# Identifying subgroups of CERME affect research papers

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*Research in mathematics related affect uses a variety of theoretical frameworks. Three different dimensions have been suggested as significant to characterize concepts in this area: (1) emotional, motivational, and cognitive aspects of affect, (2) state and trait aspects of affect, and (3) physiological, psychological, and sociological level of theorizing affect. In this study, we used the information in reference lists and graph theory to identify Graph Communities (coherent clusters) of research papers published in the affect groups of CERME conferences. The four main Graph Communities identified in the analysis were Foundation (beliefs, attitudes, emotions), Self-Efficacy, Motivation, and Teacher Development. There were six smaller Graph Communities that may suggest emerging new frameworks: Academic Emotions, Metacognition, Teacher Beliefs, Resilience, Meaning, and Identity. These results suggest that of the three possible dimensions to structure the area, the distinction between cognition (beliefs), motivation, and emotions is the most important one.*

*Keywords: Affect, educational theories, graphs, literature reviews.*

## Introduction

The affect group in CERME has spent a lot of time and energy discussing the conceptual framework and terminology, leading to more extensive theorization of the area. Three theoretical frameworks have been especially influential in CERME for structuring the area of affect. The first is McLeod's 1992 framework that identified three main topics of research in mathematics related affect: emotions, attitudes, and beliefs. Moreover, the framework suggested that emotions are the most intensive, the least stable, and the least cognitive of the three, while beliefs are at the other end of the continuum and attitudes are in the middle.

An important synthesis of discussions in the group was Op 't Eynde's graphic representation of the conceptual field at CERME 5 (Hannula, Op 't Eynde, Schlöglmann & Wedege, 2007, Figure 1). This model identifies some new constructs that had been then discussed in CERME affect group: most importantly, the model identifies motivation as a dimension separate from affect, and meta-affective constructs. Moreover, the model highlights the local (classroom) and socio-historical contexts.

The model was discussed and developed by Hannula in his CERME plenary (2011) and further elaborated in the CERME special issue of RME (2012). This cube model (Figure 2) identified three dimensions that are relevant when discussing affective constructs. The first dimension addresses cognitive, affective, and motivational types of constructs. The second dimension separates the rapidly changing state-type constructs and more stable trait-type constructs. The third dimension identifies three levels of theorizing affect: psychological, social and embodied theories.

These distinctions identify separate research areas of mathematical affect. But how separate are they? Do the studies on attitudes, emotions, and beliefs form three separate research traditions or even 18 separate research areas, as Hannula's cube model suggests? Perhaps the studies on state and trait type affects are separate? Or are the different topic areas partially overlapping with diffuse borders, as Op 't Eynde's figure depicts it?

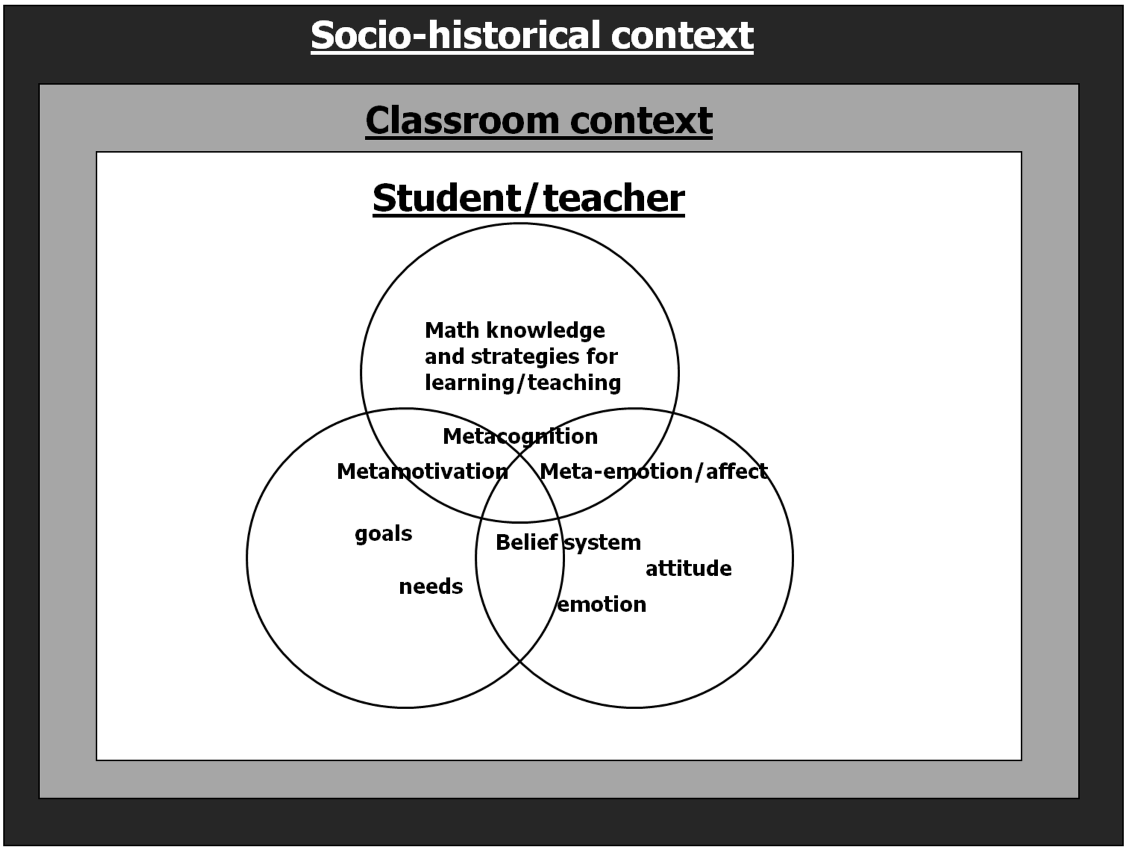



**Figure 1. A graphic representation of the different dimensions of mathematics-related affect and their relationships, presented at CERME 5 (Hannula et al., 2007, p. 204)**

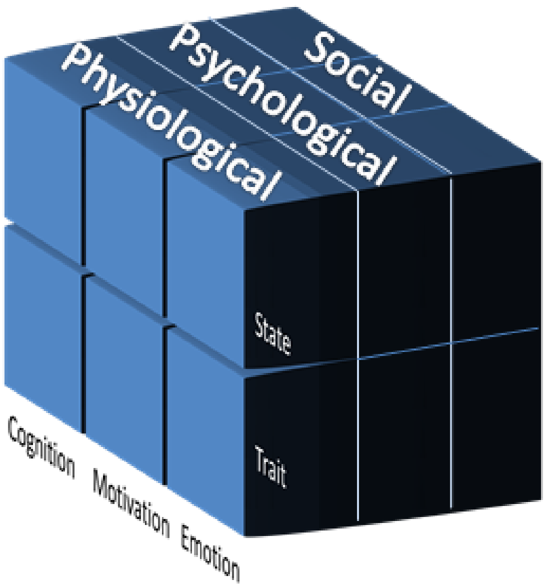



**Figure 2. Hannula's (2011; 2012) cube model of the three dimensions for affective constructs**

One way to analyze the question empirically is to analyze the references different research papers share. We are aware that there is a lot of cross-referencing between authors of CERME affect papers and that there are some fundamental articles that keep being cited often. However, there has not yet been any systematic analysis on the cohesion of the research papers. We are going to make a network analysis of CERME papers in this research area. With this analysis we can identify whether there are some clusters of different research traditions within the CERME affect group. Moreover, we expect to identify important foundation works for each research tradition.

Our research questions are: (1) What possible subgroups can we identify among CERME research papers on affect when analyzing the cited authors of each paper? (2) What are the defining characteristics of each subgroup of papers?

## Methods

### Corpus and measures

Our corpus for analysis were the 100 research reports published in the affect groups in CERME conferences four to nine. The first two CERME conferences did not have an affect group and CERME3 one was left out because of some difficulties in the formatting that we did not have time to solve. We excluded from our analysis affect papers published in other groups, for example in groups of teacher beliefs or comparative studies.

Knowing the CERME publications in affect group, we knew that some authors had published papers that might fall into different research traditions. For example, many researchers have published papers both on teacher affect and student affect, and it could be possible that these fields would be empirically separate clusters. Therefore, we decided to search for clusters of papers published in CERME rather than identifying clusters of researchers.

When analyzing the lists of references, we had two basic options to identify links between papers. We could identify whether the same reference appeared in the reference lists of two papers, or we could link them whenever they had papers by the same author in their lists of references. Because our data corpus was modest in size, there was high probability that papers using the same theoretical framework might not share the same exact references even if they use papers by the same authors. Therefore, we decided to use the authors in the reference lists rather than the exact references as the method of connecting papers.

As a conclusion, our data consisted of research reports published in the proceedings and the authors appearing in their reference lists. For each of the papers, we created a connecting link from the paper to each of the authors mentioned in the references. For analyzing the connections between papers, we identified CERME papers and cited authors as vertices of a graph. Edges of the graph are the links that connected each CERME paper with the authors mentioned in the list of references, allowing multiple edges when a paper had several references by the same author. Hence, we produced a graph connecting all papers to their cited authors and this was then subjected to a mathematical analysis.

### Analysis

The analysis included two stages: (1) To identify Graph Communities, and (2) to identify commonalities within the papers and authors of the selected Graph Communities.

We identified related papers using the FindGraphCommunities algorithm with modularity-based clustering to identify how papers and cited authors are related (Wolfram Alpha LCC, 2016). The modularity approach was originally developed by Newman and Girvan (2004) and the algorithm used in Wolfram Alpha is based on Fortunato's article (2010).

The input for the algorithm was the graph connecting research reports to authors cited in these papers. The output was subgraphs called Graph Communities, each of which consisted of some of the research reports and authors. The algorithm chooses only one Graph Community for each of the graph's vertices. In other words, although the same author may originally appear in the reference lists of papers from many graph communities, in these subgraphs each cited author belongs exclusively to one Graph Community. This accentuates the differences between Graph Communities, especially with respect to those authors who are cited in papers in several communities. In addition, we are aware that the current method does give additional weight to authors cited several times in a paper.

The next stage of analysis was to identify which Graph Communities to include in further analysis. This stage was based on a visual inspection of Graph Communities to see how well they are connected. The five biggest Graph Communities are presented as graphs (Figures 3 – 6), all affect papers published in CERME are represented as vertices with multiple edges. However, authors may have either one or multiple edges, depending on how many times they appeared in references. Another six Graph Communities are described but due to space limitation their graphs are no presented.

In the last stage of the analysis, we examined which papers and authors were represented in each of the Graph Communities.

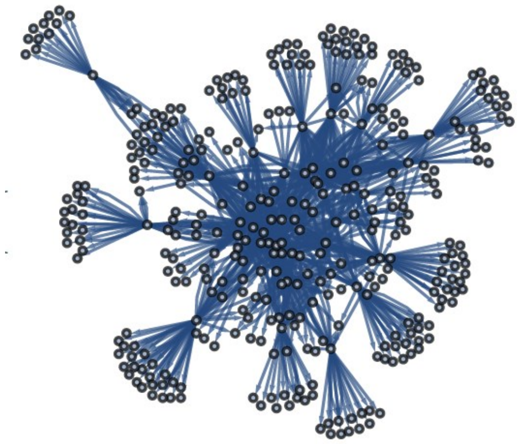
## Results

The algorithm identified 21 Graph Communities. We shall describe nine of them.

The first Graph Community 1 (29 papers; Figure 3) we call *Foundation*. It was the largest and the most cited authors in it included arguably the most influential researchers in the area of mathematical affect: McLeod (e.g. 1992), Schoenfeld (e.g. 1992) and Goldin (e.g. 2002). The most frequently cited authors in Foundation were active participants of CERME affect group: Hannula, Zan , Pehkonen, and Di Martino. Foundation is perhaps the most difficult to describe and may be best done by contrasting it with other Graph Communities. The Foundation's papers represent a large scope of research topics and theoretical frameworks, including papers that focus on beliefs, attitude, affect during problem solving, and emotions. This group also contains several papers that deal with dynamically changing affective states.

The second Graph Community (11 papers, Figure 4,) was given the label *Self-efficacy*. It had papers mostly from Cyprus and Turkey (e.g. Arslan & Bulut, 2015). The shared theoretical framework of self-efficacy was indicated by numerous references to Pajares and Bandura.

The third Graph Community (11 papers, Figure 5) we named *Motivation*. Philippou and Pantziara were influential authors in this Graph Community. Seven of the papers included at least one of them as the author (e.g. Pantziara & Philippou, 2011). Also Wæge appeared three times in this group as an author. This group shared motivation theory framework, and the most cited authors were well-known motivation theorists Midgley, Deci, Ryan, Pintrich, and Elliot.



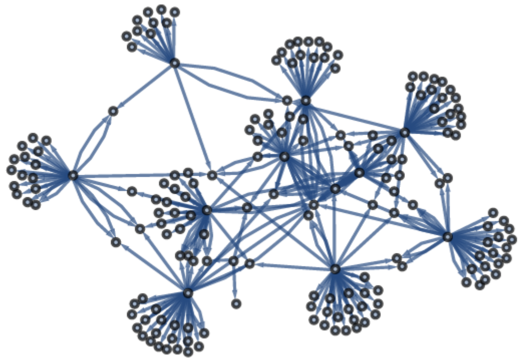
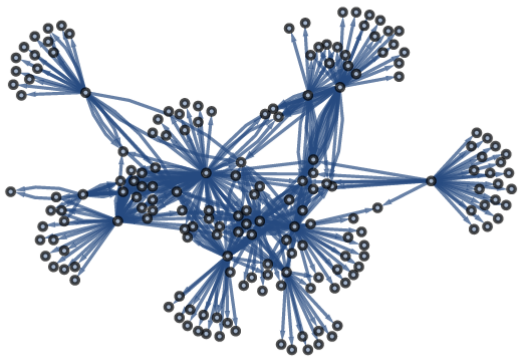


**Figure 3. The Graph Community Foundation** **Figure 4. The Graph Community Self-Efficay**

The Fourth Graph Community (8 papers, Figure 6) collected together papers on *Teacher Development.* Liljedahl was an important author in this group with four papers and the most cited authors include Liljedahl and Ball.



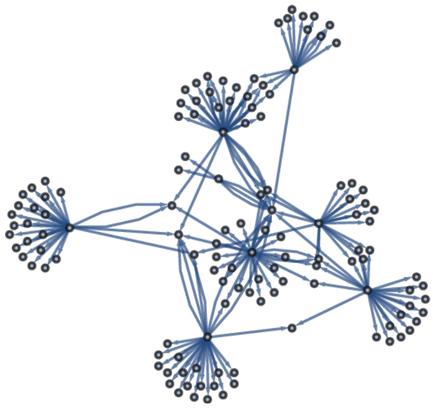


**Figure 5. The Graph Community Motivation** **Figure 6. The Graph Community Teacher Development**

The following six Graph Communities were smaller, each including 3 to 5 papers. Due to space limitations, these will be described only briefly. The four papers in *Academic Emotions* share Pekrun's (e.g. Pekrun, Goetz, Titz, & Perry, 2002) academic emotions framework and all these papers have been published in CERME8 or CERME9. All four papers in *Metacognition* were authored by Panaoura (citing e.g. Flavell, 1987). *Teacher Beliefs* had three papers from CERME8, citing, e.g. Fives & Buehl (2008). *Resilience* included 5 loosely connected papers without any frequently cited author. *Meaning* (citing e.g. Skovsmose, 2005) included four papers, and *Identity* (citing, e.g. Sfard & Prusak, 2005) four papers.

The remaining 14 Graph Communities included one or two papers each, altogether 16 papers. Nine of these papers were by authors who have ever published only once in the CERME affect group. Yet, these included also papers by frequent CERME participants (e.g. Hannula and Philippou).

## Discussion

The analysis identified nine groups of CERME affect publications. Their defining features were a shared theoretical framework and often a research team. The largest group, Foundation, did not hold a theoretical framework clearly separating it from other groups. Rather, this group seemed to rely more on the seminal works in the field of mathematics related affect and cover a variety of research topics indicating that there is much cohesion in this research field.

How much are these identified groups of papers determined by having the same authors? Most researchers with several papers in the analysis had most of their papers in a single community and only Philippou appears in three different Graph Communities. Often, having publications in different Graph Communities seems to be explained by supervisors co-authoring their students' papers that may often have quite different theoretical frameworks than their own papers.

The current method did not allow overlapping of Graph Communities, which made it difficult to identify possible authors who have a cross-cutting importance across several graph communities. However, looking at the total numbers of citations across all Graph Communities we found some such authors. The clearest examples were Ernest, who was cited 12 times in total, but not more than four times in any Graph Community, and Mason, who was cited 10 times but not more than twice in any Graph Community. Also, most authors described above as defining a Graph Community are cited in many papers of other communities. This suggests that results might identify groups more clearly, if we defined connections through specific cited research papers rather than cited authors. However, our current corpus might not be sufficiently large for that kind of analysis. Such analysis would be recommended when using a data corpus of at least a few thousand articles.

There are some methodological issues that we are aware of. We realized that summaries of the affect group from the previous CERME proceedings were cited often, inflating the number of citations by their authors. A more fundamental question is, that we have no measure for the reliability of The Graph Community analysis. With the current data corpus, our first analysis included an error that excluded 11 of the 100 papers. This was enough to produce a significantly different result: A subgroup of Foundation papers (Pehkonen and his students) was identified as a separate Graph Community and Teacher Development was not identified as a Graph Community. This suggests that the results of the analysis are quite sensitive to changes in data.

The first author of this paper has published several synthesizing articles on research in mathematics related affect. Using the graph analysis was an attempt to overcome possible personal biases in perceiving the structure of the research area. Our method of connecting CERME publications by authors appearing in their lists of references seems to have worked. It confirmed research on motivation research as a specific research domain. The analysis also identified specific research traditions on self-efficacy and academic emotions. While earlier reviews (e.g. Hannula, 2011), identified beliefs and emotions as two areas within mathematics-related affect, the current analysis identified research on beliefs in three different groups: Foundation, Self-Efficacy, and Teacher Beliefs. Similarly, the current analysis identified Academic Emotions as separate group while most emotion papers were part of Foundation. These results suggest that in the Hannula (2011; 2012) model, the distinction between cognition (beliefs), motivation, and emotions is the most important one. On the other hand, one small Graph Community, Identity, can be considered to be characterized by its theoretical background being sociological. A possible new characterizing feature for research could be focus on the dynamics of change, exemplified by the research traditions Teacher Development and Resilience.

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