

Exploring Fuzzy Set Concept in Priority Theory for Maintenance Strategy Selection Problem

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Abstract

This paper proposes and presents a different approach of choosing an appropriate maintenance strategy using Saaty's priority theory and fuzzy sets. As per the priority theory, weights are assigned to the decision criteria via pair wise comparison of criteria. Basic three types of maintenance strategies specifically corrective maintenance, preventive maintenance and predictive maintenance and eight maintenance decision criteria namely low maintenance cost, improved reliability, improved safety, high product quality, minimum inventory, return on investment, acceptance by labor, enhanced competitiveness have been considered to evaluate the most favorable strategy. Instead of usual practice of considering single value for "intensity of importance" factor, more appropriate triangular fuzzy numbers are used to represent it. A different approach using fuzzy arithmetic (α -cuts) in priority theory for the above stated problem has been investigated in this paper.

Keywords

Priority Theory, Maintenance Strategies, Multi-Criteria Decision-Making, Triangular Fuzzy Number (TFN).

Introduction

A multi-criteria decision problem generally involves choosing one out of number of alternatives based on how well those alternatives rate against a chosen set of criteria. The criteria themselves are weighted in terms of importance to the decision maker, and the overall *score* of an alternative is the weighted sum of its rating against each criteria. The ordering of the alternatives by their decision scores is taken to be their ranking by preference.

(Yager 1978) presented some ideas on the application of fuzzy sets to multi- objective decision making with particular emphasis on a means of including differing degrees of importance to different objectives. (Laarhoven 1983) presented a fuzzy method for choosing among a number of alternatives under conflicting criteria. It is a fuzzy version of Saaty's pair wise comparison method. The opinions of the decision-makers i.e. ratios are expressed in the form of fuzzy numbers with triangular fuzzy sets/ functions. First fuzzy weights for the decision criteria are found out and then fuzzy weights of alternatives under each of the decision criteria are computed. Finally, using suitable combination of these results, fuzzy scores of the alternatives are obtained based on which optimal choice is made. Zimmerman also established this fuzzy set based approach that can be found in his book (Zimmermann 1987).

(Mechefske 2001) proposed fuzzy linguistic approach to select optimum maintenance and condition-based strategy. In their paper, a heuristic algorithm is developed using the fuzzy linguistic variables to characterize the capability of available maintenance strategies to satisfy a common set of maintenance goals and to select the best strategy from those available. Importance of each maintenance goal and capability of each strategy to achieve the maintenance goals have been assessed linguistically first. Then fuzzy set concepts, some operators and distance measures have been used to decide the best strategy. The paper also further demonstrates procedure to select the correct condition monitoring technique.

Selecting optimal maintenance strategy under fuzzy environment is not a trivial task. (Verma 2007) present an illustration of multi-criteria maintenance strategy selection under fuzzy environment. Three maintenance strategies and eight maintenance decision criteria have been considered and most appropriate/ optimal strategy selection process is demonstrated using three different techniques/ methods. Fuzzy linguistic terms have been used to rate and weigh the

maintenance decision criteria. Linguistic terms/ variables are represented by triangular fuzzy sets/ number and fuzzy set operations have been carried out using α - cut method. The basic technique used is rating and ranking method using fuzzy set theory wherein ratings of alternatives/ strategies is determined first and then ranking is carried out to decide the optimal strategy. Other methods i.e. ranking fuzzy sets using *cardinal utilities* and by *maximizing and minimizing sets* are also established to confirm the choice of optimal maintenance strategy. The same problem of choosing an appropriate maintenance strategy is solved by the authors in yet another paper (Verma 2005) using the concept of suitability sets, dominance relation and preference set. (Verma² 2005) presented a case study on the maintenance of turbine. The purpose of the case study is to select the optimal technique out of three alternative techniques and nine decision criteria have been considered. Ten expert engineers judged the techniques as per the criteria. The ratings and weights have expressed by linguistic terms/ variables (i.e. as fuzzy sets). The grade membership for both the variables are considered as TFN's on the scale [0, 1]. Optimal condition monitoring technique is then found out using fuzzy multi-criteria decision-making methods as listed earlier.

Our paper explores use of fuzzy arithmetic in Saaty's priority theory to arrive at a best possible maintenance strategy depending on various criteria. Instead of single values for subjective/ linguistic term "intensity of importance", more appropriate fuzzy scale in the form of TFN's have been used and this is demonstrated with an illustration.

Saaty's Priority Theory

The Analytic Hierarchy Process (AHP) (Saaty 1982) is a powerful and flexible decision making process to help people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. By reducing complex decisions to a series of one-on-one comparisons, then synthesizing the results, AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best. Designed to reflect the way people actually think, Dr. Thomas Saaty developed AHP in the 1970's. The AHP engage decision makers in structuring a decision into smaller parts, proceeding from the goal to objectives to sub-objectives down to the alternative courses of action. Decision makers then make simple pair wise comparison judgments throughout the hierarchy to arrive at overall priorities for the alternatives. The analytic hierarchy process allows users to assess the relative

Intensity of Importance as T.F.N.'s	Definition	Explanation
[1, 1, 1] or 1	Equal Importance	Two activities contribute equally to the objective
[2, 3, 4]	Weak importance of one over another	Experience and judgment slightly favour one activity over another
[4, 5, 6]	Essential or strong importance	Experience and judgment strongly favour one activity over another
[6, 7, 8]	Demonstrated importance	An activity is strongly favoured and its dominance demonstrated in practice The evidence favouring one activity over another is of highest possible order of affirmation
[8, 9, 10]	Absolute importance	

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natives against a given criterion) in an intuitive manner. Its major innovation was the introduction of pair wise comparisons. Pair wise comparisons is a method that is informed by research showing that when quantitative ratings are unavailable and also, humans are good at recognizing whether one criteria is more important than another. Dr. Thomas Saaty, the inventor of the AHP methodology, established a consistent way of converting such pair wise comparisons (X is more important than Y) into a set of numbers (Yager 1978, Saaty 1982, Verma 2006) representing the relative priority of each of the criteria. For this, we devise a new fuzzy (in the form of TFN's) intensity scale of importance as given in Table 1.

Table 1. Intensity of Importance on Fuzzy Set Scale

Maintenance Strategy Selection Problem

The basic problem to choose between set of alternatives, given some decision criteria. Let $A = \{a_i\}; i = 1, 2, \dots, n$ be the set of decision alternatives and $C = \{c_j\}; j = 1, 2, \dots, m$ be the set of criteria according to which the desirability of an alternative is to be judged. The aim here is to obtain the optimal alternative with highest degree of desirability with respect to all relevant criteria. This problem is multi-criteria decision making problem that is tackled by many researchers working in the area of decision-making in a non-fuzzy as well as fuzzy environment (Zimmermann 1987, Zimmermann 1985).

We consider three alternatives: corrective maintenance (A_1), preventive maintenance (A_2), predictive maintenance (A_3) and eight maintenance decision criteria namely: low maintenance cost (C_1), improved reliability (C_2), improved safety (C_3), high product quality

(C_4), minimum inventory (C_5), return on investment (C_6), acceptance by labour (C_7), enhanced competitiveness (C_8) by which to judge the three alternatives.

Using Fuzzy Sets in Priority Theory

Table 2 displays a matrix of relative significance of each pair of criteria. Let r_{ij} denote the numerical value assigned to the relative significance/ importance (i.e. ratios) of criteria C_i and C_j . It is as per the “intensity of importance” fuzzy scale given in Table 1. If C_i and C_j , both are equally important, then $r_{ij} = 1$; if C_i is more important than C_j , then $r_{ij} > 1$ and if C_i is less important than C_j , then $r_{ij} < 1$. Matrix in Table 2 has positive entries everywhere and as it satisfies reciprocal property i.e. $r_{ji} = \frac{1}{r_{ij}}$. It is called a reciprocal matrix. Important point to note here is that r_{ij} 's are in the form of TFN's and therefore inverse operation on TFN's is used to get reciprocal of it. From the matrix, normalized average weights (priorities) are computed, as shown in Table 3.

(Lootsama 1980] showed that normalized column and row weights are as good enough as normalized eigen vectors. We propose the average of the two (row and column) normalized weights (Verma 2006) to be considered as final weight. Maintenance strategies are as well compared in pair wise manner under each criterion. These matrices are given in Tables 4-11. Priority of criteria and priority of maintenance strategies are then multiplied (fuzzy multiplication) as shown in Table 12 and added for each maintenance strategy to obtain the final scores. The final scores of alternatives in TFN form are: $A_1 = [0.058, 0.094, 0.162]$, $A_2 = [0.220, 0.360, 0.603]$ and $A_3 = [0.336, 0.544, 0.885]$. Thus in our illustration, we get the

highest score for predictive maintenance strategy, and therefore it is the optimal one. This is also depicted in Figure 1.

Criteria	C_1	C_2	C_3	C_4
C_1	1	[0.25,0.33,0.5]	[0.25,0.33,0.5]	[2,3,4]
C_2	[2,3,4]	1	[0.25,0.33,0.5]	[2,3,4]
C_3	[2,3,4]	[2,3,4]	1	[4,5,6]
C_4	[0.25,0.33,0.5]	[0.25,0.33,0.5]	[0.17,0.2,0.25]	1
C_5	[0.17,0.2,0.25]	[0.13,0.14,0.17]	[0.13,0.14,0.17]	[0.17,0.2,0.25]
C_6	[0.25,0.33,0.5]	[0.17,0.2,0.25]	[0.17,0.2,0.25]	[0.25,0.33,0.5]
C_7	[0.13,0.14,0.17]	[0.1,0.11,0.13]	[0.1,0.11,0.13]	[0.13,0.14,0.17]
C_8	[0.25,0.33,0.5]	[0.17,0.2,0.25]	[0.17,0.2,0.25]	[0.25,0.33,0.5]
CS	[6.05,8.33,10.92]	[4.07,5.31,6.8]	[2.24,2.51,3.05]	[9.8,13.0,16.42]

Criteria	C_5	C_6	C_7	C_8	RS
C_1	[4,5,6]	[2,3,4]	[6,7,8]	[2,3,4]	[17.5,22.66,28]
C_2	[6,7,8]	[4,5,6]	[8,9,10]	[4,5,6]	[27.25,33.33,39.5]
C_3	[6,7,8]	[4,5,6]	[8,9,10]	[4,5,6]	[31,38,45]
C_4	[4,5,6]	[2,3,4]	[6,7,8]	[2,3,4]	[15.67,19.86,24.25]
C_5	1	1	[4,5,6]	[0.17,0.2,0.25]	[6.77,7.88,9.09]
C_6	1	1	[4,5,6]	[0.25,0.33,0.5]	[7.09,8.39,10]
C_7	[0.17,0.2,0.25]	[0.17,0.2,0.25]	1	[0.13,0.14,0.17]	[1.93,2.04,2.27]
C_8	[4,5,6]	[2,3,4]	[6,7,8]	1	[13.84,17.06,20.5]
CS	[26.17,31.2,36.25]	[16.17,21.2,26.25]	[43,50,57]	[13.55,17.67,21.92]	[121.05,149.22,178.61] #

RS: Row Sum CS: Column Sum

Table 2. Matrix of Relative Significance of Decision Criteria

Criteria	C_1	C_2	C_3	C_4
RS	[17.5,22.66,28]	[27.25,33.33,39.5]	[31,38,45]	[15.67,19.86,24.25]
CS	[6.05,8.33,10.92]	[4.07,5.31,6.8]	[2.24,2.51,3.05]	[9.8,13.0,16.42]
N	[0.10,0.15,0.23]	[0.15,0.22,0.33]	[0.17,0.25,0.37]	[0.09,0.13,0.20]
IN	[0.09,0.12,0.17]	[0.15,0.19,0.25]	[0.33,0.40,0.45]	[0.06,0.08,0.10]
Average Priority of Criteria	[0.10,0.14,0.20]	[0.15,0.21,0.29]	[0.25,0.33,0.41]	[0.08,0.11,0.15]

Criteria	C_5	C_6	C_7	C_8
RS	[6.77,7.88,9.09]	[7.09,8.39,10]	[1.93,2.04,2.27]	[13.84,17.06,20.5]
CS	[26.17,31.2,36.25]	[16.17,21.2,26.25]	[43,50,57]	[13.55,17.67,21.92]
N	[0.04,0.05,0.08]	[0.04,0.06,0.08]	[0.01,0.01,0.02]	[0.08,0.11,0.17]
IN	[0.03,0.03,0.04]	[0.04,0.05,0.06]	[0.02,0.02,0.02]	[0.05,0.06,0.07]
Average Priority of Criteria	[0.04,0.04,0.06]	[0.04,0.06,0.07]	[0.02,0.02,0.02]	[0.07,0.09,0.12]

Table 3. Average Priority T.F.N.'s of Criteria

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.25,0.33,0.5]	[0.17,0.2,0.25]	[1.42,1.53,1.75]	[0.08,0.10,0.15]	[0.09,0.11,0.15]
A_2	[2,3,4]	1	[0.25,0.33,0.5]	[3.25,4.33,5.5]	[0.18,0.29,0.47]	[0.18,0.26,0.39]
A_3	[4,5,6]	[2,3,4]	1	[7,9,11]	[0.38,0.61,0.94]	[0.48,0.63,0.82]
CS	[7,9,11]	[3.25,4.33,5.5]	[1.42,1.53,1.75]	[11.67,14.86,18.25][#]		
IN	[0.09,0.11,0.14]	[0.18,0.23,0.31]	[0.57,0.65,0.70]			

Table 4. Low Maintenance Cost (C_1)

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.17,0.2,0.25]	[0.13,0.14,0.17]	[1.3,1.34,1.42]	[0.06,0.07,0.09]	[0.06,0.08,0.09]
A_2	[4,5,6]	1	[0.25,0.33,0.5]	[5.25,6.33,7.5]	[0.24,0.34,0.48]	[0.22,0.29,0.40]
A_3	[6,7,8]	[2,3,4]	1	[9,11,13]	[0.41,0.59,0.84]	[0.51,0.64,0.78]
CS	[11,13,15]	[3.17,4.2,5.25]	[1.38,1.47,1.67]	[15.55,18.67,21.92][#]		
IN	[0.06,0.08,0.09]	[0.19,0.24,0.32]	[0.60,0.68,0.72]			

Table 5. Improved Reliability (C_2)

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.17,0.2,0.25]	[0.17,0.2,0.25]	[1.34,1.4,1.5]	[0.08,0.09,0.11]	[0.08,0.09,0.11]
A_2	[4,5,6]	1	1	[6,7,8]	[0.34,0.45,0.60]	[0.39,0.45,0.53]
A_3	[4,5,6]	1	1	[6,7,8]	[0.34,0.45,0.60]	[0.39,0.45,0.53]
CS	[9,11,13]	[2.17,2.2,2.25]	[2.17,2.2,2.25]	[13.34,15.4,17.5][#]		
IN	[0.08,0.09,0.11]	[0.44,0.45,0.46]	[0.44,0.45,0.46]			

Table 6. Improved Safety (C_3)

Table 7. High Product Quality (C_4)						
Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.25,0.33,0.5]	[0.25,0.33,0.5]	[1.5,1.66,2]	[0.11,0.14,0.21]	[0.11,0.14,0.21]
A_2	[2,3,4]	1	1	[4,5,6]	[0.29,0.43,0.63]	[0.35,0.43,0.54]
A_3	[2,3,4]	1	1	[4,5,6]	[0.29,0.43,0.63]	[0.35,0.43,0.54]
CS	[5,7,9]	[2.25,2.33,2.5]	[2.25,2.33,2.5]	[9.5,11.66,14][#]		
IN	[0.11,0.14,0.2]	[0.4,0.43,0.44]	[0.4,0.43,0.44]			

Table 7. High Product Quality (C_4)

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.13,0.14,0.17]	[0.1,0.11,0.13]	[1.23,1.25,1.3]	[0.04,0.05,0.06]	[0.05,0.06,0.07]
A_2	[6,7,8]	1	[0.17,0.2,0.25]	[7.17,8.2,9.25]	[0.26,0.34,0.43]	[0.20,0.25,0.31]
A_3	[8,9,10]	[4,5,6]	1	[13,15,17]	[0.47,0.61,0.79]	[0.60,0.69,0.79]
CS	[15,17,19]	[5.13,6.14,7.17]	[1.27,1.31,1.38]	[21.4,24.45,27.55][#]		
IN	[0.05,0.06,0.07]	[0.14,0.16,0.19]	[0.72,0.76,0.79]			

Table 8. Minimum Inventory (C_5)

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.13,0.14,0.17]	[0.13,0.14,0.17]	[1.26,1.28,1.34]	[0.06,0.07,0.08]	[0.06,0.07,0.08]
A_2	[6,7,8]	1	1	[8,9,10]	[0.37,0.47,0.58]	[0.42,0.47,0.53]
A_3	[6,7,8]	1	1	[8,9,10]	[0.37,0.47,0.58]	[0.42,0.47,0.53]
CS	[13,15,17]	[2.13,2.14,2.17]	[2.13,2.14,2.17]	[17.26,19.28,21.34][#]		
IN	[0.06,0.07,0.08]	[0.46,0.47,0.47]	[0.46,0.47,0.47]			

Table 9. Return on Investment (C_6)

Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.13,0.14,0.17]	[0.1,0.11,0.13]	[1.23,1.25,1.3]	[0.05,0.06,0.07]	[0.05,0.06,0.07]
A_2	[6,7,8]	1	[0.25,0.33,0.5]	[7.25,8.33,9.5]	[0.28,0.37,0.49]	[0.24,0.31,0.41]
A_3	[8,9,10]	[2,3,4]	1	[11,13,15]	[0.43,0.58,0.77]	[0.52,0.64,0.76]
CS	[15,17,19]	[3.13,4.14,5.17]	[1.35,1.44,1.63]	[19.48,22.58,25.8][#]		
IN	[0.05,0.06,0.07]	[0.19,0.24,0.32]	[0.61,0.69,0.74]			

Table 10. Acceptance by Labour (C_7)

Alternatives/ Strategies	A_1	A_2	A_3	RS	N	Average Priority
A_1	1	[0.25,0.33,0.5]	[0.17,0.2,0.25]	[1.42,1.53,1.75]	[0.08,0.10,0.15]	[0.09,0.11,0.15]
A_2	[2,3,4]	1	[0.25,0.33,0.5]	[3.25,4.33,5.5]	[0.18,0.29,0.47]	[0.18,0.26,0.39]
A_3	[4,5,6]	[2,3,4]	1	[7,9,11]	[0.38,0.61,0.94]	[0.48,0.63,0.82]
CS	[7,9,11]	[3.25,4.33,5.5]	[1.42,1.53,1.75]	[11.67,14.86,18.25][#]		
IN	[0.09,0.11,0.14]	[0.18,0.23,0.31]	[0.57,0.65,0.70]			

Table 11. Enhanced Competitiveness (C_8)

Criteria	C_1	C_2	C_3	C_4
Average Priority of Criteria	[0.10,0.14,0.20]	[0.15,0.21,0.29]	[0.25,0.33,0.41]	[0.08,0.11,0.15]
Average Priority of Strategies	A_1 [0.09,0.11,0.15] A_2 [0.18,0.26,0.39] A_3 [0.48,0.63,0.82]	[0.06,0.08,0.09] [0.22,0.29,0.40] [0.51,0.64,0.78]	[0.08,0.09,0.11] [0.39,0.45,0.53] [0.39,0.45,0.53]	[0.11,0.14,0.21] [0.35,0.43,0.54] [0.35,0.43,0.54]
Scores of Strategies	A_1 [0.009,0.015,0.03] A_2 [0.018,0.036,0.078] A_3 [0.048,0.088,0.164]	[0.009,0.017,0.026] [0.033,0.061,0.116] [0.077,0.134,0.226]	[0.02,0.030,0.045] [0.098,0.149,0.217] [0.098,0.149,0.217]	[0.009,0.015,0.032] [0.028,0.047,0.081] [0.028,0.047,0.081]
Criteria	C_5	C_6	C_7	C_8
Average Priority of Criteria	[0.04,0.04,0.06]	[0.04,0.06,0.07]	[0.02,0.02,0.02]	[0.07,0.09,0.12]
Average Priority of Strategies	A_1 [0.05,0.06,0.07] A_2 [0.20,0.25,0.31] A_3 [0.60,0.69,0.79]	[0.06,0.07,0.08] [0.42,0.47,0.53] [0.42,0.47,0.53]	[0.05,0.06,0.07] [0.24,0.31,0.41] [0.52,0.64,0.76]	[0.09,0.11,0.15] [0.18,0.26,0.39] [0.48,0.63,0.82]
Scores of Strategies	A_1 [0.002,0.002,0.004] A_2 [0.008,0.01,0.019] A_3 [0.024,0.028,0.047]	[0.002,0.004,0.006] [0.017,0.028,0.037] [0.017,0.028,0.037]	[0.001,0.001,0.001] [0.005,0.006,0.008] [0.010,0.013,0.015]	[0.006,0.01,0.018] [0.013,0.023,0.047] [0.034,0.057,0.098]

Table 12. Computation of Scores of Maintenance Strategies

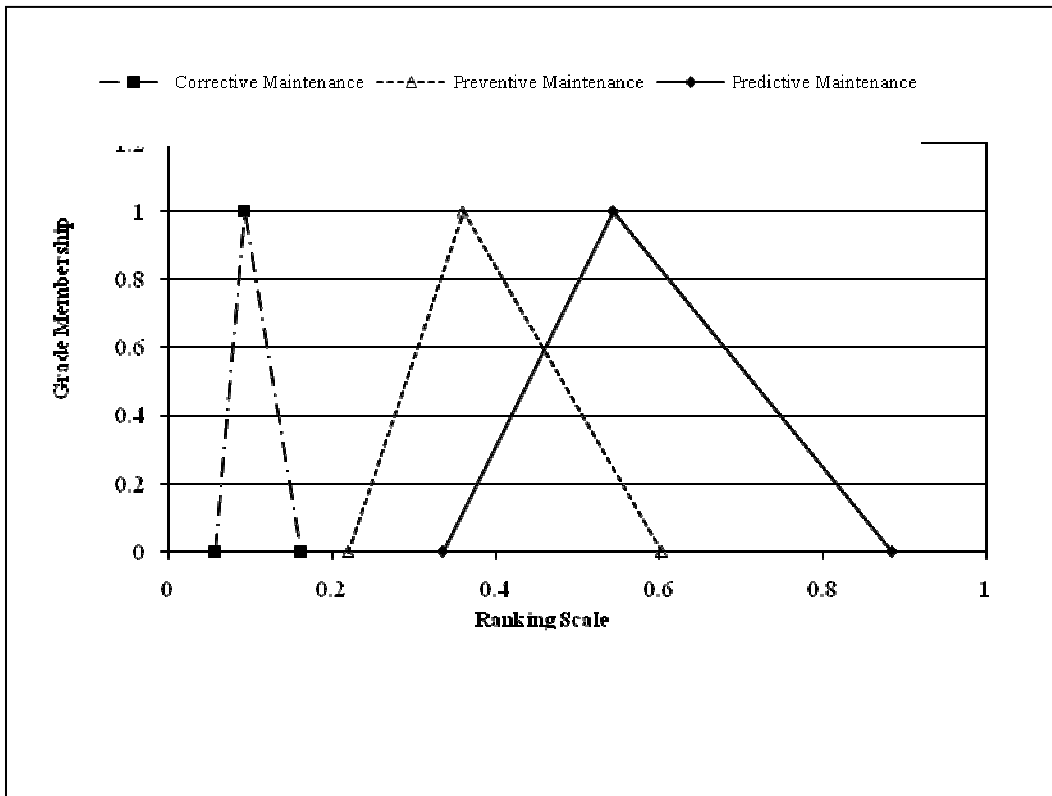


Figure 1. Final Ranking of Strategies

Conclusion

In this paper, most suitable maintenance strategy selection procedure is illustrated by incorporating fuzzy sets in Saaty's priority theory. As "importance" is more habitually and regularly expressed in subjective/ linguistic terms, "intensity of importance" scale has been fuzzified and is expressed in TFN's form. First three maintenance decision strategies and eight decision criteria have been determined and then priority theory is used. Priority theory estimates the weights (priorities) of decision criteria using pair wise comparison method. Maintenance strategies are also compared in pair wise manner under each criterion. It is worth mentioning that averaging (the row and column weights) is proposed in this paper to confirm the priorities. We obtained the final scores of each maintenance strategy by multiplying (fuzzy arithmetic) the

priorities and then adding them. In this formulated illustration, predictive maintenance strategy turned out to be the best one.

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