ N))

Fuzzy Criteria Weighting

FUZZY_WEIGHTING_CRITERIA (FUZZY_TRIPPLET)

BEGINE

FOR COLUMN in 1 TO N

CRITERIA_PCM_FUZZ[N+2][COLUMN] =

CRITERIA_PCM_FUZZ[ROW][N+1] /

CRITERIA_PCM_FUZZ[N+1][N+1] x 100/100

COLUMN = COLUMN + 1

END FOR

END

Fuzzy Criteria Time and Space Complexity

Order of time and space complexity = O(N)

Fuzzy Alternative Normalization

FOR CRITERION IN 1 TO N

FUZZY_ROW_TOTAL (FUZZY_TRIPLET)

BEGIN

FOR ROW IN 1 TO M

```
    FOR COLUMN IN 1 TO M
        (Add lower bounds for row i)
            ALTERNATIVE_FUZZY_PCM[ROW][M+1][1] +=
            ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][1]
            (Add middle value for row I)
            ALTERNATIVE_FUZZY_PCM[ROW][M+1][2] +=
            ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][2]
            (Add middle upper bound for row I)
            ALTERNATIVE_FUZZY_PCM[ROW][M+1][3] +=
            ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][3]
            COLUMN = COLUMN +1
      END FOR
      ROW = ROW + 1
END FOR
END
FUZZY_COLUMN_TOTAL(FUZZY_TRIPLET)

BEGIN
FOR COLUMN IN 1 TO M
    FOR ROW IN 1 TO M
        (Add lower bounds for column I)
```

```
        ALTERNATIVE_FUZZY_PCM[M+1][COLUMN][1] +=
        ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][1]
        (Add middle value for column I)
        ALTERNATIVE_FUZZY_PCM[M+1][COLUMN][2] +=
        ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][2]
        (Add middle upper bound for column I)
        ALTERNATIVE_FUZZY_PCM[ROW][M+1][3] +=
        ALTERNATIVE_FUZZY_PCM[ROW][COLUMN][3]
        ROW = ROW +1
    END FOR
    COLUMN = COLUMN + 1
END FOR
REPEAT UNTIL CRITERION = N
END
```

Fuzzy Alternative Time and Space Complexity

Given by O (3M x 3(M + 1) = O (9N(M^2+ M)). Note that normalization of M alternatives is similar to that of normalizing N criteria with two exceptions: replace N by M, then multiply by N criteria. The process has to be repeated N criteria times.

Criteria PCM Consistence Testing

```
BEGIN
FOR I IN 1 TO N
    FOR J IN 1 TO N
```

```
        R[I] += CRITERIA_CRISP_PCM[J]
    END FOR
END FOR
FOR K IN 1 TO N
    T += R[K]
    K = K + 1
END FOR
    LAMBDA = T / N
    CI = (LAMBDA – N) / (N – 1)
    CR = CI / RI
END
FUZZY_WEIGHTING_CRITERIA (FUZZY_TRIPPLET)
BEGIN
FOR COLUMN in 1 TO N
    CRITERIA_PCM_FUZZ[N+2][COLUMN] =
    CRITERIA_PCM_FUZZ[ROW][N+1] /
    CRITERIA_PCM_FUZZ[N+1][N+1] x 100/100
    COLUMN = COLUMN + 1
END FOR
END
```

Consistency Time and Space Complexity

Order of time and space complexity = O(N)

Model Overall Time and Space Complexity Discussion

For a complete run of the software to arrive at a classification of criteria and an alternative solution, the orders of operation derived for all the modules are added. The overall order of operation is then given by:

$O(16N^2) + O(16M^2 \text{ x } N) + 16N^2) + O\ (9(N^2 + N)) + O(N) + O(16\text{x } N \text{ x } M^2) + O(10NM^2) + O(9N(M^2 + M))$. $O(N) = O(\ 41N^2 + 3N + 26NM^2)$ where N is equal to the number of criteria and M is the number of alternative solutions. Since N and M are bounded, the performance of the application is accepted and proved to be very efficient.

Model Practical Application

Assessment of the Problem

The ABC Bank is one of the fortune 500 firms with branches throughout the North American continent. It started its networking operations early in the 1970's, but it was mainly for backing up banking information. Its use was limited to the bank's technical personnel who specialized in data communications. As data communications and networking became more advanced and their use started to provide access to many other areas that directly impact revenues, the bank installed some data centers in a few of the branches. The selection criterion involved branches with a high number of clients accessing data centers to retain the customer base they have.

With the Internet migrating from educational purposes to commercial applications in the early 1990's, the bank went through another round of upgrades including new servers, higher speed wide area networks, and 10 Megabit per second Ethernet local area networks. The new data centers were present in almost all branches in some fashion. Also, the bank leased high speed connections to handle the increased volume of traffic between the branches and the main center to handle the depository of the transactions.

By 2002, the telecommunication managers and the marketing operation managers realized the demands for e-banking were on the rise at a rate that the current data center installation would not be able to accommodate. Additionally, an environmental initiative to go green by eliminating monthly paper statements and copies of cancelled checks compounded the demands on the data centers.

Figure 16 illustrates a sample of three data centers. The bank naturally uses many more. The design is duplicated as many times as there are branches to obtain the required data from the centers. Using networking terminology, from top to bottom, this data center example is composed of a distribution layer to handle the incoming networking traffic from customers. The distribution layer bundle the traffic to what is known as the core layer. The core layer function is to direct the traffic to wide area network (WAN) interfaces. Thus, the traffic can travel from the branch to other branches or to a centralized processing center were checks get scanned and directed to enterprise data bases. The same processes are followed with other transactions, namely in-bank, on-line, or ATM (automatic teller machines). The WAN interface also allows customers to forward their requests over the Internet. The lower part of the diagram is composed of application servers dedicated to process certain client requests. The core layer also links the server area where customer information and branch systems (workgroup servers) are maintained. The outside connection (to the Internet) is also linked from the core layer.

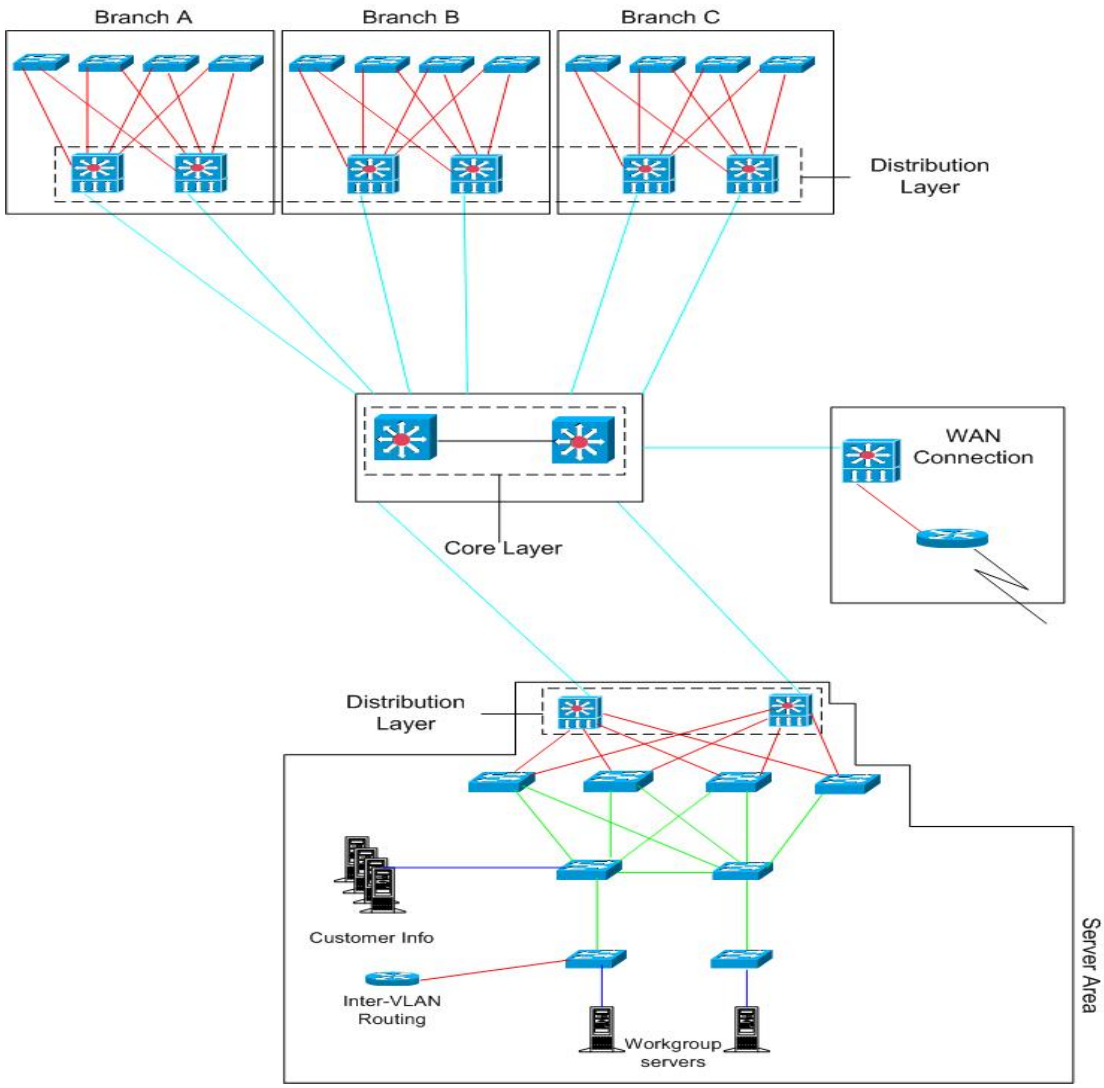


Figure 16. Current design of data centers.

The Inter-VLAN routing in the server area is used so that servers do not have to physically connected to each other on the network (communication is done logically).

Design Criteria

The networking consultants agreed with Mauricio Arregoces of Cisco Systems (2004, p. 6), Tanenbaum (2003), Stallings (2006), and Khader and Barnes (2000) that there are six high-priority design criteria that need to be taken into account when design and upgrading datacenters. Some of these criteria also have sub criteria which are important factors to be considered. The application of the sequential elimination step of the model was used to filter out criteria that must meet all designs. The primary examples of these criteria that were filtered included service contracts, payment methods, learning curves, and the reputation of vendors. All designs must meet minimum requirements. For this reason, it was not necessary to include them in the multicriteria fuzzy analysis. The main and sub criteria included in the fuzzy analysis were:

1. C1: Budgetary constraints - As noted by Kailash Jayaswal (2006), these need to be taken into account when designing/upgrading datacenters. This is a crucial factor because datacenter designers will not be able to obtain the latest and the most advanced equipment due to budget constraints. So they need to be able to work with the funds allotted to the project (p. 32).
2. C2: Security - How secure is the systems from internal and external attacks?
    a. Known security issues with current product line.
3. C3: Scalability - How much and how far can the systems expand in the future?
    a. Interoperability with existing products/vendors.
    b. Can the vendor change with needs?
    c. Life Cycle and Evolution of product line.

4. C4: Availability - Is the systems available 99.9% of the time?
    a. Redundancy features of systems to ensure uptime.
5. C5: Performance - Can the systems meet the needs of the large amount of customers/employees without sacrificing system resources?
    a. Products reporting, logging, and audits.
    b. How well does one vendor perform over another?
6. C6: Manageability - How easy or how hard is it to maintain the system for maintenance and upgrades?
    a. Learning curve to training employees on new equipment.
    b. Support and maintenance contracts.

These six design criteria are crucial to take into account for designing or upgrading a data center. After lengthy discussions with the experts, three alternative designs were agreed upon. Figure 17 illustrates the hierarchy of the design criteria and the three datacenter solutions.

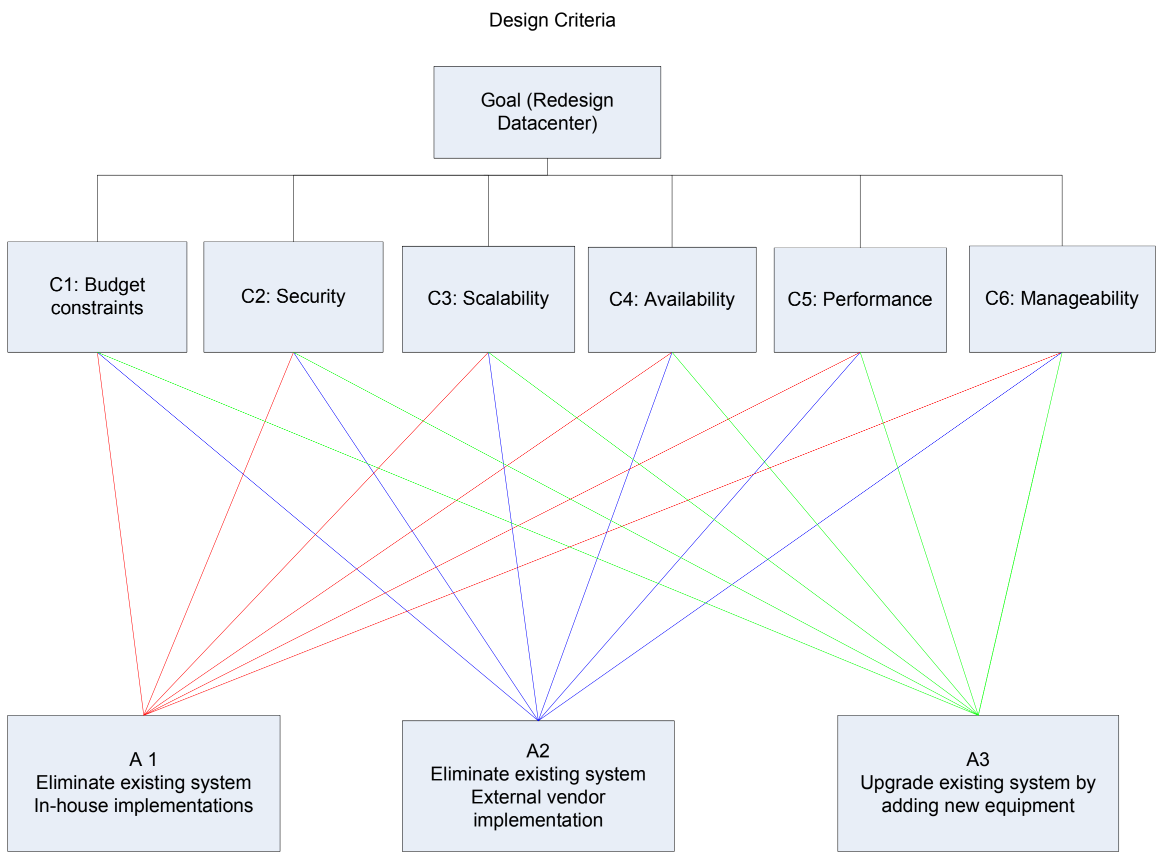


Figure 17. Hierarchy of design criteria and alternatives.

Three Solutions

According to the investigator, the experts, and the reviewed literature, there are three viable solutions to this problem that ABC Bank is having that can be implemented. However, with each solution, there are advantages and drawbacks. The advantages and drawbacks of each solution need to be weighted against each of the design criteria to arrive at an outcome that would be best for ABC Bank to implement.

Alternative A1: Eliminate current system -- start anew, in-house development

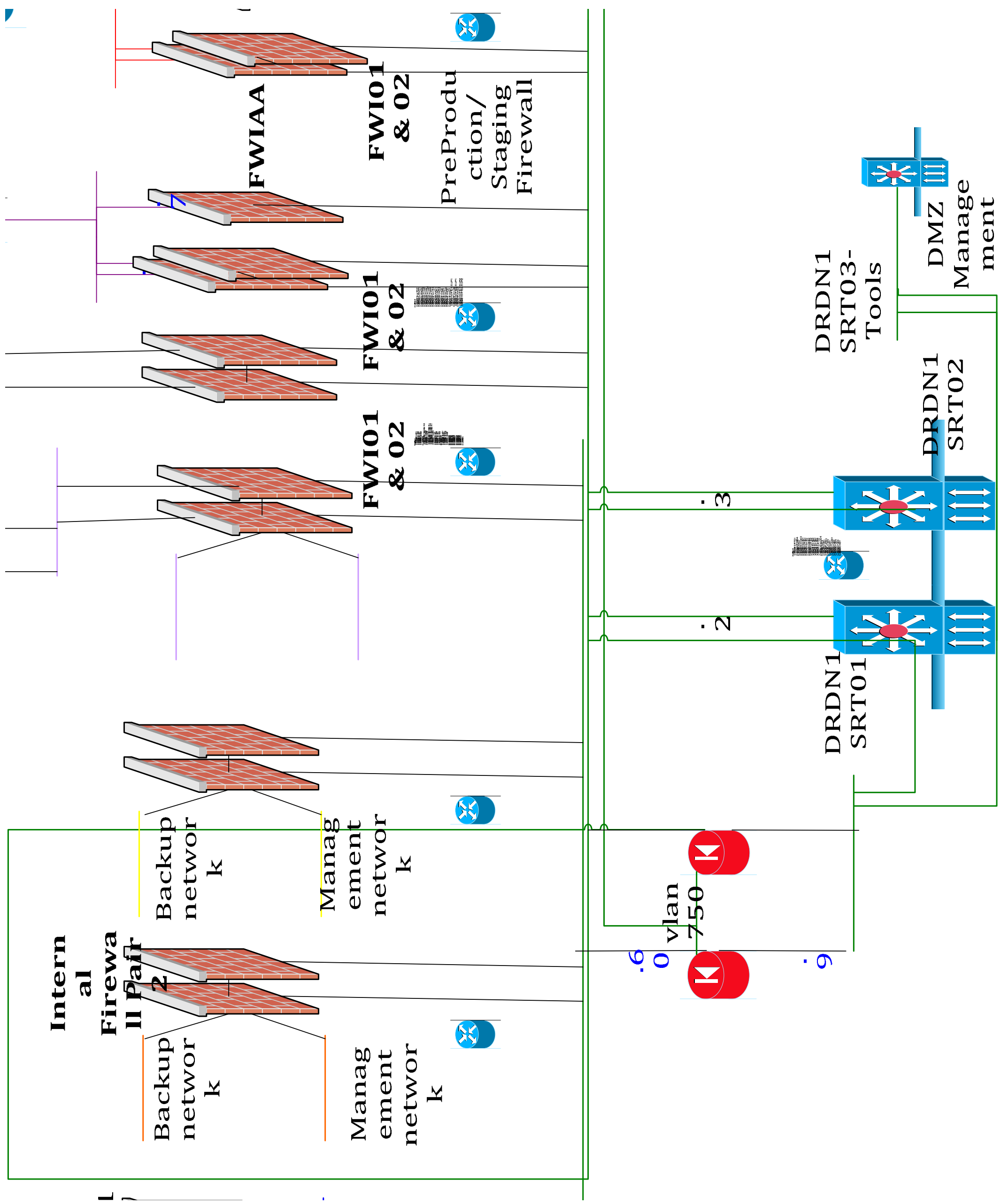
This solution involved redevelopment of the entire datacenter. The system would be completely redesigned and restructured so that growth could easily be handled in the future. However, this method would utilize the people who already work for ABC Bank to implement the new datacenter. There are three issues that need to be addressed which are: (a) Do the people who work for ABC Company have the necessary skills to implement the data center; (b) whether the costs are high enough to justify rather than expanding the old system; and (c) technicians in the current datacenter will need to be retrained on the new system, which would incur a higher cost to the overall project.

The advantage is that if the company decides to scale up its networking operations in the future, then this design will be able to handle the growth in bandwidth and services.

Figures 18 and 19 illustrate the newly redesigned data center. Figure 18 shows what the access layer would look like and figure 19 shows what the distribution/core layer would look like.

In Figure 18, the access layer (the entire picture) may look quite complicated. However, it is designed with the idea that customer information and branch systems can be separated onto different networks. This is needed so that the load is dispersed instead of having all the traffic running through one switch or router. Also, there are a lot of firewalls along with Demilitarized Zones (DMZ) so that security is ensured for the business. The Eservice and Econnectivity are networks that represent services and outside connections (to the Internet) in the old system.

Figure 18 depicts a new design of the datacenter that is capable of being duplicated at different branches. The remaining need is for a high speed wide area networking connectivity among the branches at different geographical locations.



Note, Figure 18 on next page is an extension of this part of the figure.

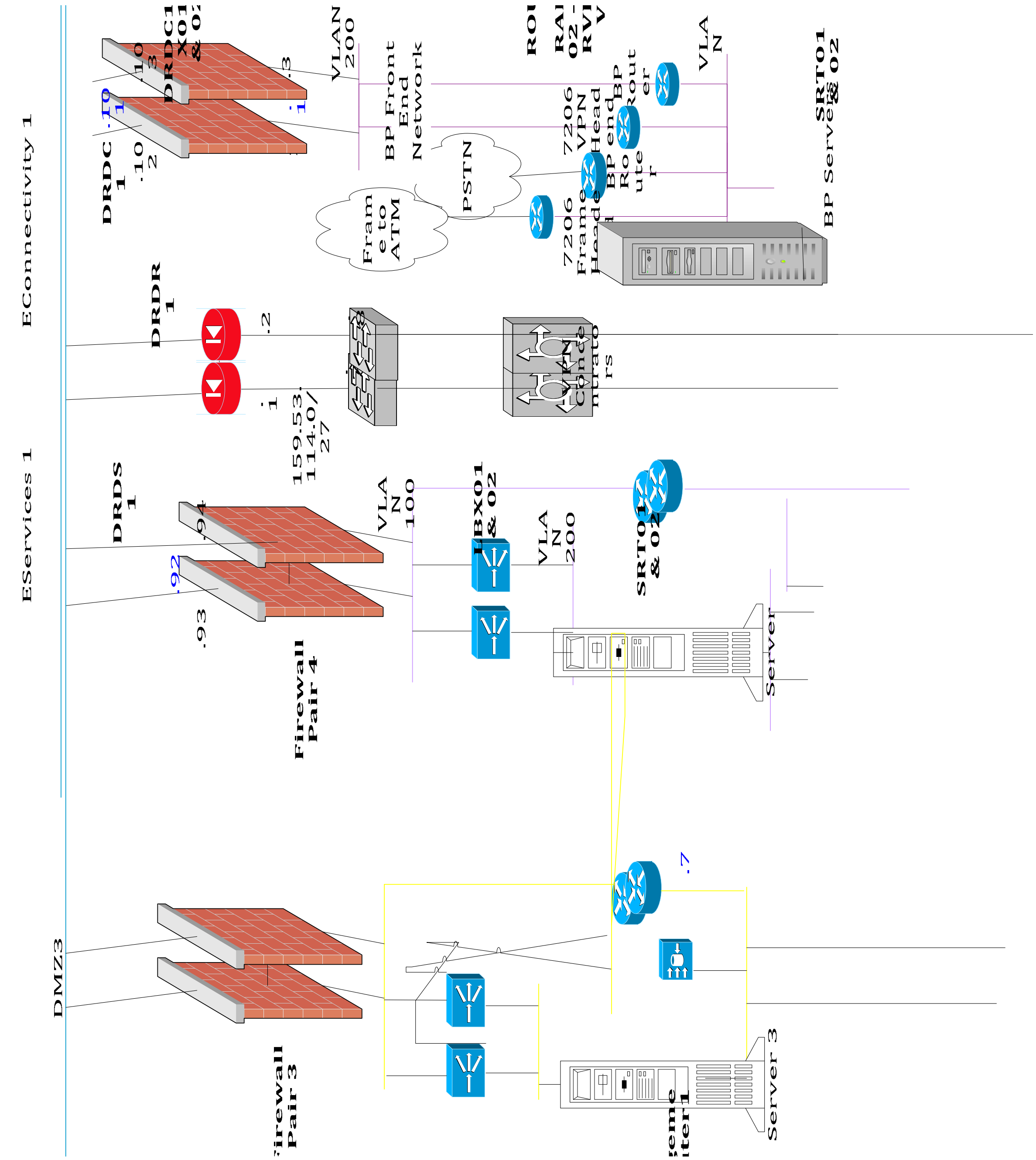


Figure 18._A new data center design for alternatives 1 and 2.

In figure 19, the distribution core layer would connect these access layers together to provide communication between branches. There are several zones that represent several distribution layers that connect access layers. The load balancer provides traffic control between the branches through the core switches.

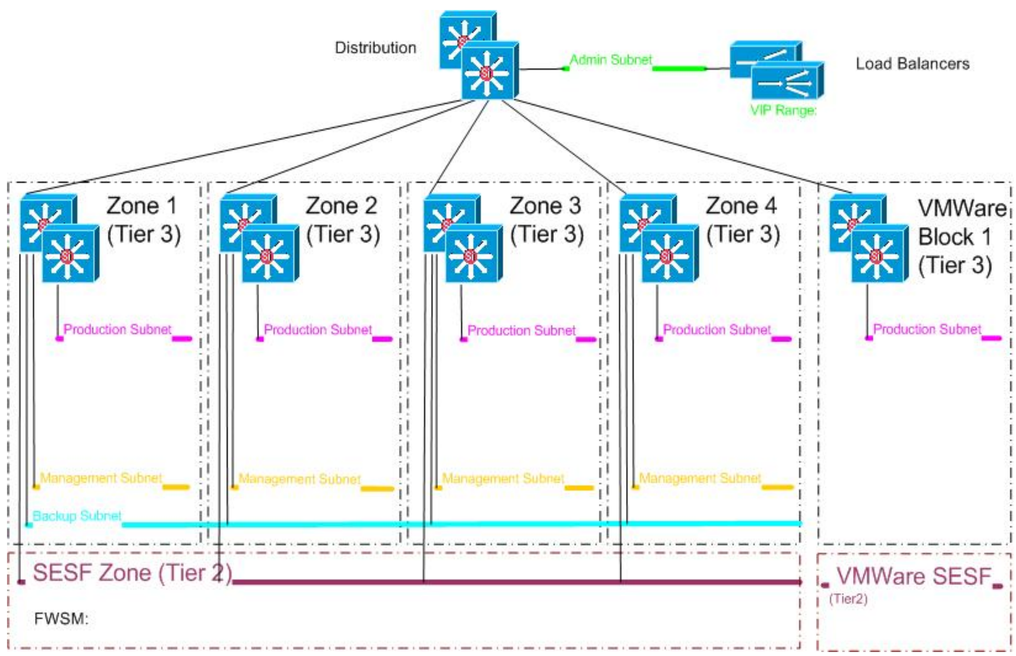


Figure 19. Distribution/core layer of new design.

Furthermore, the new design allows for more expansion in the future because of the ability to replicate it over different areas. The core layers provide ease of connectivity. Thus, the load of a single core layer of several branches will be reduced.

Alternative A2: Eliminate current system start anew with external organization to implement and operate

The second solution to ABC Bank's growth problem would be to utilize the above new datacenter design, but instead use outside people to implement it. There are several advantages to doing this as the job may get done faster since more skilled people would be contracted. If there are not enough skilled people internally, then this solution would be more feasible.

However, there are some inherent disadvantages with choosing this solution. One major issue is that security could be compromised since the outside people would be given access to the data. This could pose several problems because customer data may not be confidential anymore and could lose its integrity.

Also, this solution may cost more than alternative A2. If there are not enough inside people to complete this task than outside people may charge more. They may also buy more expensive equipment which would end up driving up the total cost. This excludes the cost of retraining technicians on the newer system as explained in alternative A2.

Alternative A3: Upgrade existing system

Since ABC Bank already has a system in place, it is possible to expand the current datacenter by adding more branches and server areas. Also, within the server areas, services can be added to allow for online banking, mobile banking, etc. in the areas where these types of services are not available. However, there would be a need to expand the core layer to handle the increased traffic on the networks. Also, compatibility between old and new equipment would be an issue.

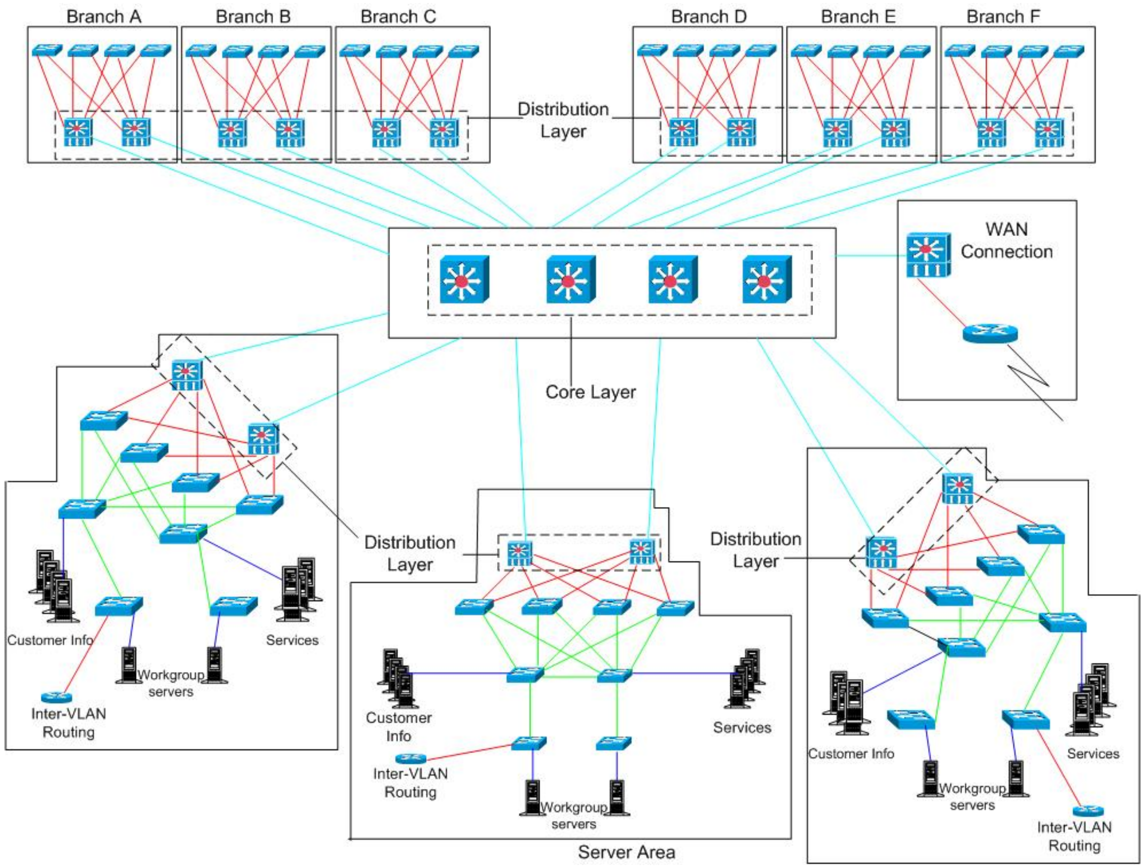


Figure 20. Upgrading existing system.

This solution would be the easiest to implement, because restructuring of the datacenter and completely new hardware would not be needed. However, as stated before in the problems section, the current system is outdated and while the expanded system may be sufficient for the present and near future, it will face expansion problems in the distant future. Not only will it face expansion problems, but also security problems. The security issues may result from the inability of the old networking equipment from handling current and feature security requirements. As hackers become more efficient and effective in the future, breaches may become much harder to stop.

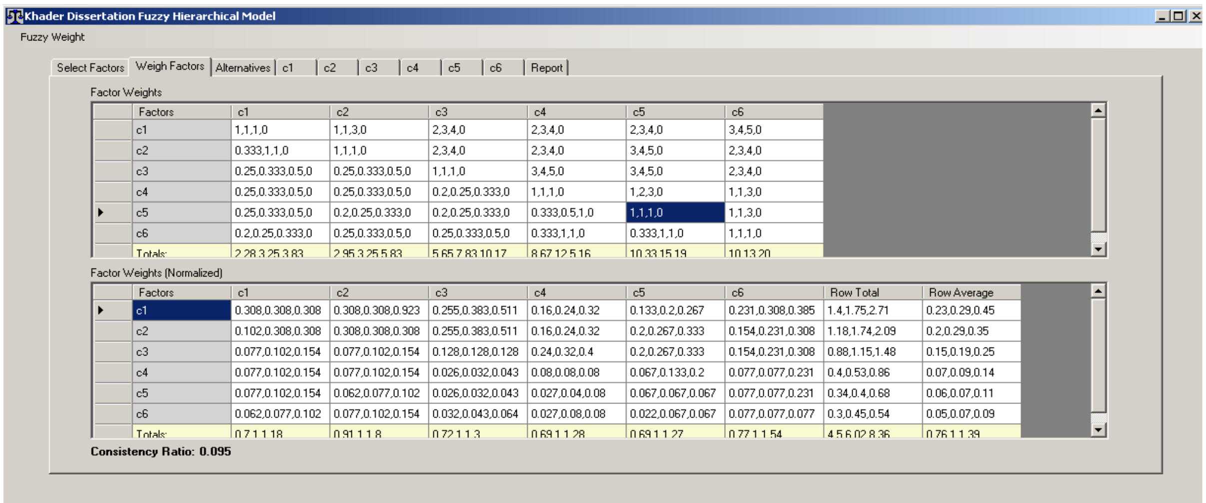
Analysis

To compare these alternatives to each other, the fuzzy hierarchical model was applied. The main goal was to solve the ABC datacenter decision problem. Crisp preference judgments of the experts were solicited and fuzzfied. The fuzzified preference judgments were entered into the upper diagonal part of the application's interface. The application filled the lower half using the reciprocal axiom of the developed model. The application also allowed an additional entry to identify the degree of uncertainty along with each fuzzified preference judgment. This uncertainty index, as discussed earlier, was embedded in the α-cut where $\alpha \in [0,1]$. In summary, each entry was in the form a fuzzy triplet (lower, middle, and upper) and an α-cut.

After all the factors were weighted against each other, the alternatives were evaluated with respect to each other under each factor. This allowed for evaluation as to how each alternative contributed to each factor. Again, the value notation was the same as comparing each factor to the main goal as stated above. However, the α-cut of the alternatives dictated the confidence level to each factor instead of to the overall goal. The analysis was carried out several times. The first round was with the maximum degree of uncertainty (α-cut = 0), the default settings. Then, a sensitivity analysis was performed to gain better insights related to economic outlooks and attitudes of decision makers. Six scenarios were executed to illustrate the ability of the developed model to assist a decision maker to gain a better understanding of the factors and the role uncertainty.

Below are the results of the analysis in a snapshot graphical format obtained from the output of the software application of the model.

Alpha-cut = 0

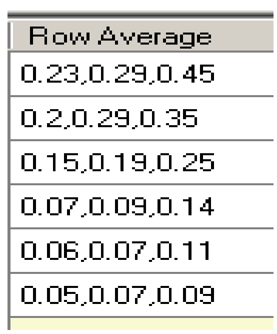


Factors	c1	c2	c3	c4	c5	c6
c1	1,1,1,0	1,1,3,0	2,3,4,0	2,3,4,0	2,3,4,0	3,4,5,0
c2	0.333,1,1,0	1,1,1,0	2,3,4,0	2,3,4,0	3,4,5,0	2,3,4,0
c3	0.25,0.333,0.5,0	0.25,0.333,0.5,0	1,1,1,0	3,4,5,0	3,4,5,0	2,3,4,0
c4	0.25,0.333,0.5,0	0.25,0.333,0.5,0	0.2,0.25,0.333,0	1,1,1,0	1,2,3,0	1,1,3,0
c5	0.25,0.333,0.5,0	0.2,0.25,0.333,0	0.2,0.25,0.333,0	0.333,0.5,1,0	1,1,1,0	1,1,3,0
c6	0.2,0.25,0.333,0	0.25,0.333,0.5,0	0.25,0.333,0.5,0	0.333,1,1,0	0.333,1,1,0	1,1,1,0
Totals:	2.28,3.25,3.83	2.95,3.25,5.83	5.65,7.83,10.17	8.67,12,5.16	10.33,15,19	10,13,20

Factor Weights (Normalized)

Factors	c1	c2	c3	c4	c5	c6	Row Total	Row Average
c1	0.308,0.308,0.308	0.308,0.308,0.923	0.255,0.383,0.511	0.16,0.24,0.32	0.133,0.2,0.267	0.231,0.308,0.385	1.4,1.75,2.71	0.23,0.29,0.45
c2	0.102,0.308,0.308	0.308,0.308,0.308	0.255,0.383,0.511	0.16,0.24,0.32	0.2,0.267,0.333	0.154,0.231,0.308	1.18,1.74,2.09	0.2,0.29,0.35
c3	0.077,0.102,0.154	0.077,0.102,0.154	0.128,0.128,0.128	0.24,0.32,0.4	0.2,0.267,0.333	0.154,0.231,0.308	0.88,1.15,1.48	0.15,0.19,0.25
c4	0.077,0.102,0.154	0.077,0.102,0.154	0.026,0.032,0.043	0.08,0.08,0.08	0.067,0.133,0.2	0.077,0.077,0.231	0.4,0.53,0.86	0.07,0.09,0.14
c5	0.077,0.102,0.154	0.062,0.077,0.102	0.026,0.032,0.043	0.027,0.04,0.08	0.067,0.067,0.067	0.077,0.077,0.231	0.34,0.4,0.68	0.06,0.07,0.11
c6	0.062,0.077,0.102	0.077,0.102,0.154	0.032,0.043,0.064	0.027,0.08,0.08	0.022,0.067,0.067	0.077,0.077,0.077	0.3,0.45,0.54	0.05,0.07,0.09
Totals:	0.7,1,1.18	0.91,1,1.8	0.72,1,1.3	0.69,1,1.28	0.69,1,1.27	0.77,1,1.54	4.5,6.02,8.36	0.76,1,1.39

Figure 21. Snapshot, fuzzy PCM weighting for design criteria of data center.



Row Average
0.23,0.29,0.45
0.2,0.29,0.35
0.15,0.19,0.25
0.07,0.09,0.14
0.06,0.07,0.11
0.05,0.07,0.09

Figure 22. Snapshot, fuzzy ranking of criteria.

Examining the ranking of the design criteria, in Figure 21, leads to the conclusion that C1 (budget), C2 (security) came in close 1st and 2nd respectively. C3 (scalability) came in close third. However, the other three criteria came in way last. Thus, their impact on the alternative selection is minimized. It is understandable that the ABC bank wants to upgrade the datacenter within the boundaries of the budget. Furthermore, security is an important issue and scalability is important considering the nature of the networking operation and environment where upgrades are almost always performed.

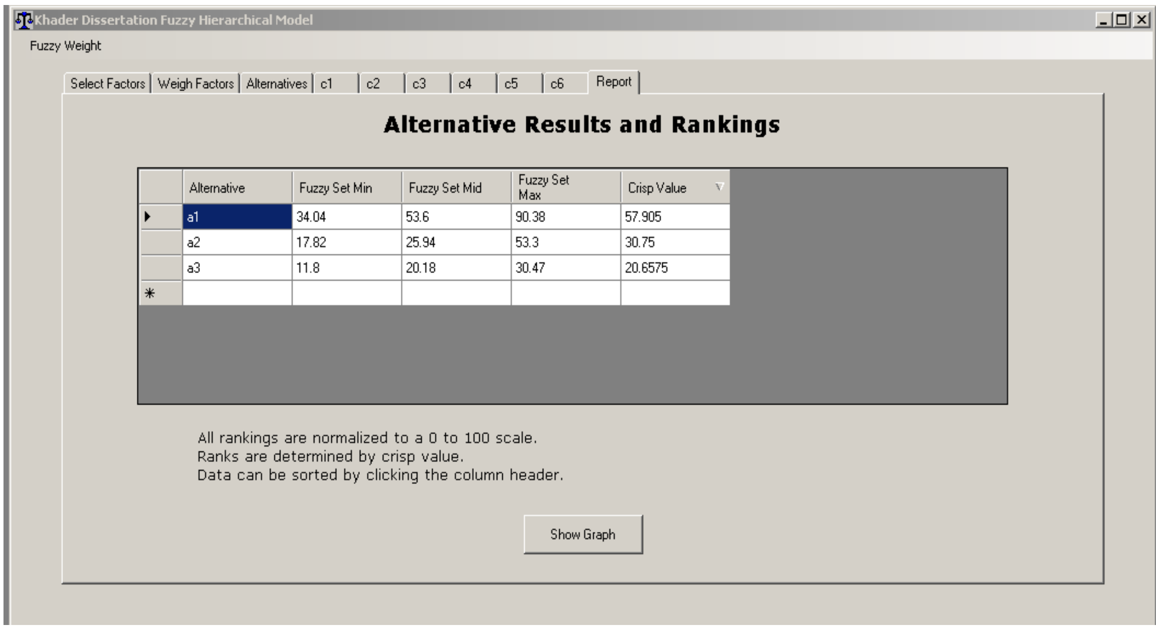


Figure 23. Snapshot, crisp ranking of alternatives.

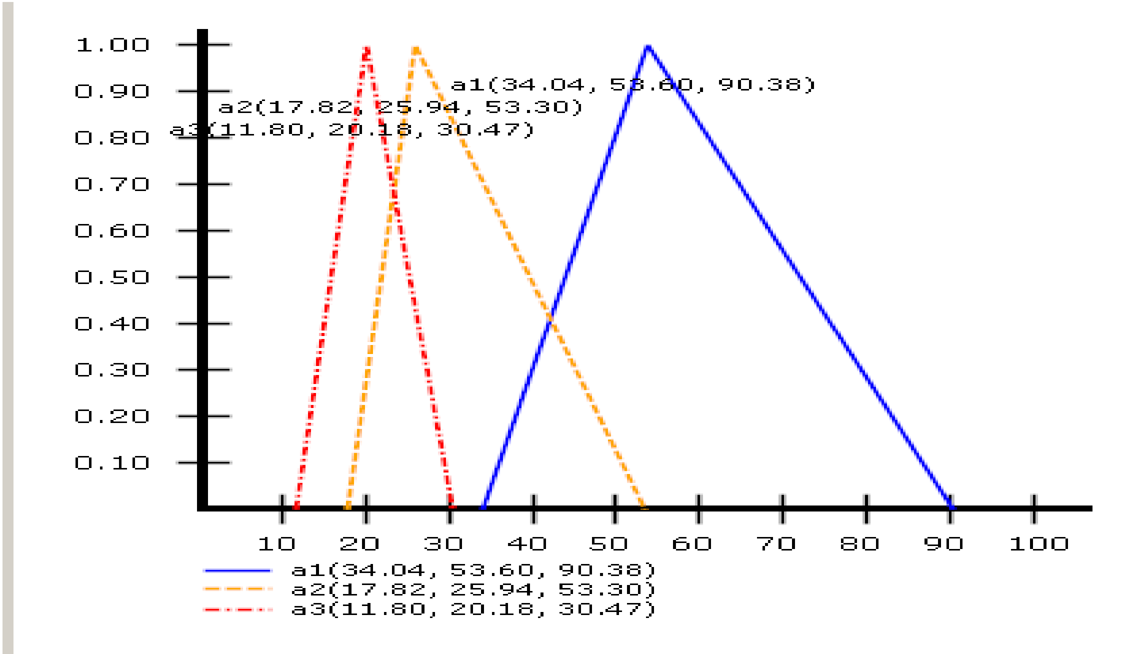


Figure 24. Snapshot, graphical fuzzy ranking of alternatives.

Examining Figure 23, the crisp ranking of the alternative gives a clear indication that A1 (start anew with in-house implementation) came in first by a good distance. However, Figure 24 tells a different story. Although A1 came in first, it carries higher risk compared to the other two alternative solutions. The risk is represented in a fuzzy interval that is about 55 points. This should give the decision maker a pause before making the selection.

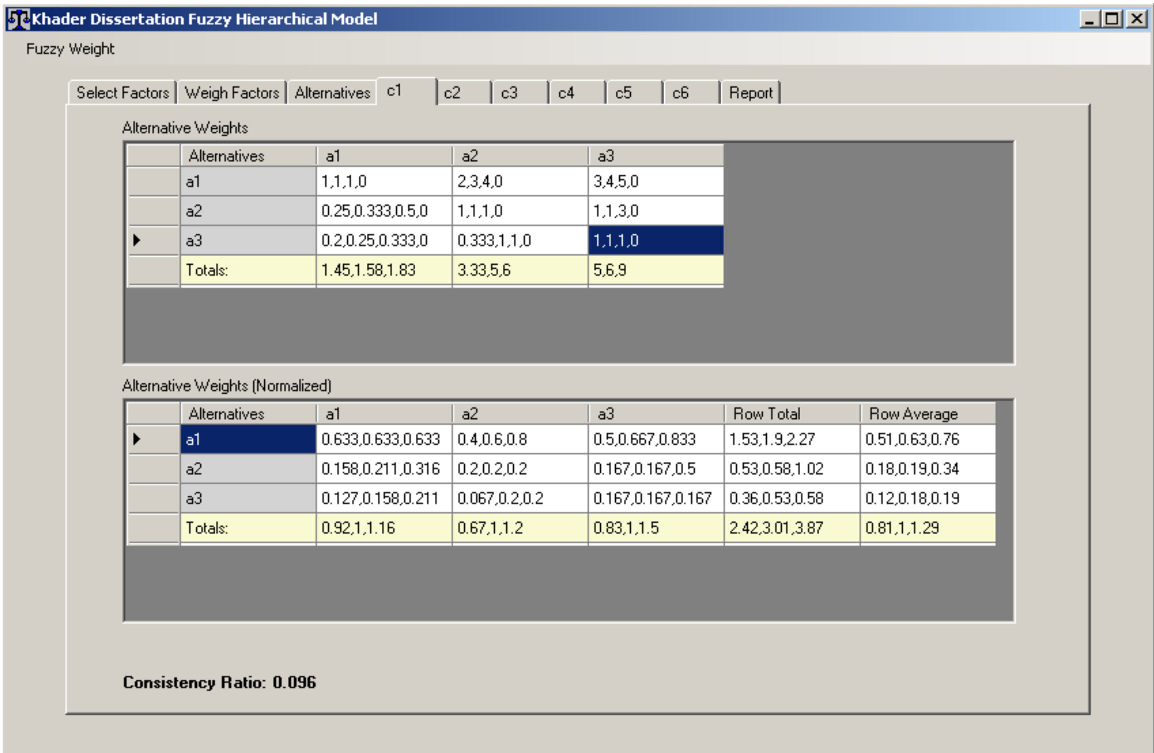


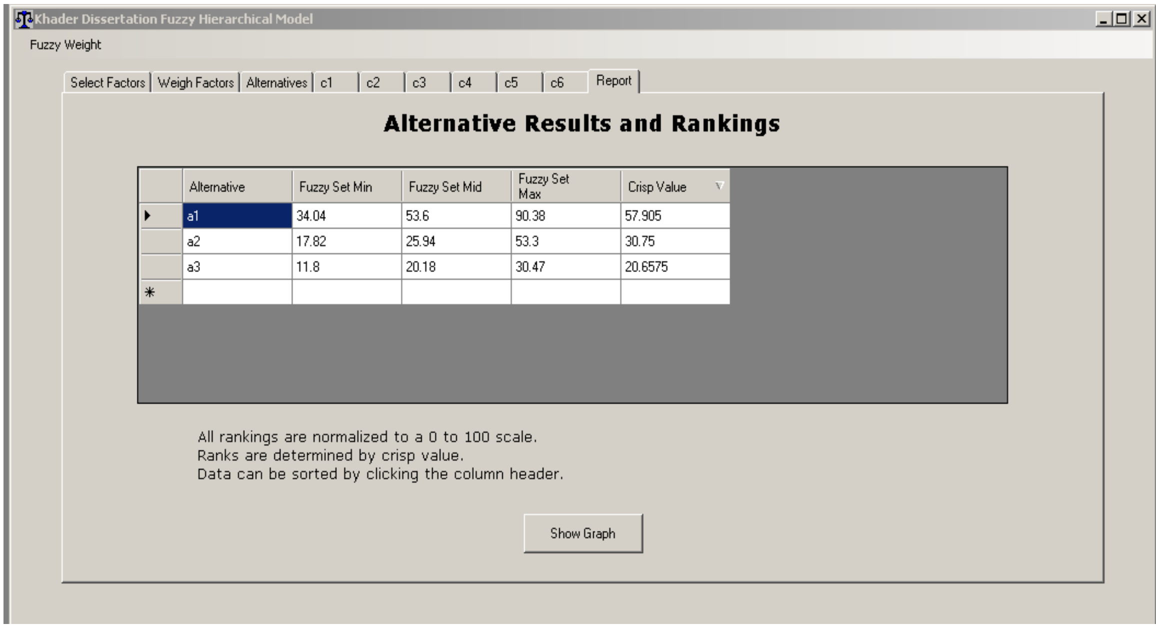
Figure 25. Snapshot, sample of weighting of alternatives with respect to criterion c1.

Figure 25 shows the evaluation of the alternatives' performance with respect to criteria C1 (Budget constraint). It appears that A1 (eliminating existing infrastructure and designing a new one while relying on in-house networking professional) is favored. It appears that the decision maker is more comfortable being in control of the expenditure than relinquishing control to an outside firm (A2). Also, A3 may break the budget since it is not clear whether the new equipment will readily work with the old. Also, it is not clear what type of training and service contracts may be required in a situation where old designs are mixed with new ones.

Each alternative solution carries with it a certain level of risk. The top ranked solution relies on the in-house professionals. Some questions may need to be asked: (1) Does the ABC bank have the required professionals to execute the task? (2) Are the in-house professionals capable of implement the project and handling its related logistics? (3) Can they manage and operate the infrastructure moving forward? (4) Can they scale the infrastructure upward to accommodate future traffic and new service needs? And, finally, (5) What about the learning curves?

These questions were shown in the risk that A1 carried in the graphical representation of the fuzzy ranking of the alternatives. The second and third solutions were separated with some fuzzy distance from the first rank. However, they carry somewhat less risk and uncertainty. A2 is ranked second and is slightly more risky than A3. But, it carries way less risk than A1. For these reasons, a sensitivity analysis to explore the impact of different degrees of uncertainty and the pessimistic and optimistic attitude of decision makers was performed.

Examining the ranking of the design criteria, in Figure 21, leads to the conclusion that C1 (budget), C2 (security) came in close 1st and 2nd respectively. C3 (scalability) came in close third. However, the other three criteria came in way last. Thus, their impact on the alternative selection is minimized. It is understandable that the ABC bank wants to upgrade the datacenter within the boundaries of the budget. Furthermore, security is an important issue and scalability is important considering the nature of the networking operation and environment where upgrades are almost always performed.



Alternative	Fuzzy Set Min	Fuzzy Set Mid	Fuzzy Set Max	Crisp Value
a1	34.04	53.6	90.38	57.905
a2	17.82	25.94	53.3	30.75
a3	11.8	20.18	30.47	20.6575

Figure 23. Snapshot, crisp ranking of alternatives.

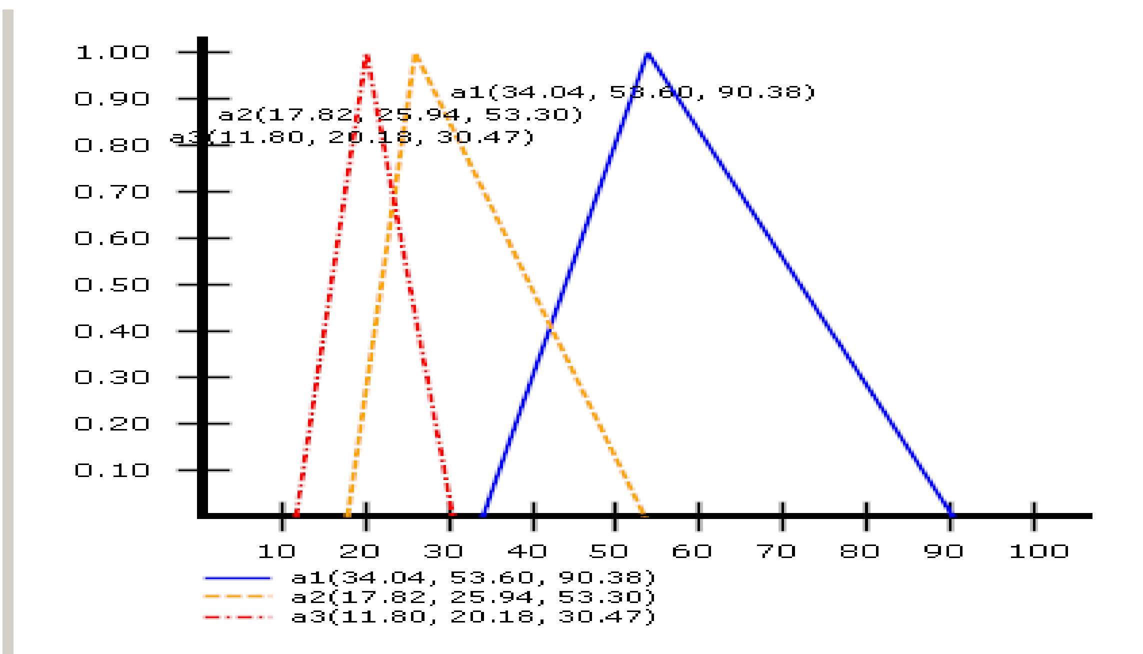


Figure 24. Snapshot, graphical fuzzy ranking of alternatives.

Examining Figure 23, the crisp ranking of the alternative gives a clear indication that A1 (start anew with in-house implementation) came in first by a good distance. However, Figure 24 tells a different story. Although A1 came in first, it carries higher risk compared to the other two alternative solutions. The risk is represented in a fuzzy interval that is about 55 points. This should give the decision maker a pause before making the selection.

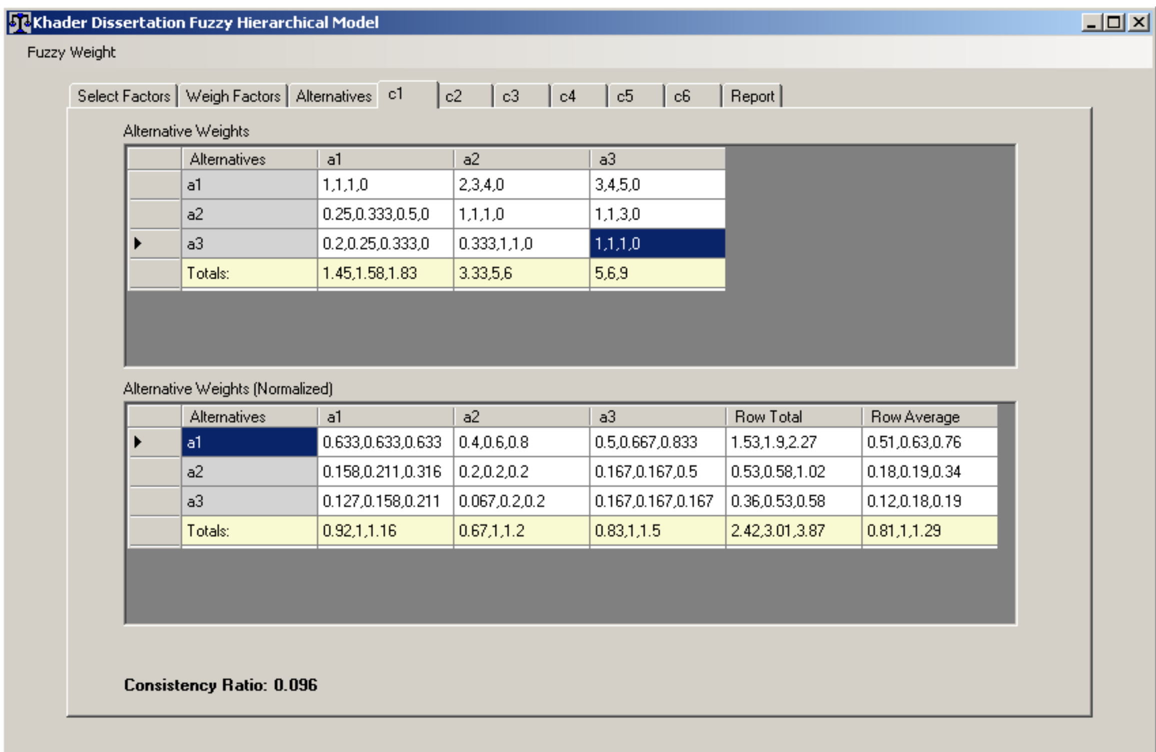


Figure 25. Snapshot, sample of weighting of alternatives with respect to criterion c1.

Figure 25 shows evaluation of the alternatives' performance with respect to criteria C1 (Budget constraint). It appears that A1 (eliminating existing infrastructure and designing a new one while relying on in-house networking professional) is favored. It appears that the decision maker is more comfortable being in control of the expenditure than relinquishing control to an outside firm (A2). Also A3 may break the budget since it is not clear whether the new equipment will readily work with the old. Also it is not clear what type of training and service contracts may be required in a situation where old designs are mixed with new ones.

Each alternative solution carries with it a certain level of risk. The top ranked solution relies on the in-house professionals. Some questions may need to be asked: (1) Does the ABC bank have the required professionals to execute the task? (2) Are the in-house professionals capable of implement the project and handling its related logistics? (3) Can they manage and operate the infrastructure moving forward? (4) Can they scale the infrastructure upward to accommodate future traffic and new service needs? And, finally, (5) What about the learning curves?

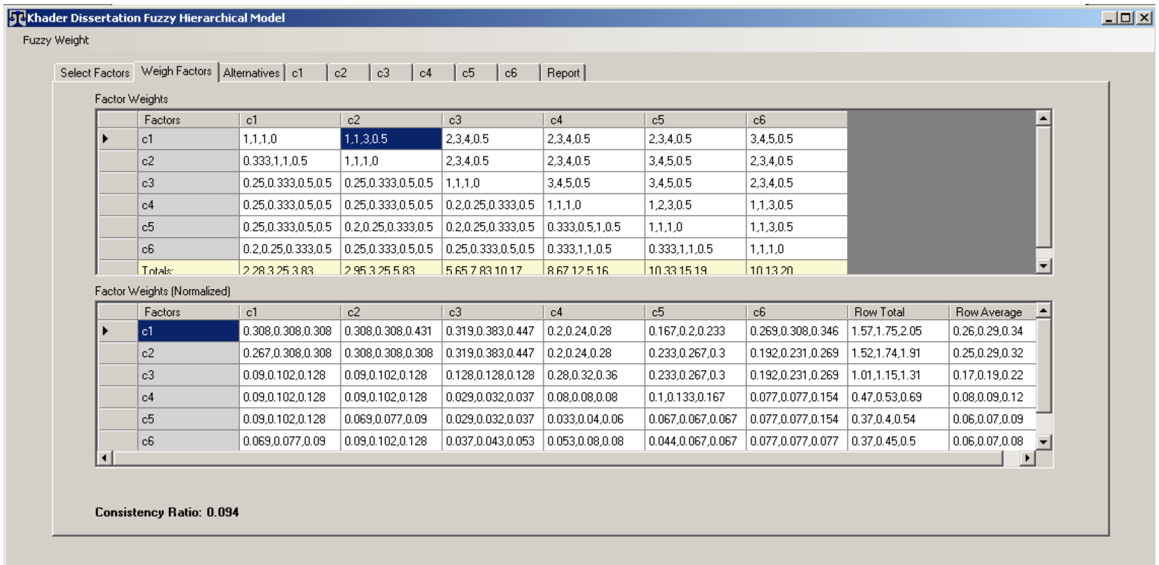
These questions were shown in the risk that A1 carried in the graphical representation of the fuzzy ranking of the alternatives. The second and third solutions were separated with some fuzzy distance from the first rank. However, they carry somewhat less risk and uncertainty. A2 is ranked second and is slightly more risky than A3. But, it carries way less risk than A1. For these reasons, a sensitivity analysis to explore the impact of different degrees of uncertainty and the pessimistic and optimistic attitude of decision makers was performed.

Sensitivity Analysis

To enlighten the effects of uncertainty in experts' knowledge, a sensitivity analysis was performed. Sensitivity analysis deals with *what-if* scenarios. The goal is to provide decision analysts with adequate information to arrive at a decision with a high degree of confidence. Arriving at an informed decision is accomplished by a reduction of the risk factors associated with each of the alternative courses of actions. Reduction in risk intervals yields a higher degree of certainty. When the analysis produces relatively large risk intervals, decision analysts may solicit additional data to reduce uncertainty. Reducing uncertainty will result in a reduction of the risk intervals.

We already have seen the most uncertain case with α-cut = 0. We have seen the risk that A1 exhibited even though it appeared first in the ranking. In this analysis, α-cut = 0.5, which represents moderately certain, and 0.8, which represents strongly certain. Then, α-cut was kept at 0.5 and changed the decision maker's attitude one time to pessimistic and another time to optimistic. This is accomplished by changing delta. To achieve the effect of changing delta, asymmetric triangular fuzzy judgment can be entered. An optimistic judgment is asymmetric to the right and a pessimistic judgment is asymmetric to the left.

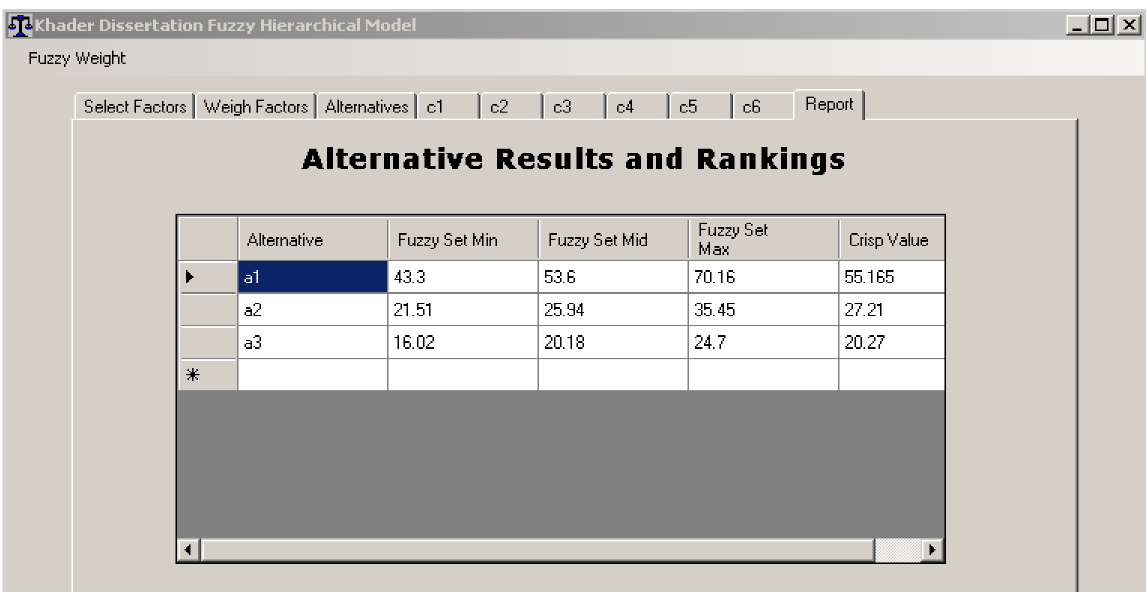
Sensitivity Analysis: α-cut of 0.5



Factors	c1	c2	c3	c4	c5	c6
c1	1,1,1,0	1,1,3,0.5	2,3,4,0.5	2,3,4,0.5	2,3,4,0.5	3,4,5,0.5
c2	0.333,1,1,0.5	1,1,1,0	2,3,4,0.5	2,3,4,0.5	3,4,5,0.5	2,3,4,0.5
c3	0.25,0.333,0.5,0.5	0.25,0.333,0.5,0.5	1,1,1,0	3,4,5,0.5	3,4,5,0.5	2,3,4,0.5
c4	0.25,0.333,0.5,0.5	0.25,0.333,0.5,0.5	0.2,0.25,0.333,0.5	1,1,1,0	1,2,3,0.5	1,1,3,0.5
c5	0.25,0.333,0.5,0.5	0.2,0.25,0.333,0.5	0.2,0.25,0.333,0.5	0.333,0.5,1,0.5	1,1,1,0	1,1,3,0.5
c6	0.2,0.25,0.333,0.5	0.25,0.333,0.5,0.5	0.25,0.333,0.5,0.5	0.333,1,1,0.5	0.333,1,1,0.5	1,1,1,0
Totals:	2.28,3.25,3.83	2.95,3.25,5.83	5.65,7.83,10.17	8.67,12.5,16	10.33,15,19	10,13,20

Factors	c1	c2	c3	c4	c5	c6	Row Total	Row Average
c1	0.308,0.308,0.308	0.308,0.308,0.431	0.319,0.383,0.447	0.2,0.24,0.28	0.167,0.2,0.233	0.269,0.308,0.346	1.57,1.75,2.05	0.26,0.29,0.34
c2	0.267,0.308,0.308	0.308,0.308,0.308	0.319,0.383,0.447	0.2,0.24,0.28	0.233,0.267,0.3	0.192,0.231,0.269	1.52,1.74,1.91	0.25,0.29,0.32
c3	0.09,0.102,0.128	0.09,0.102,0.128	0.128,0.128,0.128	0.28,0.32,0.36	0.233,0.267,0.3	0.192,0.231,0.269	1.01,1.15,1.31	0.17,0.19,0.22
c4	0.09,0.102,0.128	0.09,0.102,0.128	0.029,0.032,0.037	0.08,0.08,0.08	0.1,0.133,0.167	0.077,0.077,0.154	0.47,0.53,0.69	0.08,0.09,0.12
c5	0.09,0.102,0.128	0.069,0.077,0.09	0.029,0.032,0.037	0.033,0.04,0.06	0.067,0.067,0.067	0.077,0.077,0.154	0.37,0.4,0.54	0.06,0.07,0.09
c6	0.069,0.077,0.09	0.09,0.102,0.128	0.037,0.043,0.053	0.053,0.08,0.08	0.044,0.067,0.067	0.077,0.077,0.077	0.37,0.45,0.5	0.06,0.07,0.08

Figure 26. Snapshot , design criteria weighting with alpha = 0.5.



Alternative	Fuzzy Set Min	Fuzzy Set Mid	Fuzzy Set Max	Crisp Value
a1	43.3	53.6	70.16	55.165
a2	21.51	25.94	35.45	27.21
a3	16.02	20.18	24.7	20.27

Figure 27. Snapshot, crisp ranking of alternatives with alpha = 0.5

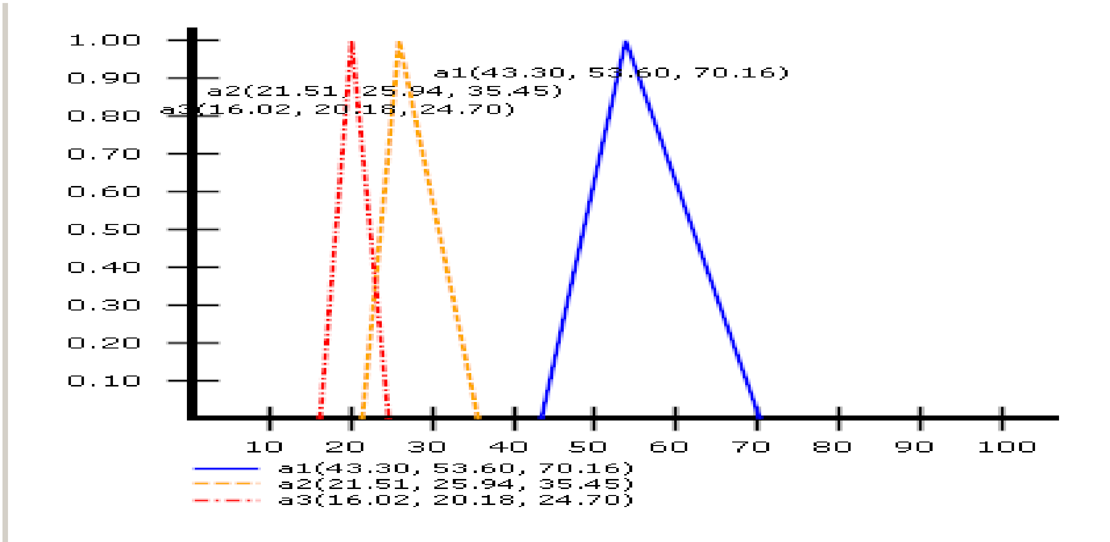


Figure 28. Snapshot, for fuzzy graphical ranking with alpha = 0.5.

Examining Figure 28 illustrates the fuzzy ranking of alternatives. It indicates shorter risk intervals when alpha was set to 0.5. Also, the fuzzy overlapping among A1 and A2 started to disappear. The risk factor was cut by almost half, from 55 points to 27. A3 exhibited the least risk, about 10 fuzzy points, while A2 carried a risk interval of about 16. A decision maker may decide based on this result that A1 is the best way to go given that the ABC bank knows something about the economic outlook over the next few years. It may also have some information about the nature of the networking development environment.

Sensitivity Analysis: α-cut of 0.8

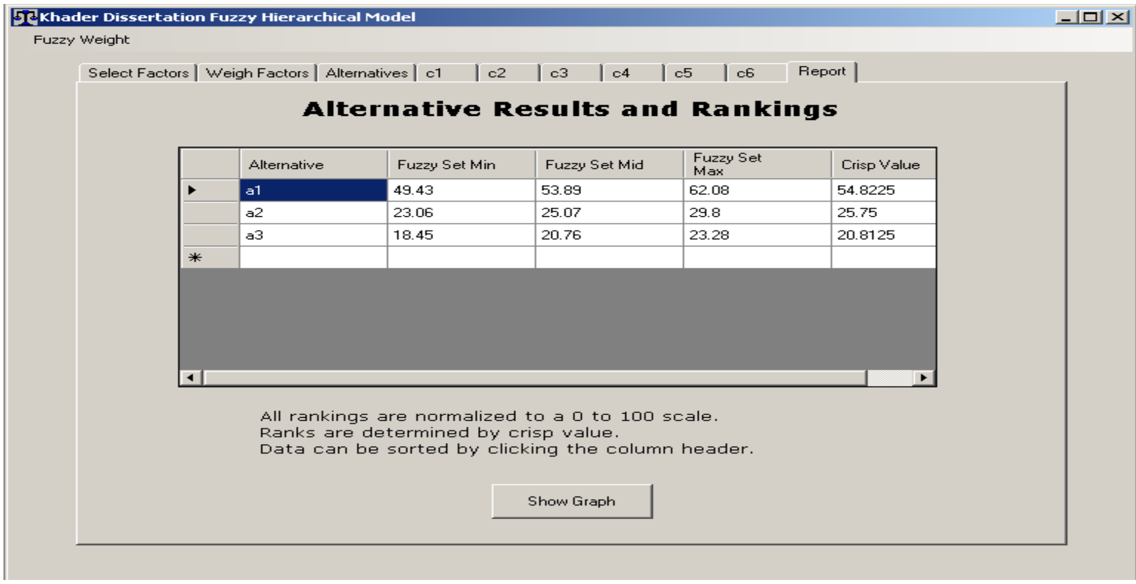


Figure 29. Snapshot of crisp ranking with alpha = 0.8.

As alpha increases, the level of certainty increases. Thus, the risk exhibited by the fuzzy ranking of the alternatives seemed to diminish, although the distances between the alternatives seem to get smaller. However, A1 still leads the other two. If a decision maker is operating in an environment that is certain, as shown here, then A1 may be selected with a high level of confidence.

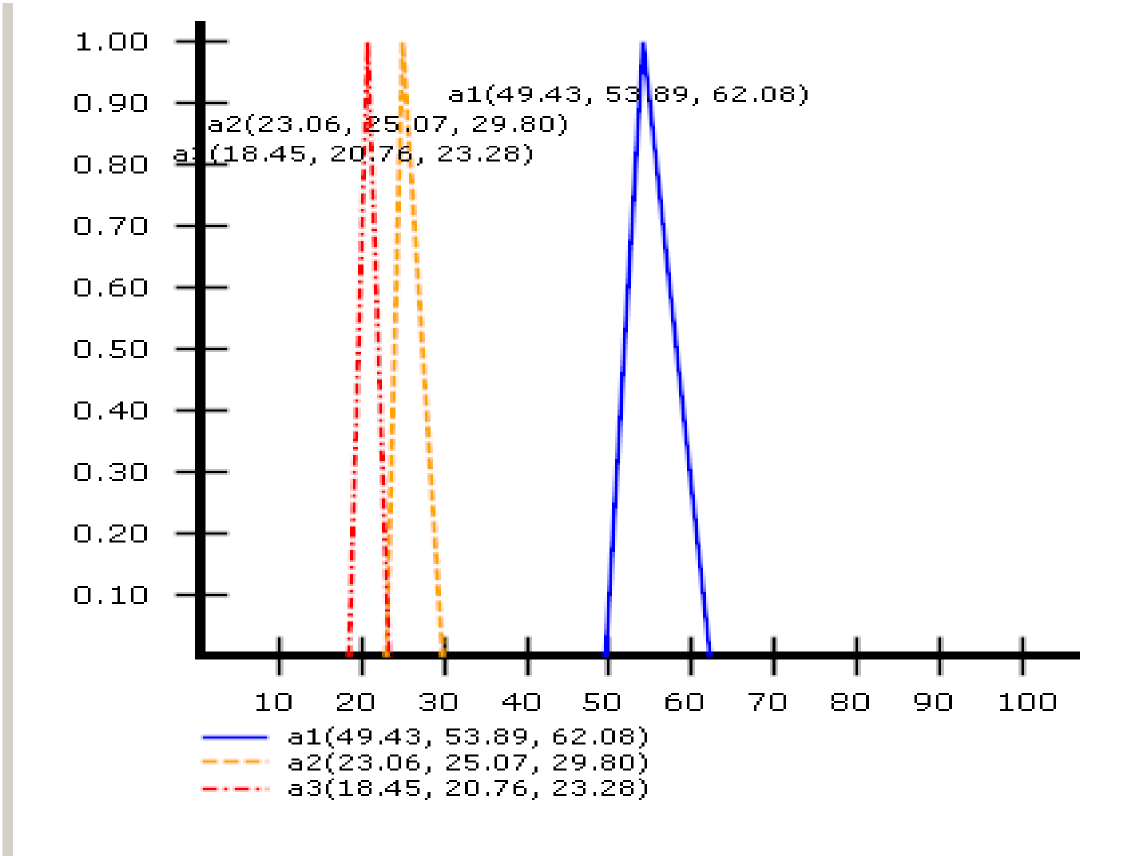
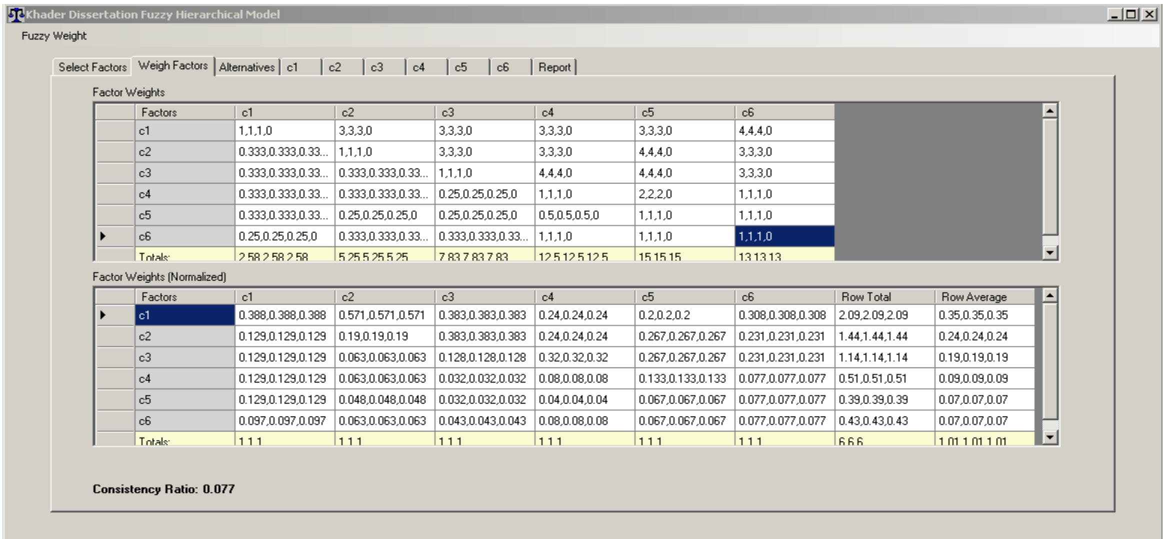


Figure 30. Snapshot for fuzzy graphical ranking with alpha = 0.8.

Figure 30 illustrates a high degree of certainty in the decision process. It depicts alternative A1 with a minimum degree of risk. It also shows that A2 and A3 are adjacent, but not overlapping. They still lag behind A1.

Fuzzy Hierarchy Model in the Classical AHP Mode

We performed the analysis in the classical AHP mode to answer research question 2 related to consistency of the developed model with the underlying framework. This mode of operation is similar to the fuzzy hierarchical model with alpha = 1. When alpha = 1, all preference judgments are crisp. Thus, we expect the output in both fuzzy and crisp graphical representations to show crisp outcomes. That is exactly what took place.



Khader Dissertation Fuzzy Hierarchical Model

Fuzzy Weight

Select Factors | Weigh Factors | Alternatives | c1 | c2 | c3 | c4 | c5 | c6 | Report

Factor Weights

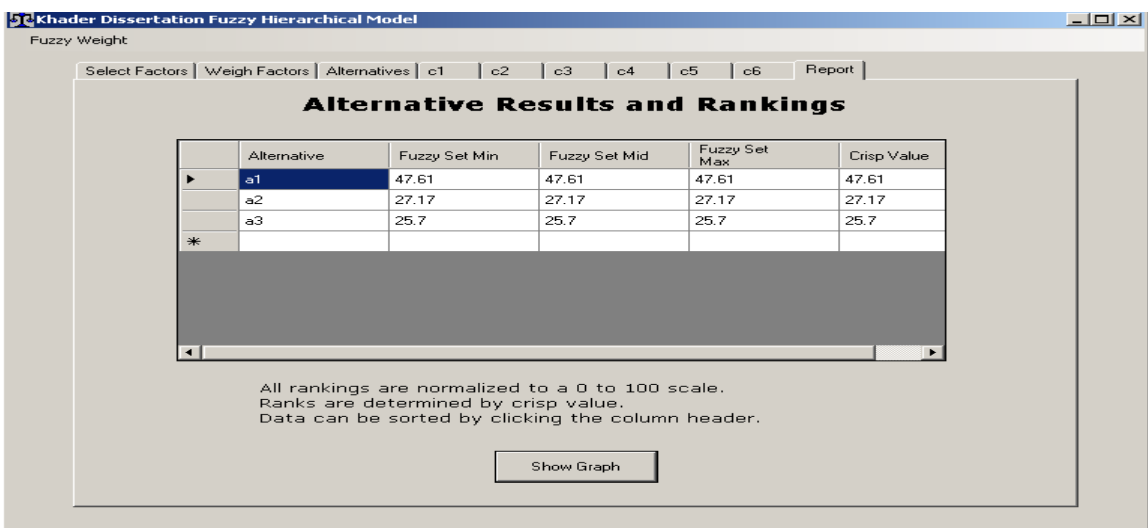
Factors	c1	c2	c3	c4	c5	c6
c1	1,1,1,0	3,3,3,0	3,3,3,0	3,3,3,0	3,3,3,0	4,4,4,0
c2	0.333,0.333,0.33...	1,1,1,0	3,3,3,0	3,3,3,0	4,4,4,0	3,3,3,0
c3	0.333,0.333,0.33...	0.333,0.333,0.33...	1,1,1,0	4,4,4,0	4,4,4,0	3,3,3,0
c4	0.333,0.333,0.33...	0.333,0.333,0.33...	0.25,0.25,0.25,0	1,1,1,0	2,2,2,0	1,1,1,0
c5	0.333,0.333,0.33...	0.25,0.25,0.25,0	0.25,0.25,0.25,0	0.5,0.5,0.5,0	1,1,1,0	1,1,1,0
c6	0.25,0.25,0.25,0	0.333,0.333,0.33...	0.333,0.333,0.33...	1,1,1,0	1,1,1,0	1,1,1,0
Totals:	2.58 2.58 2.58	5.25 5.25 5.25	7.83 7.83 7.83	12.5 12.5 12.5	15 15 15	13 13 13

Factor Weights (Normalized)

Factors	c1	c2	c3	c4	c5	c6	Row Total	Row Average
c1	0.388,0.388,0.388	0.571,0.571,0.571	0.383,0.383,0.383	0.24,0.24,0.24	0.2,0.2,0.2	0.308,0.308,0.308	2.09,2.09,2.09	0.35,0.35,0.35
c2	0.129,0.129,0.129	0.19,0.19,0.19	0.383,0.383,0.383	0.24,0.24,0.24	0.267,0.267,0.267	0.231,0.231,0.231	1.44,1.44,1.44	0.24,0.24,0.24
c3	0.129,0.129,0.129	0.063,0.063,0.063	0.128,0.128,0.128	0.32,0.32,0.32	0.267,0.267,0.267	0.231,0.231,0.231	1.14,1.14,1.14	0.19,0.19,0.19
c4	0.129,0.129,0.129	0.063,0.063,0.063	0.032,0.032,0.032	0.08,0.08,0.08	0.133,0.133,0.133	0.077,0.077,0.077	0.51,0.51,0.51	0.09,0.09,0.09
c5	0.129,0.129,0.129	0.048,0.048,0.048	0.032,0.032,0.032	0.04,0.04,0.04	0.067,0.067,0.067	0.077,0.077,0.077	0.39,0.39,0.39	0.07,0.07,0.07
c6	0.097,0.097,0.097	0.063,0.063,0.063	0.043,0.043,0.043	0.08,0.08,0.08	0.067,0.067,0.067	0.077,0.077,0.077	0.43,0.43,0.43	0.07,0.07,0.07
Totals:	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	6 6 6	1.01 1.01 1.01

Consistency Ratio: 0.077

Figure 31. Snapshot of fuzzy weighting of criteria in classical AHP mode.



Khader Dissertation Fuzzy Hierarchical Model

Fuzzy Weight

Select Factors | Weigh Factors | Alternatives | c1 | c2 | c3 | c4 | c5 | c6 | Report

Alternative Results and Rankings

Alternative	Fuzzy Set Min	Fuzzy Set Mid	Fuzzy Set Max	Crisp Value
a1	47.61	47.61	47.61	47.61
a2	27.17	27.17	27.17	27.17
a3	25.7	25.7	25.7	25.7

All rankings are normalized to a 0 to 100 scale.
Ranks are determined by crisp value.
Data can be sorted by clicking the column header.

Show Graph

Figure 32. Snapshot of crisp ranking in AHP mode.

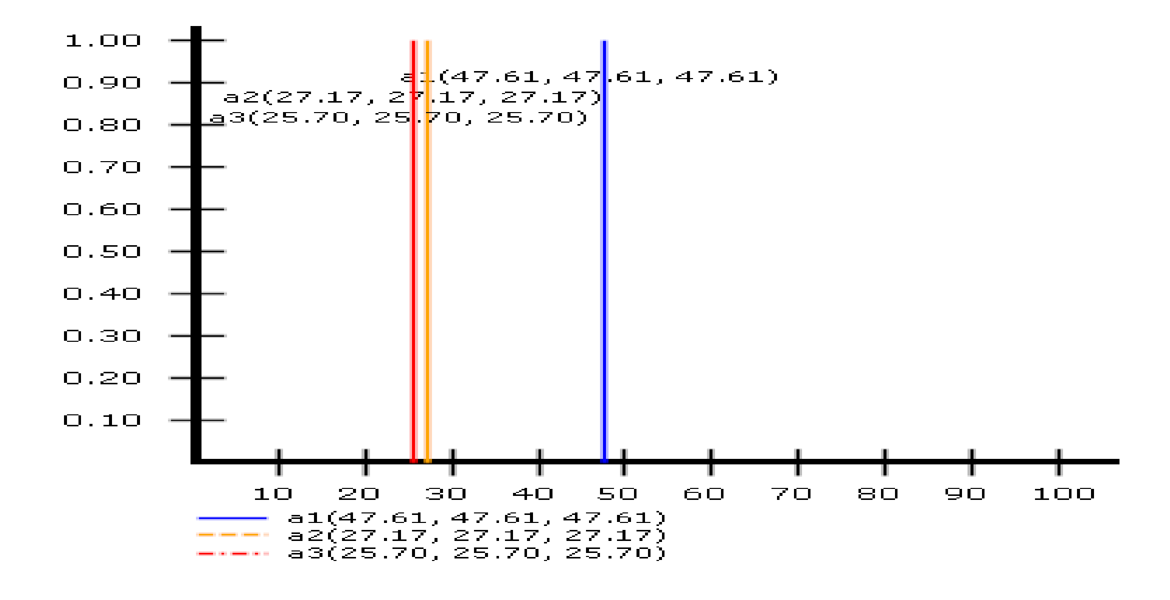


Figure 33. Snapshot of fuzzy graphical report in AHP mode, the result is a crisp ranking.

In examining Figure 33, it appears that the risk interval is reduced to 0. This means that none of the alternative carries any risk factor. This is not realistic. This will be possible only if we are absolutely certain of what we are analyzing. Also, decision equivocation with respect to certain judgment is not even considered. This situation, as was stated in chapter 1, is a major drawback of the classical AHP. This scenario showed two perspectives: the drawback of classical AHP and the consistency of the fuzzy hierarchical model developed with the underlying framework.

Uncertain and Pessimistic Decision Maker

To further evaluate the impact of the attitude of the decision makers on the outcomes of the analysis, a scenario of uncertain and pessimistic decision maker's analysis was carried out. For uncertain judgment, the decision makers used fuzzy triplets with wide fuzzy intervals. The pessimistic attitude was represented by shifting the interval to the right, toward the lower bounds. This has the effect of delta being 1. Figures 34 and 35 show an outcome that almost no decision can be taken with any degree of confidence. All alternatives came close in the fuzzy ranking and all carried a high risk factor. This is logical. When operating in a very uncertain environment and pessimism is the dominant attitude, it is difficult to make substantial decisions with a reasonable degree of confidence.

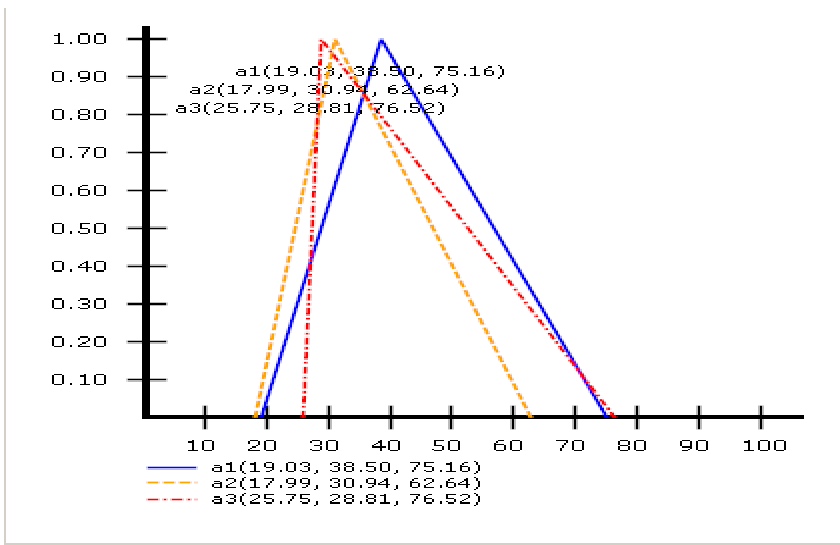
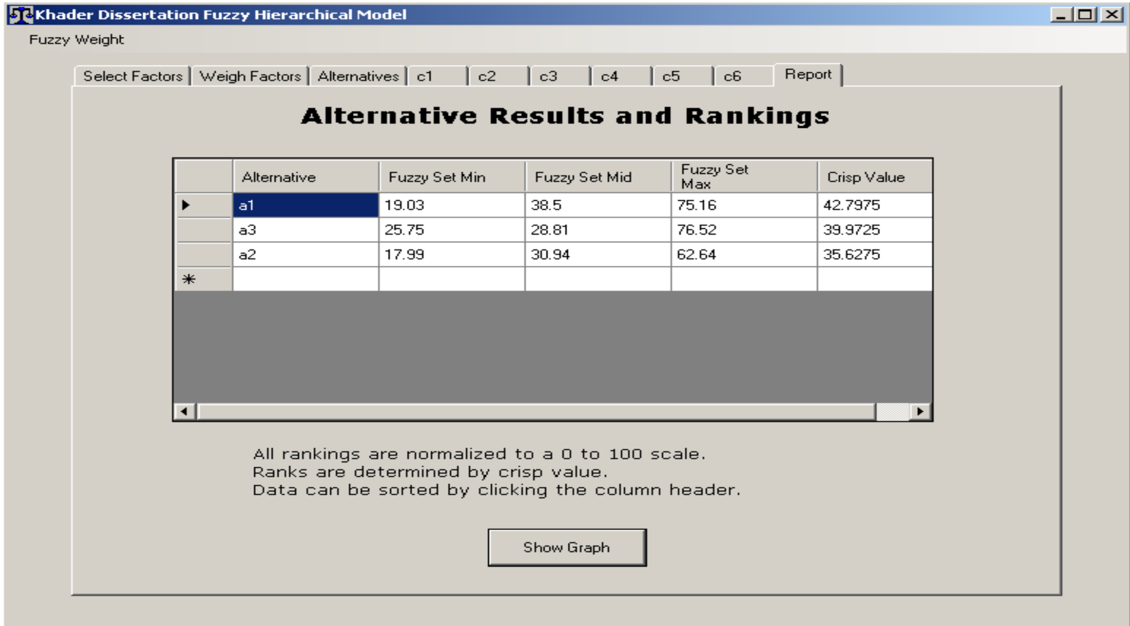
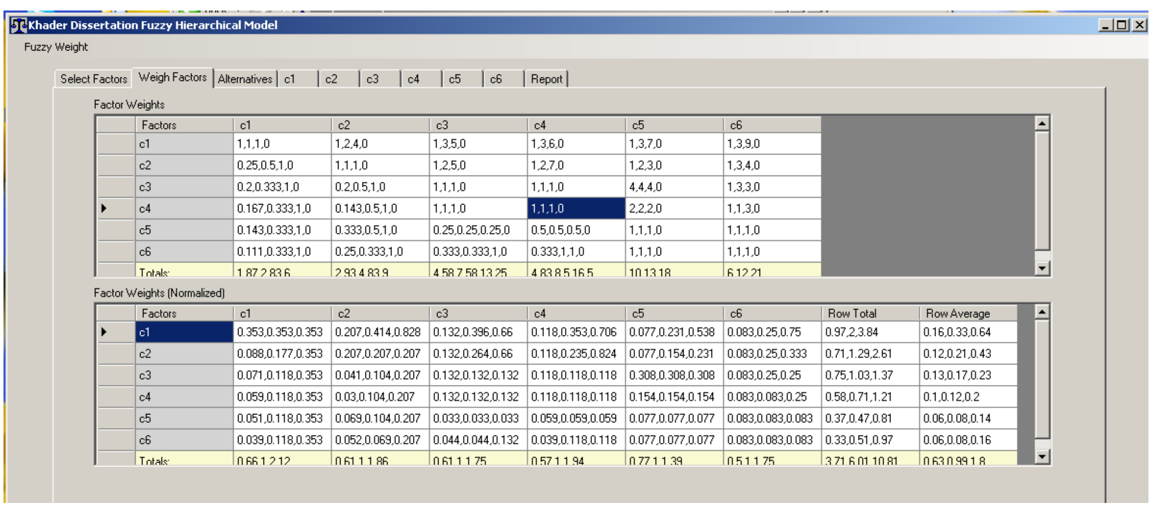


Figure 34. Snapshot of fuzzy graphical report of uncertain and pessimistic decision maker.



Alternative	Fuzzy Set Min	Fuzzy Set Mid	Fuzzy Set Max	Crisp Value
a1	19.03	38.5	75.16	42.7975
a3	25.75	28.81	76.52	39.9725
a2	17.99	30.94	62.64	35.6275

Figure 35. Snapshot of crisp ranking of uncertain and pessimistic decision maker.



Factor Weights

Factors	c1	c2	c3	c4	c5	c6
c1	1,1,1,0	1,2,4,0	1,3,5,0	1,3,6,0	1,3,7,0	1,3,9,0
c2	0.25,0.5,1,0	1,1,1,0	1,2,5,0	1,2,7,0	1,2,3,0	1,3,4,0
c3	0.2,0.333,1,0	0.2,0.5,1,0	1,1,1,0	1,1,1,0	4,4,4,0	1,3,3,0
c4	0.167,0.333,1,0	0.143,0.5,1,0	1,1,1,0	1,1,1,0	2,2,2,0	1,1,3,0
c5	0.143,0.333,1,0	0.333,0.5,1,0	0.25,0.25,0.25,0	0.5,0.5,0.5,0	1,1,1,0	1,1,1,0
c6	0.111,0.333,1,0	0.25,0.333,1,0	0.333,0.333,1,0	0.333,1,1,0	1,1,1,0	1,1,1,0
Totals:	1.87 2.83 6	2.93 4.83 9	4.58 7.58 13.25	4.83 8.5 16.5	10 13 18	6 12 21

Factor Weights (Normalized)

Factors	c1	c2	c3	c4	c5	c6	Row Total	Row Average
c1	0.353,0.353,0.353	0.207,0.414,0.828	0.132,0.396,0.66	0.118,0.353,0.706	0.077,0.231,0.538	0.083,0.25,0.75	0.97,2,3.84	0.16,0.33,0.64
c2	0.088,0.177,0.353	0.207,0.207,0.207	0.132,0.264,0.66	0.118,0.235,0.824	0.077,0.154,0.231	0.083,0.25,0.333	0.71,1.29,2.61	0.12,0.21,0.43
c3	0.071,0.118,0.353	0.041,0.104,0.207	0.132,0.132,0.132	0.118,0.118,0.118	0.308,0.308,0.308	0.083,0.25,0.25	0.75,1.03,1.37	0.13,0.17,0.23
c4	0.059,0.118,0.353	0.03,0.104,0.207	0.132,0.132,0.132	0.118,0.118,0.118	0.154,0.154,0.154	0.083,0.083,0.25	0.58,0.71,1.21	0.1,0.12,0.2
c5	0.051,0.118,0.353	0.069,0.104,0.207	0.033,0.033,0.033	0.059,0.059,0.059	0.077,0.077,0.077	0.083,0.083,0.083	0.37,0.47,0.81	0.06,0.08,0.14
c6	0.039,0.118,0.353	0.052,0.069,0.207	0.044,0.044,0.132	0.039,0.118,0.118	0.077,0.077,0.077	0.083,0.083,0.083	0.33,0.51,0.97	0.06,0.08,0.16
Totals:	0.66 1.2 12	0.61 1.1 86	0.61 1.1 75	0.57 1.1 94	0.77 1.1 39	0.5 1.1 75	3.71 6.01 10.81	0.63 0.99 1.8

Figure 36. Snapshot of criteria weighting of uncertain and pessimistic decision maker.

Similarities and Differences Between AHP and FHM

There are several similarities between classical AHP and FHM.

1. Both are multicriteria decision modeling systems.
2. Both decompose a complex decision into irreducible factors.
3. Both structure a complex decision into levels of hierarchies.
4. Both use experts' judgment to evaluate decision factors and alternative solutions.
5. Both perform consistency tests to ensure uniform logic of the analysis.
6. Both can represent data visually.

There are differences between the classical AHP and FHM. They are outlined in Table 8.

Table 8.

Summary of Differences between AHP and FHM.

	Classical AHP	FHM
1	Uses crisp judgments only.	Uses fuzzy and crisp judgment
2	Unable to handle decisions under uncertainty.	Designed to handle decisions under uncertain conditions and vague information.
3	Does not take into account decision maker's pessimistic and optimistic attitudes	Via the use of a delta function, it is capable of embedding into the decision analysis the pessimistic and optimistic attitude of the decision maker.
4	Not capable of performing sensitivity analysis with different degrees of uncertainties.	Capable of performing sensitivity analysis with varying degrees of uncertainty index and decision maker's attitude.

5	Not capable of handling decision makers equivocation with respect to individual judgments.	Capable of capturing decision maker's equivocation for individual judgments
6	Well established concept.	New advancement concept.

Summary

This chapter dealt with the results of the dissertation research. It focused mainly on the development of a fuzzy hierarchical model to embed uncertainty and the attitudes of experts into the process of solving a complex multicriteria decision project. The model dealt with major drawbacks of the classical and widely used AHP (Saaty, 1980, 1996, and 2001). The major drawbacks of the AHP are its apparent inability to handle uncertainty, ill-defined problems, and experts' pessimistic and optimistic attitude (Mikhailov & Tsvenetinove, 2004; and Tang & Beynon, 2007). The fuzzy hierarchical model (FHM) dealt with these drawbacks through the use of the fuzzy sets theory that Dr. Zadeh originally conceived in 1965. Further, a new delta-function and an alpha-cut application were introduced to take into account the decision makers' attitudes and degrees of uncertainties.

Furthermore, an important component of this dissertation research was the development of algorithmic and procedural operations of the model in both pseudo language and an actual software application. The software application proved useful in applying the model to a simulated practical datacenter multicriteria decision problem. Two networking experts collaborated with the investigator in developing the datacenter design application and the criteria for its design. Additionally, exemplars of sensitivity analysis were discussed to further illustrate the benefits of the newly developed model compared to AHP. Although the model is consistent with the AHP modeling technique, it provided many advantages in dealing with fuzzy scales, uncertainties, decision maker's attitudes, and risk factors. Consistency with the underlying framework is important in this type of this research because it lends credence to validation and acceptability (Moole, 2005). This validation was illustrated through performing decision analysis with FHM operating in the classical AHP crisp mode.

The investigator, to the best of his knowledge, is not aware of any other research that dealt with problems presented in this dissertation. The extensive search for research that treated networking design problems as fuzzy multicriteria decision problems did not yield any results. This is in spite of discussions of the multicriteria nature of this problem (Bello, 2003; Schoening, 2004; and Stallings, 2006).

Chapter 5 presented the conclusion of this research. It focused on how the research questions were dealt with in this dissertation. It summarized the new mathematical concepts that were developed to enhance the decision-making process. It also dealt with future research and implications for new techniques to further the research in the multicriteria modeling and analysis process.

CHAPTER 5: SUMMARY, CONCLUSION, AND RECOMMENDATIONS

Decision sciences focus on improving managerial decision making processes. Decision modeling is an important area of study in decision sciences. Business intelligence and decision making in today's uncertain business world require the use of tools that aid in analyzing decision problems while taking into consideration the emergent uncertain business environment. Further, decision makers' attitudes vary based on their professional background and other factors that may fall outside the scope of this research. However, in a business setting, there are decision makers who may have a predisposition toward pessimistic outlooks, and, conversely, decision makers with optimistic attitudes. Both types of decision makers may be part of a group that makes complex business decisions. A decision support system will need to take into account the variation in decision makers' predisposition toward pessimism and optimism.

Multicriteria decision making is a well established decision modeling technique that has been in use by many organizations including many of the fortune 500 firms. For example, IBM and HP are two examples of large firms that embed multicriteria decision modeling in their decision making process, especially AHP modeling (Expert choice, 2008). As was stated earlier, AHP has major drawbacks, mainly in its inability to handle ill-defined and uncertain decision problems. Also, AHP does not seem capable of integrating experts' attitudes into the decision model.

Summary

The goal of this research was to address the shortcomings of AHP while maintaining the advantages it provided. AHP shares the characteristics of multicriteria decision modeling, mainly decomposing a complex decision problem with attributes with different units of measures into irreducible decision factors. Also, AHP structures a complex multicriteria decision problem into levels of hierarchies.

FHM, the model developed in this dissertation research, maintained AHP benefits while overcoming its shortcomings. The fuzzy sets theory and its related formulae were used in the FHM modeling to treat uncertainties in multicriteria decision analysis. A new delta-function was developed to embed experts' pessimistic and optimistic attitudes into the decision modeling and analysis. A new application of alpha-cut, a technique used to present variations in the degree of uncertainty was used (Gil-Aluja, 2004). Mathematical formulations, algorithmic and procedural operation of the developed model were carried out in this research. The mathematical formulation used are well established and their development followed well established techniques similar to the techniques that were fostered by Saaty (1980, 1996, 2001), Zadeh (1965, 1975), Mikaelove (2004), and many other reputable researchers.

Conclusions

While progressing in this research, the emphasis was on answering the research questions posed in chapter 1. There are more than one technique to answer these questions. However, the investigator's belief is that answering the questions through the uses of a practical application is more beneficial because it ties the theoretical derivations to the model to its applications. In the following section, statements on how each research question was addressed are provided.

Comments on Research Question 1

Research question 1 addressed the benefits that FHM would provide compared to classic AHP: Does the model provide improvements in handling uncertainty compared to AHP? This dissertation research addressed this question in three aspects. First, FHM embedded uncertainty into complex multicriteria decision analysis through the novel techniques of using the fuzzy set theory and its associated fuzzy operations. Second, FHM used a delta-function, the investigator conceived, to integrate the experts' and the decision makers' pessimistic and optimistic attitudes into the analysis. Third, FHM provided a sensitivity analysis feature necessary to give the decision maker better insights into the problem that was highlighted. FHM allowed a decision maker to vary alpha (degree of uncertainty) on both a global level and a localized level. From a global point of view, alpha was set for all preference judgments. The localized level focused on varying alpha with regard to each individual judgment. This captured the decision maker's equivocation regarding a certain fuzzy judgment while developing confidence in other judgments. Alpha was used to embed different degrees of uncertainty into the decision analysis while delta was used to map decision makers' attitudes. Low alpha indicated highly uncertain judgments and business environment. Higher alpha values point to a more stable environment and a higher level of confidence in judgments. Delta worked in the reverse direction. A low value for delta meant a pessimistic decision maker and high value of delta indicated an optimistic decision maker. In summary, the question of the advantages of FHM was fully addressed and validated through its application to a simulated practical situation related to a complex networking decision problem.

Comments on Research Question 2

Research question 2 dealt with whether FHM is consistent with the underlying framework: To provide maximum benefits and acceptance, is the model consistent with the underlying framework (Russel & Norvig, 2003, and Moole, 2005)? Consistency was ensured through having FHM keep the heuristic properties of the underlying AHP model. The overall structure with regards to decomposing a complex multicriteria decision was maintained. To validate that FHM is consistent with the underlying platform, an additional property was added to FHM. This property permitted a decision maker to use FHM to carry the analysis in crisp AHP mode. This was illustrated in applying the model to the networking problem. The result of the analysis indicated that FHM is capable of being used as a classic crisp AHP. This was allowed through varying the input in one of two possible ways: (1) set the lower, modal, and upper values of the fuzzy triplet representation of judgments to same value (modal), or (2) fuzzy input is made in the common method with which a decision maker is familiar but set alpha to a value of one. A value one for alpha meant the decision maker was absolutely certain of the judgment rendered. This has the same effect as operating in the AHP classical mode. The analysis in the AHP mode of operation highlighted the main drawback of AHP. The result showed crisp ranking of the alternatives. In this form (AHP), the weight of each alternative exhibited zero risk factor. This is unrealistic in practical business operations and with dealing with complex multicriteria decision problems. The software application of the model proved valuable in highlighting the issues related to the posed research questions.

Comments on Research Question 3

The third research question addressed decision makers' pessimistic and optimistic attitudes: Can the developed model take into account the decision maker's pessimistic and optimistic attitudes? This question was addressed through a newly developed delta-function. The delta function was applied to the defuzzified pairwise comparison matrix. Defuzzification was carried out in one of three methods: (1) alpha-cut and delta-function manipulation, (2) central of gravity, and (3) Delphi method. A low value of delta points to an optimistic decision maker and a high value of delta move the judgment toward the pessimism domain. A scenario of an uncertain and pessimistic decision maker was presented in chapter 4. The scenario showed difficulties in these types of situation. All of the three alternative solutions exhibited a similar high degree of risks as well as indistinct fuzzy weighting where the alternatives appeared overlapping in a morphed manner. This required further analysis to arrive at a decision with a certain degree of confidence.

Comments on Research Question 4

The fourth research question dealt with whether the newly developed model improved the decision making process. The research carried in this study illustrated that this issue was dealt with from a number of angles: (1) It dealt with risk factor related to decision choices, (2) It treated uncertain economic and business environment through a degree of confidence factor coined alpha-cut, (3) It dealt with decision makers' pessimistic and optimistic attitudes via the use of the delta function, (4) It took into account groups input into the decision process through the use of the technique of geometric means. Collectively the developed model and its practical applications illustrated the improvement made to the multicriteria decision process. These improvements may improve an organization's standing in the marketplace, save a large sum of funds that can be rightfully directed and used for an overall strategic improvements and a healthy standing in a fluid environment (Nutt, 2002).

Overall, this dissertation research was conducted while anchored in the following areas: (1) applied management, (2) decision sciences, and (3) information systems. These are the three themes that define the Ph.D. program in which the investigator is enrolled. To that end, this dissertation research is directly related to the themes of specialization. A fuzzy hierarchical model was developed to capture uncertainties into the analysis of complex multicriteria decisions. The formulation of the model was logically presented. Pseudo language algorithmic and procedural operations of the model were derived from the formulation. A software application was developed following the rules of information systems and computer sciences. An analysis of time and space complexity was given for each module as well as the overall application. The application of the model was carried out to analyze a multicriteria networking decision a leading bank needed to make. Two networking experts participated in the application of the model and in defining the complex networking decision problem.

The research method used in this research is analytical. Limitations in analytical studies are usually due to interpretations, logical errors, and semantics. To minimize such limitations, substantiating claims were based on being thorough in developing formulae and adhering to well established mathematical formulations and proofs (Moole, 2005).

Recommendations for Future Research

Fuzzy decision modeling is a research area that is witnessing renewed interest in the past few years. This is evident by the work of Arslan and Khist (2007), Chan and Kumar (2005), Enea et al. (2006), and Piazza (2004). The fuzzy set theory found its way into applications in management, business, control systems, aerospace, and sophisticated military application. Some of these applications are beyond the scope of this dissertation research. One future research area that will further improve confidence in dealing with complex decisions under an uncertain and ill-defined environment is integrating the theories of probability together with fuzzy sets in multicriteria analysis. Linear programming can be also used along with fuzzy sets to improve the outcomes of multicriteria analysis. Object and dynamic programming, although hard to generalize, can also be used with fuzzy analysis to improve the analysis of complex multicriteria decision under uncertain and vague conditions.

Control systems is one of the areas that is another prime candidate for future research using the work of this dissertation, especially in areas such as real time analysis of remote images that require some immediate reaction. One example is the unmanned air vehicle. Geographical information systems (GIS) can benefit greatly from this research. Integrating GIS research and databases with this research model can contribute to many areas of research. Examples of these areas are soil suitability planning, improving traffic routing, placement of homeless people in housing according to demographic information and needs, and educational system planning.

Implications

This study used the AHP as a framework. AHP was used in the past to analyze forest management, water resource management, and renewable energy planning studies (Anada & Herath, 2007; Liebowitz, 2005; Pohekar & Ramachandran, 2004; Wang 2005). These studies are related to issues of social impacts. Since the model was developed to overcome AHP's difficulties, its use can provide improved analysis outcomes in similar socially important areas such as outsourcing decisions, poverty reduction projects, and public capital development projects.

FHM, the model developed in this dissertation research, maintained AHP benefits while overcoming its difficulties in handling uncertainty. The fuzzy sets theory and its related formulae were used in the FHM modeling to treat uncertainties in multicriteria decision analysis. A new delta-function was developed to embed experts' pessimistic and optimistic attitudes into the decision modeling and analysis. A new application of alpha-cut, a technique used to present variations in the degree of uncertainty was used. Mathematical formulations, algorithmic and procedural operation of the developed model were carried out in this research. The mathematical formulations used are well established and their development followed well established techniques embedded in AHP and fuzzy sets theory.

The model was applied to a practical application that dealt with a fortune 500 firm upgrading its datacenter infrastructure. The model used multicriteria decion-making framework including design criteria, pairwise comparison, fuzzification of preference judgments, fuzzy weighting of alternative solutions, and ranking of criteria as well as solutions. The model applied sensitivity analysis to provide the decision analyst with insight to the risk factors related to each of the proposed solutions.

Handling uncertainty and decision analysts pessimistic and optimistic outlook add tangible improvements to the decision-making process. Improved decision-making process has the potential of reducing financial losses and providing a better alignment of resources. Financial losses and misaligned resource are the primary reasons that contributed to the significance of the research problem in this dissertation.

A decision analyst using the model developed in this research does not need to be technically inclined. The traditional method used in AHP analysis which many nontechnical decision analysts use is still valid with the model developed in this research. Judgment data pertaining to a business decision are solicited in crisp numerical format. Fuzzy manipulation is transparent to the decision analysts. Viewing the results is performed either in graphical form of crisp ranking alternative. The nontechnical decision analyst selects the course of action with the highest ranking.

Chapter's Summary

Addressed in this chapter were a summary of the research, conclusions related to answering the research questions, recommendations for future research, and implications of the research. The research problem, inadequate decision-making process to acquire networking infrastructures, was identified through extensive literature review. The dissertation research focused on finding a solution. The solution materialized in the development, testing and validation of a model capable of handling uncertainty and vague information. A practical application and a software platform were used for the purpose of validation of the research. The results pointed to improvement in the decision making process in handling uncertainty and decision analysts outlook.

Some of the implications of this research are that it can be used in areas that have social impacts such as analysis of renewable energy resource, forest management, poverty reduction, and many other areas. Also this research can be coupled with research in technical areas such as control systems and geographical information systems.

In closing, this research focused on identifying a significant problem, providing a solution to the problem, and applying the results of the research to practical applications. The research questions drove the focus of this dissertation. The research effort emphasized improvements to the decision-making process, as well as consistency with underlying frameworks. The responses to the research questions were dealt with scientifically and methodologically.

REFERENCES

Arregoces, M. (2004). *Data center fundamentals*. Indiana: Cisco Systems.

Agrell, P., & Lindbroth, R. (2004). Risk, information and incentives in telecom supply chain. *International Journal of Production Economics, 90*(1), 1–16.

Anada, J., & Herath, G. (2007). Multi-attribute preference modelling and regional land-use planning. *Ecological Economics.*

Anderson, D., Sweeny, D., & Williams, T. (2004). *An introduction to management science: Quantitative approaches to decision making.* San Diego, California: Western College Publishing.

Arslan, T., & Khist, J. (2007). A rational reasoning method from fuzzy perceptions in route choice. *Journal of Fuzzy Sets and Systems, 25*(150), 419–435.

Asia Market Research (2008). Reliability. Retrieved from http://www.asiamarketresearch.com/glossary/reliability.htm

Ayag, Z. (2005). A fuzzy AHP-based simulation approach to concept evaluation environment. *IEE Transaction*, *17*(37), 827–842.

Bellman, E., & Zadeh, L. (1970). Decision making in a fuzzy environment. *Management Science, 17*(4), 141–164.

Bello, S. (2003). A case study approach to the supplier selection process. *Journal of Decision Science, 17*(25), 47-60.

Braglia, M., A., & Grassi, et al. (2004). Multi-attribute classification method for spare parts inventory management. *Journal of Quality in Maintenance Engineering, 10*(1), 55–66.

Briggs, P. (1994). Vendor assessment for partners in supply chain. *European Journal of Purchasing & Supply Management,* (15), 49-59.

Buckley, J. (1985). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems, 17*(30), 233-247.

Buckley, J., W., Buckley, H., & Chiang, H. (1976). *Research methodology & business decisions.* NY: National Association of Accountants.

Bu~yu~zkan, G. (2004). Multicriteria decision making for e-marketplace selection. *International Research: Electronic Networking Applications & Policy 14*(2), 149-154.

Buckley, J.W., Buckley, and H. Chiang. M. (1976). Research methodology and business decisions. Retrieved from http://www.maaw.info/ArtSumBuckley76.htm

Byun, D. (2002). The AHP approach for selecting an automobile purchase model: findings from consumer survey. Information and Management *38*(5), 289-297.

Chan, F. T. S., & Chan, H. K. (2004). Development of the supplier selection model – A case study in the advanced technology industry. *Journal of Engineering Manufacturing*, (218), 1807–1823.

Chan, F. T. S. (2003). Interactive selection model for supplier selection process: An analytical hierarchy process approach. *International Journal of Production Research, 41*(15), 3549–3450.

Chan, F. T. S., & Kumar, N. (2005). Global supplier development considering risk factors using fuzzy extended AHP–based approach. *Omega, 3*(4), 417–431.

Chang, D. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, *95*(3), 649 – 655.

Chen, S. J., & Hwang, C. I. (1991). Fuzzy multiple attribute decision making methods and applications. Lecture notes in economics and mathematical systems, Berlin, Germany: Springer-Verlag.

Chen, S. J. G., & Lin, L. (2004). Modeling team members' characteristics for the formation of a multifunctional team in concurrent engineering. *IEE Transaction on Engineering Management, 5*(2), 111–125.

Chen, C. T. (2000). Extensions of the TOPSIS for group decision making under fuzzy environment. *Fuzzy Sets and Systems, 114*(3), 1–9.

Cheng, S., Chen, M., Chang, H., Chen, M., Chang, Y., & Chou, S. (2007). Semantic-based facial expression recognition using analytical hierarchy process. *Journal of Expert Systems and Applications, 33*(1), 86-95.

Choi, T. Y., & Hartley, L. (1996). An exploration of supplier selection practices across the supply chain. *Journal of Operations Management,* (14), 333–343.

Cook, R. L. (1992). Expert systems in purchasing applications and development. *International Journal of Purchasing and Management, 18*(12), 20–27.

Creswell, J. (1998). *Qualitative inquiry and research design.* London: Sage Publications.

Delgado, M., Herrera, F., & Viedma, E., et al. (1998). Combining numerical and linguistic information in group decision making. *Journal of Information Sciences 107*(1998), 177–194.

Dey, D., & Sarkar, S. (2000). Modifications of uncertain data: A Bayesian framework for Belief Revision. *Information Systems Research*, *11*(1), 1-16.

Efendigil, T., Onuit, S., & Kogar, E. (2007). A holistic approach for selecting a third party reverse logistics provider in the presence of vagueness. *Journal of Computers and Industrial Engineering, 7*(9)*,* 56–67

Elisberg, R. (6/2/2007). Making the wrong decision sounds right, *Huffington Post*. Retrieved from http://www.huffington.com/robert-elisberg/making-the-wrong-decision.htm

Enea, M., & Piazza, T. (2004). Project selection by constrained fuzzy AHP, *Fuzzy Optimization and Decision Making*, *3*, 39–62.

Ertugrul, I., & Karakasglu, N. (2006). The fuzzy analytic hierarchy process for supplier selection and application in a textile company. *Proceedings of 5th International Symposium on Intelligent Manufacturing Systems*, 195–207.

Expert choice, (2008). Retrieved from http://www.expertchoice.com/solutions

Fahmy, H. (2001). Reliability evaluation in distributed computing environment using the AHP. *Journal of Computer Networks*, *36*(5), 597-615.

Fishburn, P. (1993). *The foundations of expected utility*. NJ: Prentice Hall.

Fitzgerald, J., & Dennis, A. (2005). *Business communications and networking*. NY: Wiley and Sons.

Ghodsypour, H., & O'Brien C. (1998). A decision support system for supplier selection. *Journal of Production Economics*, *56*(57), 199–212.

Gil-Aluja, J. (2004). *Fuzzy sets in the management of uncertainty*. Berlin: Springer Verlag.

Granat, J., & Wiercbicki, A. (2004). Multicriteria analysis in telecommunications. *37th International Conference on System Sciences, 1*(4), 25-37.

Guh, Y., & Po, R. (2004). Establishing a multiple structures analysis model for AHP. *International Journal of Information and Management Sciences, 15*(2), 35–42.

Kahraman, C., Cebeci, U., & Ruan, D. (2004). Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. *International Journal of Production Economics*, *87*(2), 171-184.

Kahraman, C., Cebeci, U., & Ulukan, Z. (2003). Multicriteria supplier selection of catering service companies using fuzzy AHP. *European Journal of Operational Research*, *20*(124), 102–113.

Kahraman, C., Ruan, D., & Ibrahim, D. (2003). Fuzzy group decision making for facility location selection. *Information Sciences*, *157,* 135-153.

Khaled, S. (2006). On deployment of VOIP in Ethernet networks: Methodology and case study. *Journal of Computer Communications*, *29*(8), 1039–1054.

Khader, M., & Barnes, W. (2000). *Telecommunications systems and technology*. Saddle River, NJ: Prentice Hall.

Kothari, C. (1990). *Research Methodology*. New Age International Publications. Retrieved from http://books.google.com/books?id=8c6gkbKi-F4C&pg=PA2&lpg=PA2&dq=benefits+of+analytical+research+method&source=web&ots=iFhGsUN8mN&sig=gOCm5hqKBdI4JhG5O-pw6Tm7vyk&hl=en&sa=X&oi=book_result&resnum=6&ct=result#PPR14,M1

Kuo, Y., & Chen, P. (2007). Constructing performance appraisal indicators for mobility of the service industries using Fuzzy Delphi Method. *Expert Systems with Applications, 19*(28), 39–54.

Kuo, Y., & Yu, C. (2006). 3G telecommunication operators' challenges and roles: A perspective of mobile commerce value chain. *Technovation, 26*(12), 1256-1347.

Kuo, R. J., Chi, S. C., & Kao S. S. (1999). A decision support system for locating convenience store through fuzzy AHP. *Computers & Industrial Engineering, 37*(1), 323-326.

Hui, S. C., & Foo, S. (1998). Towards a standard-based Internet telephony system. *Journal of Computer standards & Interfaces*, (19), 89–103.

Jayaswal, K. (2006). *Administering data centers*. Indianapolis, IN: Wiley Publishing Inc.

Lai, V. S., Wong, B. K., & Cheung W. (2002). Group decision making in a multiple criteria environment: A case using AHP in software selection. *European Journal of Operational Research*, *137*(1), 134-144.

Lang, H., & Merino, D. (1996). *The selection process for capital project.* NYC*,* NY: Wiley and Sons.

Laarhoven, V., & Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems*, *11*, 229-241.

Liebowitz, J. (2005). Linking social network analysis with the analytic hierarchy process for knowledge mapping in organizations. *Journal of Knowledge Management, 9*(1), 76–98.

Leung, L. C., & Cao, D. (2000). On consistency and ranking of alternatives in fuzzy AHP. *European Journal of Operational Research*, *124*(1), 102-113.

Lichtenthaler, E. (2007). Managing technology intelligence processes in situations of radical technological change. *Technological Forecasting and Social Change*, *7,* 1109-1136.

Lindroth R., & Agrell, N. (2004). Risk, information and incentives in telecom supply chains. *International Journal of Production Economics*, *90*(1), 1-16.

Laarhoven, J., & Pedrycz, W. (1983). Techniques in the design with fuzzy models. *Fuzzy Theory and Systems*, 3 – 25.

Martin, J. R. (2/8/2004). Summary of research methods. Retrieved from http://maaw.info/R.Martin/summary.htm

Mikhailov, L., & Tsvenetinove, P. (2004). Evaluation of services a fuzzy analytical hierarchy process. *Applied Soft Computing*, *5*(1), 23–33.

Mikhailov, L. (2003). Deriving priorities from fuzzy pairwise comparison judgments. *Fuzzy Sets and Systems,* (134), 365–385.

Moole, B. (2005). A probabilistic multidimensional data model and its applications in business (Doctoral dissertation, Walden University School of Applied Management and Decision Sciences, 2005. (Proquest No. AAT 3180107)

Morris, P., & Hough G. (1987). *The anatomy of major projects*. New York: John Wiley.

Nagi, E. (2003). Selection of web sites for online advertising using AHP. *Information and Management, 40*(4), 233-242.

Nutt, P. (2005). *Why decisions fail* [audio seminar]. Retrieved from www.bettermanagement.com

Nutt, P. (2002). *Why decisions fail.* NY: McGraw-Hill, Inc.

Parakash, T. N. (2005). Land stability analysis: A fuzzy multicriteria decision approach. *Geo Information Science*, *13*, 112 -129.

Peng, L., Chen, Z., & Qi, L. (2006). Model and method for evaluating credibilityof C2C trade. *International Conference of Electronic Commerce*, *3*(5), 244– 249.

Poh, K., & Ang, W. (199). Transportation fuels and policy for Singapore: an AHP planning approach. *Computer and Industrial Engineering*, *40*(3), 507-525.

Pohekar, S. D., & Ramachandran, M. (2004). Application of multicriteria decision making to sustainable energy planning - A review. *Renewable and Sustainable Energy Reviews, 8*(4), 365–389.

Raftery, J. (1994). *Risk analysis in project management*. London: E & FN Spon.Rommelfanger, H. J. (2003). A fuzzy logic approach to multicriteria decisionmaking. Retrieved from wiwi.unifrankfurt.de/prefessren/rommelfanger/index/dokumente/

Ross, T. (1995). *Fuzzy logic with engineering applications*. NY: McGraw-Hill, Inc.

Russel, S., & Norvig, P. (2003). *Artificial intelligence: A modern approach* (2nd ed.), NJ: Prentice Hall.

Saaty, T. (1980). *The analytic hierarchy process*. NY: McGraw Hill International Book ccompany.

Saaty, T. (1996). *The analytical network process*. Pittsburgh, PA: RWS Publications.

Saaty, T. (2000). *Decision making for leaders: The analytic hierarchy process for decisions in a complex world.* RWS Publications.

Sharifi, M. A. (2003). Application of GIS and multicriteria evaluation in locating sustainable boundary of the tunari national park and Cochabamba city (Bolivia). *Journal of Multicriteria Analysis*, *11,* 151—164.

Sarkis, J. (2003). Quantitative models for performance measurement systems –Alternate considerations. *International Journal of Production Economics, 86*(1), 81–103.

Saltelli, A., & Tarantola, S. (2004). *In practice: A guide to assessing scientific models*.

NYC, NY: John Wiley and Sons.

Samuel, B. (1999). *Risk aversion: Lecture notes*. University of South Carolina.

Schoening, H. (2004*). Business management of telecommunications*. Saddle River, NJ: Prentice Hall.

Singleton, R., & Straits, B. (2005). *Approaches to social research methods* (4th ed.). NY: Oxford University Press.

Stallings, W. (2003*). Data Communications and architecture*. NJ: Pearson Prentice Hall.

Stallings, W. (2006) *Computer and network organization and architecture: Designing for performance.* NJ: Pearson Prentice Hall.

Stam, A., & Duarte, S. (2003). On multiplicative priority rating methods for the AHP. *European Journal of Operational Research*, *148*(1), 92-108.

Tam, M., & Tummala, V. (2001). An application of the AHP in vendor selection of a telecommunications system. *Omega*, *29*(2), 171-182.

Tanenbaum, A. (2003). *Computer networks and data communications*. NJ: Prentice Hall.

Tang, Y., & Beynon, M. (2007). Application and development of a fuzzy analytic hierarchy process within a capital investment study. *Journal of Economic and Management*, *1*(2), 207–230.

Tolga, E., Demircan, M. L., & Kahraman, C. (2005). Operating system selection using fuzzy replacement analysis and analytic hierarchy process. *International Journal of Production Economics, 97*(1), 89-117.

Triantaphyllou, E. (2001). *Multicriteria decision making methods: A comparative study*. Boston: Kluwer Academic Publishers.

Verma, R., & Pullman, M. E. (1998). An analysis of the supplier selection process. *Omega*, *26*(6), 739-750.

Vertical Markets (2007). *IDC IT Spending forecast, FY05 – FY07, Corporate market intelligence* [Power Point slides]. Retrieved from www.verticalmarkets.com

Wang, G. (2005). Manufacturing supply chain design and evaluation. *International*

Journal of Advanced Manufacturing Technology, 25(2), 93–110.

Wang, Q. (2005). Delphi-AHP method for allocation of waste loads in a region. *Journal of Harbin Institute of Technology, 37*(1), 84-103.

Zadeh, L. A. (1975). *Fuzzy sets and their applications to cognitive and decision process*. NY: Academic Press.

Zadeh, L. A. (1965). Fuzzy sets. *Information & Control*, *8*, 338-353.

Zimmerman, H (1986). *Fuzzy sets and its applications*. Boston: Kluwer-Nijoff Publishing.

Zhu, K. J., Jing, Y., & Chang, D-Y. (1999). A discussion on extent analysis method and applications of fuzzy AHP. *European Journal of Operational Research*, *116*(2), 450-56.

CURRICULUM VITAE

Experience
Academic:

New Jersey Institute of Technology, 1999 – present: Associate Professor

Teaching courses in the electrical and computer engineering technology areas:

- applied Telecommunications
- Networking Design and Protocols
- Object Oriented Design and Programming
- Microcontroller Embedded Applications
- Circuit Systems and Signals
- Electrical Measurements
- Senior Projects

Responsible for services that includes student advisement, faculty council, and departmental services.

New Jersey Institute of Technology, 1993 – 1999, Assistant Professor

- Taught many of the classes offered in the electrical and engineering technology area. These course were in areas that included telecommunications, programming, circuit analysis, and digital design.
- Principle investigator: Distance learning approach to teaching laboratory based course.
- Principle Investigator: A view with a room: an approach to distance learning in electrical engineering and tele-manufacturing.

- Investigator: Multimedia gateway project.

Industrial:

Lucent Technology, 2002 – present.

- Consulting for Lucent Technology on projects in asynchronous transfer modes, multi layer protocols, Internet telephony, and voice over IP.
- Developed numerous system requirements for video and mobile products.
- Performed performance analysis using queuing theory constructs on products in area such as VPN, backbone networking, and SONET.

Lucent Technology, 1989 – 2002. Member of Technical Staff

- Responsibilities encompassed a wide range of activities including project management, hardware development, software development, system testing, and system integration on many of successful multi-million products.
- System engineered a video conference system intended to operate over the wide area networks. It was the first of its type when it was developed.
- Conceived and lead many projects that resulted in successful business to business products.

Harris Corporation, 1983 – 1989, Senior Engineer.

- Worked on the development and integration of many military avionic projects including the F111, F18A. These projects included design of armament computers, and intra-fighter communication systems.

Publication

- Khader, M, & Barnes, W. (2000). *Telecommunications systems and technologies*, NJ: Prentice Hall.
- Numerous peered reviewed conference and journal papers in distance learning and teaching methods.

Education

- M.S., Computer Science, Stevens Institute of Technology, 1993.
- B.S., Electrical Engineering, Polytechnic Institute of Ney York, 1983.
- B.S., Biomedical Engineering, Cairo University, 1980.