

Mixed-Integer Linear Programming for Vehicle Routing Problem with Simultaneous Delivery and Pick-Up with Maximum Route-Length

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Abstract

The vehicle routing problem with simultaneous delivery and pick-up (VRPSDP) is the problem of optimally assimilating goods collection and distribution, when no priority constraints are imposed on the order in which the vehicle must perform the operations. This paper considers an additional constraint of maximum route length in VRPSDP. We develop a mixed-integer linear programming model for VRPSDP with an additional constraint of maximum route length. The results are encouraging for a sample benchmark data set.

Keywords

Vehicle Routing; Simultaneous Delivery and Pick-up; Maximum Route Length; Mixed-Integer Linear Programming

Introduction

A key element of many distribution systems is the routing and scheduling of vehicles through a set of nodes requiring service. The Vehicle Routing Problem (VRP) involves the design of a set of minimum-cost vehicle routes, originating and terminating at a central depot, for a fleet of vehicles that services a set of nodes with known demands. Each node is serviced exactly once and, furthermore, all nodes must be assigned to vehicles without exceeding vehicle capacities (Bodin *et al* 1983).

Vehicle routing is defined by decisions, objectives and constraints. Fundamentally, the decisions of vehicle routing are to assign a group of nodes to depot(s) and to groups of drivers and vehicles, and to sequence and schedule their visits. The objective of vehicle routing is to provide a

high level of customer service while keeping the operating and investment costs as low as possible. VRP consists of two sub-problems: the nodes grouping to clusters and finding the best tour for every cluster. Therefore, route is the total number of deliveries made by a single vehicle and tour is their sequence. The solution of these sub-problems results to the routes and tours that minimize the total transportation cost. VRP has received considerable attention over the last two decades. Many efforts in the literature have been established to extend the basic VRP model to incorporate additional constraints or different objective functions.

Many applications of VRP involve delivery and pick-up services between the depot and peripheral locations. Delivery refers to transportation of goods from depot to nodes, and pick-up refers to shipment in the opposite direction (to the depot). The delivery and pick-up problem is a generalization of the VRP, which is a generalization of the traveling salesman problem (TSP), the well-known hard combinatorial optimization problem (Mosheiov, 1994). Considering also that the problem in practice is, usually, of a large-scale, it is obvious why the problem is a challenge for both researchers and practitioners. Many researchers in the last two decades worked on the problem and the significant achievements are reached. Still, there are areas and sub problems, yet, to be researched. The delivery and pick-up problem is a problem of finding a set of optimal routes for a fleet of vehicles in order to serve both delivery and pick-up requests. Each delivery and pick-up request is defined by delivery and pick-up location and a load. If the delivery and pick-up location is the same, the problem is known as the simultaneous delivery and pick-up problem (Min, 1989). One of the extensions of the VRP considered in this paper is that accommodate the environment where vehicle is not only responsible for the distribution but also the collection of the goods. This is named VRP with Simultaneous Delivery and Pick-up (VRPSDP).

The VRPSDP can be stated as follows: A set of N nodes with deterministic demands for delivery and pick-up services have to be visited by a fleet of homogeneous vehicles, all of which originate and terminate at the same node (depot). Every node must be visited exactly once which implies that the delivery and pick-up services occur at the same stopover. The objective is to minimize the sum total distance of all the routes.

The VRPSDP can be constrained by maximum route length. Since the traditional VRP itself is a known NP -hard problem, the additional feature of simultaneous delivery and pick-up clearly rules out the use of conventional optimization methods.

The remainder of the paper is organized as follows: In Section 2, we present a review of literature. In Section 3, the complexity of VRPSDP is explained with examples. The problem description is given in Section 4. In Section 5, the assumptions and the feasibility constraints are discussed. The mathematical programming formulation is presented in Section 6. In Section 7 the computational experiments are discussed. Conclusions are presented in Section 9.

Literature Review

The basic version of VRP is a pure delivery (pick-up) problem, and has been studied extensively in the literature. VRPSDP envisages receipt and dispatch of goods at the same point of stop over. Min (1989) was the first to tackle this version, solving a practical problem faced by a public library, with one depot, two vehicles and 22 nodes. The nodes were first clustered into groups and for each group, a TSP was solved. The infeasible arcs were penalized (their lengths set to infinity), and the TSPs solved again. Salhi and Nagy (1999) proposed four insertion heuristics based on the methodology proposed by Golden *et al.* (1985) and Casco *et al.* (1988). Dethloff (2001) and Dethloff (2002) introduced insertion-based heuristics for the problem. Angelelli and

Mansini (2002) solved the problem using pricing and branching strategy. Tang and Galvao (2002) developed two local search heuristics based on Beasley (1983) and Gillet and Miller (1974). Nagy and Salhi (2004) proposed local search heuristics which consider the degree of infeasibility. Tang and Galvao (2006) developed a TS for VRPSDP which uses three types of movements such as relocation, interchange and crossover movements to obtain inter-route adjacent solutions. Bianchessi and Righini (2007) proposed TS algorithm based on complex and variable neighborhoods. They combine arc-exchange-based and node-exchange-based neighborhoods, employing different and interacting tabu lists. Alshamrani *et al* (2007) addressed a dynamic version of VRPSDP to the application of blood bank logistics. The hybrid of meta-heuristics approaches remains untried for the variant VRPB.

More work has been presented on related decision situations. One of them is the delivery and pick-up problem (DPP). In this situation transportation requests have to be carried out, where the origin as well as the destination of each of these requests can be the locations other than the depot. Another variant is the vehicle routing problem with backhauls (VRPB) with an important assumption that deliveries must precede pick-ups on each route (Goetschalckx and Jacobs-Blecha,, 1989).

Another variant of VRP with the concept of mixed loads termed as Vehicle Routing Problem with Mixed delivery and pick-up (VRPMDP) problem. VRPMDP is an extension of the general pick-up and delivery problem, where linehauls and backhauls can occur in any sequence on a vehicle route (Wade and Salhi, 2002). The VRPMDP can be considered the special case of the VRPSDP where either the delivery demand or the pick-up demand of each customer equals zero. Even the VRPMDP is closely related to the VRPSDP, none of the solution approaches towards the VRPMDP can be used 'directly' for the strict VRPSDP, but some basic ideas can be transferred (Dethloff, 2001).

Complexity of VRPSDP.

The VRPSDP complexity is shown with illustrative examples

Example No.1

Consider the data in Table 1 showing the distance matrix, and Table 2 with the delivery and pick-up demand. The node 0 is depot and the nodes 1 to 13 are customers.

Table 1**Table 2**

There are three vehicles with capacities equal to 150 units. We first solve the problem using the nearest neighbour heuristic (Reference) to create sub-cycles that are both delivery and pick-up feasible. This gives the following three sub-cycles:

Route 1: 0 – 13 – 12 – 8 – 9 – 10 – 3 – 0

Distance Traveled = 327, Capacity of vehicle = 130 units

Route 2: 0 – 11 – 2 – 1 – 5 – 4 – 7 – 0

Distance Traveled = 583, Capacity of vehicle = 130 units

Route 3: 0 – 11 – 2 – 1 – 5 – 4 – 7 – 0

Distance Traveled = 318, Capacity of vehicle = 30 units

It is easily checked that all three sub-cycles are load feasible and can be used as a solution for VRPSDP. The total distance traveled cost is 1228.

If the Fisher and Jaikumar (FJ) (Fisher and Jaikumar, 1981) approach is used for the same, we will get two clusters

{0, 1, 4, 5, 9, 12, 13} and {0, 2, 3, 6, 7, 8, 10, 11}

By construction, these two clusters are both delivery and pick-up feasible with vehicle capacity 145 units in both cases.

Making sub-cycles from the first cluster, the cheapest feasible Hamiltonian cycle is

0 – 4 – 5 – 9 – 12 – 13 – 1 – 0

Distance Traveled = 518; Capacity of vehicle = 145 units

For the second cluster, the cheapest feasible Hamiltonian cycle is

0 – 2 – 3 – 10 – 6 – 7 – 8 – 11 – 0

Distance Traveled = 493; Capacity of vehicle = 145 units

It is easily checked that all two sub-cycles are load feasible and can be used as a solution for VRPSDP. The total distance traveled cost is 1011.

But, if one-stop constraint is violated for the above tour, and the example problem is solved by FJ approach then the feasible tour is:

The sub-cycles from the first cluster, the solution is

0 – 13 – 12 – 9 – 5 – 4 – 1 – 13 – 0

Distance Traveled = 498, Capacity of vehicle = 145 units

For the second cluster, the cheapest feasible Hamiltonian cycle is

0 – 2 – 3 – 10 – 6 – 7 – 8 – 11 – 0

Distance Traveled = 493, Capacity of vehicle = 145 units

It is easily checked that all two sub-cycles are load feasible and can be used for delivery and pick-up problem. The total distance traveled cost is 991. But there is a stop constraint in the first sequence. So, there is a need of heuristic approach to solve the problem with single visit constraint.

Example 2

Consider the data in Table 3 showing the distance matrix, and Table 4 with the delivery and pick-up demand. The total number of vehicles is three. The node 0 is depot and the nodes 1 to 13 are customers.

Table 3

Table 4

There are set of homogeneous fleet of vehicles with capacities equal to 180 units. It is obvious that there is a need of two vehicles or more to handle the planning situation.

We first solve the problem using the nearest neighbor heuristic to create sub-cycles that are both delivery and pick-up feasible. This gives the following three sub-cycles:

Route 1: 0 – 13 – 12 – 8 – 9 – 10 – 3 – 0

Distance Traveled = 34, Capacity of vehicle = 160 units

Route 2: 0 – 11 – 2 – 1 – 5 – 4 – 7 – 0

Distance Traveled = 83, Capacity of vehicle = 180 units

It is easily checked that all two sub-cycles are load feasible and can be used for delivery and pick-up problem. The total distance traveled cost is 117.

If the FJ approach is used for the same, we will get two clusters

{ 0, 1, 4, 5, 9, 12, 13 } and { 0, 2, 3, 6, 7, 8, 10, 11 }

By construction, these two clusters are both delivery and pick-up feasible with vehicle capacity 145 units for first case and 180 units for second case.

Making sub-cycles from the first cluster, the cheapest feasible Hamiltonian cycle is

0 – 4 – 5 – 9 – 12 – 13 – 1 – 0

Distance Traveled = 53, Capacity of vehicle = 175 units

For the second cluster, the cheapest feasible Hamiltonian cycle is

0 – 2 – 3 – 10 – 6 – 7 – 8 – 11 – 0,

Distance Traveled = 49, Capacity of vehicle = 180 units

It is easily checked that all two sub-cycles are load feasible and can be used for delivery and pick-up problem. The total distance traveled cost is 102.

But, if one one-stop constraint is violated for the above tour, and the example problem is solved by FJ approach then the feasible tour is:

The sub-cycles from the first cluster, the solution for VRPMDP is

0 – 13 – 12 – 9 – 5 – 4 – 1 – 13 – 0

Distance Traveled = 51, Capacity of vehicle = 175 units

For the second cluster, the cheapest feasible Hamiltonian cycle is

0 – 2 – 3 – 10 – 6 – 7 – 8 – 11 – 0,

Distance Traveled = 49, Capacity of vehicle = 180 units

It is easily checked that all two sub-cycles are load feasible and can be used for delivery and pick-up problem. The total distance traveled cost is 100. But there is a stop constraint in the first sequence.

Problem Description

Formally, the VRPSDP is defined as follows:

Data: A fleet of V vehicles with identical capacity Q , servicing a set of N customers with integer weights representing demands (demand for pick-up and demand for delivery), from / to a depot, the distances between the locations where the customers and depot are placed. The problem can be formulated on a graph (oriented or not) as well as in the Euclidean plane.

Variables: A solution is a set of V tours visiting the customers, starting from and returning to the depot. When a vehicle visits a customer it is supposed to deliver and to pick-up a variable amount of load.

Constraints: Each customer must be completely served, that is the vehicles visiting customer must pick-up and deliver an overall quantity equal to the customer demand. Each vehicle is supposed to start from the depot carrying an amount of goods equal to the total amount it must deliver and to return to the depot carrying an amount of goods equal to the total amount it picked-up. In each point along its tour each vehicle cannot carry a total load greater than its capacity. The major constraint is the maximum route length for each vehicle.

Objective: The goal is to minimize the total distance traveled.

Assumptions and Feasibility of VRPSDP with Maximum Route Length

The following assumptions apply for VRPSDP with maximum route length.

- all routes start and end at the node of origin, also known as depot.
- each node in N is visited exactly once.
- demand at any node shall never exceed the vehicle capacity Q .
- all vehicles have the same capacity and are stationed at the node of origin.
- split delivery is not permitted.
- each vehicle makes exactly one trip.
- all delivery quantities are loaded at the depot; all quantities picked up must be unloaded at the depot.

The following assumptions are underlying our model.

1. Within the planning horizon all orders are known. Thus, planning can be based on full order information. All orders are known in advance, the basic problem is static. Thus, planning can be based on full order information. However, this assumption is omitted and the model and the solution heuristic can treat dynamic problems as well, if it is used on a rolling basis and using advanced information systems for transportation order planning.

2. All orders are consolidated to full vehicle loads. We do not consider crossdocking options. There is at most one order on a vehicle at any given time, which is transported directly from its CBB to RBB.

3. Each vehicle is assigned to a specific CBB, where it has to return to after each tour. The motivation of this assumption is that vehicle drivers can't stay away from CBB too long, but have to come back to CBB on a regular basis.

4. The fleet consists of homogeneous vehicles operating from a number of depots.

5. Each vehicle can be used repeatedly within the planning horizon.

The task is to determine a *route* for each vehicle so as to serve a set of nodes such that the total distance traversed is minimal. The problem is considered with the constraint on maximum route length (*TL*).

The feasibility of a route depends on the total quantity of goods loaded for delivery at the depot, the cumulative load picked up and the transit load between consecutive nodes visited. Suppose a vehicle starts from the depot ($j=0$) and travels along a certain path until it reaches node j_k , ($0, j_1, j_2, j_3, \dots, j_j, \dots, j_k$). The cumulative loads to be delivered and to be collected up to the point j_k of the path are

$$U_d(j_k) = \sum_{j \in P(0, j_k)} d_j \text{ and } U_p(j_k) = \sum_{j \in P(0, j_k)} p_j \quad (1)$$

$P(0, j_k)$ denotes the nodes along the path. The path becomes infeasible if either of these cumulative loads exceeds Q , i.e., when

$$\sum_{j \in P(0, j_k)} d_j > Q \text{ or } \sum_{j \in P(0, j_k)} p_j > Q \quad (2)$$

Each feasible route is formed such that

$$U_d(j_k) \leq Q \text{ and } U_d(j_{k+1}) > Q \quad (3)$$

$$U_p(j_k) \leq Q \text{ and } U_p(j_{k+1}) > Q \quad (4)$$

A third check for feasibility is whether any net load in transit between two consecutive nodes exceeds capacity. After a vehicle visits node j_k , its net load

$$D_p(j_k) = U_p(j_k) + D(0) - U_d(j_k) \quad (5)$$

If the vehicle cannot pick the load up at the next node j_{k+1} ,

i.e., if $D_p(j_{k+1}) > Q$, then the path becomes infeasible.

$$D_p(j_{k+1}) > Q$$

Thus the feasibility of a route in VRPSDP also depends on the sequence of nodes.

$$D_p(j_k) \leq Q \quad \forall j_k \tag{6}$$

A route is feasible if and only if it satisfies the conditions (3), (4) and (6).

Mathematical Programming to solve VRPSDP with Maximum Route Length

VRPSDP with maximum route length is modeled as a mixed integer linear programme (MILP).

Notations

G = Symmetric Graph; $G = (T, A)$

T = Set of Nodes; $T = [N \cup \{0, n+1\}]$

A = Set of Arcs linking any pair of nodes, $(i,j) \in A$

V = Total Number of Vehicles; $v = \{1, 2, \dots, V\}$

y_{ij} = Cost of travel from node i to node j

d_i = Delivery requests of node i , $i = 1, \dots, N$

p_i = Pickup requests of node i , $i = 1, \dots, N$

Q = Capacity of Vehicle

TL = Maximum Route Length for any Vehicle v

Decision Variables

D_{iv} = The load remaining to be delivered by vehicle v when departing from node i

P_{iv} = The cumulative load picked by vehicle v when departing from node i

$$X_{ijv} = \begin{cases} 1 & \text{if vehicle } v \text{ travels from } i \text{ to } j, \\ 0 & \text{otherwise;} \end{cases}$$

P_i, d_i, Q, y_{ij} are non-negative integers

Note:

- (i) The distance matrix y_{ij} satisfies triangular inequality
- (ii) At every node in a path, the sum of the loads picked up and the quantities remaining to be delivered must not exceed the vehicle capacity.
- (iii) Formulation

$$\text{Minimize } \sum_{v=1}^V \sum_{i=0}^N \sum_{j=0}^N y_{ij} X_{ijv} \quad (7)$$

Subject to

$$\sum_{v=1}^V \sum_{i=0}^N X_{ijv} = 1 \quad \forall j=1, \dots, N \quad (8)$$

$$\sum_{j=1}^N X_{0jv} \leq 1 \quad \forall v=1, \dots, V \quad (9)$$

$$\sum_{i=0}^N X_{ijv} - \sum_{i=0}^N X_{jiv} = 0 \quad \forall j=0, \dots, N \text{ and } v=1, \dots, V \quad (10)$$

$$D_{iv} + P_{iv} \leq Q \quad \forall i=0, \dots, N \text{ and } v=1, \dots, V \quad (11)$$

$$D_{0v} = \sum_{i=1}^N \sum_{j=1}^N X_{ijv} d_i = 0 \quad \forall v=1, \dots, V \quad (12)$$

$$P_{0v} = 0 \quad \forall v=1, \dots, V \quad (13)$$

$$(D_{iv} - d_j - D_{jv}) X_{ijv} = 0 \quad \forall v=1, \dots, V \text{ and } i, j=1, \dots, N \quad (14)$$

$$(P_{iv} + p_j - P_{jv}) X_{ijv} = 0 \quad \forall v=1, \dots, V \text{ and } i, j=1, \dots, N \quad (15)$$

$$\sum_{i=0}^N \sum_{j=0}^N y_{ij} X_{ijv} \leq TL \quad \forall v=1, \dots, V \quad (16)$$

$$D_{iv} \geq 0 \quad \forall v=1, \dots, V \text{ and } i=1, \dots, T \quad (17)$$

$$P_{iv} \geq 0 \quad \forall v=1, \dots, V \text{ and } i=1, \dots, T \quad (18)$$

$$X_{ijv} \in \{0,1\} \quad \forall v=1, \dots, V \text{ and } i, j=1, \dots, T \quad (19)$$

Objec

Objective Function and Constraints

The objective function (7) seeks to minimize the total cost of travel.

Constraint (8) stipulates that each node must be visited by exactly once.

(9) ensure that each vehicle is used, at most, once.

(10) ensure that same vehicle arrives and departs from each node it serves.

(11) ensures that the load on vehicle v , when departing from node i , is always lower than the vehicle capacity.

(12) and (13) ensures that the total delivery load for a route is placed on the vehicle v , embarking on each trip, at the starting node itself.

(14) and (15) are transit load constraints i.e., if arc (i, j) is visited by the vehicle v , then the quantity to be delivered by the vehicle has to decrease by d_j while the quantity picked-up has to increase by p_j .

(16) ensure the maximum route length for any vehicle v .

Lower Bound for Number of Vehicles

For a homogeneous vehicle fleet, it is possible to find a lower bound for the number of vehicles required for a solution and thus ensure efficient utilization of vehicles capacities. If assume that all vehicle are identical with the same capacity Q , a simple formula for the minimal number v of vehicles necessary to serve all customers in VRPSDP can be suggested as follows:

Notations

N = Number of nodes

d_i = Delivery demand

p_i = Pick-up demand

Q = Vehicle capacity

$$v = \max \left\{ \left\lfloor \frac{\sum_{i=1}^N d_i}{Q} \right\rfloor, \left\lfloor \frac{\sum_{i=1}^N p_i}{Q} \right\rfloor \right\} \quad (20)$$

Computational Experiments

MILP is solved using CPLEX and run on a PC Pentium IV 1.70 GHz processor for benchmark data set and randomly generated data sets. We also report the computing times but do not use them for comparison owing to possible variations in the configurations of hardware and software employed.

(i) Data-set of Min (1989)

This is from a real-life problem with 22 nodes, vehicle capacity of 10500, total delivery load of 20300 and total pick-up load of 19950. It requires, at least, two vehicles. Table 5 shows the result of MILP with other reported solutions, including the best-known. The optimal solution obtained by MILP shows an improvement of 6.38 % over Min's (1989) result and 3.30 % over Dethloff's (2001) when compared with all the proposed heuristics.

Table 5

(ii) Randomly Generated TSDP Instances

Random test data-sets with 7 to 15 nodes are generated under the following assumptions:

The coordinates of the nodes are uniformly distributed over a square spreading from (0,0) to [100,100]. The quantities to be delivered at the nodes are uniformly distributed over the interval [0,100]. The pick-up demand (p_i) is computed as a fraction of the delivery demand (d_i), using a random number m that is uniformly distributed over the interval [0,1] such that $p_i = (0.5 + m) d_i$. For each of the resulting configurations, four experiments are performed. The results are shown in Table 6 in which the last digit of the instance descriptor denotes the number of experiment.

The results obtained by MILP Model on the randomly generated instances is provided in Table 6.

Table 6

Performance of MILP

The solutions are compared on the basis of relative percentage deviation (RD).

$$RD = \left(\frac{\text{Solution of MILP} - \text{Best Known Solution}}{\text{Best Known Solution}} \right) 100$$

An average of the “RD’s” is then calculated for the best solutions and presented in last row of each table.

Conclusions

This study has addressed the vehicle routing problem with simultaneous delivery and pick-up (VRPSDP) with maximum tour length constraint. For this np -hard problem, we have developed a mixed-integer linear programming model. The MILP model is tested for standard data-set of VRPSDP. As can be seen from the results, proposed MILP promises to be a useful model for solving VRPSDP. Further research can consider constraints on precedence and time windows of nodes.

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Table 1. Distance Matrix for the graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	55	75	115	170	148	145	159	98	101	95	50	58	20
1	55	0	43	115	165	155	170	198	142	138	115	75	105	75
2	75	43	0	80	125	120	142	181	135	123	91	64	106	95
3	115	115	80	0	56	41	72	124	102	80	40	66	100	124
4	170	165	125	56	0	41	85	145	145	120	88	121	151	178
5	148	155	120	41	41	0	45	105	105	80	55	96	120	153
6	145	170	142	72	85	45	0	60	75	53	53	98	103	145
7	159	198	181	124	145	105	60	0	63	60	91	123	104	150
8	98	142	135	102	145	105	75	63	0	25	60	72	40	88
9	101	138	123	80	120	80	53	60	25	0	40	65	50	95
10	95	115	91	40	88	55	53	91	60	40	0	45	64	98
11	50	75	64	66	121	96	98	123	72	65	45	0	46	58
12	58	105	106	100	151	120	103	104	40	50	64	46	0	47
13	20	75	95	124	178	153	145	150	88	95	98	58	47	0

Table 2. Delivery and pick-up demand for the nodes

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Delivery	0	20	25	15	40	20	10	30	30	25	20	15	20	20	290
Pick-up	0	30	30	15	30	15	15	20	35	10	15	15	30	30	295

Table 3. Distance Matrix for the graph

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	6	8	12	17	15	15	16	10	10	10	5	6	2
1	6	0	4	12	17	16	17	20	14	14	12	8	11	8
2	8	4	0	8	13	12	14	18	14	12	9	6	11	10
3	12	12	8	0	6	4	7	12	10	8	4	7	10	12
4	17	17	13	6	0	4	9	15	15	12	9	12	15	18
5	15	16	12	4	4	0	5	11	11	8	6	10	12	15
6	15	17	14	7	9	5	0	6	8	5	5	10	10	15
7	16	20	18	12	15	11	6	0	6	6	9	12	10	15
8	10	14	14	10	15	11	8	6	0	3	6	7	4	9
9	10	14	12	8	12	8	5	6	3	0	4	7	5	10
10	10	12	9	4	9	6	5	9	6	4	0	5	6	10
11	5	8	6	7	12	10	10	12	7	7	5	0	5	6
12	6	11	11	10	15	12	10	10	4	5	6	5	0	5
13	2	8	10	12	18	15	15	15	9	10	10	6	5	0

Table 4. Delivery and pick-up demand for the nodes

Node	0	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Delivery	0	25	30	20	45	25	15	35	35	30	25	20	25	25	355
Pick-up	0	35	35	20	35	20	20	25	40	15	20	20	35	40	360

Table 5: Comparison of MILP with different heuristics for Data Set of Min (1989)

Data-set	N	M			D			MILP			RD from M	RD from D
		V	S	CPU	V	S	CPU	V	S	CPU		
1	22	2	94	18	2	91	-	2	88	6	-6.38	-3.30

Legends:

- N** : Number of nodes
- M** : Heuristics of Min (1989)
- D** : Heuristic of Dethloff (2001)
- MILP** : Mixed Integer Linear Programming Model
- V** : Number of vehicles
- S** : Solution
- CPU** : Computing times
- RD** : Relative % deviation

Table 6. MILP Model Solution for Randomly Generated VRPSDP Data-sets with Maximum Route Length

Data-set	N	MILP Approach
VSDP11	7	94
VSDP12	7	142
VSDP13	7	166
VSDP14	7	212
VSDP21	9	110
VSDP22	9	212

VSDP23	9	234
VSDP24	9	304
VSDP31	11	174
VSDP32	11	220
VSDP33	11	280
VSDP34	11	320
VSDP41	13	180
VSDP42	13	224
VSDP43	13	366
VSDP44	13	456
VSDP51	15	264
VSDP52	15	321
VSDP53	15	423
VSDP54	15	512