Constrained paths based on Farey sequence in learning to juggle

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Abstract

In this paper, we report on the results of a study conducted to describe the learning dynamics of three-ball juggling from the perspective of frequency locking. The theoretical prediction about coordination patterns that could appear in learning process of three-ball juggling from the principle of frequency locking based on Farey sequence showed the existence of several stable coordination patterns denoted by dwell ratios of 0.83, 0.75, 0.67, and 0.50. We observed the changes in the coordination patterns during actual learning processes based on task performance, and compared them with the predictive coordination patterns. Consequently, we discovered that individuals acquired their own coordination patterns in the early stage of learning, and those coordination patterns did not change in subsequent learning processes. The observed coordination patterns corresponded with theoretical prediction, and imply that these observed patterns have a stable coordination structure that shows strong frequency locking among the temporal variables

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that comprise juggling. This implies that the learning dynamics of three-ball juggling can be described as a process in which a learner acquires one of several stable coordination patterns based on the principle of frequency locking during the exploratory process in the early stage of learning. In other words, there may be several paths through the learning process that utilize different attractors.

Keywords: Learning dynamics, Coordination pattern, Frequency locking, Farey sequence, Task performance-based

1. Introduction

In various sport, there are many instances in which new motor skills are learned. This learning process is one of the important themes in the study of motor learning. The motor learning process seen in the real world comprises various factors and develops over a period of time, making it difficult to describe the overall aspect. However, in recent years, learning dynamics have been clarified by a dynamical system approach and now serves as the key to understanding various motor learning processes. For example, it has been reported that the learning dynamics of bimanual coordination skill is the process of acquiring new attractors from an intrinsic attractor by analyzing the coordination pattern of the movement (Zanone & Kelso, 1992).

Learning the coordination pattern crucial in three-ball juggling is a practical example of a real-world learning process. It is relatively easy to throw a single ball up into the air and catch it. In contrast, it is difficult to maintain three or more balls in the air at the same time because spatial and temporal constraints abruptly increase and the degree of difficulty is higher.

Consequently, the learning process essential for skilled performance requires a relatively long time scale (Hashizume & Matsuo, 2004).

In previous research, various aspects of cascade juggling have been investigated, including of optical information pickup (Huys & Beek, 2002; Dessing et al., 2012), skill automatization (Bebko et al., 2003), learning by visual and auditory information (Zelic et al., 2012), and change of postural sway through learning (Huys et al., 2003, 2004a, 2004b; Leroy et al., 2008). However, in learning to juggle the critical problem is how to acquire appropriate timing. Beek (1988, 1989) presented a mathematical explanation for the temporal structure of coordination patterns. Mathematically, juggling can be described using five basic variables(Fig. 1). The number of balls juggled (N), the number of hands doing the juggling (H), the time the ball is in the air between a throw by one hand and a catch by the contralateral hand (TF: Time Flight), the time the ball is held in a hand between a catch and a throw (TL: Time Loaded), and the time the hand is empty (TU: Time Unloaded). Shannon (see Beek and Lewbel, 1995) defined the temporal constraint in juggling in the well-known "Shannon's equation" as B/H=(TL + TF) / (TL + TU). In order to perform B × H juggling, it is necessary to fulfill the constraints of this equation. For juggling to be periodic with time T, the juggling hands and juggled objects must satisfy, on average, a general timing requirement in which the ratio of the object cycle time to the hand cycle time equals the ratio of the number of objects to number of hands(Beek & Turvey, 1992).

To examine how the juggler controls this temporal structure, the proportion of TL to the cycle of a hand, which is a variable reflecting the temporal structure of juggling was focused on dwell ratio = TL/(TL + TU) (Beek, 1989). If TL is extended, it approaches HCT (Hand Cycle Time = TL + TU), that is, the duration of the hand cycle time in which a ball is present will be long, whereas the duration without a ball will become short. This juggling pattern is called "delayed juggling." Conversely, if TL becomes short, the duration after catching a ball, in which a ball is in the hand cycle, will become short, i.e., it will be thrown immediately. This juggling pattern is called "hot potato juggling" (Beek & Turvey, 1992). Thus, dwell ratio is a key variable that describes the coordination pattern of juggling.

Beek (1989) showed that the dwell ratio is theoretically fixed by the constraint of the temporal structure of juggling, which was shown in Shannon's equation and the principle of frequency locking. This is expressed as the "tiling principle," which states the ratio at which to distribute the individual temporal variables of the juggling "TL/TF." That is, when TL/TF is distributed by the ratio of the integer that causes strong frequency locking, it is thought that the pattern is most stable. In three-ball juggling, when TL/TF = 1/1, it is set to TU/TF = 1/3, and is set to a dwell ratio TL/(TL+TL) = 3/4; that is, individual temporal variables may show strong locking at a dwell ratio of 0.75. Hence, it has been reported that a coordination pattern with a dwell ratio of 0.75 is an attractor in the coordination structure of three-ball juggling.

Although it was shown clearly that a dwell ratio of 0.75 theoretically exists as an attractor in the coordination structure of juggling, it does not necessarily converge on a dwell ratio of 0.75. Beek (1989) examined three-ball juggling performed by four skilled jugglers at three kinds of juggling speeds, and reported that the dwell ratio was varied from 0.54 to 0.83, with a mean value of 0.71.

Further, Beek & van Santvoord (1992) sought to clarify the change in the dwell ratio through learning by conducting an experiment with 20 novice three-ball jugglers. They used the results of data collected at the end of the 4th, 7th, and 10th sessions of 10 thirty-minute sessions and discovered that the dwell ratio decreased significantly during the learning process. The mean dwell ratio of the 4th session was 0.77, that of the 7th session was 0.76, and that of the the 10th session was 0.74. On the basis of experimental results obtained from learners (Beek & van Santvoord, 1992) and expert jugglers (Beek, 1989), the learning process of three ball juggling came to described as comprising a three-stage model of learning. The first stage consists of learning to accommodate the real-time requirements of juggling, as expressed in Shannon's equation of juggling. The second stage of learning consists of discovering the primary frequency locking, that is, the dwell ratio of 0.75. The third stage of learning consists of discovering frequency modulation from the dwell ratio of 0.75 to lower values. That is, the learning dynamics of three-ball juggling is viewed as acquisition of and deviation from a first attractor to acquisition of a new attractor.

Subsequent to the study conducted by Beek & van Santvoord (1992), Hashizume & Matsuo (2004) conducted a detailed analysis of the spatiotemporal variables during learning to juggle. They used virtually the same procedure as Beek & van Santvoord (1992). Their experiment comprised 10 thirty-minute learning sessions from which they took data at the end of the 3rd, 7th, 10th sessions. Their results indicated that learners choose one of two coordination patterns of dwell ratio in the early stage of learning; that is, after the end of the 3rd session. One pattern showed a dwell ratio less than 0.80 and the other showed a dwell ratio greater than 0.83. However, in their study, two of eight learners had decreased dwell ratios around 0.70. Their results serve to corroborate those obtained by Beek & van Santvoord(1992), which indicate that learners acquire another stable attractor from the once-convergent attractor as learning progresses. That is, the learning dynamics of three-ball juggling involves acquiring another attractor in the late stage of learning from one of two attractors in the early stage of learning. In other words, their results suggest that there are two paths to another attractor in the late stage of learning. Further, the analysis conducted by in Hashizume & Matsuo(2004) showed that when the dwell ratio is 0.83, all events timing of the throw in right hand, catch in the right hand, zenith of the ball, throw in the left hand, catch in the left hand, zenith of the ball, and again throw in the right hand occur in sequence within the same interval (see Fig.3(a)). This suggests that when the dwell ratio is 0.83, the juggling pattern has a stable temporal structure from stable event timing. They suggested that an attractor with a dwell ratio of 0.83 exists in addition to the attractor with dwell ratio 0.75 discovered by Beek & van Santvoord (1992).

However, to the best of our knowledge, theoretical evidence pointing to other stable coordination patterns has not been provided. Our study theoretically predicts other stable coordination patterns in addition to the dwell ratio of 0.75 by focusing on the temporal structure based on a principle of frequency locking, and examines the validity of the theoretical prediction through empirical analysis of actual learning processes.

Previous studies examined the learning process in terms of sessions using a physical time scale. However, there are individual differences in the amount of learning, that is, task performance, even in the same amount of learning time. Therefore, in our study, in order to take into account individual differences in the amount of learning, we examined the change in the coordination pattern on the basis of the task performance.

We observed the change in the coordination patterns constituting three-ball juggling through learning process based on the number of consecutive catches. Subsequently, by comparing the coordination patterns that appear through learning process with the coordination patterns based on theoretical prediction, we describe the learning dynamics of three-ball juggling.

2. Predictive coordination patterns derived from frequency locking based on farey sequence

We investigated the coordination pattern of three-ball juggling by focusing on frequency locking between individual temporal factors TL, TU, and TF, and theoretically predicted a stable coordination pattern. It is well-known that frequencies locking become stronger between small integers according to Farey sequence, Fig.2 (Peper et al., 1995). The ratio between each frequency located on the upper level signifies stronger frequency locking. Therefore, we determined the ratio between two variables, TL/TF, TU/TF, and TU/TL, and the integer in the mode relatively located on the upper level (from F1 to F5 in Fig.2) of the Farey sequence (Table 1). From Shannon's equation, B/H = (TL + TF)/(TL + TU), for the case where B = 3 and N = 2, this equation provides TL + 3TU = 2TF. Hence, the value of the third variable can be calculated when two variables have been determined. There are two possible relationships between TL and TF either TL is less than TF or TL is greater than TF. We considered these possible relationships separately.

As a result, the dwell ratio that appeared in the three combinations TL/TF, TU/TF, and TU/TL until level F5 in the Farey sequence were calculated as 0.83, 0.75, 0.67, and 0.50. This implies that these patterns are stable in the coordination structure of the juggling because three temporal factors, TL, TU, and TF, constituting juggling, show strong frequency locking with each other. These four coordination patterns are different in the event timing. Figure 3 shows the schematic image of the event timing of throw and catch for both hands in coordination patterns with dwell ratios 0.83, 0.75, 0.67, and 0.50.

From Fig.3, the coordination pattern with dwell ratio 0.83, in which the loaded time occupies 5/6 of the hand cycle, and TU : TL : TF $= 1 : 5 : 4$, the interval between the throw and catch events in the same hand is relatively short, and the interval between the throw event in one hand and the catch event in the other hand is relatively long. This is the pattern called “delayed juggling.”

In the coordination pattern with dwell ratio 0.75, in which the loaded time occupies 3/4 of the hand cycle, and TU : TL : TF $= 1 : 3 : 3$, and with dwell ratio 0.67, in which the loaded time occupies 2/3 of the hand cycle, and TU : TL : TF $= 2 : 4 : 5$, the interval between the throw and catch events is relatively long, and the interval between events of both hands is relatively short. In the dwell ratio of 0.75, because the interval between the throw and catch events in both hands becomes equal, it was thought that it is a stable pattern on the part of regularity of event timing.

Furthermore, a dwell ratio of 0.50, in which the loaded time occupies 1/2 of the hand cycle, and TU : TL : TF = 1 : 1 : 2, is the lowest dwell ratio for which three-ball juggling can be achieved, the timing of the catch in one hand and the throw in the other hand occurs at the same moment. This is the pattern called "hot potato juggling."

The differences in dwell ratios constitute the difference in timing of key events in juggling; that is, these coordination patterns are different patterns in the spatiotemporal structure of juggling. Thus, it was supposed that four attractors could exist theoretically from the principle of frequency locking based on Farey sequence. We compared these predictive coordination patterns with observed coordination patterns during the learning of three-ball juggling.

3. Methods

3.1. Participants

Eight volunteers (four males and four females) between 20 and 25 years old (mean age 22.2 years) participated in our experiment. All participants were right handed, and had no prior juggling experience. In consideration of individual differences in the ability to handle a ball, person who had experienced playing baseball or softball were excluded. In addition, the purpose of the experiment was explained prior to the start of the experiment, and the participants signed informed consent forms. The procedures were approved by the Internal Review Board at Aichi University of Education.

3.2. Procedure

The participants practiced three-ball cascade juggling–the most famous fashion in ball juggling. At the beginning of the first session, the task was outlined to the participants by means of a video presentation. After the explanation, participants attempted to perform juggling as long as possible without receiving any additional instructions. No learning aids, such as a metronome were used. The participants performed all trials in a standing position.

One learning session was carried out for 60 minutes (four fifteen-minute sets) for a day, with a break of five minutes between the sets. Participants were required to achieve 150 consecutive catches as the task goal. In Hashizume & Matsuo (2004), participants who arrived at the third stage of learning (see Beek & van Santvoord, 1992) performed more than 150 consecutive catches. In addition, in Zelic et al. (2012), the intermediate juggler was defined as "the person who can juggle more than 20 seconds and less than 60 seconds in a circle of 2m in diameter." An exploratory experimental result based on this definition showed that the number of consecutive catches varied between 50 and 150 catches. This suggests that 150 catches are an appropriate criterion for the to-be-learned goal. However, to ensure this achievement did not occur by accident, the learning session was finished only after the participant had achieved 10 trials of 150 catches. After the learning session, participants were given a retention period of one week, after which they performed a retention test comprising three trial for 20 seconds juggling at their favorite tempo.

3.3. Data acquisition

An optical motion capture system with four cameras (250 Hz, OQUS, Qualysis Inc.) was used to record the participant ' s movement during all trials in the learning session and the retention test. Eleven spherical reflective markers (2.5cm in diameter) were attached with double-sided tape to the right and left shoulders, elbows, wrists and middle fingers, while the head was covered with a tight swimming cap to which three markers were attached. Three balls were (6.6 cm in diameter and mass 130 g) covered with reflective tape. The cameras were placed around the participant, so that the participant and the balls being juggled were all in view. The three-dimensional coordinates of the markers (x-axis: anterior-posterior, y-axis: lateral-medial, z-axis: vertical) were calculated using a Qualysis Tracking Manager (QTM). Reconstruction of the known marker positions on calibration frame prior to each experiment yielded residual errors of reconstruction of less than 1mm in each coordinate.

3.4. Data reduction

Digitized coordinates of 14 markers, including three balls, were identified and tracked using the QTM. Marker switching, or the misidentification of two adjacent markers during automatic tracking, was corrected manually. Missing data points due to a short occlusion were interpolated automatically by spline method using QTM. The raw displacement data were filtered using a second-order Butterworth digital filter, with cutoff frequency defined using residual analysis by Winter (2005) in each marker. The filtered displacement values along the z-axis were differentiated to obtain the velocity of the ball in the vertical direction. The velocity profile was used to identify the moments at which the throws, catches, and arrival points at the zenith occurred. The moment of throw was defined as the time at which the positive peak of the ball velocity occurred and the moment of catches was defined as the time the negative peak of the measured velocity of the ball occurred. The moment of arrival at the ball's zenith was defined as the time the highest location in the vertical direction was reached.

3.5. Achievement level

The number of consecutive catches in each trial was recorded as the resulting performance. We defined the achievement level as the 1st, 2nd, 3rd, 4th, and 5th by the achievement of 30, 60, 90, 120, and 150 catches respectively. When participants achieved each consecutive catches for the first time, we analyzed these trials as each achievement level.

3.6. Temporal and spatial variables

Each cycle of the 40 cycles from the 1st to 5th levels and the trial in the retention test were used to calculate the temporal and spatial variables.

Using throw and catch events in both hands, the following temporal variables were calculated: HCT, TU, TL, TF, and BCT along with dwell ratio(see Fig. 1). The mean and coefficient of variance (CV) of these variables within trial were then calculated. The spatial variables, such as the position of catches (PC), throws (PT) and zeniths (PZ) were obtained using the 3D coordinates of the ball at the moments that these events occurred, and standard deviation of the positions of each event within trial were calculated. We also calculated the CV of the horizontal distance between throw and catch positions by the same hand(HD : Hand Distance) and by different hands(BD

: Ball Distance). We conducted one-way repeated ANOVA to reveal the effect of the achievement level in those temporal and spatial variables.

4. Results

4.1. Change in coordination patterns through a learning process

We examined the mean value of the dwell ratio in each achievement level to clarify the change in the coordination pattern through learning three-ball juggling. One-way repeated-measures ANOVA on the mean value of the dwell ratio showed no significant effect of achievement level($F(4, 28) = 0.30$, n.s.). The overall mean of the dwell ratios did not change throughout the learning process.

Next we examined in detail the dwell ratio at each achievement level, In the 1st level, three participants presented higher dwell ratios around 0.83, four participants presented intermediate dwell ratios around 0.75, and one participant presented a lower dwell ratio around 0.65. A participant who presented a dwell ratio of 0.75 in the 1st level changed to a dwell ratio of 0.83 in the 2nd level. From the 2nd level, the dwell ratio of each participant appeared to constitute two clusters showing a relatively higher ratio group and a relatively lower ratio group, which were maintained during the learning process after the 2nd level.

We classified participants into two groups by dwell ratio in the retention test for each participant, with criterion based on a dwell ratio of 0.75, which is the attractor used for three-ball juggling in the previous study. Randomization test on the mean dwell ratio of each of the four participants of the group higher than 0.75 and the group lower than 0.75 showed a significant effect of group(p=0.03),participants were significantly classified into the group having a higher dwell ratio(HDR group) and a group having lower dwell ratio(LDR group) in the retention test.

We also examined the change in the dwell ratio in each achievement level for posteriori divided groups in the retention test. Randomization tests on mean value of the dwell ratio in each level of each participant of each of the two groups showed a significant effect of groups(p=0.03, respectively), except for the 1st level.

This suggests that the coordination patterns seen in the retention test were already acquired when the participants achieved 60 catches, and participants learned juggling without any change in coordination patterns after they first acquired the coordination patterns.

4.2. Theoretical prediction versus observed coordination pattern

We compared the coordination patterns observed during the learning process with the theoretical prediction derived from Farey sequence. Figure 5 shows a histogram of the dwell ratio of all cycles for each groups in the 2nd, 3rd, 4th, and 5th level. It shows that the peak of the dwell ratio appears around 0.83 in the HDR group , and around 0.75 and 0.67 in the LDR group. The distributions of the dwell ratios in each group did not change in each level.

The coordination pattern with dwell ratio of 0.83 in the HDR group shows a coordination pattern in which the rate of ball in hand to the hand cycle is relatively long. That is, the patterns of the HDR group were the same as “delayed juggling” in each level. On the other hand, the coordination pattern with dwell ratio of 0.75 and 0.67 in the LDR group shows coordination patterns in which the rate of ball in hand to the hand cycle is relatively short compared with the dwell ratio of 0.83. The dwell ratio of the LDR group also appeared around 0.50. That is, a pattern such as “hot potato juggling” was also observed.

Thus, during the actual learning process, several coordination patterns with different timing of key events appeared, and it is seen that the coordination structure of each pattern comprises a stable time interval of key events. The attractors that were theoretically predicted from the principle of frequency locking on the basis of Farey sequence corresponded with several observed patterns through the learning process.

4.3. Changes in variability of temporal and spatial variables

The coordination patterns observed through learning process were consistent with the theoretical prediction. It can be considered that the participants fixed a temporal variable in the early stage of learning, and then decreased they would decrease spatial variability to improve performance further on in their own temporal coordination pattern. We therefore examined the changes in the variability of the spatial and temporal variables that constitute juggling.

For temporal variability, a one-way ANOVA on the CV of HCT, BCT, TU, TL, and TF showed no significant effect of achievement level (HCT: $F(4{,}28) = 0.73$, n.s., BCT: $F(4{,}28) = 0.78$, n.s., TU: $F(4{,}28) = 0.22$, n.s., TL: $F(4{,}28) = 0.26$, n.s., TF: $F(4{,}28) = 1.06$, n.s.). The temporal variability that constitutes juggling did not change through learning.

For spatial variability, a one-way ANOVA on the CV of Hand Distance(HD), and Ball Distance(BD) and on the SD of Position of Throw (PT), Catch (PC), and Zenith (PZ) showed a significant effect of achievement level on some spatial variables as follows: y-direction of HD ($F(4{,}28) = 5.21$, $p<0.05$, $\eta^2 = .39$, Fig.6A); y-direction of BD ($F(4{,}28) = 3.89$, $p<0.05$, $\eta^2 = .20$, Fig.6B); y-direction of PT ($F(4{,}28) = 5.40$, $p<0.05$, $\eta^2 = .23$, Fig.6C); y-direction of PC ($F(4{,}28) = 4.18$, $p<0.05$, $\eta^2 = .27$, Fig.6D); and, z-direction of PC ($F(4{,}28)=4.54$, $p<0.05$, $\eta^2=.28$, Fig.6D). These results show that the spatial variability decreased through learning.

5. Discussion

The problem of motor control in juggling is expressed as the "tiling principle," which relates to how the individual temporal variables that constitute juggling are distributed, and it have been shown that a coordination pattern with dwell ratio 0.75 is a stable coordination pattern (Beek, 1989). However, as the theoretical prediction in this study from the principle of frequency locking based on Farey sequence suggests, in addition to 0.75, dwell ratios of 0.83, 0.67, and 0.50 could also be stable coordination patterns. If the individual temporal variables of TL, TU, and TF could be locked to each other by ratios between relatively small integers, the coordination patterns in the temporal structure of juggling would become stable.

To examine whether these predictive coordination patterns exist as attractors, we observed the coordination patterns observed in an actual learning process of three-ball juggling. From analysis based on the number of consecutive catches, at achievement of 60 catches, participants were classified into two groups: higher dwell ratio, more than 0.75, and lower dwell ratio, less than 0.75. Those groups kept their own dwell ratio during the achievement of 90, 120, and 150 catches, and in the retention test. Thus, we consider that in the early stage of learning, acquired coordination patterns are classified into two patterns: lower dwell ratio, so-called "hot potato juggling," and higher dwell ratio, so-called "delayed juggling." After participants acquire coordination patterns, they advance learning without changing their own coordination patterns even when the number of consecutive catches increases.

The comparison of theoretical prediction with the actual dwell ratio observed in the learning process indicates that the peak of the dwell ratio is approximately 0.83 in the HDR group, and approximately 0.75 and 0.67 in the LDR group. This suggests that the theoretical prediction corresponds with the observed patterns, and further confirms the existence of several attractors in the coordination structure of three-ball juggling.

These several attractors may exist as a result of the task goal, which increases the number of consecutive catches, and the task constraint of three-ball juggling. Previous research on motor learning from a dynamical approach examined the acquisition process of the coordination pattern in bimanual hands or fingers coordination(e.g., Zanone & Kelso, 1992). This task requires the acquiring of a phase shift pattern of 90 degrees from in-phase and anti-phase patterns. In this case, since it is a task goal to gain a new attractor itself, the layout of the coordination pattern changes throughout the learning process. However, the goal of three-ball juggling in this study was the achievement of consecutive catches, and not the acquisition of a coordination pattern with dwell ratio of 0.75. In three-ball juggling, spatial and temporal constraints are not rigidly limited. This leads to a coordination pattern that can achieve a task goal and which is not determined uniquely.

In other words, participants could achieve the required number of consecutive catches using different coordination patterns.

However, the most important thing is that the coordination patterns observed in this experiment converged to only several patterns from various possible patterns. Furthermore, those coordination patterns had temporal stability in the coordination structure that were explained with a theoretical evidence. That is, this could be evidence that several coordination patterns with a stable temporal structure exist as an attractor in three-ball juggling. Moreover, since participants did not change the coordination patterns they acquired in the early stage of learning, it suggests that these patterns are attractive and appropriate for the achievement of a task's goals.

Although, Hashizume & Matsuo (2004) saw changes in dwell ratio during learning, no such changes manifested in our study. This suggests that the changes were caused by the individual differences in learning amount caused by the use of a physical time scale in the previous study. In the motor learning process, there are individual differences in the amount of learning, in other words, the same time does not result in the same learning amount. In fact, in our study, the range of the total time required to achieve a task goal was from 3.5 h to 22 h. This suggests that the data obtained at the same time might contain both the data about the exploratory process in the early stage of learning and the data about the stabilization process after the acquisition in the middle or late stage of learning. With consideration of the individual differences of the learning amount, it suggests that the pattern acquired in the early stage could not change for the achievement of task goal in the middle or late stage of learning.

However, Hanshizume & Matsuo(2004) showed that the two learners who decreased their dwell ratios through the learning process achieved over 150 catches in more than 80 % of all trials in one session. It may be that those two learners were at a more advanced stage than participants in our study. Therefore, decreasing the dwell ratio would become the prerequisite for advancement to the next stage. However, we suggest that during learning process of three-ball juggling, another learning path would not be required to decrease the dwell ratio for achievement of 150 consecutive catches which is the criterion to become an intermediate juggler. In other words, there may be several paths that keep different attractors through the learning process.

On the other hand, from the analysis of variability in temporal and spatial variables, the variability in temporal variables did not change and variability in some spatial variables decreased through learning. The decrease in variability of some spatial variables suggests that the decrease in the variability of the horizontal distance of the thrown ball corresponded to an increase in the number of consecutive catches. It might also be caused by the decrease in the variability of the horizontal position of throw and catch.

Because a dwell ratio is related to event timing, a dwell ratio would depict the rhythm of juggling. Both results in which participants already acquired their own coordination patterns in the early stage of learning and variability in temporal variables did not change through learning, imply that participants already acquired their own rhythm of juggling in the early stage of learning and changed their spatial stability. That is, participants in our study selected the strategy that kept their own preferred temporal patterns and acquired the spatial stability to continue juggling as long as possible rather than to explore more stable patterns by changing their own preferred temporal patterns.

In summary, there are several stable attractors that have stable temporal structures according to strong frequency locking between temporal variables in the coordination structure of three-ball juggling. The learning dynamics of three-ball juggling is described as the process in which a learner acquires one attractor from several stable attractors that have stable temporal structures. In other words, during learning process of juggling to increase the number of consecutive catches, the learner could choose several optimal paths to the achievement of that goal, but is however theoretically constrained. That is, it suggests that several paths that keep different attractors exist through the learning process. After an attractor is acquired, however learners has decreased spatial variability to increase the number of consecutive catches.

6. Conclusion

Our objective in the study reported in this paper was to describe the learning dynamics of three-ball juggling from the perspective of frequency locking. The prediction from the principle of frequency locking based on Farey sequence indicated that there are several stable coordination patterns that have stable temporal structures in three-ball juggling. Those predictive stable patterns corresponded with the observed coordination patterns in the actual learning process. In addition, the coordination patterns that were acquired in the early stage of learning did not change in the subsequent learning process. Thus, learners learned with the coordination pattern acquired in the early stage. On the other hand, the variability of only some spatial variables decreased, and the variability of temporal variables did not change.

In summary, the learning dynamics of three-ball juggling can be described as a process in which a learner acquires one attractor from several stable attractors that have stable temporal structures during the exploratory process in the early stage of learning, and after an attractor is acquired, the learner emphasizes movement stability to increase the number of consecutive catches by decreasing spatial variability.

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Figure captions

Fig. 1. Schematic representation of the temporal sequence of events in three-ball cascade juggling. Circles denote the balls. T and C indicate the moments at which a ball was thrown and caught, respectively.

Fig. 2. The Farey tree that denotes the relation of the Farey sequence. Here, "1 / 1" means frequency locking of 1 : 1, and the ratio located in higher level (F1) shows strong locking.

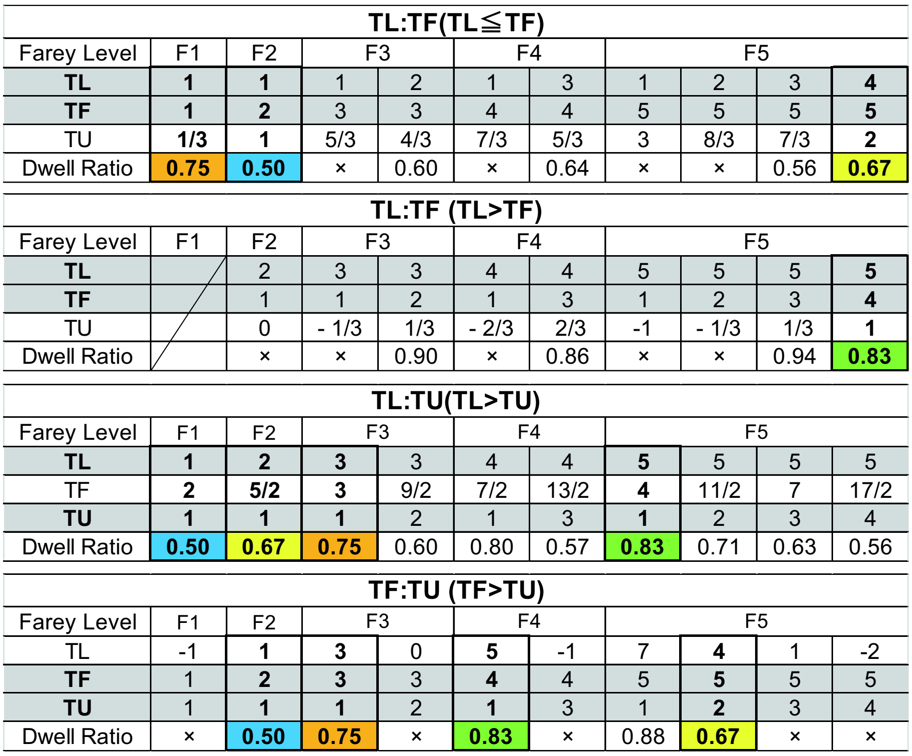
Fig. 3. The relative timing of throw and catch events in right and left hand in dwell ratios of (a) 0.83, (b) 0.75, (c) 0.67, and (d) 0.50. T and C indicate the moment of throw and catch, open circles indicate right hand and filled circles indicate left hand. The gray filled circles indicate the moment when a ball reaches the zenith. The numbers in the circles indicate the ratio of the frequency locking of individual temporal variables in each dwell ratio. The parabolas represent the trajectories of the ball.

Fig. 4. The mean dwell ratio of all eight participants at the trial in which participants achieved 1st, 2nd, 3rd, 4th, and 5th achievement levels, and at the retention test trial. The black filled symbols signify the values for the participants in the low dwell ratio group. Conversely, the gray filled symbols signify the values for the participants in the high dwell ratio group. The dashed line signifies dwell ratio of 0.75, which is stated as the attractor of three-ball juggling in the previous study.

Fig. 5. Histogram of the dwell ratio of all cycles for each group in 2nd, 3rd, 4th, and 5th achievement levels. The gray lines signify the histogram for the HDR group, and the black lines signify the histogram for the LDR group. The dashed lines indicate the values of the four dwell ratios that show the stable coordination patterns that were predicted from the principle of frequency locking based on the Farey sequence in Section 2.

Fig. 6. Change in some spatial variables in all eight participants, which decreased through the learning process. (A) shows the change in CV for hand distance in the horizontal direction, (B) shows the change in CV for ball distance in the horizontal direction, (C) shows the change in SD for the position of throw in the horizontal direction, and (D) shows the change in SD for the position of catch in the horizontal direction (filled symbols), and in the vertical direction (open symbol), respectively.

Table 1: The theoretical prediction from frequency locking between individual temporal variables based on Farey sequence. The gray color denotes the combination of individual temporal variables fitting by Farey sequence.The four colors(blue, green, yellow, and orange) on the Dwell Ratio row signify the dwell ratio that appeared in the three combinations of TL/TF, TU/TF, and TU/TL.



TL:TF(TL≦TF)

Farey Level	F1	F2	F3		F4		F5			
TL	**1**	**1**	1	2	1	3	1	2	3	**4**
TF	**1**	**2**	3	3	4	4	5	5	5	**5**
TU	**1/3**	**1**	5/3	4/3	7/3	5/3	3	8/3	7/3	**2**
Dwell Ratio	**0.75**	**0.50**	×	0.60	×	0.64	×	×	0.56	**0.67**

TL:TF (TL>TF)

Farey Level	F1	F2	F3		F4		F5			
TL		2	3	3	4	4	5	5	5	**5**
TF		1	1	2	1	3	1	2	3	**4**
TU		0	- 1/3	1/3	- 2/3	2/3	-1	- 1/3	1/3	**1**
Dwell Ratio		×	×	0.90	×	0.86	×	×	0.94	**0.83**

TL:TU(TL>TU)

Farey Level	F1	F2	F3		F4		F5			
TL	**1**	**2**	**3**	3	4	4	**5**	5	5	5
TF	**2**	**5/2**	**3**	9/2	7/2	13/2	**4**	11/2	7	17/2
TU	**1**	**1**	**1**	2	1	3	**1**	2	3	4
Dwell Ratio	**0.50**	**0.67**	**0.75**	0.60	0.80	0.57	**0.83**	0.71	0.63	0.56

TF:TU (TF>TU)

Farey Level	F1	F2	F3		F4		F5			
TL	-1	**1**	**3**	0	**5**	-1	7	**4**	1	-2
TF	1	**2**	**3**	3	**4**	4	5	**5**	5	5
TU	1	**1**	**1**	2	**1**	3	1	**2**	3	4
Dwell Ratio	×	**0.50**	**0.75**	×	**0.83**	×	0.88	**0.67**	×	×

Figure1

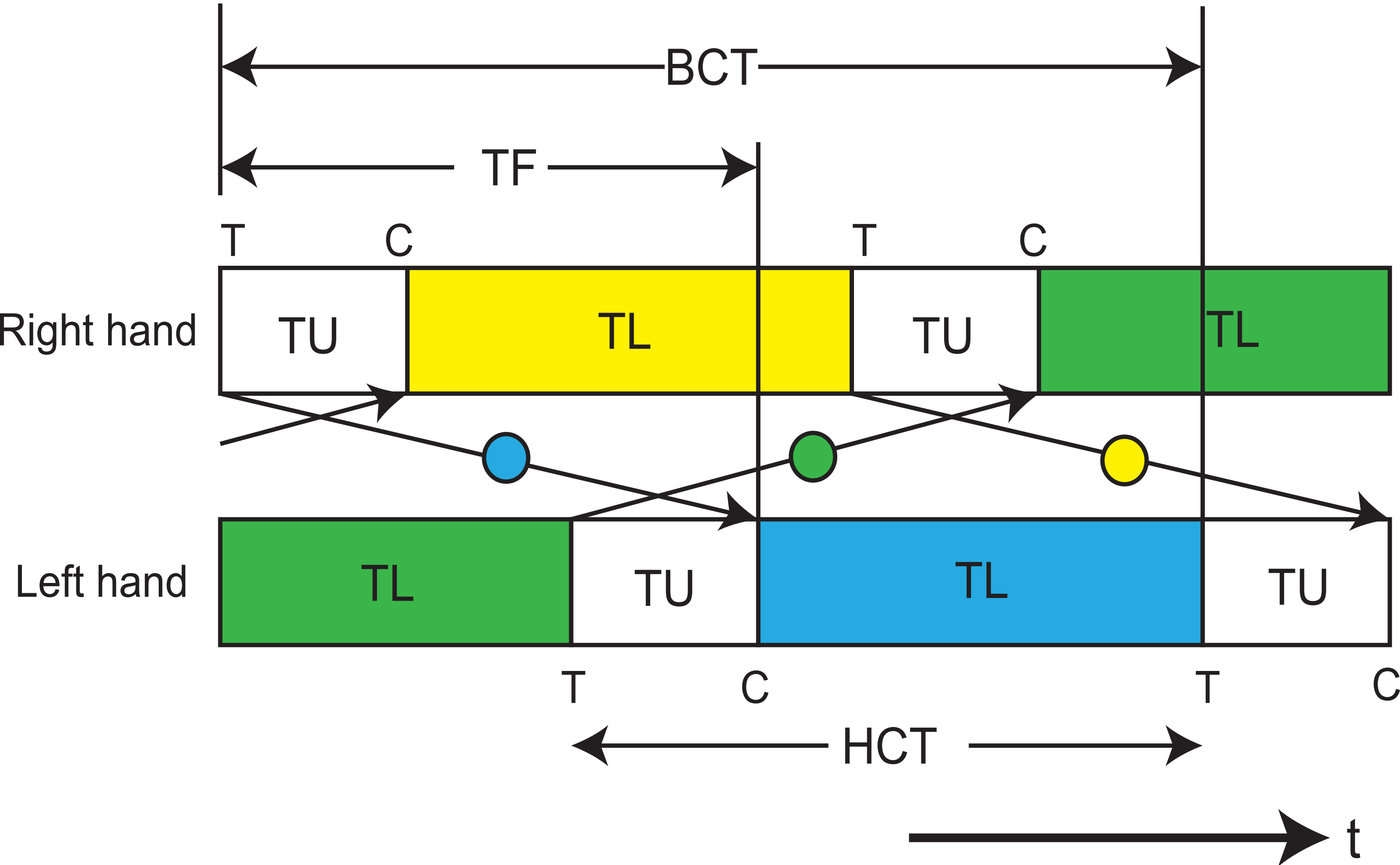

BCT
TF
T
C
T
C
Right hand
TU
TL
TU
TL
Left hand
TL
TU
TL
TU
T
C
T
C
HCT
t

Figure2

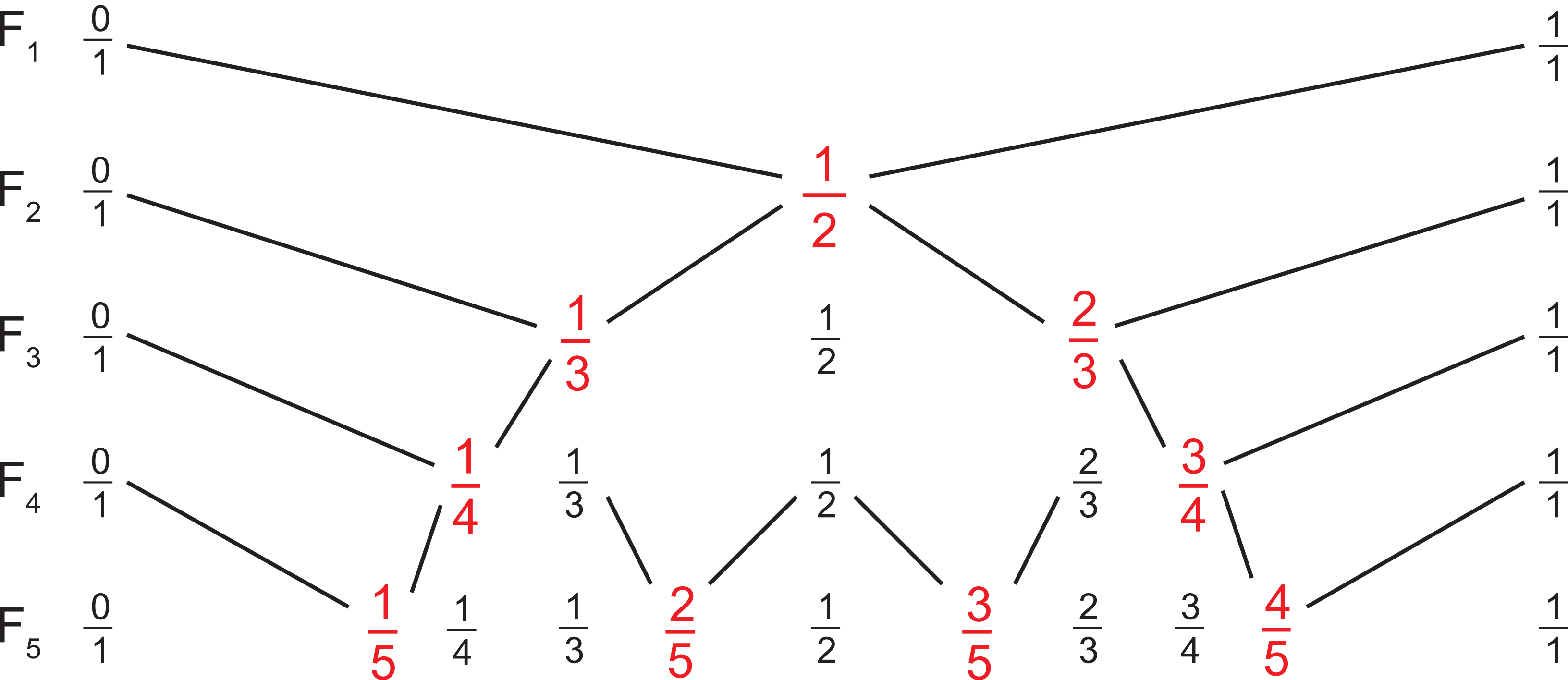

F1 0/1 1/1
F2 0/1 1/2 1/1
F3 0/1 1/3 1/2 2/3 1/1
F4 0/1 1/4 1/3 1/2 2/3 3/4 1/1
F5 0/1 1/5 1/4 1/3 2/5 1/2 3/5 2/3 3/4 4/5 1/1

Figure3

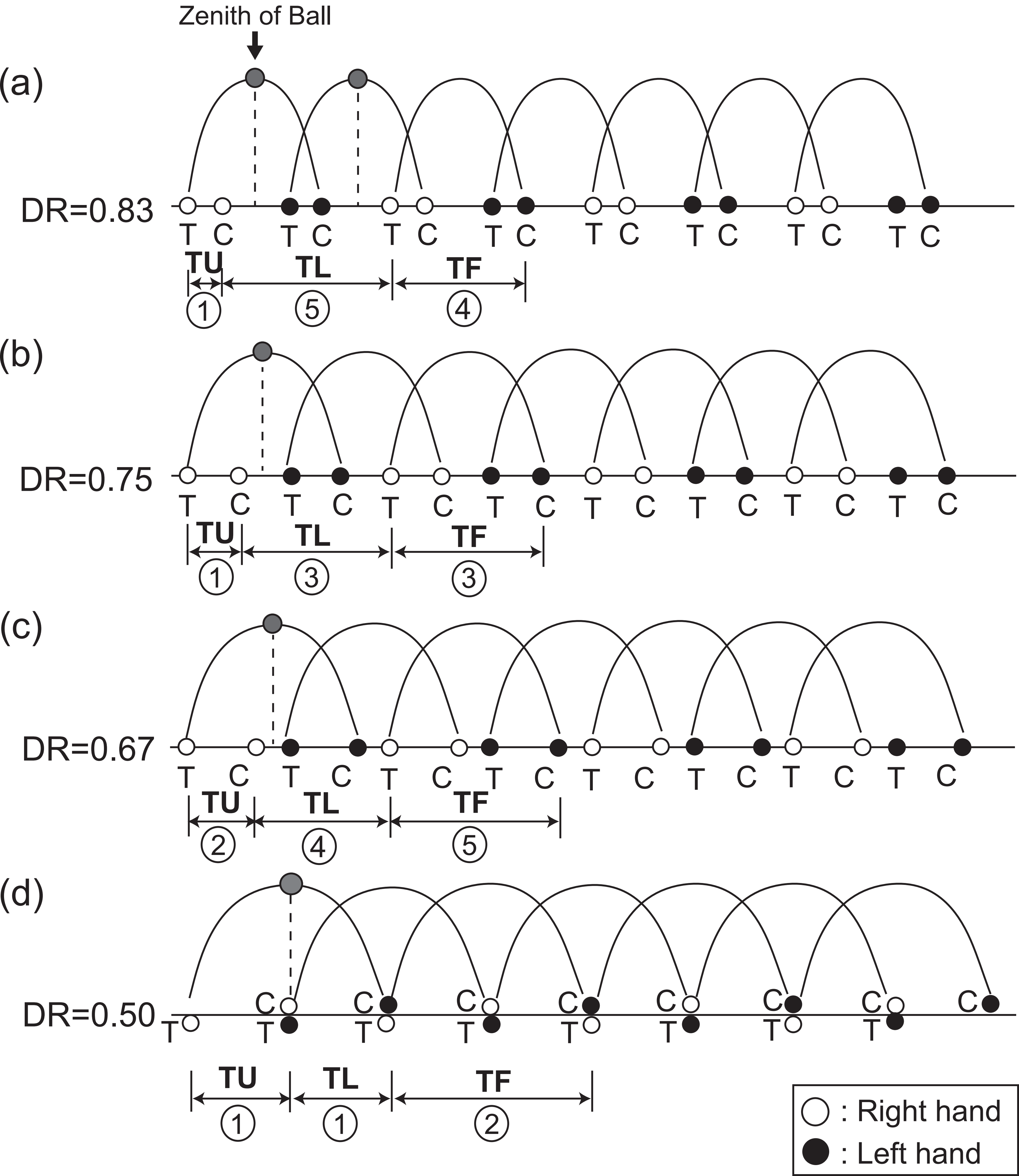

Zenith of Ball
(a)
DR=0.83
T C T C T C T C T C T C T C T C
TU TL TF
① ⑤ ④
(b)
DR=0.75
T C T C T C T C T C T C T C T C
TU TL TF
① ③ ③
(c)
DR=0.67
T C T C T C T C T C T C T C T C
TU TL TF
② ④ ⑤
(d)
DR=0.50
T C T C T C T C T C T C T C T C
TU TL TF
① ① ②
○ : Right hand
● : Left hand

Figure4

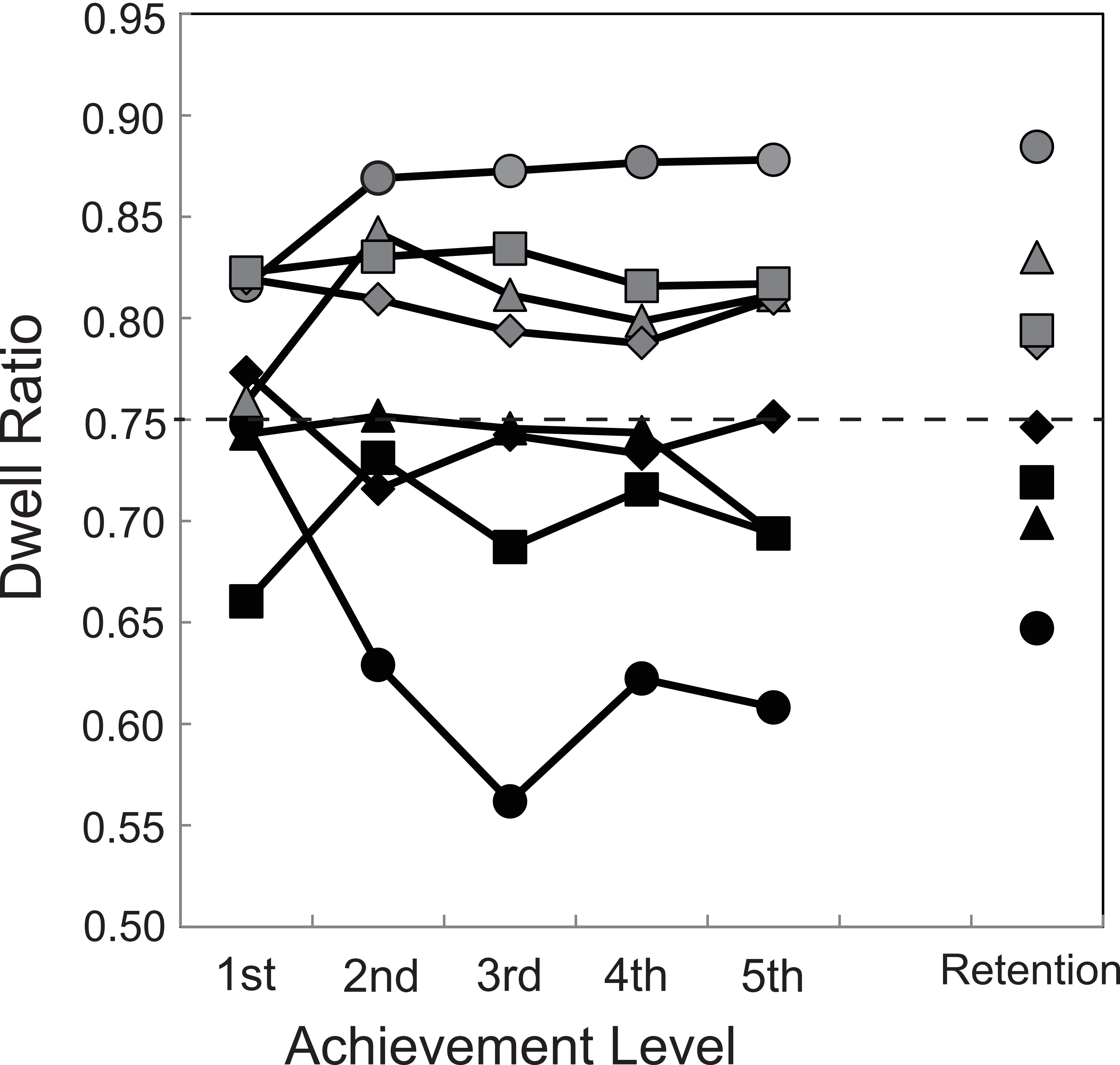
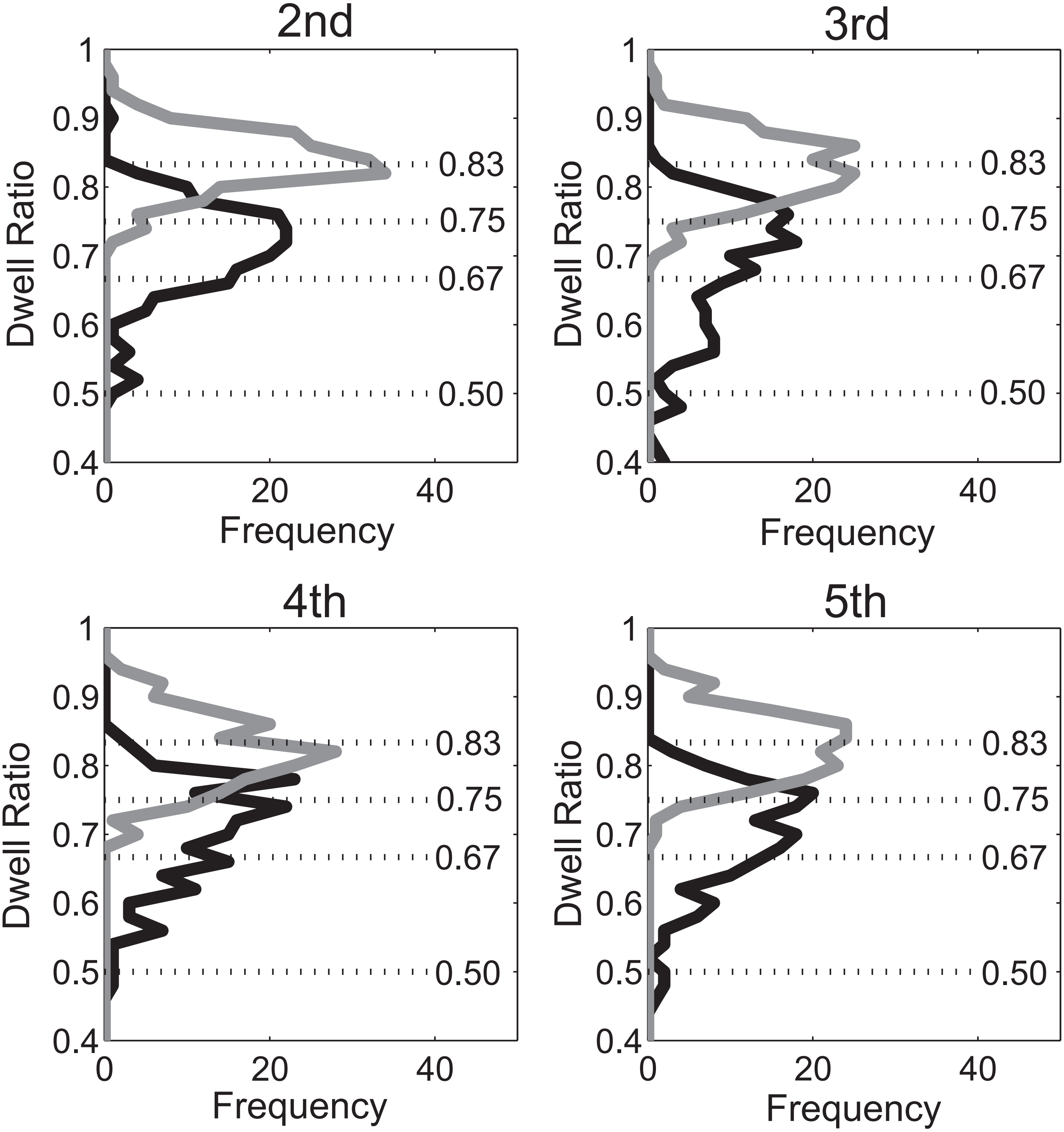
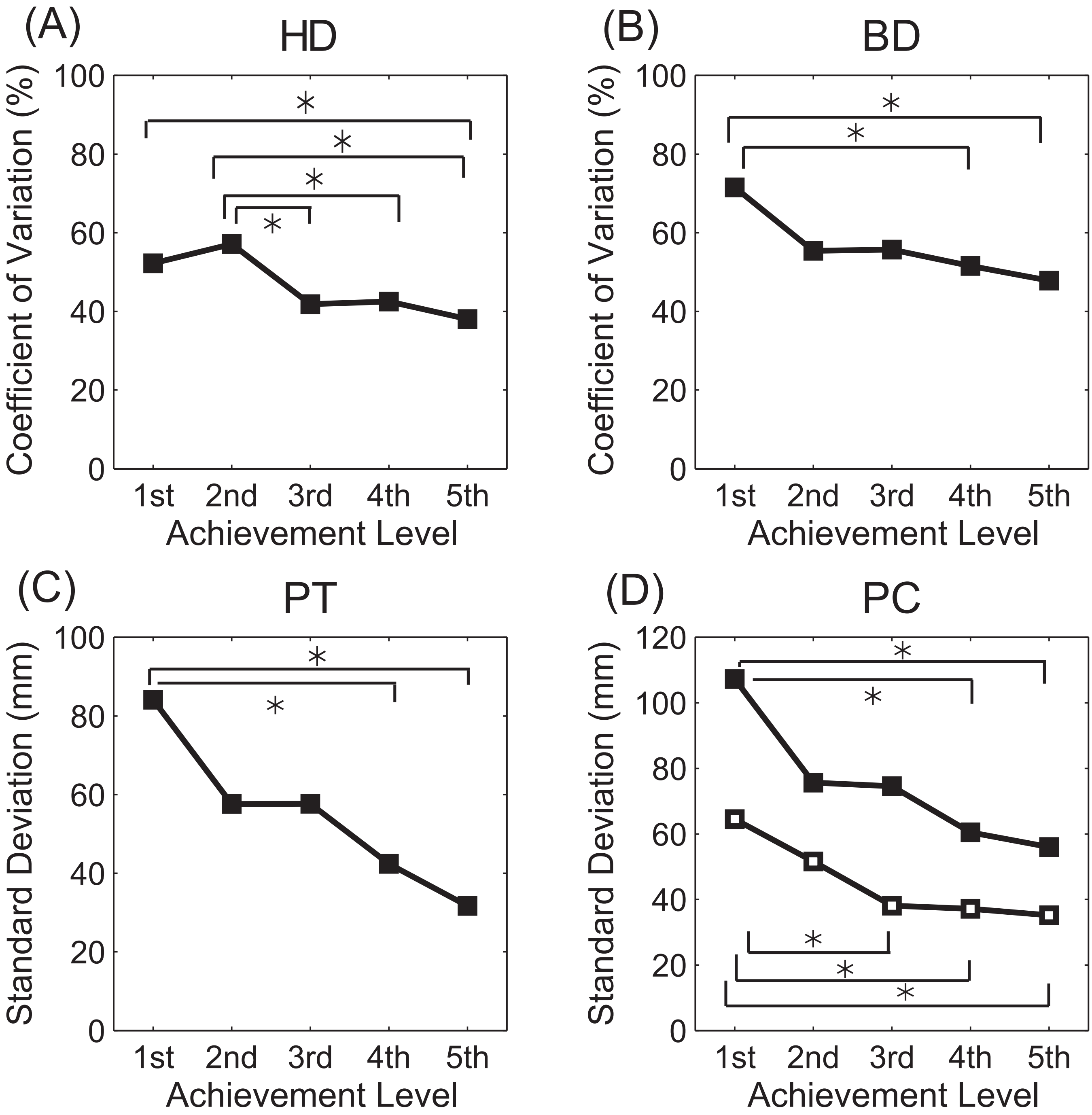

0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
Dwell Ratio
1st
2nd
3rd
4th
5th
Retention
Achievement Level

Figure5



2nd
3rd
4th
5th
Dwell Ratio
Frequency
1
0.9
0.8
0.7
0.6
0.5
0.4
0
20
40
0.83
0.75
0.67
0.50

Figure6


(A)
HD
Coefficient of Variation (%)
100
80
60
40
20
0
1st
2nd
3rd
4th
5th
Achievement Level
*
*
*
*
(B)
BD
Coefficient of Variation (%)
100
80
60
40
20
0
1st
2nd
3rd
4th
5th
Achievement Level
*
*
(C)
PT
Standard Deviation (mm)
100
80
60
40
20
0
1st
2nd
3rd
4th
5th
Achievement Level
*
*
(D)
PC
Standard Deviation (mm)
120
100
80
60
40
20
0
1st
2nd
3rd
4th
5th
Achievement Level
*
*
*
*
*

Highlights

- The prediction from the principle of frequency locking based on Farey sequence shows the existence of several stable coordination patterns with stable temporal structures in three-ball juggling.
- We found that these stable patterns correspond with observed coordination patterns in the actual learning processes based on task performance.
- Further, the coordination patterns acquired in the early stage of learning do not change during subsequent learning processes.
- The variability of spatial variables decrease, but the variability of temporal variables does not change throughout the learning process.
- Thus, the learning dynamics of three-ball juggling can be described as a process in which a learner acquires one attractor from several stable attractors during the exploratory process in the early stage of learning.