LETTERS TO THE EDITOR

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Observation of thermoacoustic shock waves in a resonance tube (L)

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(Received 10 May 2014; revised 14 July 2014; accepted 24 July 2014)

This paper reports thermally induced shock waves observed in an acoustic resonance tube. Self-sustained oscillations of a gas column were created by imposing an axial temperature gradient on the short stack of plates installed in the resonance tube filled with air at atmospheric pressure. The tube length and axial position of the stack were examined so as to make the acoustic amplitude of the gas oscillations maximum. The periodic shock wave was observed when the acoustic pressure amplitude reached 8.3 kPa at the fundamental frequency. Measurements of the acoustic intensity show that the energy absorption in the stack region with the temperature gradient tends to prevent the nonlinear excitation of harmonic oscillations, which explains why the shock waves had been unfavorable in the resonance tube thermoacoustic systems. © *2014 Acoustical Society of America*. [http://dx.doi.org/10.1121/1.4892782]

PACS number(s): 43.35.Ud, 43.25.Gf [PBB] Pages: 965–968

I. INTRODUCTION

When a gas-filled tube is subjected to a steep axial temperature gradient, the gas column begins to oscillate spontaneously at the natural frequency. Such thermoacoustic oscillations are often enhanced by using a high-Q resonator, while installing a short *stack* of plates, a porous material having many narrow flow passages, in the region with the temperature gradient.[1] The oscillation modes can be classified into two types, depending on the resonator shape: One is the traveling wave mode[2] induced in a looped tube, and the other is the standing wave mode[3] excited in a resonance tube. In both types, high amplitude but shock-free oscillations have been observed.[4–7] These facts present clear contrast to forced acoustic gas oscillations in a *consonant* resonator; acoustic pressure at a few percent of the mean pressure creates periodic shock waves having discontinuous wave fronts,[8–12] because of unavoidable nonlinear excitation of harmonic modes.

Thermoacoustic systems can be seen as intrinsically *dissonant* even when the resonance tube or the loop has a uniform cross section, because at least the temperature gradient makes the overtones to have non-integral multiple frequencies of the fundamental through temperature dependent gas properties like sound speed. This would explain generation of quasiperiodic oscillations in some of thermoacoustic systems.[13,14] However, in the traveling wave mode oscillations, intense oscillations with shock fronts were observed recently,[15] when the pressure amplitude reached about 3% of the mean pressure. This fact indicates that the shock can be excited even if a resonator is dissonant. However, in the standing wave mode oscillations, generation of shock wave has not been observed until now although the acoustic pressure can reach about 10% of the mean pressure.[6,7] In this Letter, we report the shock wave generation in the standing wave mode oscillations and present the mechanism that tends to suppress it.

Figure 1 shows the resonance tube thermoacoustic system employed in this study. The tube was made of transparent acrylic circular tubes with an inner radius of $R = 20.5$ mm. Use of the transparent tube was to adopt a Laser Doppler velocimeter (LDV) for measurement of axial acoustic particle velocity. The length L of the tube was arbitrarily changed using two movable heavy brass plugs sealed with O-rings. A 20-mm-long ceramic catalyst support containing many square pores with the size $2r_0 \times 2r_0$ ($2r_0 = 1.49$ mm) was installed in the tube as a stack. The axial coordinate x is taken to the right along the tube axis with $x = 0$ at the surface of the left plug. The central position of the stack is denoted as $x = a$, and it was varied independently of L. The axial temperature gradient with a negative sign was created along the stack using hot and cold heat exchangers located at each side of the stack. The heat exchangers were made of

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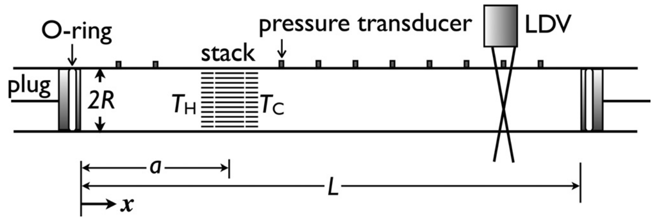

FIG. 1. Experimental setup.

0.5-mm-thick and 10-mm-long brass plates aligned in parallel with 1-mm spacing. An electrical heater was wound around the hot heat exchanger to control its temperature T_H. The cold heat exchanger was kept at room temperature T_C ($= 293\,\mathrm{K}$) by using circulating water. Air at atmospheric pressure ($p_m = 1.0 \times 10^5$ Pa) and at ambient temperature was filled in the tube as the working gas.

Many small ducts were mounted on the tube wall with an axial spacing of 50 mm for connecting pressure transducers (XTL190; Kulite Semiconductor Inc., with a natural frequency of 350 kHz) through them. When exploring the thermoacoustic shock waves, we monitored the acoustic pressure p at the duct position nearest to the surface of the solid plug at the right. The acoustic signals were recorded with a spectrum analyzer (DS-2000, Ono-Sokki), and the Fourier spectra were obtained using 16 384 points sampled at 20 kHz, from which the fundamental acoustic pressure amplitude P_{e1} was determined.

It has been reported that periodic shock waves in the driven resonance tube are induced through nonlinear effects inherent in high amplitude acoustic waves.[8,16] So we tried to make the oscillation amplitude as large as possible by changing a and L at a constant input heat power of $Q = 250$ W, expecting that the nonlinear effects would also dominate in the resonance tube thermoacoustic system. First, the stack position a was changed while keeping $L = 1.40$ m. Periodic oscillations of the fundamental mode were consistently observed with various a/L values tested from 0.08 to 0.29. The fundamental pressure amplitude P_{e1} read off from the amplitude spectrum was always beyond 3 kPa, and reached 5.7 kPa at the maximum when $a/L = 0.14$. But the shock wave was not observed. Next, we changed the resonance tube length L in the range of $0.7\,\mathrm{m} < L < 1.6$ m while fixing the ratio of $a/L = 0.14$. When $L = 0.84$ m, the pressure amplitude increased to 5.9 kPa, which is almost twice as large as the acoustic pressure amplitudes to cause shock waves in the driven resonance tube[9–12] and in the looped tube thermoacoustic system.[15] Nevertheless, the pressure waveform was continuous, as shown by open circles in Fig. 2. Finally, while fixing $L = 0.84$ m and $a/L = 0.14$, we increased the input heat power to $Q = 550$ W, being the maximum applicable heat power in the setup, and succeeded in observing periodic shock waves shown by solid circles in Fig. 2 when P_{e1} reached 8.3 kPa corresponding to 8.3% of the mean pressure.

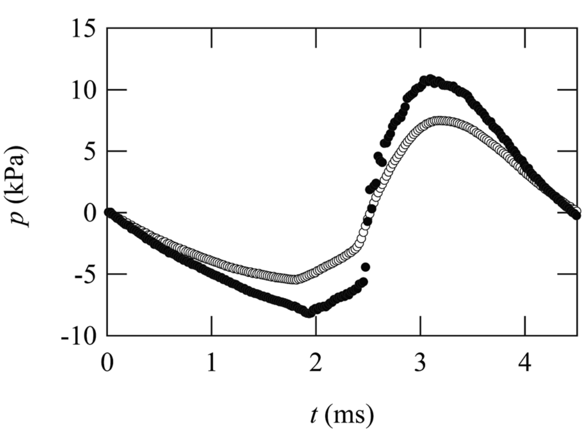

FIG. 2. Acoustic pressure waveforms observed near the end of the resonance tube when input heat power $Q = 250$ W (○) and when $Q = 550$ W (•).

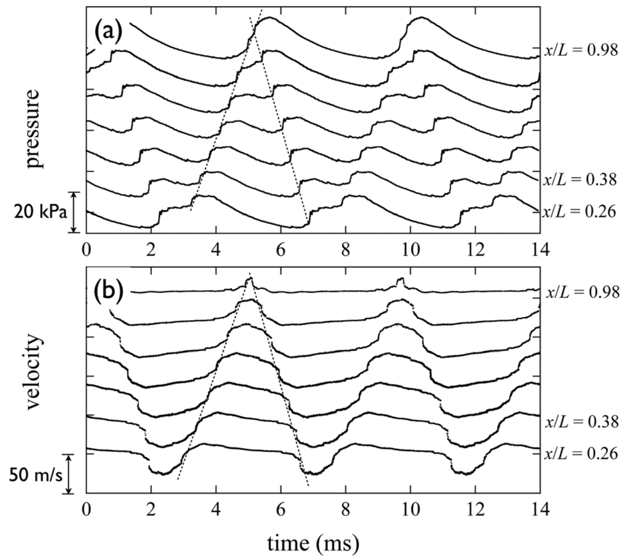

FIG. 3. Spatio-temporal evolution of thermoacoustic shock wave observed with $Q = 550$ W; acoustic pressure (a) and axial velocity on the central axis (b).

Figures 3(a) and 3(b) show the spatiotemporal evolutions of thermally induced shock waves thus obtained, where the acoustic pressure p and the axial acoustic particle velocity u on the central axis of the tube are shown in Fig. 3(a) and Fig. 3(b), respectively, and they are plotted with vertical offsets proportional to the measured axial position. The velocity was measured on the central axis of the resonance tube by using the LDV with cigarette smoke as seeding particles, and the pressure was measured simultaneously with the velocity by using the A/D converter installed in the signal processor. The dashed lines show how the shock front goes back and forth in the resonance tube. The propagation speed of shock front, determined from the slope of these lines, was found to be close to the speed of sound ($\approx 340\,\mathrm{m/s}$). In order to discover the mechanism responsible for excitation or suppression of shock waves in the resonance tube thermoacoustic system, we determined the axial distribution of the acoustic intensity I from the pressure and velocity shown in Figs. 3(a) and 3(b).

For periodic shock waves shown in Figs. 3(a) and 3(b), $p(t)$ and $u(t)$ can be decomposed as

$$p(t) = \sum_{n=1} p_n \cos(2n\pi f t) \tag{1}$$

and

$$u(t) = \sum_{n=1} u_n \cos(2n\pi f t + \phi_n), \tag{2}$$

where f represent the fundamental frequency ($f = 212.5$ Hz when $Q = 250$ W and $f = 215.6$ Hz when $Q = 550$ W), p_n and u_n the amplitudes and ϕ_n the phase difference of the nth harmonic component of p and u, $n = 1$ corresponds to the fundamental mode, $n = 2$ corresponds to the second harmonic. After resampling the measured p and u, the frequency components were determined via a fast-Fourier transform. Resampling procedure was necessary because p and u were randomly accumulated in time when the seeding particle passed through the measurement volume of the LDV. The magnitude and phase of frequency components of the central velocity u were converted to those of the cross sectional average velocity v by using the result of the laminar oscillating flow theory.[17]

The acoustic intensity I is given as

$$I = \langle p(t)v(t)\rangle, \tag{3}$$

where angular brackets indicate taking the time average. Substitution of Eqs. (1) and (2) into (3) yields

$$I = \sum_{n=1} I_{2n}, \tag{4}$$

where I_{2n} is the intensity component associated with the nth harmonic oscillation given as

$$I_{2n} = \frac{1}{2} p_n v_n \cos\phi_n. \tag{5}$$

Shown in Fig. 4(a) is the acoustic intensity I_2 of the fundamental frequency component when $Q = 250$ W and 550 W. The intensity I_2 flows out of the cold end of the stack and decreases to zero as it flows to the right end of the resonance tube. The magnitude of the negative slope observed in the uniform temperature region ($0.2 < x/L < 1.0$) represents the local energy dissipation rate per unit volume. Thus, the stack region with the nonuniform temperature plays a role of the *energy source* from which the acoustic power is supplied to sustain the lossy acoustic field. In the shock free oscillations with $Q = 250$ W, I_2 almost linearly decreases as expected for the linear acoustic waves,[17] but when the shock wave is present, a slight fluctuation seems to be superimposed on the linear decrease, which suggests the manifestation of nonlinearity associated with the high-amplitude acoustic wave.

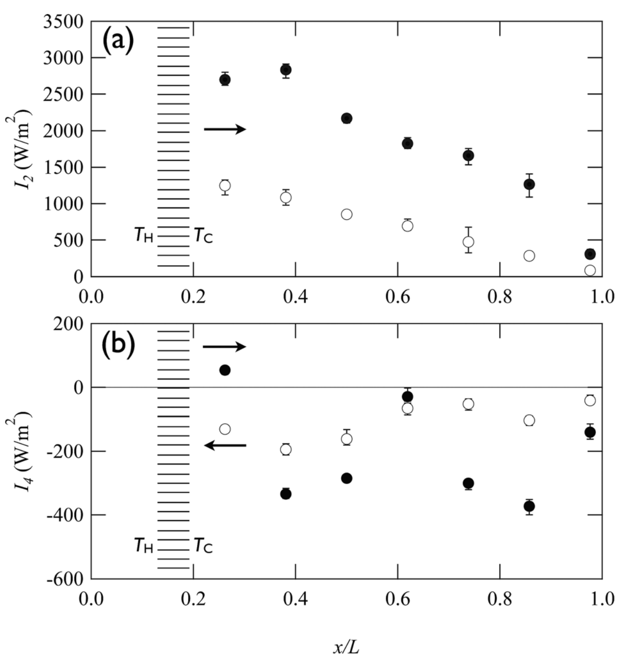

FIG. 4. Axial distribution of acoustic intensities (a) I_2 and (b) I_4 when the shock is absent with $Q = 250$ W (○), and when the shock is present with $Q = 550$ W (•). The region shown by horizontal lines represents the stack location. Arrows are shown to emphasize the flow direction of I_2 and I_4.

The intensity I_4 of the second harmonic component is shown in Fig. 4(b). The positive slopes are observed at $x \approx 0.5$ and 0.9 in both cases with and without the shock. Since thermoacoustic energy conversions to create the acoustic intensity need temperature gradients,[18,19] nonlinear energy pumping from the fundamental to the second harmonic oscillations[16] should be responsible for the positive slopes in the region without the temperature gradient. Indeed, the magnitude of the slopes becomes greater with increasing the acoustic amplitude caused by the change of Q from 250 W to 550 W. As a result, the stack changes its role from the *energy sink* to the energy source as evidenced by the change of the sign of I_4 from negative to positive at the cold side of the stack. Namely, when $Q = 250$ W I_4 goes into the stack, as observed in the previous study,[15] whereas it is emitted from the stack when the shock wave is created with $Q = 550$ W. This is a remarkable difference from the looped tube thermoacoustic system, where I_4 is observed to be amplified in the stack region with a positive temperature gradient at the moderate acoustic amplitude of about 3 kPa. Such a difference would explain why the shock waves had been inhibited in the resonance tube thermoacoustic systems and more easily excited in the looped tube system. In the case of the driven resonance tube, the shock is induced at the lower acoustic amplitudes than in the resonance tube thermoacoustic system. Installation of the stack will provide a method to prevent the shock formation in the driven resonance tube, as well as the change of the resonator shape[20] and the introduction of wave dispersion.[21]

In summary, we reported the observation of thermally induced shock waves that had been inhibited in the resonance tube thermoacoustic system. Measurements of the acoustic intensity showed that the acoustic amplitude reaching 8.3 % of the mean pressure was necessary for the nonlinear energy pumping to overcome the energy absorption in the stack region.

ACKNOWLEDGMENTS

This work is supported by KAKENHI 23360039.

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