Viability Analysis of an Enterprise I

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Abstract

The study derives the conditions for the profitability of any enterprise. So long as all the products produced by an enterprise are sold or will be bought, in theory, there are no bounds on the profit an enterprise can make. In practice bounds exists, and it is shown that the region of profitability declines if some products are not sold or bought and that an optimal profit can still be attained in this case with proper combination of products produced.

Keywords

Profitability region, sustainability condition, viability condition.

1. Introduction

An enterprise would be viable if the total wealth invested in the enterprise is less than the total wealth accruing from the sale of the products of the enterprise in any given time horizon. A living standard can be sustained or improved upon if total income exceeds total expenditure in any given time horizon.

People do all sorts of business without recognizing either at the start or in the process of the venture whether the enterprise is succeeding or failing. Even if they recognize the success or failure, by how much precisely is the enterprise succeeding or failing could be difficult to determine. It would be nice for managers, or business operators to know in advance or during the process of operation whether the enterprise is failing or not and if so by how much. Bank managers sometimes find it difficult to decide whether to grant a loan to a small business applicant. A proprietor of a school would want to know at the start of business the minimal number of pupils (or students) to admit for the school's operational cost to be met. Every worker would want to know whether the current living standard can be maintained or improved upon. Likewise, some people lead all kinds of lifestyles without recognizing whether the chosen standard of life can be improved or sustained over time within the environment in which they find themselves. A private clinic would want to know the minimal number of patients to report to a clinic for the clinic's operations to thrive or even how much to charge a patient given that a certain number of patients reports at the clinic. A washing bay or company would want to know the minimal number of cars to wash a day to make profit for the day's operations. A petty trader selling at the roadside would want to know how much items to sell a day to realize some profit or even if there are leftovers, the minimal price to sell the remaining items for sustainability. A farmer specializing in the production of certain crops would want to know how much to sell the produce for subsequent survival of the farming enterprise. This study addresses these issues by deriving mathematically the conditions for profitability of any enterprise. It has to be emphasized that the analyses incorporates issues such as enterprise personnel management, enterprise environment, workers satisfaction and performance that enhances a firm's value and for that matter increase profitability.

The paper is organized as follows: section 2 discusses the number of products to produce for an already set price. It formulates the general problem for an enterprise that specializes in the production of any number of products. On the other hand, section 3 discusses the price for a given number of products produced by an enterprise specializing in the production of one or two products and formulates the general problem for the price of an enterprise that specializes in any number of products. Section 4 applies the results to living standards and section 5 concludes.

2 Number of products for a given price

Some enterprise require that a certain minimal number of products be produced for the enterprise to thrive given that the selling price has already been set based on the cost incurred or by prevailing market conditions. This is especially true for enterprises in the service industry for which the selling price is set and the enterprise has to produce or obtain a certain number of products (or people for sustainability.

2.1 An enterprise specializing in the production of one product only: all products sold

Suppose an enterprise produces a single product and sells it for s_p for each unit of item produced. Whether the enterprise will thrive or not depend on control of production cost and the ability to sell the products. For simplicity, let's assume there are two types of cost involved in the production process: variable cost (labor, materials, price of fuel, etc.) that do vary directly with the level of output and fixed cost (renting, administration, lighting) that do not vary directly with output Curwin and Slater (1996), pg 528.

Let the number of products produced by the enterprise be n_p and suppose that t_r is the total revenue accruing from the sale of the products of the enterprise. Then provided every product is sold within a given time horizon

$$t_r = n_p\, s_p$$

Let the cost of producing each unit of item be v_c. Then the total variable cost of production is $n_p\, v_c$. If t_c is the total cost of production, then t_c is:

$$t_c = n_p\, v_c + t_f$$

where t_f is the fixed cost of production. The profit, p_f, from the enterprise operations will be

$$p_f = t_r - t_c$$

$$p_f = n_p\,(s_p - v_c) \qquad (1)$$

The term $(s_p - v_c)$ is the contributory factor to profit Curwin and Slator (1996), pg 530. It has to be emphasized that equation (1) is true provided all the products produced by the enterprise are sold. For the enterprise to survive obviously $p_f > 0$ and consequently

$$n_p \in \left(\frac{t_f}{s_p - v_c}, \infty\right), s_p > v_c \qquad (2)$$

Definition 1 An enterprise is viable (or sustainable) if, and only if $p_f > 0$. The condition $p_f > 0$ would be called the viability condition.

Observe that

$$\frac{t_f}{s_p - v_c} \in \mathbb{R}^+$$

and so long as □□exceeds $\frac{t_f}{s_p - v_c}$, there are no bounds on the number of products an enterprise can produce for maximum profit. In practice, however, bounds exist because of time constraint, storage facility constraint or some other form of constraint on production. Thus there exists an □□ $\mathbb{R}^+$ such that

$$n_p \in \left(\frac{t_f}{s_p - v_c}, m\right]$$

2.2 An enterprise specializing in the production of one product only: not all products sold

Suppose that some of the products produced by the enterprise are not sold within a given time horizon. Then one of the following conditions occurs: the enterprise has already made profit, broken even or is losing. For the first two cases, the enterprise could choose to sell the remaining products at any price and still thrive. In the third case, the remaining items have to be sold at a certain minimum for the enterprise to survive.

Let n_k be the number of items that are not sold. Then

$$t_r = (n_p - n_k)s_p$$

and so

$$p_f = n_p\left(s_p - v_c\right) - n_k s_p - t_f \qquad (3)$$

Call the quantity $n_k s_p$ in equation (3), the strain on profit. Thus for viability of the enterprise, $p_f > 0$ and

$$n_k \in [0, \frac{n_p\left(s_p - v_c\right) - t_f}{s_p})$$

Hence, so long as the items that remain is less than $\frac{n_p\left(s_p - v_c\right) - t_f}{s_p}$, the enterprise can thrive. If the number of items that remain is exactly $\frac{n_p\left(s_p - v_c\right) - t_f}{s_p}$, then the enterprise has broken even. In the case where the remaining items exceeds $\frac{n_p\left(s_p - v_c\right) - t_f}{s_p}$, what minimum price can the enterprise sell the remaining items so as to thrive? This is, particularly, true for an enterprise specializing in the production of perishable goods where time is of much concern.

Let the remaining items be sold at a reduced price $s_k < s_p$(this is no restriction as for prices $s_k \geq s_p$the strain on profit declines faster and for that matter profitability increases). Then

$$t_r = \left(n_p - n_k\right)s_p + s_k n_k$$

and consequently

$$p_f = n_p\left(s_p - v_c\right) - n_k s_p + s_k n_k - t_f \qquad (4)$$

Observe, by comparing equation (3) and (4) that the strain on profit is reduced by the additional term $s_k n_k$.From the viability condition,

$$s_k \in (s_p + \frac{t_f - n_p\left(s_p - v_c\right)}{n_k}, \infty)$$

If $v_c = s_p$ then $s_k \in (s_p + \frac{t_f}{n_k}, \infty)$. Thus each unit of the remaining item has to be sold at a price greater than the cost price per unit of production for sustainability.

2.3 One product: an enterprise involving fixed assets that has to be paid over a number of years

At times the set up of an enterprise requires an initial capital investment in some fixed assets (say buildings, machinery, etc.) that has to be paid over a number of years. Suppose that the interest rate in the economy is $(r \in (0,1])$ and that the enterprise invested initially an amount T_f in some fixed asset that has to be paid over a period of years. Then in the first year, the amount that has to be paid off must be $\frac{T_f}{n}(1+r)$ accounting for the time value of money, Higgins (2004), pg 234 (i.e. the fact that the amount could have been invested in some safe asset to yield profit). The analysis can be adjusted for any given time horizon. On a monthly basis, the amount that has to be paid off must be $\frac{T_f}{12n}(1+r)$.

Let the yearly, fixed total cost be t_f. For year 1,

$$p_{f_1} = n_p (s_p - v_c) - \frac{T_f}{n}(1+r) - t_f \tag{5}$$

The expression $\frac{T_f}{n}(1+r)$ acts as the strain on first year profit. If all products are sold in the first year, the number of products to produce for the enterprise to thrive has to exceed $\frac{t_f + \frac{T_f}{n}(1+r)}{s_p - v_c}$, $s_p > v_c$. For year 2,

$$p_{f_2} = n_p\left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^2 - t_f$$

Hence, by induction for □years,

$$p_{f_n} = n_p\left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^n - t_f$$

and from the viability condition:

$$n_p \in \left(\frac{T_f(1+r)^2}{n(s_p - v_c)} + \frac{T_f}{(s_p - v_c)}, \infty\right), s_p > v_c \qquad (6)$$

At the end of □years when all T_f have been paid, observe that the (6) reduces to the (2).

Remark 2 If there is a reason to believe that the asset will depreciate (or appreciate) and affect n_p negatively (or positively) over the □years, then T_f has to be weighted. In this case, the amount that has to be paid in the i^{th} year has to be:

$$w_i T_f (1+r)^i, i = 1, \ . \ . \ . \ , n, 0 < w_i < 1, \sum_{i=1}^{n} w_i = 1$$

□□□□□ w_i □□□□□□□□□□□□□□□□□□ i^{th} year. Accountants incorporates weighting on

the income statement as depreciation, Higgins (2004), pg 10.

If n_{k_1} products are not sold in the first year, there is a strain $n_{k_1} s_p$ on the enterprise profit. Consequently

$$p_{f_1} = n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r) - n_{k_1} s_p - t_f$$

Thus for the enterprise to thrive for the first year

$$n_{k_1} \in [0, \frac{n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r) - t_f}{s_p})$$

For the second year if n_{k_2} items are not sold, then

$$p_{f_2} = n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^2 - n_{k_2} s_p - t_f$$

By induction, for □years if n_{k_n}items are not sold

$$p_{f_n} = n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^n - n_{k_n} s_p - t_f$$

For the enterprise to thrive for the n^{th} year,

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$$n_{k_n} \in [0, \frac{n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^n - t_f}{s_p})$$

2.3.1 Minimal selling price for remaining items

Suppose that some items remains and that the enterprise has not already made profit or broken even. What minimal price should the remaining items be sold for sustainability?

Let n_{k_1} be sold at a price $s_{k_1} < s_p$in the first year. Then

$$p_{f_1} = n_p \left(s_p - v_c\right) - \frac{T_f}{n}(1+r) - n_{k_1}(s_p - s_{k_1}) - t_f$$

Viability condition imply that for the first year,

$$s_{k_1} \in (s_p - \frac{n_p\left(s_p - v_c\right) - \frac{T_f}{n}(1+r) - t_f}{n_{k_1}}, \infty)$$

In the second year, if the n_{k_2} items are sold at a price $s_{k_2} < s_p$ then

$$s_{k_2} \in (s_p - \frac{n_p\left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^2 - t_f}{n_{k_2}}, \infty)$$

By induction for the n^{th} year, if n_{k_n} items are sold at a price $s_{k_n} < s_p$ then

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$$s_{k_n} \in (s_p - \frac{n_p\left(s_p - v_c\right) - \frac{T_f}{n}(1+r)^n - t_f}{n_{k_n}}, \infty)$$

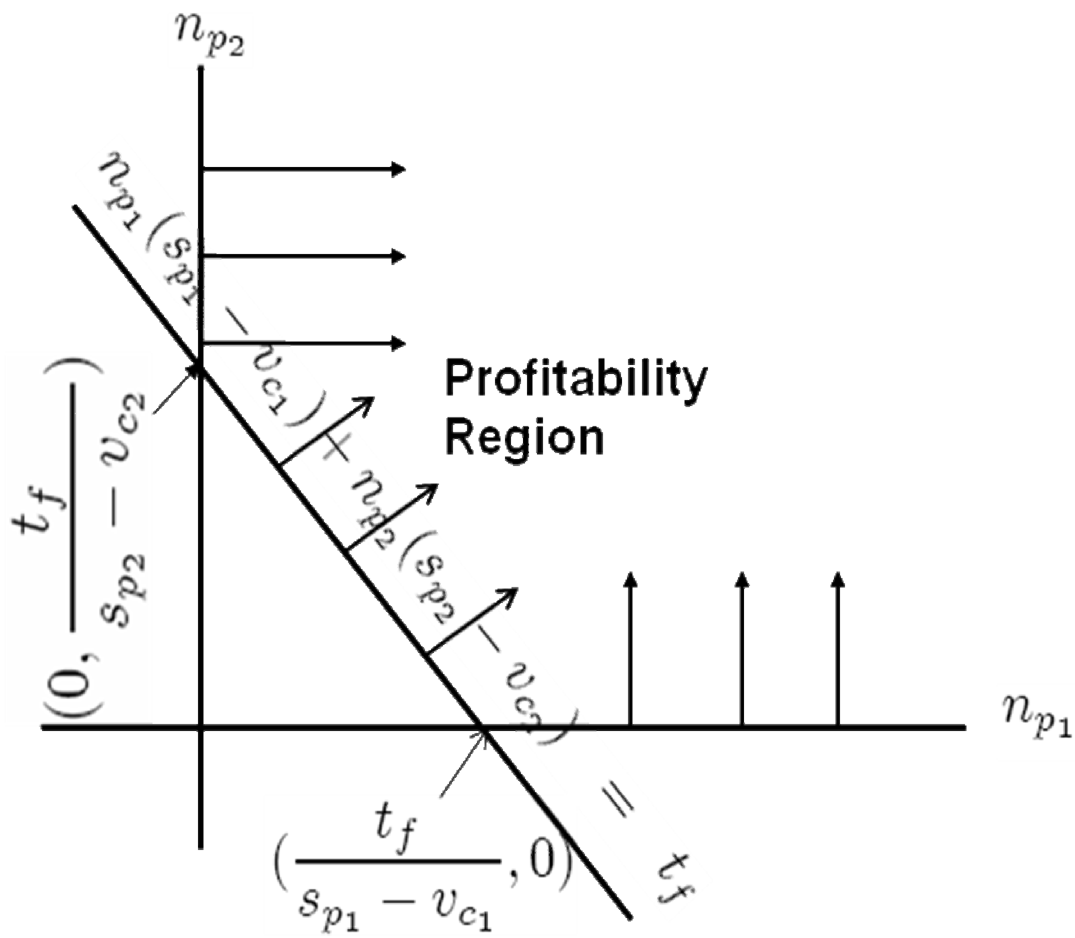

Figure 1: A graph showing the combinations of two products n_{p_1} and n_{p_2} to produce for sustainability of an enterprise

2.4 An enterprise specializing in the production of two products only: all products sold

Pretend the enterprise specializes in the production of two products only. Using the same notation as the case for one product but with subscripts 1 and 2 indicating product one and product two respectively, it can be deduced as in one product case that

$$p_f = n_{p_1}\left(s_{p_1} - v_{c_1}\right) + n_{p_2}\left(s_{p_2} - v_{c_2}\right) - t_f \tag{7}$$

provided all products produced are sold. The viability condition, $p_f > 0$ imply that

$$n_{p_1}\left(s_{p_1} - v_{c_1}\right) + n_{p_2}\left(s_{p_2} - v_{c_2}\right) > t_f$$

The goal is to determine the number of products n_{p_1} and n_{p_2} to produce for maximum profit. This can stated mathematically as

$$\max(p_f)$$

$$\textit{subject to} \quad n_{p_1}\left(s_{p_1} - v_{c_1}\right) + n_{p_2}\left(s_{p_2} - v_{c_2}\right) > t_f$$

$$n_{p_1} \geq 0, n_{p_2} \geq 0$$

The proper combination of products to products for the enterprise to thrive is shown in figure 1. It can be seen from the graph that the there are no limits to the profit an enterprise can make so long as the combination n_{p_1} and n_{p_2} of the product lies in the profitability region. In reality bounds exists and there exists $m_1, m_2 \in \mathbb{R}^+$ such that

$$\frac{t_f}{s_{p_1} - v_{c_1}} \leq n_{p_1} \leq m_1$$

$$\frac{t_f}{s_{p_2} - v_{c_2}} \leq n_{p_2} \leq m_2$$

Consequently, in real life the objective is to:

$$\max(p_f)$$

$$\textit{subject to} \quad n_{p_1}\left(s_{p_1} - v_{c_1}\right) + n_{p_2}\left(s_{p_2} - v_{c_2}\right) > t_f$$

$$\frac{t_f}{s_{p_1} - v_{c_1}} \leq n_{p_1} \leq m_1$$

$$\frac{t_f}{s_{p_2} - v_{c_2}} \leq n_{p_2} \leq m_2$$

$$n_{p_1} \geq 0, n_{p_2} \geq 0$$

The region of profitability is depicted in figure 2.

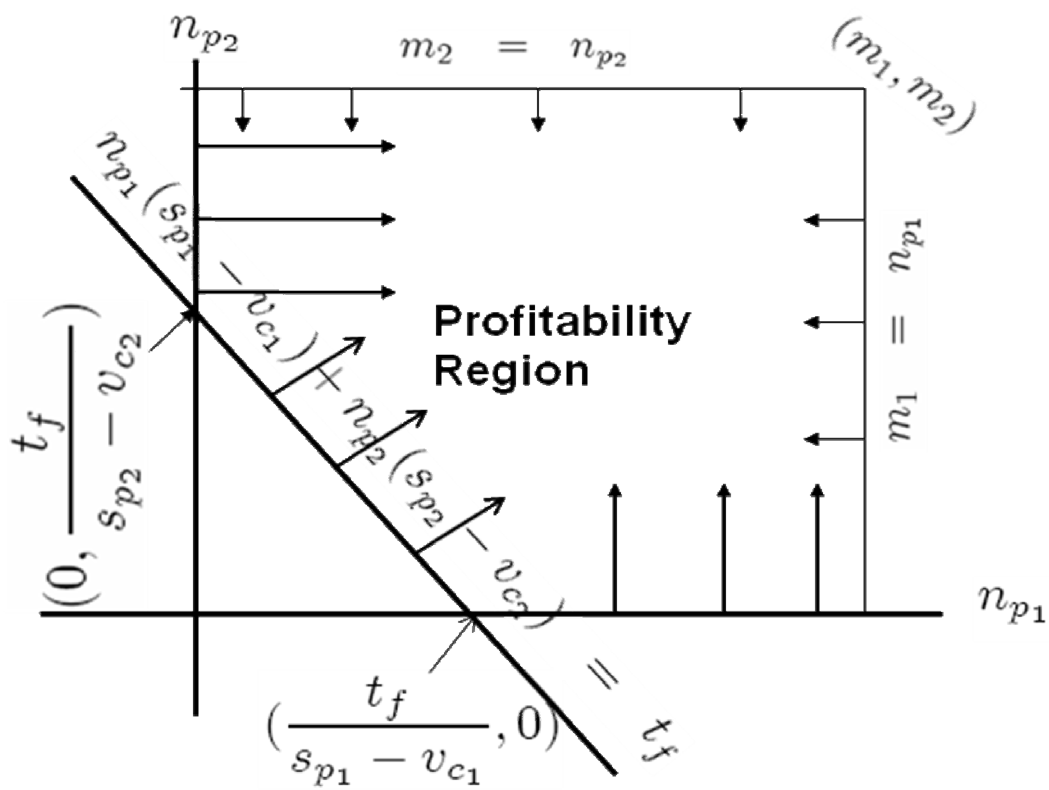

Figure 2: A graph showing the combinations of two products n_{p_1} and n_{p_2} to produce with profitability region bounded.

The level curves of the objective function are:

$$n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - t_f = c, c \in \mathbb{R}$$

Consequently, the maximum profit can be obtained at the unique point (m_1, m_2) on the graph.

Remark 3 The optimal profit is rich with information about the state of an enterprise. An enterprise composed of highly inefficient systems and poor working environment would be far from the optimal profit.

Definition 4 An enterprise that achieves the optimal profit is said to be a 'perfect enterprise'. That is the operational systems (workforce and machinery) are efficient and the working environment is serene.

How do an enterprise determine (m_1, m_2) in practice? One way is for managers to engage in different units of the production process intermittently and observe the number of products produced within the working hours. In the absence of the manager, any shortfall from the optimal can be detected and problem areas traced and rectified.

2.5 An enterprise specializing in the production of two products only: not all products sold

Suppose that n_{k_1} of n_{p_1} and n_{k_2} of n_{p_2} are not sold within a given time horizon. Then

$$p_f = n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - (n_{k_1}s_{p_1} + n_{k_2}s_{p_2}) - t_f$$

The goal is to maximize p_f by minimizing $(n_{k_1}s_{p_1} + n_{k_2}s_{p_2})$. That is if some items n_{k_1} and n_{k_2} remain, how can an enterprise determine whether it is thriving. The problem may be stated as

$$\min(n_{k_1}s_{p_1} + n_{k_2}s_{p_2})$$

$$subject\ to\ n_{k_1}s_{p_1} + n_{k_2}s_{p_2} < \tilde{k}$$

$$n_{k_1} \geq 0, n_{k_2} \geq 0$$

where $\tilde{k} = n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - t_f$.

Hence if (n_{k_1}, n_{k_2}) lies in or on the boundary of the profitability region, the enterprise can sell the remaining items at any price. Observe that the level curves of the objective function is

$$(n_{k_1}s_{p_1} + n_{k_2}s_{p_2}) = c', c' \in \mathbb{R}$$

(see figure 3). Clearly, the minimum of $(n_{k_1}s_{p_1} + n_{k_2}s_{p_2})$ occurs at the origin when all products are sold. For any combination of n_{k_1} and n_{k_2} that lie on the straight line joining $(\frac{\tilde{k}}{s_{p_1}}, 0)$ and $(0, \frac{\tilde{k}}{s_{p_2}})$, the enterprise breaks even. For

combinations of n_{k_1} and n_{k_2} that lie outside the profitability region, the remaining items n_{k_1} and n_{k_2} has to be sold at a certain minimal price for the enterprise to thrive.

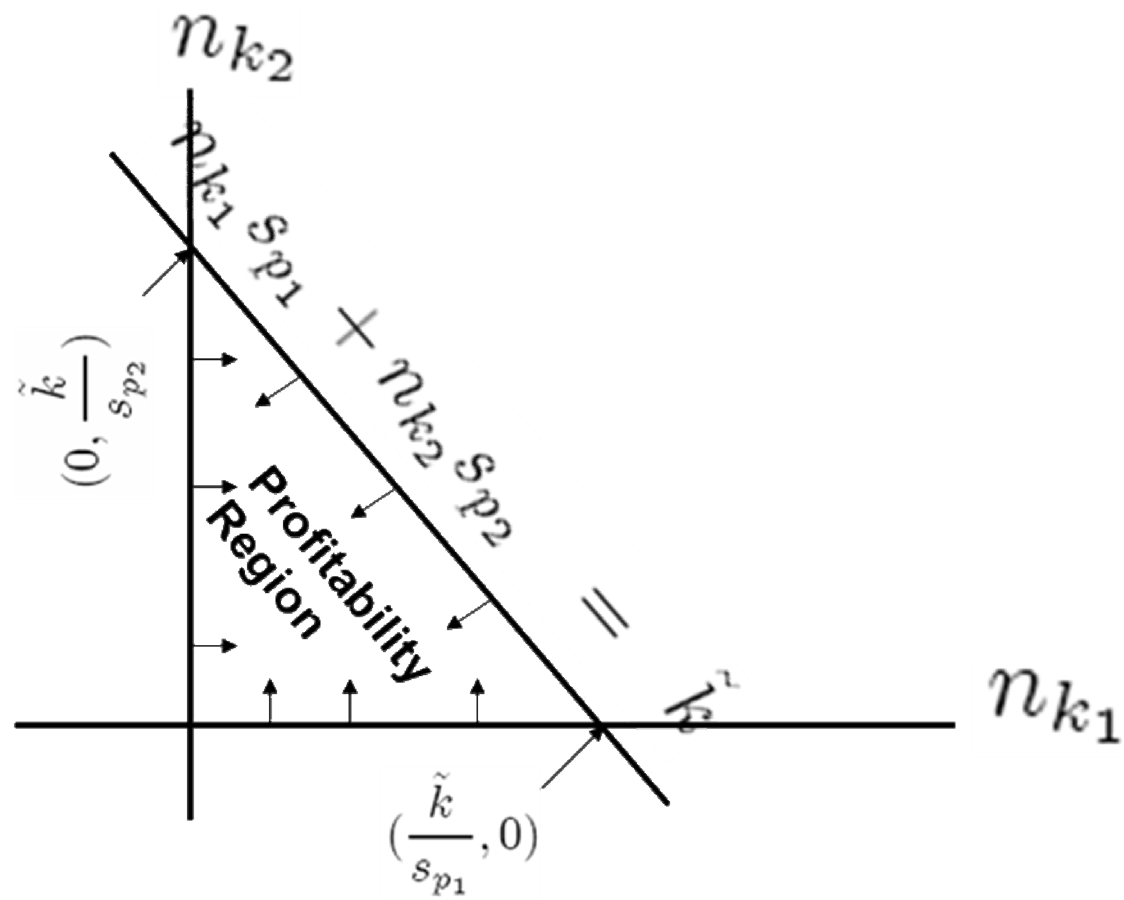

Figure 3: Profitability region for an enterprise producing two products n_{p_1} and n_{p_2} with number of products n_{k_1} and n_{k_2} not sold

In practice, minimizing $(n_{k_1} s_{p_1} + n_{k_2} s_{p_2})$ involves having a look at the marketing department or the marketing strategy of the enterprise. The manager of the enterprise may have a word or motivate the marketing division a bit more or promote the enterprise's products. The manager may also have to check whether the division has the needed resources to carry out its functions. Are there some redundancies in the productions process that can be removed to reduce cost? Could the selling price be increased? These are issues the manager of an enterprise is worth considering.

2.5.1 Minimal selling price of n_{k_1} and n_{k_2}

Suppose that the n_{k_1} and n_{k_2} items are sold respectively at s_{k_1} and s_{k_2}. Then

$$p_f = n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - (n_{k_1}s_{p_1} + n_{k_2}s_{p_2}) - t_f + n_{k_1}s_{k_1} + n_{k_2}s_{k_2}$$

The goal is to determine the minimum price to sell the remaining items n_{k_1} and n_{k_2} for □□□ (p_f) so the enterprise can thrive. Mathematically, this may be stated as

max(p_f)

$$\mathit{subject\ to}\ n_{k_1}s_{k_1} + n_{k_2}s_{k_2} > \bar{k}$$

$$s_{k_1} \geq 0, s_{k_2} \geq 0$$

where $\bar{k} = -[n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2})] + (n_{k_1}s_{p_1} + n_{k_2}s_{p_2}) + t_f$

Alternatively, the problem may be stated as:

max$(n_{k_1}s_{k_1} + n_{k_2}s_{k_2})$

$$\mathit{subject\ to}\ n_{k_1}s_{k_1} + n_{k_2}s_{k_2} > \bar{k}$$

$$s_{k_1} \geq 0, s_{k_2} \geq 0$$

Claim 5 $\bar{k}$□0

Proof. Observe that

$$\bar{k} = -n_{p_1}(s_{p_1} - v_{c_1}) - n_{p_2}(s_{p_2} - v_{c_2}) + (n_{k_1}s_{p_1} + n_{k_2}s_{p_2}) + t_f$$

$$= s_{p_1}\underbrace{(-n_{p_1} + n_{k_1})}_{<0} + s_{p_2}\underbrace{(-n_{p_2} + n_{k_2})}_{<0} + n_{p_1}v_{c_1} + n_{p_2}v_{c_2} + t_f$$

As s_{p_1} and s_{p_2} are positive, $\bar{k} > 0$ if it can be shown that

$$s_{p_1}(-n_{p_1} + n_{k_1}) < n_{p_1}v_{c_1}$$

$$s_{p_2}(-n_{p_2} + n_{k_2}) < n_{p_2}v_{c_2}$$

(i) is true as $s_{p_1}\underbrace{(-1 + \frac{n_{k_1}}{n_{p_1}})}_{<0} < v_{c_1}$. Similarly, (ii) is true as $s_{p_2}\underbrace{(-1 + \frac{n_{k_2}}{n_{p_2}})}_{<0} < v_{c_2}$.

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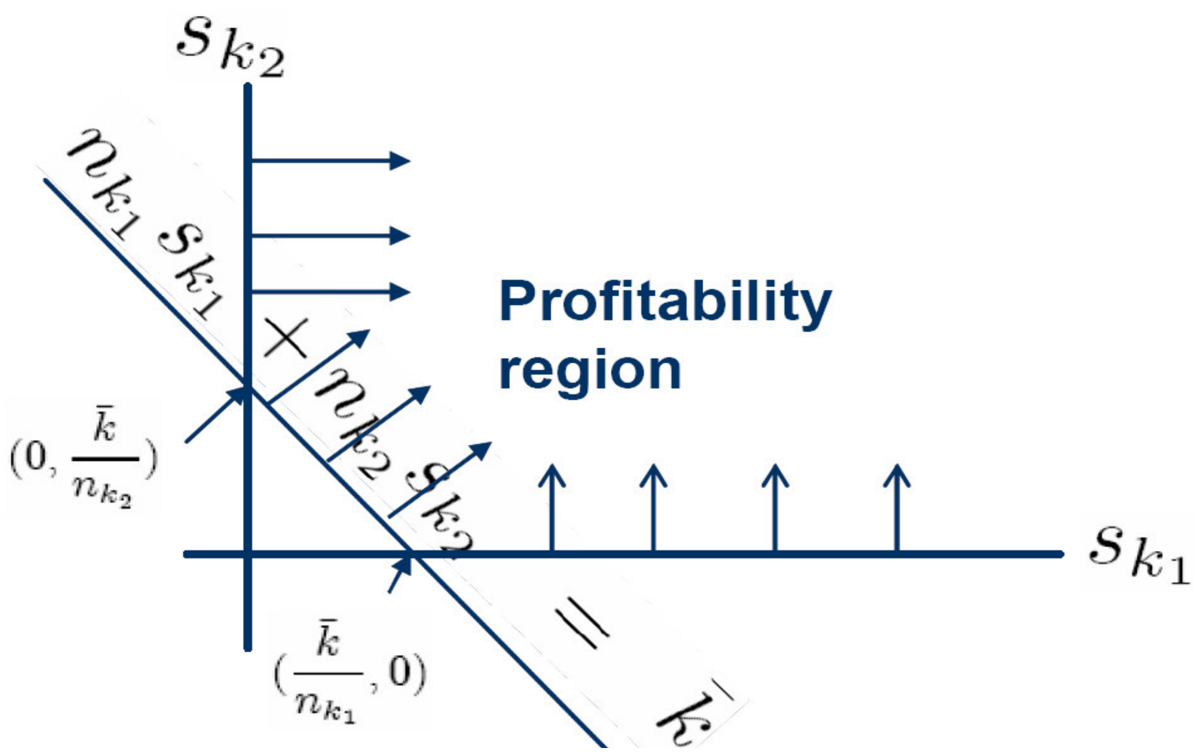

Figure 4: Selling price s_{k_1} and s_{k_2} for the remaining items n_{k_1} and n_{k_2} for the two products n_{p_1} and n_{p_2}

It is clear from the figure 4 that any combination of selling price s_{k_1} and s_{k_2} that lie in the profitability region should lead to sustainability. In reality, there exists λ_1, $\lambda_2 \in \mathbb{R}^+$ such that

$$\frac{\bar{k}}{n_{k_1}} < s_{k_1} < \lambda_1$$

$$\frac{\bar{k}}{n_{k_2}} < s_{k_2} < \lambda_2$$

Incorporating these constraints on the graph yields figure 5. It clear from the figure 5 that an optimal selling price for the remaining item occurs at the unique point (λ_1, λ_2).

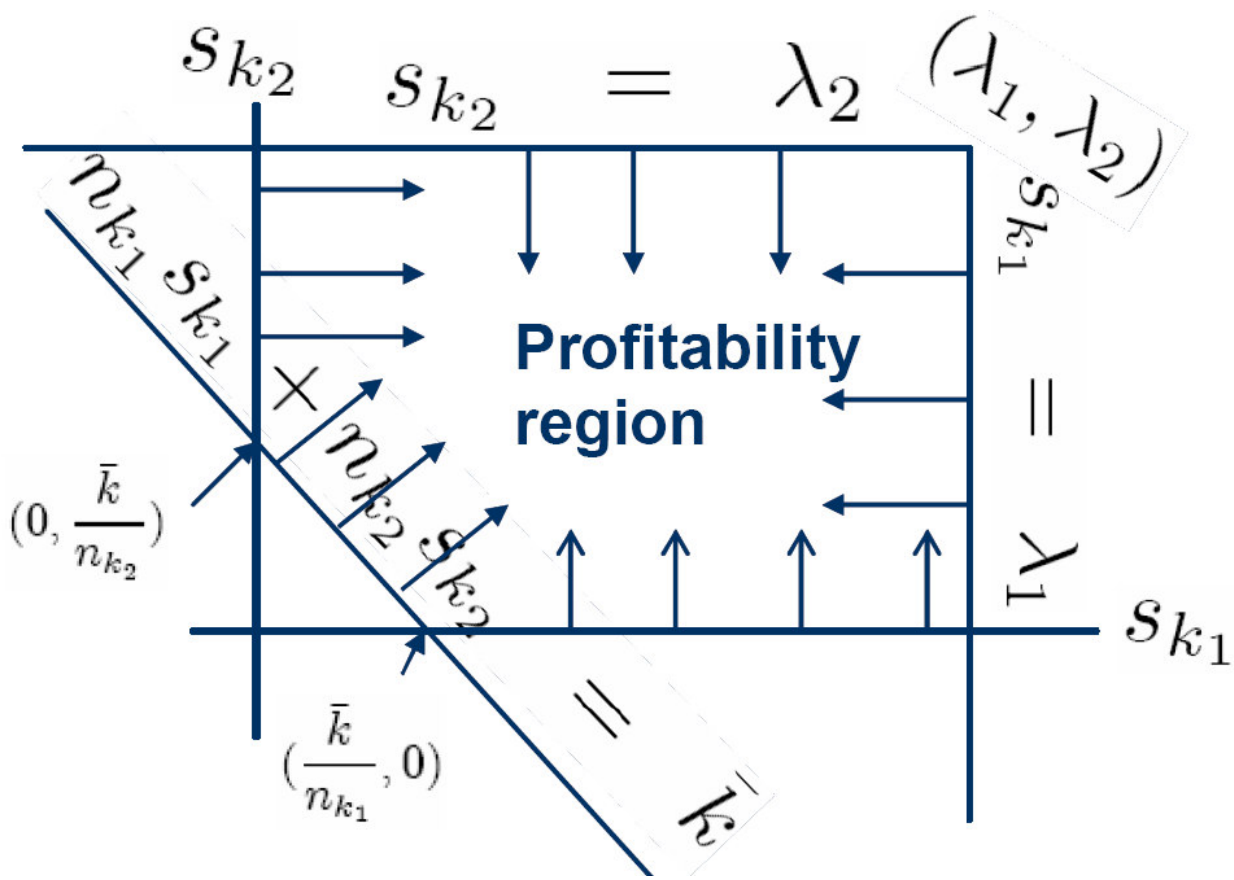

Figure 5: Selling price s_{k_1} and s_{k_2} for the remaining items n_{k_1} and n_{k_2} with bounded profitability region

2.6 An enterprise specializing in any number of products: all products sold

Suppose an enterprise specializes in the production of products where $m \in \mathbb{N}$. Then

$$p_f = \sum_{r=1}^{m} n_{p_r}(s_{p_r} - v_{c_r}) - t_f$$

The goal is to determine $n_{p_r}, r = 1, \ldots, m$ for maximum p_f. Mathematically, this may be stated as

$\max(p_f)$

$$\mathit{subject\ to} \sum_{r=1}^{m} n_{p_r}(s_{p_r} - v_{c_r}) > t_f$$

$$n_{p_1} \geq 0, n_{p_2} \geq 0, \ldots, n_{p_m} \geq 0$$

In reality, there are constants $\lambda_i \in \mathbb{R}^+, \mathrm{i} = 1, \ldots, \mathrm{m}$ such that

$$\frac{t_f}{s_{p_1} - v_{c_1}} \leq n_{p_1} \leq \lambda_1$$

$$\frac{t_f}{s_{p_2} - v_{c_2}} \leq n_{p_2} \leq \lambda_2$$

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$$\frac{t_f}{s_{p_m} - v_{c_m}} \leq n_{p_m} \leq \lambda_m$$

Consequently, the problem is:

max(p_f)

$$subject\ to \sum_{r=1}^{m} n_{p_r} (s_{p_r} - v_{c_r}) > t_f$$

$$\frac{t_f}{s_{p_1} - v_{c_1}} \leq n_{p_1} \leq \lambda_1$$

$$\frac{t_f}{s_{p_2} - v_{c_2}} \leq n_{p_2} \leq \lambda_2$$

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$$\frac{t_f}{s_{p_m} - v_{c_m}} \leq n_{p_m} \leq \lambda_m$$

$$n_{p_1} \geq 0, n_{p_2} \geq 0, \ . \ . \ . \ , n_{p_m} \geq 0.$$

The solution to the general case would be discussed in the next research.

2.7 Any number of products: not all products sold

Suppose that n_{k_r} of $n_{p_r}, r = 1, \ . \ . \ . \ , m$ of products are not sold. Then

$$p_f = \sum_{r=1}^{m} n_{p_r} (s_{p_r} - v_{c_r}) - \sum_{r=1}^{m} n_{k_r} s_{p_r} - t_f$$

If some items $n_{k_r}, r = 1, \ . \ . \ . \ , m$ are not sold how does an enterprise recognize it is thriving? The problem may be stated as

$$\min\left(\sum_{r=1}^{m} n_{k_r} s_{p_r}\right)$$

$$\textit{subject to} \sum_{r=1}^{m} n_{k_r} s_{p_r} > t_f + \sum_{r=1}^{m} n_{p_r} (s_{p_r} - v_{c_r})$$

$$s_{k_1} \geq 0, s_{k_2} \geq 0, \ . \ . \ . \ , s_{k_m} \geq 0$$

If $n_{k_r}, r = 1, \ . \ . \ . \ , m$ items remain, then the enterprise has already made profit, broken even or is losing. In the latter case, what minimal price should the $n_{k_r}, r = 1, \ . \ . \ . \ , m$ items be sold in this case.

2.7.1 Minimal selling price for $n_{k_r}, r = 1, \ . \ . \ . \ , m$

Let n_{k_r} be sold at a price $s_{k_r} < s_{p_r}$ for $r = 1, \ . \ . \ . \ , m$. Then

$$p_f = \sum_{r=1}^{m} n_{p_r} (s_{p_r} - v_{c_r}) - \sum_{r=1}^{m} n_{k_r} s_{p_r} + \sum_{r=1}^{m} n_{k_r} s_{k_r} - t_f \tag{8}$$

The goal is to determine the minimal selling price for max(□□) for sustainability of the enterprise. Mathematically, the problem may be stated

$$\max(p_f)$$

$$\textit{subject to} \sum_{r=1}^{m} n_{k_r} s_{k_r} > t_f - \sum_{r=1}^{m} n_{p_r} (s_{p_r} - v_{c_r}) + \sum_{r=1}^{m} n_{k_r} s_{p_r}$$

$$s_{k_1} \geq 0, s_{k_2} \geq 0, \ . \ . \ . \ , s_{k_m} \geq 0$$

Remark 6 The case where an enterprise invested an initial amount in some fixed asset, can be derived analogously by the addition of the term $-w_i T_f (1+r)^i \ , i = 1, \ \ . \ . \ . \ \ , n, \ \ 0 < w_i \leq 1, \sum_{i=1}^{n} w_i = 1$ to equation (8), if an initial investment T_f is made, the interest rate in the economy is □and □is the number of years the asset is to be paid off.

3 Price for a given number of products produced

With some enterprises, a given number of products are known and one wishes to determine the selling price for sustainability. This is, particularly, true for primary products such as farm produce, mining products, enterprises engaged in the collection of items like snails, tortoise in certain times of the year, etc where an initial cost has been incurred and the enterprise has to price products appropriately for sustainability.

3.1 An enterprise specializing in the production of a single product: all products will be sold

Suppose that a given number n_p of the enterprise products are known and that there is ready market for the products. What minimal price should an enterprise charge to ensure profitability? From the viability condition, $p_f > 0$, it follows from equation (1) that

$$s_p \in \left(v_c + \frac{t_f}{n_p}, \infty \right) \quad (9)$$

Thus, so long as the products are sold at a price greater than $v_c + \frac{t_f}{n_p}$, the enterprise can thrive. The inequality (9) indicates that there is no upper bound on the selling price for a given number n_p of products produced. In practice, however, bounds exists by virtue of competition in the market or even government regulation on the selling price of products produced. Consequently, there

is a $n \in \mathbb{R}^+$ such that $s_p \in \left(v_c + \frac{t_f}{n_p}, n\right]$. Observe that (9) is valid if, and only if all products produced would be sold.

3.2 An enterprise specializing in the production of a single product: not all products will be sold

Typically, not all products produced would be sold within a given time horizon. How then should an enterprise price its products with the possibility that some may remain and at the same time thrive. Recall from equation (3) that

$$p_f = n_p\left(s_p - v_c\right) - n_k s_p - t_f$$

where n_k is the number of products that are not sold. Therefore, for viability

$$s_p > \frac{n_p v_c - t_f}{n_p - n_k}$$

It is difficult to determine n_k as n_k is not known in advance and so one can perform sensitivity analysis on n_k knowing v_c, and t_f.

Example 7

Suppose $n_p = 50, v_c = 2000, t_f = 10000$□□□□□□□□□□□□□□□□□□□□□□□□□□□□
□□□□□□□□□□□□□□□□□□□□. Then

$$s_p > \frac{50(200) + 10000}{50 - n_k}$$

Figure 6 shows a plot of the selling price s_p versus the items that remain n_k. If all products will be bought then $s_p > 2200$ for the enterprise to thrive. If for example half the products are not bought then s_p has to be greater than 4400 for the enterprise to thrive. Observe from the figure 6 that for $n_k \geq 45,\ s_p > 22000$ (i.e. ten times as high as when all products were to be sold) for the enterprise to thrive.

One way to set prices is to examine or investigate the selling price of the product with competitors assuming the production cost is same for all competitors. If the competitors are selling at a price 2200 (or slightly greater than 2200) as in the example, then competitors have not accounted for the fact that some products can remain within a given time horizon. Such a competitor cannot thrive with time. On the other hand, if competitors are selling at a price 4400 per unit as in the example, then competitors have accounted for the fact that some products could remain within a given time horizon. Here, the enterprise could set price lower than 4400 per unit. On the other hand, if the enterprise's product is the only one on the market, prices could be set such that quarter of the products will not be bought within a given time horizon.

Another way is to use historical data of items that remained in previous years (or any reasonable time horizon) to assist in the price setting. Alternatively, one can take insurance of items that might remain within a given time. This increases the selling price of the product as premium has to be paid to the insurer with a resulting moral hazard for the insured.

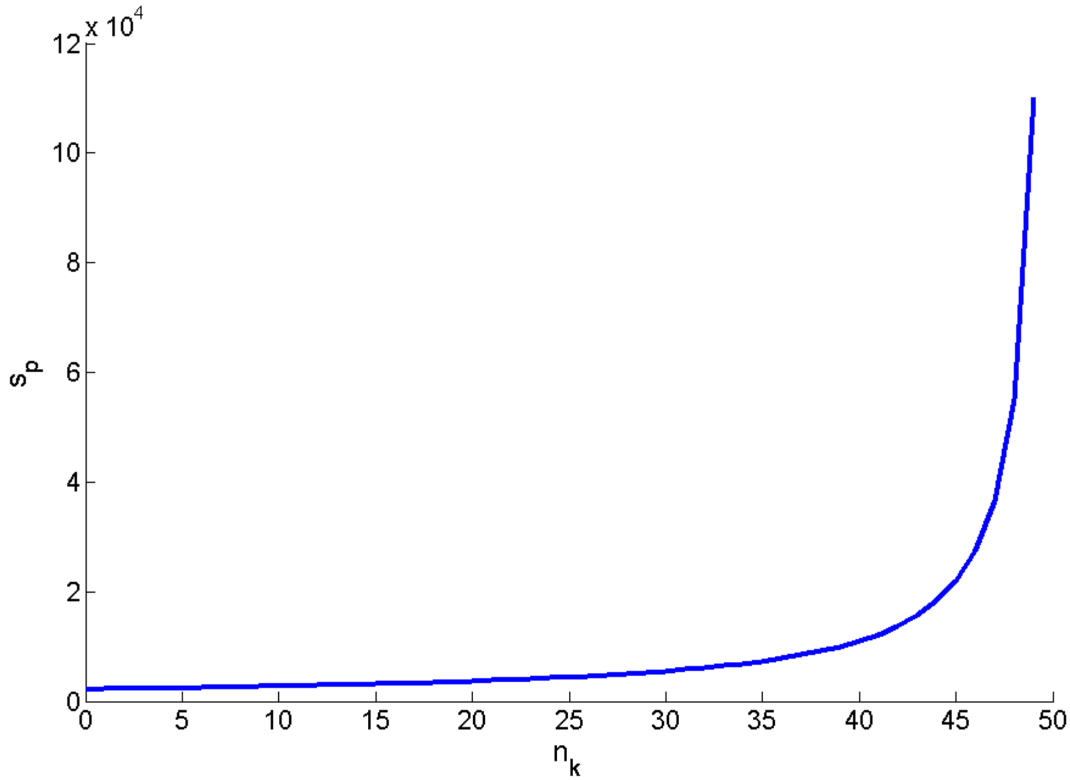

Figure 6: Graph of s_p versus n_k

3.3 One product: an enterprise involving fixed assets that has to be paid over a number of years

Here, for the first year if all products are bought, from equation (5)

$$p_{f_1} = n_p\left(s_p - v_c\right) - w_1 T_f (1 + r) - t_f$$

Viability condition implies $s_p \in (v_c + \frac{1}{n_p}[w_1 T_f(1+r) + t_f], \infty)$ for the first year. In the second year, if all products would be sold $s_p \in (v_c + \frac{1}{n_p}[w_2 T_f(1+r)^2 + t_f], \infty)$. Hence by induction, for the n^{th} year if all products would be sold then $s_p \in (v_c + \frac{1}{n_p}[w_n T_f(1+r)^n + t_f], \infty)$, w_i is the weight for the i^{th} year. See remark 2.

Remark 8 Where some items remain, analysis of section 3.2 applies.

3.4 An enterprise specializing in the production of two products: all products would be sold

Suppose an enterprise produces two products n_{p_1} and n_{p_2} and that customers are readily available to purchase all the products. From equation (7),

$$p_f = n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - t_f$$

and so for sustainability of an enterprise

$$s_{p_1} n_{p_1} + s_{p_2} n_{p_2} > t_f + n_{p_1} v_{c_1} + n_{p_2} v_{c_2}$$

The goal is determine the price to sell the products n_{p_1} and n_{p_2} for maximum profit. Mathematically, this may be stated as

$$\max(p_f)$$

$$subject\ to\ s_{p_1} n_{p_1} + s_{p_2} n_{p_2} > t_f + n_{p_1} v_{c_1} + n_{p_2} v_{c_2}$$

$$s_{p_1} \geq 0,\ \ s_{p_2} \geq 0.$$

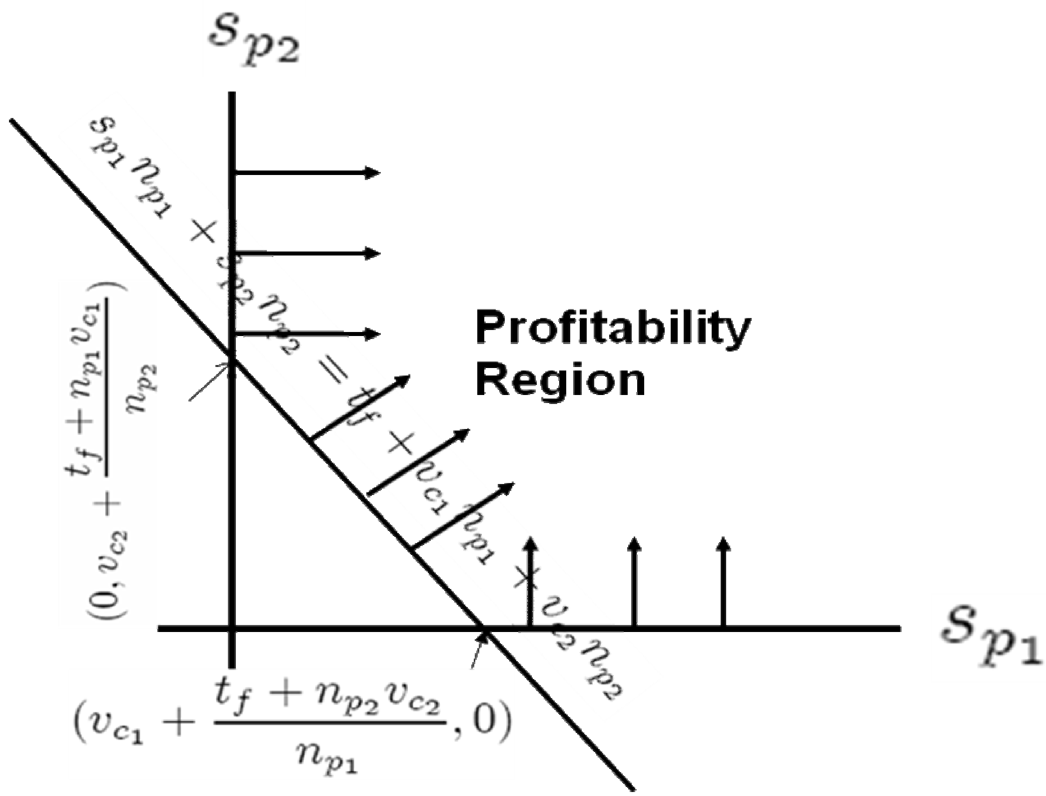

Figure 7: A graph showing the how much to sell the two products n_{p_1} and n_{p_2} for sustainability of an enterprise

It clear from figure 7 that in theory there are no upper bounds on the selling price for the two products produced by an enterprise. In practice, however, there exists n_1, $n_2 \in \mathbb{R}^+$ such that

$$v_{c_1} + \frac{t_f + n_{p_2} v_{c_2}}{n_{p_1}} < s_{p_1} \leq n_1$$

$$v_{c_2} + \frac{t_f + n_{p_1} v_{c_1}}{n_{p_2}} < s_{p_2} \leq n_2$$

Consequently, the objective is to:

$\max(p_f)$

$$subject\ to\ s_{p_1}n_{p_1} + s_{p_2}n_{p_2} > t_f + n_{p_1}v_{c_1} + n_{p_2}v_{c_2}$$

$$v_{c_1} + \frac{t_f + n_{p_2}v_{c_2}}{n_{p_1}} < s_{p_1} \leq n_1$$

$$v_{c_2} + \frac{t_f + n_{p_1}v_{c_1}}{n_{p_1}} < s_{p_2} \leq n_2$$

$$s_{p_1} \geq 0, \ \ s_{p_2} \geq 0.$$

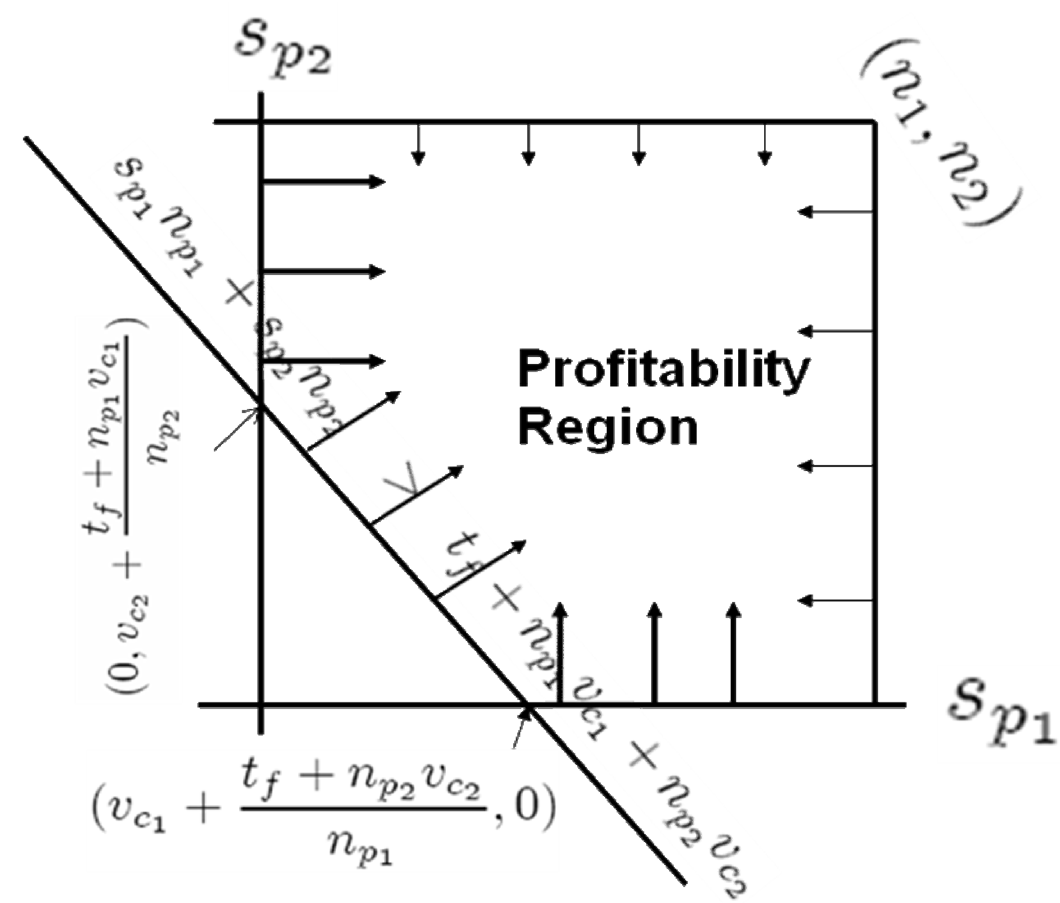

Figure 8: A graph showing the price to sell the two products n_{p_1} and n_{p_2} for sustainability of an enterprise with region of profitability bounded

The level curves of the objective function are:

$$s_{p_1} n_{p_1} + s_{p_2} n_{p_2} = t_f + n_{p_1} v_{c_1} + n_{p_2} v_{c_2} + d$$

where $d \in \mathbb{R}$. It is clear from figure 8 that an optimal selling price occurs at the unique point (n_{p_1}, n_{p_2}) for the two products.

3.5 An enterprise specializing in the production of two products: not all products would be sold

If some items n_{k_1} of n_{p_1} □□□ n_{k_2} of n_{p_2} remain, then there is a strain, then there is a strain $n_{k_1} s_{p_1} + n_{k_2} s_{p_2}$ on profit p_f. That is

$$p_f = n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - (n_{k_1} s_{p_1} + n_{k_2} s_{p_2}) - t_f$$

For the enterprise to know that it has broken even, has already made a profit and for that matter can sell the remaining items at any price

$$(n_{k_1} s_{p_1} + n_{k_2} s_{p_2}) < n_{p_1}(s_{p_1} - v_{c_1}) + n_{p_2}(s_{p_2} - v_{c_2}) - t_f$$

In the case where the enterprise is losing, the minimal selling price for sustainability is as given in section 2.5.1.

3.6 An enterprise specializing in any number of products: all products will be sold

Assume that the enterprise specializes in the production of □products where □□ N. Here the goal is to determine $s_{p_r}, r = 1, \ldots, m$ for maximum profit. This may be stated mathematically (Brandimarte(2001), pg 124) as

-min(p_f)

$$subject\ to \sum_{r=1}^{m} n_{p_r}(s_{p_r} - v_{c_r}) > t_f$$

$$s_{p_1} \geq 0, s_{p_2} \geq 0, \ . \ . \ . \ , s_{p_m} \geq 0$$

The general solution would be discussed in the next research.

4 Application to living standards

Assume for simplicity that the living expenses of an individual involve two types of cost: variable cost (utility bills, amount spend on food, etc.) that do vary directly with a person needs and fixed cost, t_f,(renting, nondurable consumer goods, etc.) which are fixed within a given time horizon. If t_c is the total fixed cost incurred by an individual in a given time horizon, then letting v_c be the variable cost yields

$$t_c = v_c + t_f$$

By letting t_r be the total revenue accrued over a given time horizon, for an individual living standard to be sustained, the quantity

$$s_f = t_r - (v_c + t_f) \qquad (10)$$

has to be positive. Call s_f the survival function.

Definition 9 A living standard is sustainable if $s_f > 0$. A living standard is not sustaining if $s_f \leq 0$. The inequality $s_f > 0$ would be called the sustainability condition. If $s_f = 0$ a person or an individual is said to be just surviving.

Observe that as t_f is fixed an individual can only sustain or improve upon a current living standard provided v_c is minimized. Now suppose to improve upon the living standard, an individual acquires a durable consumer good (such as automobile, television set, fridge, etc.) at a price T_f that has to be paid over a period of n years. If r is the interest rate in the economy, then for the first year the amount that has to be paid off must be $\frac{T_f}{n}(1+r)$ accounting for the time value of money. Therefore, an individual living standard is sustainable in the first year provided

$$s_f = t_r - v_c - t_f - \frac{T_f}{n}(1+r)$$

is positive. The quantity $\frac{T_f}{n}(1+r)$ acts as a strain on an individual living standard for the first year. For two years,

$$s_f = t_r - v_c - t_f - \frac{T_f}{n}(1+r)^2$$

By induction for n years,

$$s_f = t_r - v_c - t_f - \frac{T_f}{n}(1+r)^n \tag{11}$$

Observe that at the end of n years when all T_f has been paid, equation (11) reduces to equation (10).

4.1 Acquisition of an asset for an individual with a positive s_f

Suppose an individual with a positive s_f is interested in acquiring an asset at a price T_f in n years time. If r is the interest rate in the economy, the total wealth accrued at the end of the period is

$$\frac{s_f}{r}[(1+r)^n - 1]$$

Hence $T_f = \frac{s_f}{r}[(1+r)^n - 1]$ from which

$$n = \frac{\log\left(\frac{rT_f}{s_f} + 1\right)}{\log(1+r)} \qquad (12)$$

Equation (12) gives the number of years an asset could be acquired provided s_f is invested yearly in a safe asset at an interest rate r.

Example 10 Suppose that $T_f = 10000, s_f = 3600$, then

$$n = \frac{\log\left(\frac{250r}{9} + 1\right)}{\log(1+r)}$$

As r typically varies, a plot of n against r is given in figure 9 below:

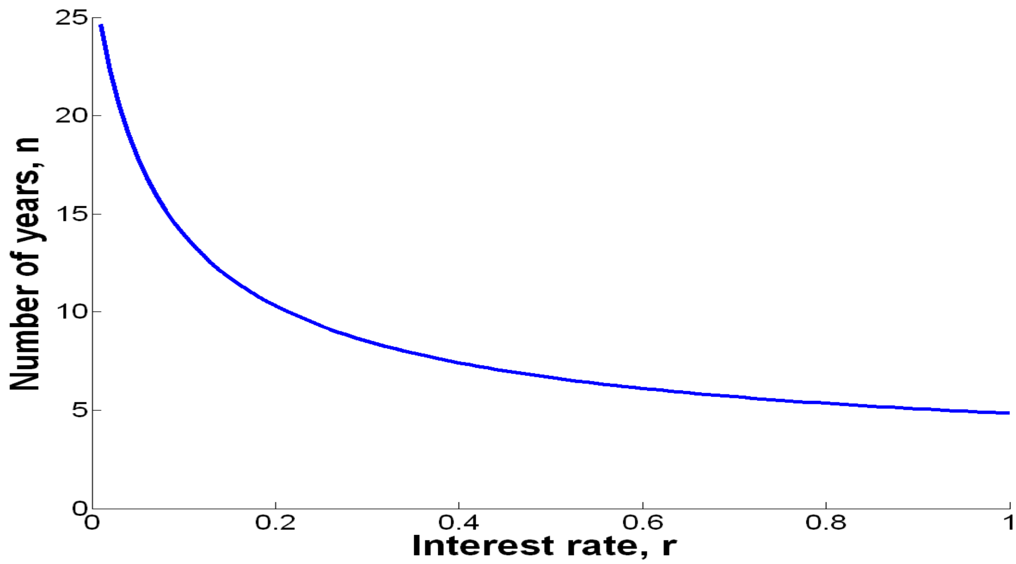

Figure 9: Number of years taken versus interest rate for an individual with $s_f = 3600$ to acquire an asset at a cost $T_f = 10000$

It is clear from the figure 9 that there is an inverse relation between the years to acquire T_f and the interest rate, □. As □increases, it takes a shorter time to acquire T_f.

The best time an individual in this example can acquire $T_f = 10000$ is approximately 4.9 years when the interest rate is 100%.The worse time is about 27.8 years when the interest rate is 0.000001%. When the interest rate is 12%this individual can acquire the asset in about 13 years.

Observe that an individual whose chooses to keep her positive s_f under the "bed" takes a longer period to acquire T_f.The time taken for such an individual is

$$n = \frac{T_f}{s_f}$$

It follows that

$$\frac{\log\left(\frac{rT_f}{s_f}+1\right)}{\log(1+r)} \leq \frac{T_f}{s_f}$$

As it is to be expected, it takes less time to acquire the asset if the s_f is invested in a safe asset.

5. Conclusions

The study has addressed the number of products an enterprise can produce for a given price for sustainability of an enterprise that specializes in the production of one or two products. It also considered the case where some products are not sold and how to set prices for sustainability. The general problem of an enterprise that specializes in any number of products was formulated. It also addressed the selling price for a given number of products for an enterprise that specializes in the production of one or two products. In addition, it discussed how to set prices with the possibility that some items will not be sold within a given time horizon. The general problem of how to price products of an enterprise that specializes in any number of products was formulated. Finally, application of the results to an individual standard of living was discussed. In the next research, the general solution of how to price and the number of products to produce would be discussed as well as the possibility that some items may not be sold within a given time horizon.

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