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# New Possibility of Solving the Problem of Lifetime Ratio $\tau(\Lambda_b)/\tau(B_d)$

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**Abstract**

We discuss the problem of the large discrepancy between the observed lifetime ratio of $\Lambda_b$ to $B_d$ and the theoretical prediction obtained by the heavy quark effective theory. A new possibility of solving this problem is proposed from the viewpoint of operator product expansion and the lifetime ratio of $\Omega_b$ to $B_d$ is predicted.

The heavy quark effective theory (HQET) is successful in explaining various nature of the hadrons containing a heavy quark. The HQET has been extensively applied to the meson system containing a heavy quark and has brought about many remarkable results. The HQET has been also applied to the system of baryons containing a heavy quark and has led to some interesting results. However, when applying the HQET to the heavy baryon systems, two problems are encountered. One is that the experimental value of the mass difference between $\Sigma_b$ and $\Sigma_b^*$ is quite large in comparison with the predicted value from the mass relation in the HQET as

$$\begin{aligned} m_{\Sigma_{b^*}} - m_{\Sigma_b} &= 56 \pm 13\,\mathrm{MeV} \quad (Exp.)\,[1], \\ &= 15.8 \pm 3.3\,\mathrm{MeV} \quad (Theory)\,[2]. \end{aligned} \tag{1}$$

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The other one is that the experimental lifetime ratio of $\Lambda_b$ to $B_d$ [3]

$$\frac{\tau\,(\Lambda_b)}{\tau\,(B_d)} = 0.78 \pm 0.06 \tag{2}$$

is quite small compared with the theoretical predictions[4]–[7].

In this paper, we concentrate our attention to the latter problem and propose a possible solution to this problem from the viewpoint of operator product expansion(OPE). Moreover, we point out the importance of measuring the lifetime of $\Omega_b$ to select out the models.

The inclusive decay width of a hadron $H_b$ containing a bottom quark can be written as [5]

$$\Gamma(H_b \to X_f) = \frac{1}{m_{H_b}}\,\mathrm{Im}\,\langle H_b|\,\hat{T}\,|H_b\rangle, \tag{3}$$

where the transition operator $\hat{T}$ is given by

$$\hat{T} = i\int d^4x\, \mathrm{T}\{\,\mathcal{L}_W(x)\,\mathcal{L}_W(0)\,\}\,. \tag{4}$$

The effective Lagrangian $\mathcal{L}_W$ for the weak decay $H_b \to X_f$ is

$$\begin{aligned}\mathcal{L}_W \quad = \quad & -\frac{4G_F}{\sqrt{2}}\,V_{cb}\Big\{c_1(m_b)\left[\bar{d}'_L\gamma_\mu u_L\,\bar{c}_L\gamma^\mu b_L + \bar{s}'_L\gamma_\mu c_L\,\bar{c}_L\gamma^\mu b_L\right] \\ & \qquad + c_2(m_b)\left[\bar{c}_L\gamma_\mu u_L\,\bar{d}'_L\gamma^\mu b_L + \bar{c}_L\gamma_\mu c_L\,\bar{s}'_L\gamma^\mu b_L\right] \\ & \qquad + \sum_{\ell=e,\mu,\tau}\bar{\ell}_L\gamma_\mu\nu_\ell\,\bar{c}_L\gamma^\mu b_L\Big\} + \mathrm{h.c.}\,,\end{aligned} \tag{5}$$

where $q_L$ denotes a left-handed quark field and $d'$ and $s'$ stand for the weak eigenstates. We neglect the CKM suppressed transitions $b \to u$, because this effect is negligibly small in the present analyses. Up to the leading order, the combinations $c_\pm = c_1 \pm c_2$ are given by

$$c_\pm(m_b) = \left(\frac{\alpha_s(m_W)}{\alpha_s(m_b)}\right)^{a_\pm}\,, \qquad a_- = -2a_+ = -\frac{12}{33-2n_f}\,. \tag{6}$$

In order to pursue the calculation of the inclusive decay width in eq.(3), we must evaluate the matrix element of the nonlocal operator $\hat{T}$ in eq.(4). Although we don't know how to evaluate such the matrix element of the nonlocal operator directly, it is efficient for us to use the method of the OPE for the $\hat{T}$. This is because the energy release is quite large compared with the QCD scale in the bottom quark decay. Using the OPE, the total decay width of a hadron $H_b$ can be written in the form [8]

$$\Gamma(H_b \to X_f) = \frac{G_F^2 m_b^5}{192\pi^3}\,\frac{1}{2m_{H_b}}\left\{ c_3(f)\,\langle H_b|\,\bar{b}b\,|H_b\rangle + c_5(f)\,\frac{\langle H_b|\,\bar{b}\,g_s\sigma_{\mu\nu}G^{\mu\nu}b\,|H_b\rangle}{\Delta^2} \right.$$
$$\left. + \sum_i c_6^i(f)\frac{\langle H_b|\,(\bar{b}\,\Gamma_i q)\,(\bar{q}\Gamma_i b)\,|H_b\rangle}{\Delta^3} + \ldots \right\}, \qquad (7)$$

where $c_n(f)$ are dimensionless coefficient functions depending on the quantum numbers of the final state and including the renormalization group and phase factors, $\Gamma_i$'s denote the combinations of $\gamma$-matrices and $\Delta$ is an expansion parameter with the mass dimension, which should be much larger than the scale of $\Lambda_{QCD}$ to justify the OPE.

The Dirac spinor $b(x)$ in eq.(7), which is an operator of QCD, can be expressed as

$$b\,(x) = e^{-im_b v\cdot x}\,\{h_b(x) + \chi_b(x)\,\}\ , \qquad (8)$$

where $h_b(x)$ and $\chi_b(x)$ are the large and small component of the spinor $b(x)$ respectively and $v$ stands for the velocity of the hadron containing a bottom quark[9]. The $\chi_b(x)$ can be written in terms of $h_b(x)$ by using the equation of motion. Therefore the first and second matrix elements in eq.(7) are expanded as [4, 5]

$$\frac{1}{2m_{H_b}}\langle H_b|\,\bar{b}b\,|H_b\rangle \;=\; 1 - \frac{\mu_\pi^2(H_b) - \mu_G^2(H_b)}{4m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right)\ , \qquad (9)$$

$$\frac{1}{2m_{H_b}}\langle H_b|\,\bar{b}\,g_s\sigma_{\mu\nu}G^{\mu\nu}b\,|H_b\rangle \;=\; 2\mu_G^2(H_b) + \mathcal{O}\left(\frac{1}{m_b}\right)\ , \qquad (10)$$

with $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ defined by

$$\mu_\pi^2(H_b) \;\equiv\; -\frac{1}{2m_{H_b}}\langle H_b|\,\bar{h}_b\,\left(iD_\perp\right)^2 h_b\,|H_b\rangle\,, \qquad (11)$$

$$\mu_G^2(H_b) \;\equiv\; \frac{1}{4m_{H_b}}\langle H_b|\,\bar{h}_b\,g_s\sigma_{\mu\nu}G^{\mu\nu}h_b\,|H_b\rangle\,, \qquad (12)$$

where $D_\perp^\mu = \partial^\mu - v^\mu(v\cdot D)$ and $D^\mu$ is the covariant derivative of QCD. The parameters $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ represent the matrix elements of the kinetic energy operator and the chromo-magnetic one, respectively. These parameters can be estimated by using the mass spectra of heavy hadron states and are of $\mathcal{O}(\Lambda_{QCD}^2)$.

The third matrix element in eq.(7) can be parameterized in the model-independent way[5]. For the meson matrix elements of local four-quark operators, we use the same parameters $B_i$ and $\varepsilon_i$ as Ref.[5] such that

$$\frac{1}{2m_{B_q}}\,\langle B_q|\,O_{V-A}^q\,|B_q\rangle \;\equiv\; \frac{f_{B_q}^2 m_{B_q}}{8}\,B_1\,,$$

$$
\begin{aligned}
\frac{1}{2m_{B_q}}\langle B_q|\,O^q_{S-P}\,|B_q\rangle &\equiv \frac{f^2_{B_q} m_{B_q}}{8}\,B_2\,,\\
\frac{1}{2m_{B_q}}\langle B_q|\,T^q_{V-A}\,|B_q\rangle &\equiv \frac{f^2_{B_q} m_{B_q}}{8}\,\varepsilon_1\,,\\
\frac{1}{2m_{B_q}}\langle B_q|\,T^q_{S-P}\,|B_q\rangle &\equiv \frac{f^2_{B_q} m_{B_q}}{8}\,\varepsilon_2\,,
\end{aligned}
\tag{13}
$$

where $f_{B_q}$ is the decay constant of $B_q$ meson. The local four-quark operators defined by

$$
\begin{aligned}
O^q_{V-A} &= \bar{b}_L\gamma_\mu q_L\,\bar{q}_L\gamma^\mu b_L\,,\\
O^q_{S-P} &= \bar{b}_R\,q_L\,\bar{q}_L\,b_R\,,\\
T^q_{V-A} &= \bar{b}_L\gamma_\mu t_a q_L\,\bar{q}_L\gamma^\mu t_a b_L\,,\\
T^q_{S-P} &= \bar{b}_R\,t_a q_L\,\bar{q}_L\,t_a b_R\,,
\end{aligned}
\tag{14}
$$

where $t_a = \lambda_a/2$ are the generators of color SU(3). For the baryon matrix elements of local four-quark operators, we use the same parameters as Ref.[5, 10] such that

$$
\begin{aligned}
\langle \Lambda_b|\,\widetilde{O}^q_{V-A}\,|\Lambda_b\rangle &\equiv -\widetilde{B}_{\Lambda_b}\,\langle \Lambda_b|\,O^q_{V-A}\,|\Lambda_b\rangle\,,\\
\frac{1}{2m_{\Lambda_b}}\langle \Lambda_b|\,O^q_{V-A}\,|\Lambda_b\rangle &\equiv -\frac{f^2_{B_q} m_{B_q}}{48}\,r_{\Lambda_b}\,,\\
\langle \Omega_b|\,\widetilde{O}^q_{V-A}\,|\Omega_b\rangle &\equiv -\widetilde{B}_{\Omega_b}\,\langle \Omega_b|\,O^q_{V-A}\,|\Omega_b\rangle\,,\\
\frac{1}{2m_{\Omega_b}}\langle \Omega_b|\,O^q_{V-A}\,|\Omega_b\rangle &\equiv -\frac{f^2_{B_q} m_{B_q}}{8}\,r_{\Omega_b}\,,
\end{aligned}
\tag{15}
$$

where the local four-quark operator $\widetilde{O}^q_{V-A}$ is defined by

$$
\widetilde{O}^q_{V-A} = \bar{b}^i_L\gamma_\mu q^j_L\,\bar{q}^j_L\gamma^\mu b^i_L\,. \tag{16}
$$

By using these parameters, the $1/\Delta^3$-term of each bottom hadron can be written as follows.

$$
\begin{aligned}
&\frac{1}{2m_B}\sum_i c^i_6(f)\langle B^-|\,(\bar{b}\,\Gamma_i q)\,(\bar{q}\Gamma_i b)\,|B^-\rangle\\
&\quad = \eta\,(1-z)^2\,\left\{(2c^2_+ - c^2_-)\,B_1 + 3(c^2_+ + c^2_-)\,\varepsilon_1\right\}\,,\\
&\frac{1}{2m_B}\sum_i c^i_6(f)\langle B_d|\,(\bar{b}\,\Gamma_i q)\,(\bar{q}\Gamma_i b)\,|B_d\rangle\\
&\quad = -\eta\,(1-z)^2\left\{\frac{1}{3}\,(2c_+ - c_-)^2\,\left[\left(1+\frac{z}{2}\right)\,B_1 - (1+2z)\,B_2\right]\right.\\
&\qquad \left. + \frac{1}{2}\,(c_+ + c_-)^2\left[\,\left(1+\frac{z}{2}\right)\,\varepsilon_1 - (1+2z)\,\varepsilon_2\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&- \eta\, \sqrt{1-4z}\, \frac{|V_{cd}|^2}{|V_{ud}|^2} \left\{ \frac{1}{3}\, (2c_+ - c_-)^2\, \left[ (1-z)\, B_1 - (1+2z)\, B_2 \right] \right. \\
&\left. + \frac{1}{2}\, (c_+ + c_-)^2\, \left[ (1-z)\, \varepsilon_1 - (1+2z)\, \varepsilon_2 \right] \right\}, \\
&\frac{1}{2m_{B_s}} \sum_i c_6^i(f) \langle B_s|\, (\bar{b}\, \Gamma_i q)\, (\bar{q} \Gamma_i b)\, |B_s\rangle \\
&= -\eta'\, (1-z)^2\, \frac{|V_{us}|^2}{|V_{cs}|^2} \left\{ \frac{1}{3}\, (2c_+ - c_-)^2\, \left[ \left(1 + \frac{z}{2}\right)\, B_1 - (1+2z)\, B_2 \right] \right. \\
&\left. + \frac{1}{2}\, (c_+ + c_-)^2\, \left[ \left(1 + \frac{z}{2}\right)\, \varepsilon_1 - (1+2z)\, \varepsilon_2 \right] \right\} \\
&- \eta'\, \sqrt{1-4z} \left\{ \frac{1}{3}\, (2c_+ - c_-)^2\, \left[ (1-z)\, B_1 - (1+2z)\, B_2 \right] \right. \\
&\left. + \frac{1}{2}\, (c_+ + c_-)^2\, \left[ (1-z)\, \varepsilon_1 - (1+2z)\, \varepsilon_2 \right] \right\}, \\
&\frac{1}{2m_{\Lambda_b}} \sum_i c_6^i(f) \langle \Lambda_b|\, (\bar{b}\, \Gamma_i q)\, (\bar{q} \Gamma_i b)\, |\Lambda_b\rangle \\
&= \eta\, \frac{r_{\Lambda_b}}{16} \left\{ 8(1-z)^2\, \left[ (c_-^2 - c_+^2) + (c_-^2 + c_+^2)\, \widetilde{B}_{\Lambda_b} \right] \right. \\
&- \left[ (1-z)^2 (1+z)\, + \sqrt{1-4z}\, \frac{|V_{cd}|^2}{|V_{ud}|^2} \right] \\
&\left. \times \left[ (c_- - c_+)(5c_+ - c_-) + (c_- + c_+)^2\, \widetilde{B}_{\Lambda_b} \right] \right\}, \\
&\frac{1}{2m_{\Omega_b}} \sum_i c_6^i(f) \langle \Omega_b|\, (\bar{b}\, \Gamma_i q)\, (\bar{q} \Gamma_i b)\, |\Omega_b\rangle \\
&= -\eta'\, \frac{r_{\Omega_b}}{24} \sqrt{1-4z}\, \left[ (c_- - c_+)(5c_+ - c_-) + (c_- + c_+)^2\, \widetilde{B}_{\Omega_b} \right], \qquad (17)
\end{aligned}
$$

where $z \equiv m_c^2/m_b^2$ (we set z equal to 0.083 [5]) and $\eta$ and $\eta'$ are defined by

$$
\begin{aligned}
\eta &\equiv 16\pi^2 f_B m_B |V_{cb}|^2 |V_{ud}|^2\,, \\
\eta' &\equiv 16\pi^2 f_{B_s} m_{B_s} |V_{cb}|^2 |V_{cs}|^2\,, \qquad (18)
\end{aligned}
$$

respectively. We set both of $f_B$ and $f_{B_s}$ equal to 0.18 GeV [11].

By using these model independent parameters $B_i$, $\varepsilon_i$, $\widetilde{B}_{\Lambda_b}$ and $r_{\Lambda_b}$, the lifetime ratio of $\Lambda_b$ to $B_d$ can be written as

$$
\begin{aligned}
\frac{\tau\,(\Lambda_b)}{\tau\,(B_d)} &= 1 + \frac{\mu_\pi^2\,(\Lambda_b) - \mu_\pi^2\,(B_d)}{4m_b^2} + \left( \frac{1}{4} + c_M \frac{m_b^2}{\Delta^2} \right) \frac{\mu_G^2\,(B_d) - \mu_G^2\,(\Lambda_b)}{m_b^2} \\
&+ \frac{1}{\Delta^3} \left\{ k_1 B_1 + k_2 B_2 + k_3 \varepsilon_1 + k_4 \varepsilon_2 + \left( k_5 + k_6 \widetilde{B}_{\Lambda_b} \right) r_{\Lambda_b} \right\}, \qquad (19)
\end{aligned}
$$

where $c_M$ and $k_i$'s are the coefficients including the renormalization group factors $c_\pm$ and phase factor $z$. By using the same definition of $A_i$ and $z_i$ as in Ref.[12], $c_M$ is given by

$$
\begin{aligned}
c_M &\equiv \frac{2c_5(f)}{c_3(f)} && (20)\\
&\cong -\frac{2\left(N_c A_0 z_1 + 4N_c A_2 z_2 + 2z_1\right)}{N_c A_0 z_0 + 2z_0}\,. && (21)
\end{aligned}
$$

At the heavy quark limit $m_b \to \infty$, the lifetime ratio (19) approaches unity. However, the recent experiment (2) shows that the large discrepancy exists between the $B_d$ and $\Lambda_b$ lifetimes. Recently several literatures [4]–[7] have been devoted to study this problem. To seek a solution that can reconcile this discrepancy, the contributions of the $\mathcal{O}\left(1/\Delta^3\right)$ term in eq.(7) have been estimated under the condition that the expansion parameter $\Delta$ is equal to $m_b$. This condition might be natural because the physical scales in the system are only $\Lambda_{QCD}$ and $m_b$. The parameters $\mu_\pi^2(H_b)$ and $\mu_G^2(H_b)$ in eq.(19) can be estimated from the mass formulae of the hadrons containing a bottom quark as follows

$$
\begin{aligned}
\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B_d) &= -(0.01 \pm 0.03)\ (\mathrm{GeV})^2\,,\\
\mu_G^2(B_d) &= \frac{3}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.36\ (\mathrm{GeV})^2\,,\\
\mu_G^2(\Lambda_b) &= 0\,. && (22)
\end{aligned}
$$

The estimation of the parameters $B_i$, $\varepsilon_i$, $\widetilde{B}_{\Lambda_b}$ and $r_{\Lambda_b}$ has only been carried out in the model-dependent ways. The parameter $\widetilde{B}_{\Lambda_b}$ is equal to 1 in valence quark approximation. Because the matrix elements of $O_{V-A}^q$ and $\widetilde{O}_{V-A}^q$ differ only by a sign in this approximation, since the color wave function for a baryon is totally antisymmetric. The parameters $B_i$ and $\varepsilon_i$ have been estimated by using QCD sum rules[13]. The parameter $r_{\Lambda_b}$ have been estimated by using both QCD sum rules[7] and non-relativistic quark model[6, 10]. These analyses give the result in the ratio

$$
\frac{\tau\left(\Lambda_b\right)}{\tau\left(B_d\right)} \geq 0.94. \tag{23}
$$

Thus the problem remains unsolved at this stage.

One possibility of solving this problem was proposed in Ref.[14]. This proposal contains the insistence that the mass $m_b$ in the factor $m_b^5$ in eq.(7) should be replaced by the mass of the parent hadron $m_{B_d}$ or $m_{\Lambda_b}$. If this insistence is correct, the lifetime ratio of $B_d$ to $\Lambda_b$ is almost determined by the ratio of the decaying hadron masses;

$$
\frac{\tau\left(\Lambda_b\right)}{\tau\left(B_d\right)} \sim \left(\frac{m_B}{m_{\Lambda_b}}\right)^5 = 0.73 \pm 0.01\,. \tag{24}
$$

When this proposal is applied to charmed hadrons lifetime, the results are in good agreement with the experiment [10].

In this paper, we discuss another possibility of solving the problem of the lifetime ratio of $\Lambda_b$ to $B_d$. In the literatures [4]–[7], the expansion parameter $\Delta$ in eq.(7) is taken as $m_b$. However, the physical reason of the setting $\Delta = m_b$ is not so obvious. Therefore the value of $\Delta$ should be determined such that it is able to reproduce the experimental results of all lifetime ratios of bottomed hadrons decaying only through the weak interactions. For bottomed hadrons, the other lifetime ratios which have ever been measured by experiment [3] are

$$\frac{\tau\left(B^-\right)}{\tau\left(B_d\right)} = 1.09 \pm 0.02\,, \tag{25}$$
$$\frac{\tau\left(B_s\right)}{\tau\left(B_d\right)} = 0.97 \pm 0.05\,. \tag{26}$$

If the $B_i$, $\varepsilon_i$ and $r_{\Lambda_b}$ of the $1/\Delta^3$ order terms are treated as completely free parameters, we can not obtain any restriction to the value of $\Delta$. However the $B_i - 1$ and $\varepsilon_i$ shows how large non-factorizable effects contribute to the meson matrix elements of local four-quark operators, since if the factorization hypothesis is valid, $B_i = 1$ and $\varepsilon_i = 0$. Therefore these parameters should be given some physical restriction. In order to restrict the value of these parameters, we use the estimations by QCD sum rules[7, 13] and non-relativistic quark model[6, 10]. For $B_i$ and $\varepsilon_i$, the estimations by QCD sum rules[13]

$$\begin{aligned} |B_i - 1| &\sim 10^{-2}\,, \\ |\varepsilon_i| &\sim 10^{-2}\,, \end{aligned} \tag{27}$$

indicate that non-factorizable contributions to the meson matrix elements of local four-quark operators are not so large. Therefore we give restrictions to these parameters as follows,

$$\begin{aligned} |B_i - 1| &\le 0.1\,, \\ |\varepsilon_1| &\le 0.1\,, \\ |\varepsilon_2| &\le 0.05\,. \end{aligned} \tag{28}$$

For the parameter $r_{\Lambda_b}$, $r_{\Lambda_b} \sim 0.1 - 0.3$ has been obtained by using QCD sum rules[7] and $r_{\Lambda_b} \sim 0.6$ by using non-relativistic quark model[10]. Therefore we give restriction to the parameter as follows,

$$0.1 \le r_{\Lambda_b} \le 0.6\,. \tag{29}$$

Under setting the regions (28), (29) to parameters $B_i$, $\varepsilon_i$, and $r_{\Lambda_b}$ and setting $\widetilde{B}_{\Lambda_b}$ equal to 1, we calculate the OPE expansion parameter $\Delta$ to satisfy the experimental results of the lifetime ratios of $B^-$, $B_s$ and $\Lambda_b$ to $B_d$. As the result, we obtain

$$\Delta = 3.25 \pm 0.67\,\mathrm{GeV}\,. \tag{30}$$

This shows that the parameter $\Delta$ is smaller than the pole mass of bottom quark $m_b = 4.8 \pm 0.2\,\mathrm{GeV}$[5].

Let us consider the physical meanings of $\Delta$. Blok and Shifman[8] have discussed the role of the expansion parameter $\Delta$ to study the effects of the subleading operators in the inclusive heavy hadron decays. In Ref.[8], the parameter $\Delta$ has been taken as

$$\Delta = m_b - m_c \tag{31}$$

and the analyses have been carried out at large $m_b$ limit

$$\Delta = m_b - m_c \sim m_b \gg \Lambda_{QCD}\,. \tag{32}$$

This limit corresponds to neglect the mass ratio $m_c/m_b$. However this ratio is preserved under taking the heavy quark limit $m_{b,c} \to \infty$. Therefore we can not neglect it when considering the higher order corrections in $1/m_Q$ expansion. In fact, the pole mass of charm quark $m_c = 1.4\,\mathrm{GeV}$ [5] is not so small compared to $m_b$. The difference between quark pole masses $m_b - m_c = 3.40 \pm 0.06\,\mathrm{GeV}$ [5] well corresponds to the $\Delta$ given in eq.(30). This result indicates that we can not neglect charm quark mass for calculating the lifetime ratios of bottomed hadrons.

If the lifetime difference of $B_d$ and $\Lambda_b$ can be explained on the basis of the present approach, its applicability to other heavy hadrons should be investigated. Here we take the baryon $\Omega_b$ as a good candidate to implement our purpose. The baryon $\Xi_b$ is also a candidate which decays only through the weak interaction. For $\Xi_b$, however, we have to solve the mixing problem between $\Xi_b$ and $\Xi_b'$ [15]. Thus it is difficult to expect that we obtain the meaningful result for $\Xi_b$.

The ratio $\tau(\Omega_b)/\tau(B_d)$ will give the important clue to the lifetime problem in $B_d$-meson and $\Lambda_b$-baryon. By using the model independent parameterization similar to eq.(19), this lifetime ratio can be written in the form,

$$\begin{aligned} \frac{\tau(\Omega_b)}{\tau(B_d)} &= 1 + \frac{\mu_\pi^2(\Omega_b) - \mu_\pi^2(B_d)}{4m_b^2} + \left(\frac{1}{4} + c_M \frac{m_b^2}{\Delta^2}\right) \frac{\mu_G^2(B_d) - \mu_G^2(\Omega_b)}{m_b^2} \\ &\quad + \frac{1}{\Delta^3}\left\{k_1 B_1 + k_2 B_2 + k_3 \varepsilon_1 + k_4 \varepsilon_2 + \left(k_7 + k_8 \widetilde{B}_{\Omega_b}\right) r_{\Omega_b}\right\}\,, \end{aligned} \tag{33}$$

where $c_M$ and $k_{1-4}$ are same coefficients as eq.(19), and $k_{7,8}$ are coefficients of matrix elements between $\Omega_b$ states of the local four-quark operators. In order to estimate $1/m_b^2$ and $1/\Delta^2$ terms, we need to know the values of

$$\begin{aligned} &\mu_\pi^2(\Omega_b) - \mu_\pi^2(B_d), \\ &\mu_G^2(\Omega_b) - \mu_G^2(B_d)\,. \end{aligned} \tag{34}$$

Since the baryon $\Omega_b$ has not been found yet experimentally, we take the relations

$$\begin{aligned} \mu_\pi^2(\Omega_b) &\simeq \mu_\pi^2(\Sigma_b), \\ \mu_G^2(\Omega_b) &\simeq \mu_G^2(\Sigma_b)\,, \end{aligned} \tag{35}$$

which are followed by $SU(3)$ light flavor symmetry. The mass formulae of the HQET are translated into the forms

$$\begin{aligned} &\left\{\frac{1}{3}\left(2m_{\Sigma_b^*}+m_{\Sigma_b}\right)-\frac{1}{3}\left(2m_{\Sigma_c^*}+m_{\Sigma_c}\right)\right\}-\left\{\frac{1}{4}\left(3m_{B^*}+m_B\right)-\frac{1}{3}\left(2m_{D^*}+m_D\right)\right\} \\ &\qquad\qquad = \left\{\mu_\pi^2(B_d)-\mu_\pi^2(\Sigma_b)\right\}\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)+\mathcal{O}\left(\frac{1}{m_Q^2}\right) \end{aligned} \tag{36}$$

and

$$\mu_G^2(\Sigma_b) \simeq \frac{1}{6}\left(m_{\Sigma_b^*}^2 - m_{\Sigma_b}^2\right)\,. \tag{37}$$

From eqs.(22), (35) (36) and (37), we obtain

$$\begin{aligned} \mu_\pi^2(\Omega_b)-\mu_\pi^2(B_d) &\sim 0.03\,(\mathrm{GeV})^2, \\ \mu_G^2(\Omega_b)-\mu_G^2(B_d) &\sim -0.25\,(\mathrm{GeV})^2\,. \end{aligned} \tag{38}$$

Here we take $m_b = 4.8\,\mathrm{GeV}$, $m_c = 1.4\,\mathrm{GeV}$ and the values of the hadron masses given by Ref.[1] for $m_{\Sigma_b}$ and $m_{\Sigma_b^*}$ and Ref.[2, 16, 17] for the others. Although this estimation is influenced by the mass difference between $\Sigma_b$ and $\Sigma_b^*$, this uncertainty hardly influences to the lifetime ratio $\tau(\Omega_b)/\tau(B_d)$. For the parameters of $1/\Delta^3$ terms, we use same parameter region for $B_i$ and $\varepsilon_i$ as the case of lifetime ratio of $\Lambda_b$ to $B_d$ and set $\widetilde{B}_{\Omega_b} = 1$ (valence quark approximation) and $r_{\Omega_b} = 0.53$ (non-relativistic quark model[10]). By substituting these values and the $\Delta$ given by eq.(30), we have the ratio

$$\frac{\tau(\Omega_b)}{\tau(B_d)} = 1.10 \pm 0.06\,. \tag{39}$$

If we take the method in Ref.[10, 14] that $m_b^5$ in front of eq.(7) is replaced with $m_{H_b}^5$ while keeping $\Delta = m_b$, the ratio becomes

$$\frac{\tau(\Omega_b)}{\tau(B_d)} \sim \left(\frac{m_{B_d}}{m_{\Omega_b}}\right)^5 \simeq 0.55\,, \tag{40}$$

where we use the mass $m_{\Omega_b} = 6.06\,\mathrm{GeV}$ which is derived from the mass relations given by Ref.[2].

The discrepancy between the two predictions of the lifetime ratio $\tau(\Omega_b)/\tau(B_d)$ is quite large in contrast with that in the ratio $\tau(\Lambda_b)/\tau(B_d)$. The present approach implies

$$\tau(\Omega_b) > \tau(B_d)\,, \tag{41}$$

whereas the approach of Ref.[10, 14] leads to

$$\tau(\Omega_b) < \tau(B_d)\,. \tag{42}$$

It should be emphasized that the hierarchy of the lifetime becomes entirely opposite among the both approaches. The hierarchy obtained from the conventional approach [4]–[7] is the same as the present result (41).

In this short note, we proposed a new approach to the problem of the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$. The main point of this approach is that the expansion parameter $\Delta$ of OPE is taken to be smaller than the pole mass of bottom quark $m_b = 4.8\,\mathrm{GeV}$, numerically $3.25 \pm 0.67\,\mathrm{GeV}$. This approach could well reproduce the experimental lifetime ratio $\tau(\Lambda_b)/\tau(B_d) \cong 0.78$ with keeping the lifetime ratio $\tau(B^-)/\tau(B_d) \cong 1.09$. The large ambiguity of $\Delta$ mainly comes from the estimations of the $1/\Delta^3$ term contributions. The operator product expansion (7) is defined at scale $\mu = m_b$. Therefore when the expansion parameter $\Delta$ set different value from $m_b$, we must consider the operator rescaling effects which come from the renormalization group running $m_b$ to $\Delta$. We include these effects in our calculation. However the effects are very small since the difference between $m_b$ and $\Delta$ is small.

As for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$, the present approach and the one of Ref.[10, 14] lead to almost the same result. To discriminate the approaches, it is important to measure the lifetime of $\Omega_b$ since the prediction of the hierarchy of the lifetimes of $\Lambda_b$ and $\Omega_b$ is opposite between these approaches. Therefore the future experiment of the lifetime of $\Omega_b$ will be able to test the models clearly.

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