# 修 士 論 文 の 和 文 要 旨

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| 研究科・専攻 | 大学院　情報理工　学研究科　　情報．通信工学　　専攻　博士前期課程 | | |
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| 論 文 題 目 | Channel Estimation for SC-FDMA in LTE Uplink using Data-Domain Pilot Signals and B-Spline Approximation<br>データ領域パイロット信号と B スプライン近似を用いた LTE 上り回線のための通信路推定 | | |

要　　旨

LTE アップリンク上でシングルキャリア周波数分割多元接続（SC-FDMA）において、通信路速い変動があると間干渉（ICI）が生ずる。そのため、チャネル推定は、LTE アップリンクシステムの堅牢な性能を得るために、特別な注意が必要である。

通信路推定のために、我々は、高速フェージング下での SC-FDMA システムにおけるデータ領域におけるパイロット信号の挿入を提案する。我々は、通信路推定のために支援するためのデータ領域で、パイロット信号を使用して数学的モデルを開発した。

移動体通信では、ドップラー効果がよく Rayleigh fading のモデルによって記述さるが、このモデルは 3 次 B-スプライン関数で近似することができることが知られている。そこで、LTE-アップリンクにおける通信路推定を支援するためにデータ領域内のパイロット信号と 3 次 B-スプライン近似適用を提案する。

コンピュータシミュレーションは、他の方式では、ツイスト-OFDMA で実施されている。これは、パイロットが挿入された提案されたスキーム B スプライン近似の性能を示すために行わ。結果は、MMSE チャネル推定による SC-FDMA システムとの性能を比較した。ドップラースプレッドに対する弱点に、正規化されたドップラー周波数が増加するにつれて、SC-FDMA の性能低下となる。

結果は、B スプライン近似が SC-FDMA システムにおいて実現することが非常に有望な候補であることを示している。私たちは、今後の研究への SC-FDMA システムにおける B スプライン近似の C モデルの実装のままにしておきます。

# Channel Estimation for SC-FDMA in LTE Uplink using Data-Domain Pilot Signals and B-Spline Approximation


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A DISSERTATION PRESENTED TO THE
DEPARTMENT OF COMMUNICATION ENGINEERING AND INFORMATICS
IN CANDIDACY FOR THE MASTER'S DEGREE

RECOMMENDED FOR ACCEPTANCE BY THE
DEPARTMENT OF COMMUNICATION ENGINEERING AND INFORMATICS

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JANUARY 30, 2015

## Abstract

In Single Carrier Frequency Division Multiple Access (SC-FDMA) on the LTE uplink, fast variations of channel result in inter-carrier-interference(ICI). Therefore, the channel estimation needs a special care in order to robust performance of LTE uplink system.

In order to estimate the channel, we propose pilot signals insertion in data-domain, for design of SC-FDMA systems under fast fading. We developed a mathematical model which uses pilot signals in data domain to aid for channel estimation.

In mobile communication the Doppler effect is well described by Rayleigh fading model, and this model can be accurately approximated by the cubic B-spline function. Therefore, we propose cubic B-spline approximation and pilot signals in data domain to aid for channel estimation in LTE-Uplink.

The computer simulations are conducted in other scheme, Twisted-OFDMA. This done to illustrate the performance of the proposed scheme B-spline approximation with pilot insertion. The results then compared to the performance with SC-FDMA system with MMSE channel estimation. Due to its weakness against Doppler spread, as the normalized Doppler frequency increased, so become the degradation in the performance of SC-FDMA.

The result shows that B-Spline approximation is very promising candidate to be implemented in SC-FDMA system. We leave the C model implementation of B-Spline approximation in SC-FDMA system to future research.

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# Chapter 1

# Introduction

LTE Uplink uses SC-FDMA to keep a low peak to average power ratio (PAPR). Since the radio channel is highly dynamic, the transmitted signal travel by undergoing many effects that corrupt the signals and lower the performance of the system. SC-FDMA has similar throughput performance and complexity as OFDMA and is highly sensitive the Doppler spread of the channel [1],[2]. Doppler spread caused frequency offset which leads to Inter Carrier Interference (ICI) [3] and results in the degradation of bit error rate (BER). Channel estimation techniques allow the receiver to approximate the impulse response of the channel, therefore choosing the best channel estimation technique is essential in order to robust performance of LTE uplink system.

There have been several researches concerning in improving channel estimation in LTE uplink system over fast fading channel [3]-[7]. In [3] and [4], time domain Kalman filter is introduced to mitigate ICI in high mobility environment, but this only applied in single user system, and also these techniques involve high complexity. Thus in [7] channel estimation is conducted using Basis Expansion Model (BEM) which easy to use and no need of prior channel statics if compared to Kalman filter, however for mobility more than 350 km/h, least square proposition has no solution. Zhang in [6] shows that the poor performance in LTE uplink is mainly caused by limited time-domain pilot symbols. Zhang in [6] also describes that using cubic spline interpolation for channel estimation improve bit error rate (BER) in fast fading channel.

In mobile communication the Doppler effect which causes time-varying fading of multipath components, is well described by Clarke's model. This model can be accurately approximated by the cubic B-spline function [8]. Therefore we propose cubic B-spline approximation and pilot signals in data domain to aid for channel estimation in LTE-Uplink.

The rest of the paper is organized as follows:

**Chapter 2** introduces SC-FDMA system model and the performance over fast fading channels. Theoretical analyses are conducted to help understand the impacts of Doppler spreads in SC-FDMA system for determining optimal solutions.

**Chapter 3** presents the theory of cubic B-spline approximation in pilot signals for channel estimation in SC-FDMA system, and the mathematical solution of equivalent channel is presented.

**Chapter 4** shows the performance of cubic B-spline approximation in Twisted-OFDMA to illustrate the proposed scheme, and then the performance of SC-FDMA with MMSE channel estimation is conducted and compared to that of the Twisted-OFDMA performance.

**Chapter 5** conludes the thesis and gives study for the further research.

# Chapter 2

# SC-FDMA in LTE Uplink

SC-FDMA is utilized in LTE uplink which has similarity multiple access scheme with the downlink, has some desirable attributes, such as orthogonal uplink transmission by different user, flexibility to support a wide range of data ranges, sufficiently low Peak to Average Power Ratio (PAPR), exploit the frequency diversity even at low data rates transmission, etc [9]. In order to estimate the channel, we propose pilot signals insertion in data-domain, for design of SC-FDMA systems under fast fading.

## 2.1 SC-FDMA System Model

Transmitter and receiver structure in LTE Uplink is shown in Fig. 2.1. The transmitted signal is distorted by multipath propagation in the channel and is received together with an additive white Gaussian noise (AWGN). In this research we only focused on single antenna case. Assuming an $N$-point DFT for spreading the $u$th users time domain signal $d(n)$ into frequency domain

$$D_{(u)}(\kappa) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{(u)}(n) e^{\frac{-j2\pi n\kappa}{N}}, \tag{2.1}$$

After spreading, then $D_{(u)}(\kappa)$is mapped onto the $k^{th}$ subcarrier $S_{(u)}(k)$ given by

$$S_{(u)}(k) = \begin{cases} D_{(u)}(\kappa), & k = \Gamma_{N,(u)}(\kappa) \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

where $\Gamma_{N,(u)}(\kappa)$ denotes $N$-element mapping set of $u^{th}$user. Consecutively $\Gamma_{N,(u)}(\kappa) = \kappa + uN$ it called localized mapping, and if it is not consecutive $\Gamma_{N,(u)}(\kappa) = \kappa U + u$, it called distributed mapping.

Using $M$-point IDFT, where $M$ is the minimum power of 2 and larger than $N$, the transmitted signal is then expressed by

$$s_{(u)}(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} S_{(u)}(k) e^{\frac{j2\pi mk}{M}}, \tag{2.3}$$

Assuming a quasi-static multipath fading channel with CIR $\mathbf{h} = [h_0, h_1, \cdots, h_L]^{\mathrm{T}}$, the signal received $r(m)$, at the output of the channel after CP removal is given by

$$r(m) = \sum_{u=0}^{U-1} \sum_{\ell=0}^{L-1} h_{(u)}(m,\ell) s_{(u)}(m-\ell) + w(m) \tag{2.4}$$

where $h_{(u)}(m,\ell)$ is sample spaced channel response of $\ell^{th}$ path during time $m$ of $u^{th}$ user, and $w(m)$ denotes the white Gaussian noise. From (2.3) and (2.4) we can get

$$\begin{aligned} r(m) &= \sum_{u=0}^{U-1} \sum_{\ell=0}^{L-1} \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} S_{(u)}(k) h_{(u)}(m,\ell) e^{\frac{j2\pi k(m-\ell)}{M}} + w(m) \\ &= \frac{1}{\sqrt{M}} \sum_{u=0}^{U-1} \sum_{k=0}^{M-1} S_{(u)}(k) H_{(u)}(k,m) e^{\frac{j2\pi km}{M}} + w(m) \end{aligned} \tag{2.5}$$

where

$$H(k,m) = \sum_{\ell=0}^{L-1} h(m,\ell) e^{\frac{-j2\pi k\ell}{M}} \tag{2.6}$$

The FFT output at $k^{th}$ subcarrier can be expressed as

$$
\begin{aligned}
R(k) &= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} r(m) e^{-j\frac{2\pi mk}{M}} \\
&= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \left( \frac{1}{\sqrt{M}} \sum_{u=0}^{U-1} \sum_{k'=0}^{M-1} S_{(u)}(k') H_{(u)}(k',m) e^{\frac{j2\pi k'm}{M}} + w(m) \right) e^{\frac{-j2\pi mk}{M}} \\
&= \frac{1}{M} \sum_{m=0}^{M-1} \sum_{u=0}^{U-1} \sum_{k'=0}^{M-1} S_{(u)}(k') H_{(u)}(k',m) e^{\frac{j2\pi m(k'-k)}{M}} \\
&\qquad + \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} w(m) e^{\frac{-j2\pi mk}{M}} \\
&= \frac{1}{M} \sum_{u=0}^{U-1} S_{(u)}(k) \sum_{m=0}^{M-1} H_{(u)}(k,m) \\
&\qquad + \frac{1}{M} \sum_{u=0}^{U-1} \sum_{k'\neq k, k'=0}^{M-1} S_{(u)}(k') \sum_{m=0}^{M-1} H_{(u)}(k',m) e^{\frac{j2\pi m(k'-k)}{M}} \\
&\qquad\qquad + \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} w(m) e^{\frac{-j2\pi mk}{M}} 
\end{aligned} \tag{2.7}
$$

Data obtained after subcarrier demapping block with $k = \kappa + uN$ for localized mapping and $k = \kappa U + u$ for interleaved mapping, is defined by

$$
\begin{aligned}
\hat{D}_{(u)}(\kappa) &= R(k) \\
&= \frac{1}{M} \sum_{u'=0}^{U-1} S_{(u')}(k) \sum_{m=0}^{M-1} H_{(u')}(k,m) \\
&\qquad + \frac{1}{M} \sum_{k'\neq k, k'=0}^{M-1} S_{(u)}(k') \sum_{m=0}^{M-1} H_{(u)}(k',m) e^{\frac{j2\pi m(k'-k)}{M}} \\
&\qquad + \frac{1}{M} \sum_{u'\neq u, u'=0}^{U-1} \sum_{k'\neq k, k'=0}^{M-1} S_{(u')}(k') \sum_{m=0}^{M-1} H_{(u')}(k',m) e^{\frac{j2\pi m(k'-k)}{M}} \\
&\qquad + \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} w(m) e^{\frac{-j2\pi mk}{M}}
\end{aligned} \tag{2.8}
$$

We can also write equ. (2.8) as

$$
\hat{D}_{(u)}(\kappa) = D_{(u)}(\kappa) H_{(u)}(k,m) + I_{MUI}(k) + W(k) \tag{2.9}
$$

where $H_{(u)}(k,m)$ represents frequency domain channel response, $I_{ISI}(k)$and $I_{MUI}(k)$ represents inter-symbol interference and multi-user interference respectively, and $W(k)$ as is Fourier transform of noise $w(m)$.

After N-point IDFT, data for each user can be expressed by

$$\begin{aligned}
\hat{d}_{(u)}(n) &= \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}\hat{D}_{(u)}(\kappa)e^{\frac{j2\pi n\kappa}{N}} \\
&= \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}e^{\frac{j2\pi n\kappa}{N}}\left(D_{(u)}(\kappa)H_{(u)}(k,m)+I_{MUI}(k)+W(k)\right) \\
&= \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}e^{\frac{j2\pi n\kappa}{N}}D_{(u)}(\kappa)H(k)+\frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}e^{\frac{j2\pi n\kappa}{N}}(I_{MUI}(k)+W(k)) \qquad (2.10)
\end{aligned}$$

let the inverse Fourier of inter-symbol interference, the inverse Fourier of multi-user interference, and noise in data domain as

$$\imath_{MUI}(n) = \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}I_{MUI}(k)e^{\frac{j2\pi n\kappa}{N}}$$

$$w(n) = \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}W(k)e^{\frac{j2\pi n\kappa}{N}}$$

then from (2.1) and (2.6) we get

$$\begin{aligned}
\hat{d}_{(u)}(n) &= \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}e^{\frac{j2\pi n\kappa}{N}}D_{(u)}(\kappa)H(k)+\imath_{MUI}(n)+w(n) \\
&= \frac{1}{\sqrt{N}}\sum_{\kappa=0}^{N-1}e^{\frac{j2\pi n\kappa}{N}}\left(\frac{1}{\sqrt{N}}\sum_{n'=0}^{N-1}d_{(u)}(n')e^{-\frac{j2\pi n'\kappa}{N}}\right)\left(\sum_{\ell=0}^{L-1}h(m,\ell)e^{-\frac{j2\pi kl}{M}}\right) \\
&\qquad + \imath_{MUI}(n)+w(n) \\
&= \frac{1}{N}\sum_{n'=0}^{N-1}d_{(u)}(n')\sum_{\kappa=0}^{N-1}e^{-\frac{j2\pi\kappa(n'-n)}{N}}\sum_{\ell=0}^{L-1}h(m,\ell)e^{-\frac{j2\pi kl}{M}}+\imath_{MUI}(n)+w(n) \\
&= d_{(u)}(n')\hat{h}(n,n')+\imath_{MUI}(n)+w(n) \qquad (2.11)
\end{aligned}$$

where we let

$$\hat{h}(n,n') = \frac{1}{N}\sum_{\kappa=0}^{N-1}e^{-\frac{j2\pi\kappa(n'-n)}{N}}\sum_{\ell=0}^{L-1}h(m,\ell)e^{-\frac{j2\pi kl}{M}}, \qquad (2.12)$$

as apparent channel response.

## 2.2 Pilot Aided System Model

For channel estimation purposes, pilot is added after constellation mapping block (see Figure 2.2) with $N_p$ as number of pilots in a symbol, and $\lambda$ as length of pilot guards, to avoid mutual interference between data and pilot.

In cubic b-spline interpolation there is a need for equally distance knots (pilots). Then the pilot $d_p(n)$ with $p = (0, 1, 2, ..., N_p - 1)$ are spreaded by

$$d_p(n) = b_p, \tag{2.13}$$

Then, the number of data in a block (symbol) of SC-FDMA is

$$N_d = N - N_p - \lambda, \tag{2.14}$$

where $N$ is number of point DFT in the following block. Then, the position of pilots in a symbol can be shown in the Figure 2.3.

Assuming that with an appropriate pilot guard $\lambda$, there are no interference between the pilot and the data signals, the received pilot of the $\xi$th SC-FDMA symbol (for $\xi = 0, 1, ..., B - 1$) is given by

$$\begin{aligned} \hat{d}_p^{\xi} &= d_{p,(u)}(n')\hat{h}(n, n') + \imath_{MUI}(n) + w(n) \\ &= b_p\hat{h}(n, n') + z_m, \end{aligned} \tag{2.15}$$

where we let $z_m = \imath_{MUI}(n) + w(n)$.

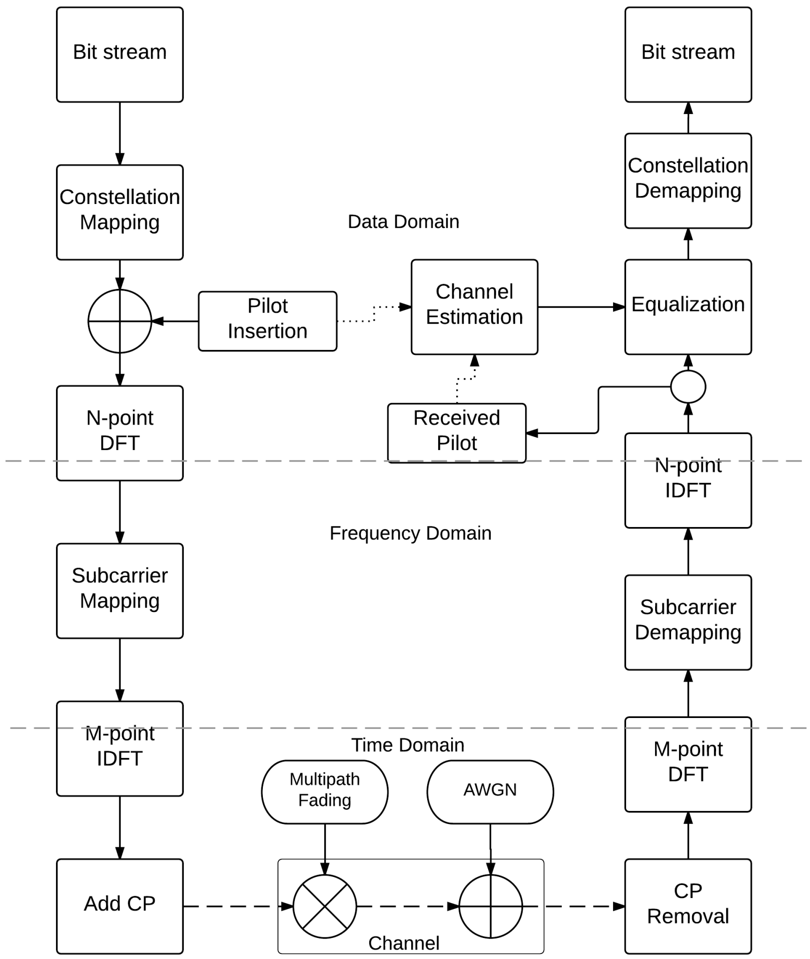


Figure 2.1: Uplink transmission - receiver model for LTE

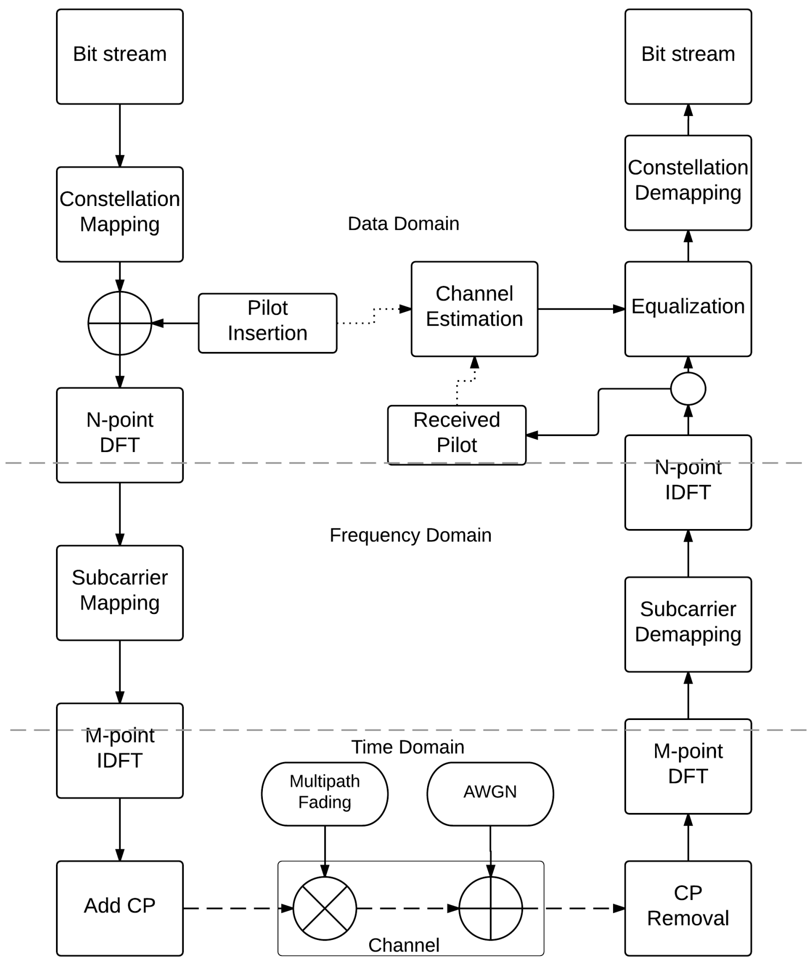


Figure 2.2: LTE Uplink Transmitter - Receiver Block

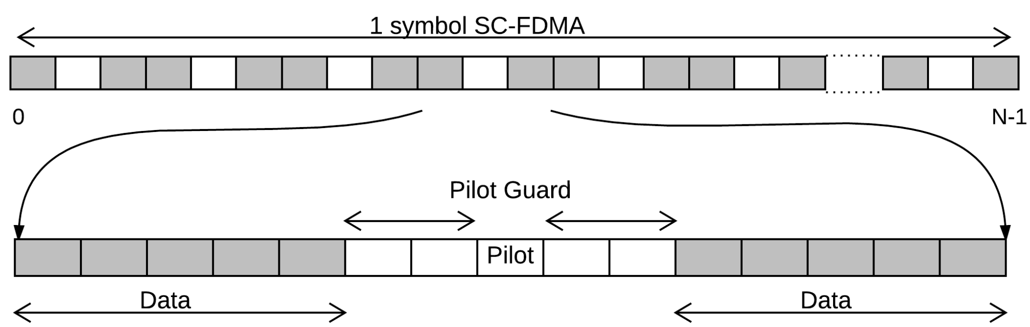


Figure 2.3: Pilot Insertion

# Chapter 3

# Channel Estimation

In mobile communication the Doppler effect which causes time-varying fading of multipath components, is well described by Clarke's model. This model can be accurately approximated by the cubic B-spline function [8]. We propose channel estimation using pilot signals and cubic B-spline approximation for SC-FDMA system in LTE uplink.

## 3.1 Response Model

To estimate the inter-shift response $\hat{h}(n, n')$, first we estimate the physical channel impulse response $h(m, \ell)$ which is approximated by a series of cubic B-splines

$$h_{m,\ell} = \sum_{\gamma=0}^{B+2} a_{\ell,\gamma} \varphi_\gamma(m), \tag{3.1}$$

where $\varphi_\gamma(m)$ denotes the B-spline basis located at knot $\gamma$. The whole transmission duration $BN_s$ is divided into $B$ intervals by equally-distanced knots $\varepsilon_\gamma$, $\gamma = 0, 1, \cdots, B-2$, and 4 additional knots, $\varepsilon_{B-1}, \cdots, \varepsilon_{B+2}$, are introduced in order to generate the necessary B-spline basis (note that in each spline base function, there are at most four polynomial terms (segment)). $\varphi_\gamma(m)$ can be calculated using the recurrence relations, and for $\gamma = 3, 4, \cdots, B-1$, we have

$$\varphi_\gamma(m) = \varphi(m - \gamma N_s + 3N_s) \tag{3.2}$$

for

$$\varphi(m) = \begin{cases} \frac{m^3}{6N_s^3}, & 0 \le m < N_s \\ -\frac{m^3}{2N_s^3} + \frac{2m^2}{N_s^2} - \frac{2m}{N_s} + \frac{2}{3}, & N_s \le m < 2N_s \\ \frac{m^3}{2N_s^3} - \frac{4m^2}{N_s^2} + \frac{10m}{N_s} - 7\frac{1}{3}, & 2N_s \le m < 3N_s \\ \frac{(4N_s-m)^3}{6N_s^3}, & 3N_s \le m < 4N_s \\ 0, & \text{otherwise} \end{cases} . \tag{3.3}$$

where $N_s = M+L$ is symbol length of SC-FDMA including CP. The first three B-spline base functions which are prototype base functions, are given by [11]

$$\varphi_0(m) = \begin{cases} \frac{(N_s-m)^3}{N_s^3}, & 0 \le m < N_s \\ 0, & \text{otherwise} \end{cases} , \tag{3.4}$$

$$\varphi_1(m) = \begin{cases} \frac{7m^3}{4N_s^3} - \frac{9m^2}{2N_s^2} + \frac{3m}{N_s}, & 0 \le m < N_s \\ \frac{(2N_s-m)^3}{4N_s^3}, & N_s \le m < 2N_s \\ 0, & \text{otherwise} \end{cases} , \tag{3.5}$$

$$\varphi_2(m) = \begin{cases} -\frac{11m^3}{12N_s^3} + \frac{3m^2}{2N_s^2}, & 0 \le m < N_s \\ \frac{7m^3}{12N_s^3} - \frac{3m^2}{N_s^2} + \frac{9m}{2N_s} - \frac{3}{2}, & N_s \le m < 2N_s \\ \frac{(3N_s-m)^3}{6N_s^3}, & 2N_s \le m < 3N_s \\ 0, & \text{otherwise} \end{cases} , \tag{3.6}$$

and the last three base functions are given as

$$\begin{aligned} \varphi_B(m) &= \varphi_2(BN_s - M), \\ \varphi_{B+1}(m) &= \varphi_1(BN_s - M), \\ \varphi_{B+2}(m) &= \varphi_0(BN_s - M), \end{aligned} \tag{3.7}$$

$$\varphi_B(m) = \begin{cases} \frac{(m-(B-3)N_s)^3}{6N_s^3}, & (B-3)N_s \le m < (B-2)N_s \\ -\frac{7(m-(B-2)N_s)^3}{12N_s^3} + \frac{(m-(B-2)N_s)^2}{2N_s^2} + \frac{m-(B-2)N_s}{2N_s} + \frac{1}{6}, & (B-2)N_s \le m < (B-1)N_s \\ \frac{(11(m-(B-1)N_s)+7N_s)(N_s-m+(B-1)N_s)^2}{12N_s^3}, & (B-1)N_s \le m < BN_s \\ 0, & \text{otherwise} \end{cases}, \tag{3.8}$$

$$\varphi_{B+1}(m) = \begin{cases} \frac{(m-(B-2)N_s)^3}{4N_s^3}, & (B-2)N_s \le m < (B-1)N_s \\ -\frac{7(m-(B-1)N_s)^3}{4N_s^3} + \frac{3(m-(B-1)N_s)^2}{4N_s^2} + \frac{3(m-(B-1)N_s)}{4N_s} + \frac{1}{4}, & (B-1)N_s \le m < BN_s \\ 0, & \text{otherwise} \end{cases}, \tag{3.9}$$

$$\varphi_{B+2}(m) = \begin{cases} \frac{(m-(B-1)N_s)^3}{N_s^3}, & (B-1)N_s \le m \le BN_s \\ 0, & \text{otherwise} \end{cases}. \tag{3.10}$$

For the pilot signal in symbol $\xi = 0, 1, ..., B-1$, the channel output at the receiver with CP removed is

$$\begin{aligned} r_p^{\xi}(m) &= \sum_{\ell=0}^{L-1} h_{\ell,t} s_p(m-\ell) + z_p \\ &= \sum_{\ell=0}^{L-1} h_{\ell,m+\xi N_s+L} s_p(m-\ell) + z_p \end{aligned} \tag{3.11}$$

where we define $t = \ell, m + \xi N_s + L$. Then, the corresponding matched filter output is

$$\begin{aligned} r_p^{\xi} &= b_p e^{-j\frac{2\pi pk}{N}} \sum_{\ell=0}^{L} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{m=0}^{M-1} h_{\ell,m+\xi N_s+L} + z_p \\ &= b_p e^{-j\frac{2\pi pk}{N}} \sum_{\ell=0}^{L} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{\gamma=0}^{B+2} a_{\ell,\gamma} \sum_{m=0}^{M-1} \varphi_{\gamma}(m+\xi N_s+L) + z_p \\ &= b_p e^{-j\frac{2\pi pk}{N}} \sum_{\ell=0}^{L} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{d=0}^{3} a_{\ell,\xi+d} \sum_{m=0}^{M-1} \varphi_{\xi+d}(m+\xi N_s+L) + z_p \\ &= b_p \sum_{\ell=0}^{L} \sum_{d=0}^{3} a_{\ell,\xi+d} \beta_{\ell,d}^{\xi}(p) + z_p \end{aligned} \tag{3.12}$$

where we define

$$\beta_{\ell,d}^{\xi}(p) = e^{-j\frac{2\pi pk}{N}} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{m=0}^{M-1} \varphi_{\xi+d}(m + \xi N_s + L) \tag{3.13}$$

Employing the prototype base function in (3.2), we can express (3.13) based on the prototype base functions where

$$\beta_{\ell,d}^{\xi}(p) = e^{-j\frac{2\pi pk}{N}} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{m=0}^{M-1} \varphi_{\xi+d}(m + \xi N_s + L) \tag{3.14}$$

for $0 \leq \xi + d \leq 3$,

$$\beta_{\ell,d}^{\xi}(p) = e^{-j\frac{2\pi pk}{N}} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{m=0}^{M-1} \varphi(m + [3-d]N_s + L) \tag{3.15}$$

for $3 \leq \xi + d \leq B$, and

$$\beta_{\ell,d}^{\xi}(p) = e^{-j\frac{2\pi pk}{N}} e^{j\frac{2\pi k(m-\ell)}{M}} \sum_{m=0}^{M-1} \varphi_{\xi+d}(m + [3-d]N_s + L) \tag{3.16}$$

for $B \leq \xi + d \leq B + 3$.

## 3.2 Vector Matrix Model

Let $\mathbf{r}_p^{\xi} = \left[r_{p1,\zeta_s}^{\xi}, r_{p2,\zeta_s}^{\xi}, \cdots, r_{p1,\zeta_e}^{\xi}, r_{p2,\zeta_e}^{\xi}\right]^{\mathrm{T}}$, shows the pilot in $\xi$th symbol of SC-FDMA has output from $\zeta_s$ to $\zeta_e$, we can introduce the following vectors.

$$\begin{aligned}
\mathbf{r} &= \left[\mathbf{r}_p^{0\mathrm{T}}, \mathbf{r}_p^{1\mathrm{T}}, \cdots, \mathbf{r}_p^{B-1\mathrm{T}}\right]^{\mathrm{T}}, \\
\mathbf{a}_{\ell} &= \left[a_{\ell,0}, a_{\ell,1}, \cdots, a_{\ell,B+2}\right]^{\mathrm{T}}, \\
\mathbf{a} &= \left[\mathbf{a}_0^{\mathrm{T}}, \mathbf{a}_2^{\mathrm{T}}, \cdots, \mathbf{a}_L^{\mathrm{T}}\right]^{\mathrm{T}}
\end{aligned} \tag{3.17}$$

then, with the response matrix $\mathbf{\Psi}$ we can express $\mathbf{y}$ as

$$\mathbf{r} = \mathbf{\Psi}\mathbf{a} + \mathbf{z} \tag{3.18}$$

where $\mathbf{\Psi} = [\mathbf{\Psi}_0 \mathbf{\Psi}_1 \cdots \mathbf{\Psi}_L]$ yields $L$ path response matrix.

From this expression, the maximum likelihood (ML) estimator is given by

$$\hat{\mathbf{a}} = \left(\mathbf{\Psi}^{\mathrm{H}} \mathbf{\Psi}\right)^{-1} \mathbf{\Psi}^{\mathrm{H}} \mathbf{r}. \tag{3.19}$$

## 3.3 Equivalent Channel

Since the calculation of apparent channel response $\hat{h}(n, n')$ is difficult to obtain, therefore as stated before we estimate the physical channel impulse response $h_{\ell,m}$ then calculate $\hat{h}(n, n')$ from the estimated $h_{\ell,m}$. From equation (2.12) and (3.1) we can express

$$\hat{h}(n, n') = \frac{1}{N} \sum_{\kappa=0}^{N-1} e^{-\frac{j2\pi\kappa(n'-n)}{N}} \sum_{\ell=0}^{L-1} \sum_{d=0}^{3} \hat{a}_{\ell,d+3} \varphi_{d+3}(m + \xi N_s + L) e^{-\frac{j2\pi kl}{M}}, \tag{3.20}$$

The apparent channel response is obtained by using pilot signals which transmitted in data domain parallel with data signals. With this proposed scheme, the mathematical solution of the apparent channel response is presented.

# Chapter 4

# Performance Evaluation

The performance of B-spline approximation of pilot signals which modeled in 3 has been developed in Twisted-OFDMA [12]. In this chapter we present the performance of B-Spline approximation in Twisted-OFDMA, compared to SC-FDMA with MMSE channel estimation.

## 4.1 Simulation Setups

The performance of the proposed B-Spline approximation is evaluated using pilot signals in Twisted-OFDMA. This is done, in order to gives an overview of B-spline approximation capability in estimating fast-fading channel transmission. Twisted-OFDMA itself is a scheme, that is introduced in [12] to suppress the effect of Doppler Spread. The transmitted signal of Twisted-OFDMA is solely an OFDM signal, which are "twisted" over the full frequency band.

The performance characteristic of proposed B-Spline approximation in Twisted-OFDMA then compared with SC-FDMA system, which uses MMSE channel estimation. Bit Error Rate (BER) to SNR characteristic of both systems is compared in various Doppler frequency with various number of users. The system parameter for both Twisted-OFDM and SC-FDMA is presented in Table 4.1.

Table 4.1: Simulation parameters

| Parameters | Twisted-OFDMA | SC-FDMA |
|---|---|---|
| Carrier Frequency | $f_c$ =2.0 GHz | |
| Chip Rate | $R_c$ =3.84 Mcps | |
| Chip Duration | $T_c$ =260.417 ns | |
| Number of Pilot | $N_p^{\xi}$ =8 | |
| Channel Estimation | B-Spline Approx. | MMSE |
| Normalized Doppler Freq. | $f_D T$ =0, 0.05, 0.1, 0.2, 0.3 | |
| Number of User | 7, 14 | |

## 4.2 Simulation Result

Here, computer simulations are conducted to illustrate the performance of the proposed scheme B-spline approximation with pilot insertion in Twisted-OFDMA, and to compare the performance with SC-FDMA system with MMSE channel estimation. Figure 4.1 compares bit error rate (BER) performance in normalized Doppler frequency $f_D T = 0.05$. We can see that the performance of SC-FDMA with MMSE channel estimation is inferior if compared to the performance of Twisted-OFDMA with B-Spline approximation. The increase of number of user in SC-FDMA is effecting to lower performance due to the Doppler spread. However in Twisted-OFDMA, the increase of the number of user seems does not infecting the performance.

In Figure 4.2, Figure 4.3, and Figure 4.4 respectively, shows the BER performance comparison with increasing of normalized Doppler frequency for $f_D T = 0.1$, $f_D T = 0.2$, and $f_D T = 0.3$. Clearly the performance of Twisted-OFDMA is stable in each normalized Doppler frequency with various number of user. However for SC-FDMA we can see the degradation of performance, especially for number of user $N_u ser = 7$, starting from normalized frequency $f_D T = 0.2$.

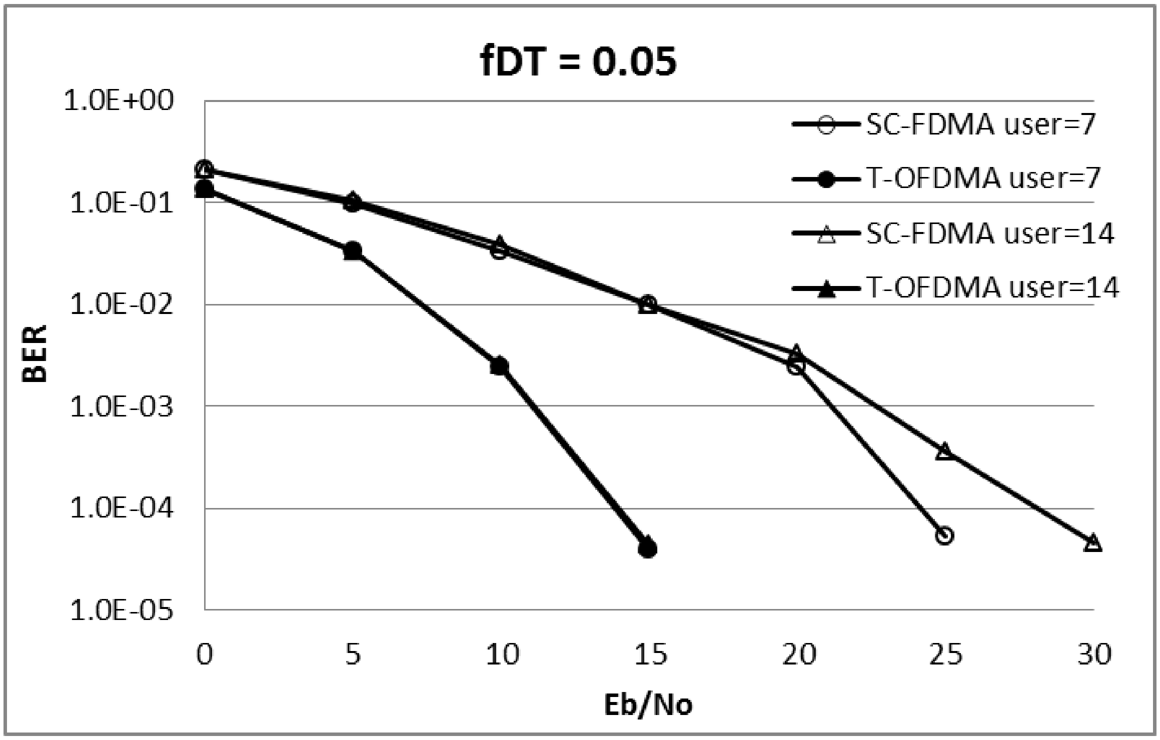


Figure 4.1: BER performance comparison of the Twisted-OFDMA system and the SC-FDMA system for user = 7, 14, $f_{dt} = 0.05$

From the performance in Twisted-OFDMA system, it can be concluded that, B-Spline approximation method using pilot signals can be used as channel estimator in system with fast fading channel transmission. Such system is not limited to Twisted-OFDMA. Other system, such as SC-FDMA also can take advantage of B-Spline approximation method to deal with fast fading channel transmission.

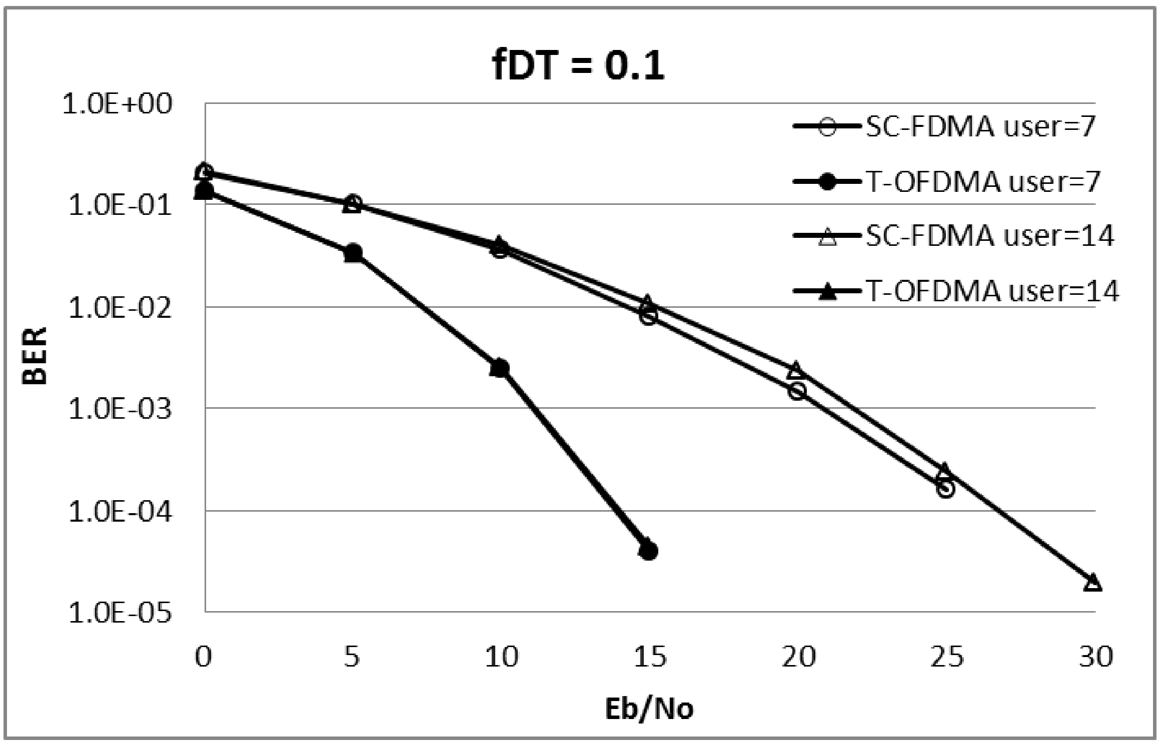


Figure 4.2: BER performance comparison of the Twisted-OFDMA system and the SC-FDMA system for user = 7, 14, $f_{dt} = 0.1$

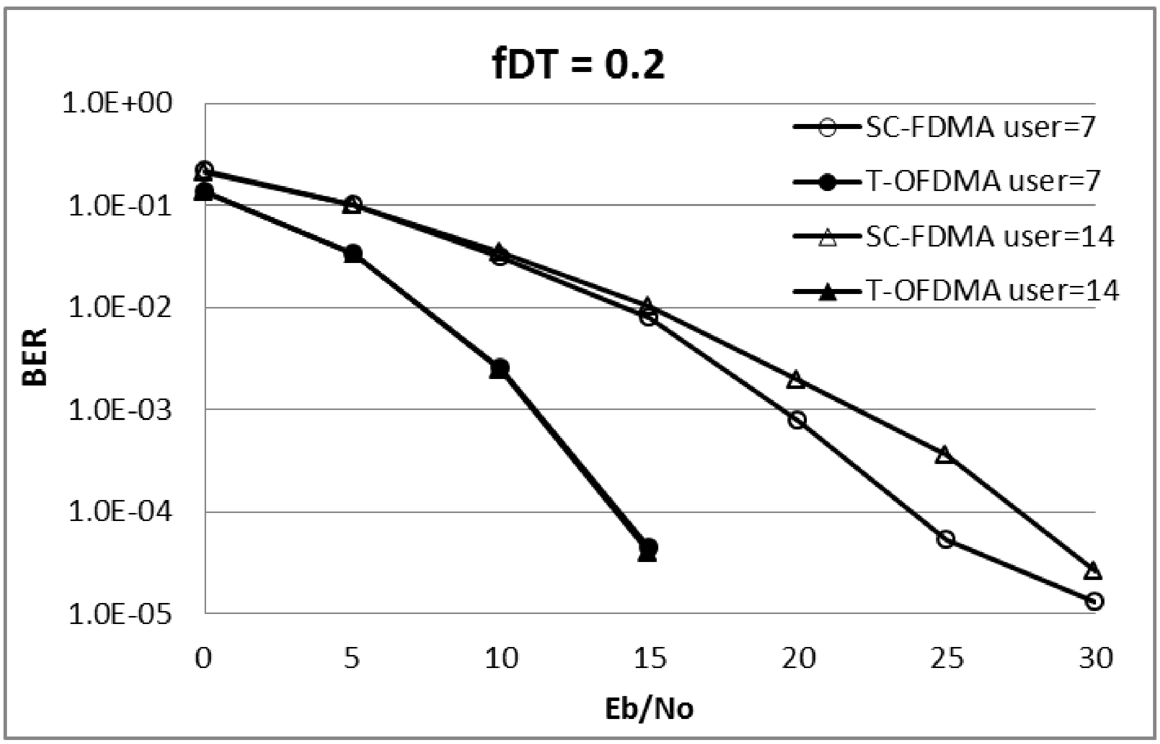


Figure 4.3: BER performance comparison of the Twisted-OFDMA system and the SC-FDMA system for user = 7, 14, $f_{dt} = 0.2$

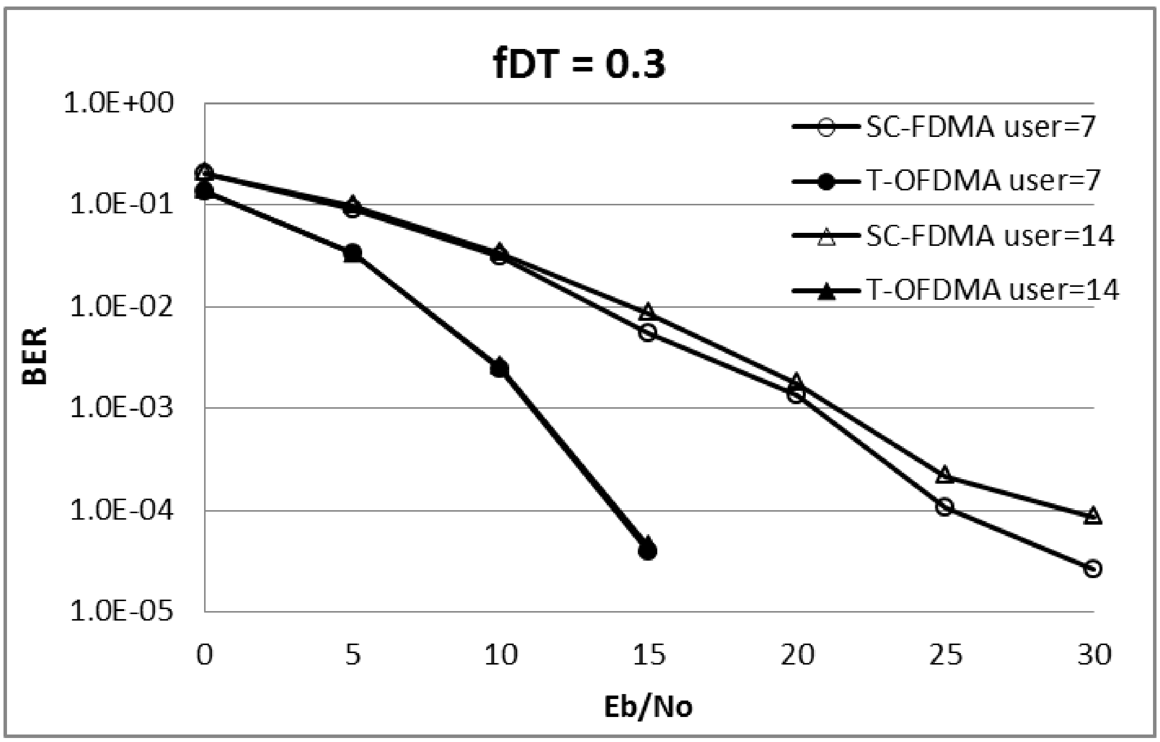


Figure 4.4: BER performance comparison of the Twisted-OFDMA system and the SC-FDMA system for user = 7, 14, $f_{dt} = 0.3$

# Chapter 5

# Conclusion

This paper has proposed a scheme to implement B-Spline approximation method as channel estimator in SC-FDMA system. We developed a mathematical model of B-Spline approximation for SC-FDMA system. This mathematical model uses pilot signal in data domain to aid for channel estimation. The calculation of apparent channel response is difficult to obtain, therefore we estimate the physical channel impulse response using B-Spline approximation then calculate the apparent channel response.

We also introduced C model simulation of B-Spline approximation in Twisted-OFDMA system. It showed that B-Spline approximation able to estimate fast fading channel channel transmission. We compare our result with existing SC-FDMA C model simulation, which uses MMSE as chnanel estimator. From the simulation we can see that the performance of SC-FDMA with MMSE channel estimation is inferior if compared to the performance of Twisted-OFDMA with B-Spline approximation, due to its weakness against Doppler spread. As the normalized Doppler frequency increased, so become the degradation in the performance of SC-FDMA.

The result shows that B-Spline approximation is very promising candidate to be implemented in SC-FDMA system. We leave the C model implementation of B-Spline approximation in SC-FDMA system to future research.

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