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The Aligned $SU(5) \times U(1)^2$ Model

Naoyuki HABA[1], Chuichiro HATTORI[2], Masahisa MATSUDA[3],
Takeo MATSUOKA[1] and Daizo MOCHINAGA[1]

[1] *Department of Physics, Nagoya University*
Nagoya, JAPAN 464-01

[2] *Science Division, General Education*
Aichi Institute of Technology
Toyota, Aichi, JAPAN 470-03

[3] *Department of Physics and Astronomy*
Aichi University of Education
Kariya, Aichi, JAPAN 448

Abstract

In Calabi-Yau string compactification, it is pointed out that there exists a new type of $SU(5) \times U(1)^2$ model (the aligned $SU(5) \times U(1)^2$ model) in which the $SU(5)$ differs from the standard $SU(5)$ and also from the flipped $SU(5)$. With the aid of the discrete symmetry suggested from Gepner model, we construct a simple and phenomenologically interesting three-generation model with the aligned $SU(5) \times U(1)^2$ gauge symmetry. The triplet-doublet splitting problem can be solved. It is also found that there is a realistic solution for solar neutrino problem and for the μ-problem. At low energies this model is in accord with the minimal supersymmetric standard model except for the existence of singlet fields with masses of $O(1)$TeV.

1 Introduction

It is very plausible that the Planck scale($M_{\rm Pl}$) is the fundamental scale of the theory which unifies all fundamental interactions. The only known candidate of the consistent Planck scale theory is the heterotic superstring theory. On the other hand, the standard model is consistent with many of observations at low energies. How does the superstring theory connect with the standard model ? How does the hierarchical ramification of the unified interaction occur ? Especially, it is important to clarify the energy scale of the ramification into $SU(3)_c$ and $SU(2)_L$ gauge interactions. If $SU(3)_c$ and $SU(2)_L$ gauge interactions are unified at the Planck scale, the ramification must have its origin in the flux breaking associated with the multiply-connectedness of the compactified manifold. If we have GUT types of gauge group such as $SU(5), SO(10)$ at the scale smaller than $M_{\rm Pl}$, the ramification into $SU(3)_c$ and $SU(2)_L$ needs to occur at an intermediate energy scale through Higgs mechanism. The scale of the ramification into $SU(3)_c$ and $SU(2)_L$ is closely related to the longevity of proton. For superstring models to be consistent with proton stability, it is required that $SU(3)_c$-triplet and $SU(2)_L$-doublet gauge bosons in **78**-representation of E_6 get masses of $O(\gtrsim 10^{16})$GeV. On the other hand, it is commonly considered that in superstring models Higgs mechanism can hardly occur at a scale of $O(\gtrsim 10^{16})$GeV. For this reason, until now many authors have preferred the case in which the ramification into $SU(3)_c$ and $SU(2)_L$ is due to flux breaking at the Planck scale. However, if there appear mirror chiral superfields in the effective theory and if an appropriate discrete symmetry restricts nonrenormalizable interactions to a special form, it is theoretically possible that Higgs mechanism occurs at a scale of $O(\gtrsim 10^{16})$GeV [1].

The purpose of this paper is to study the GUT type scenario with $SU(5)$ gauge symmetry in Calabi-Yau string compactification. In this scenario Higgs mechanism should occur at a scale M_X with $M_{\rm Pl} > M_X \gtrsim 10^{16}$GeV. As a result, we find a new type of $SU(5) \times U(1)^2$ model, which is named the aligned $SU(5) \times U(1)^2$ model by the reason shown later. As is well known, there is a disparity between $M_{\rm Pl}$ and the unification scale $O(10^{16})$GeV of gauge couplings in the minimal supersymmetric standard model [2]. In the scenario with the aligned $SU(5) \times U(1)^2$ it is possible to solve such a disparity. In this paper we construct a realistic three-generation model with the aligned $SU(5) \times U(1)^2$ gauge symmetry. In the model $SU(3)_c$ and $SU(2)_L$ gauge couplings come together at the scale $O(10^{17.5})$GeV, while the aligned $SU(5)$ and $U(1)^2$ gauge interactions are unified at the Planck scale.

In the four-dimensional effective theory from Calabi-Yau compactification the gauge symmetry G at the Planck scale becomes a subgroup of E_6. Phenomenologically it is required that the standard gauge group $G_{st} = SU(3)_c \times SU(2)_L \times U(1)_Y$ is contained in G. As an example of GUT types of the G there is an $SU(5) \times U(1)^2$ group. When we embed G_{st} into $SU(5) \times U(1)^2$, we obtain different types of $SU(5)$ according as the $SU(5)$ entirely contains $U(1)_Y$ or not. On the other hand, we assign matter fields to **27** of E_6 so as to connect the effective theory with the standard model. In the standard model Y-charges are settled for $SU(2)_L$-doublet superfields Q, L of quarks and leptons, singlet superfields U^c, D^c, E^c and Higgs-doublet superfields H_u, H_d. Then $U(1)_Y$ should be embedded into E_6 so that we can reproduce Y-charges of these matter fields. Furthermore, it is plausible for us to require that in the effective theory there appear the Yukawa interactions QU^cH_u, QD^cH_d, LN^cH_u and LE^cH_d to get Dirac masses of quarks and leptons, where N^c represents a superfield of conjugate neutrino. We study GUT types of model under these constraints on the effective theory. For illustration we take up an $SU(5) \times U(1)^2$ gauge group.

In the case $U(1)_Y \subset SU(5)$ quark and lepton superfields in **27**-representation of the E_6 belong to **10** and $\mathbf{5}^*$ representations of $SU(5)$ as

$$\begin{array}{ll} \mathbf{10}: & Q, \quad U^c, \quad E^c, \\ \mathbf{5}^*: & D^c, \quad L. \end{array}$$

The $SU(5)$ of this case is the standard $SU(5)$ [3]. Hereafter we denote this $SU(5)$ as $SU(5)_S$. In superstring models, however, we have no Higgs superfields in an adjoint representation. Therefore, the $SU(5)_S$ symmetry can not be broken spontaneously into the standard gauge group G_{st} through Higgs mechanism [4] [5]. Thus GUT type of models with $SU(5)_S$ are excluded in the scheme of Calabi-Yau compactification.

In the case $U(1)_Y \not\subset SU(5)$ we have two different types of the assignment of matter fields. The situation is as follows. In addition to the above-mentioned matter fields, in **27** of E_6 we have an extra G_{st}-neutral superfields S which is a standard $SO(10)$-singlet, and extra colored superfields g, g^c. Among these matter fields (D^c, g^c), (L, H_d) and (S, N^c) are indistinguishable with respect to G_{st}, respectively. Therefore, at first sight it seems that we may interchange the assignment of these fields to the **27** states at will. However, under the requirement that we get the Yukawa interactions LN^cH_u and QD^cH_d, it is only possible for us to interchange these fields in sets of (D^c, L, S) and (g^c, H_d, N^c). These assignments implies that extra colored superfields g and g^c mediate proton decay and then hereafter we refer g and g^c as leptoquark superfields. These leptoquark fields get masses through the Yukawa interaction Sg^cg with a non-zero VEV of S. Depending on whether the S resides in **10** or **1** of $SU(5)$, we have two different types of $SU(5)$ in the case $U(1)_Y \not\subset SU(5)$. In the case that the S belongs to **10** of $SU(5)$, a non-zero VEV of S results in the spontaneous breaking of the $SU(5)$ symmetry. While, in the case the S resides in **1** of $SU(5)$, the $SU(5)$ is unbroken even with a non-zero VEV of S.

In the case that the S belongs to **1** of $SU(5)$, quarks and leptons are assigned as

$$\begin{array}{rlll} \mathbf{10}: & Q, & D^c, & N^c, \\ \mathbf{5}^*: & U^c, & L, & \\ \mathbf{1}: & E^c. & & \end{array}$$

This assignment of matter fields to the representations of $SU(5)$ is the same in the case of the flipped $SU(5) \times U(1)$ model [6][7]. Then we denote the $SU(5)$ of this case as $SU(5)_F$. The so-called flipped $SU(5) \times U(1)$ model is derived from the compactification in which the holonomy group is $SO(6)$ [8]. On the other hand, in Calabi-Yau compactification there is a possibility of the flipped $SU(5) \times U(1)^2$ model. An extra $U(1)$ ($U(1)_\psi$) gauge symmetry distinguishes the flipped $SU(5) \times U(1)^2$ model from the flipped $SU(5) \times U(1)$ model. From the study of mass spectra it turns out that the flipped $SU(5) \times U(1)^2$ model is not realistic.

The case of $SU(5)$'s that the S resides in $\mathbf{10}$ of $SU(5)$ is a new type of $SU(5)$. In this case matter fields are assigned as

$$
\begin{array}{rlll}
\mathbf{10}: & Q, & g^c, & S, \\
\mathbf{5}: & L, & g, & \\
\mathbf{5}^*: & D^c, & H_u, & \\
\mathbf{5}^*: & U^c, & H_d, & \\
\mathbf{1}: & E^c, & & \\
\mathbf{1}: & N^c. & &
\end{array}
$$

In this case quark and lepton superfields belong separately to six irreducible representations and *are aligned in the front row on the above list*. Then a new type of $SU(5)$ is named the aligned $SU(5)$ and denoted as $SU(5)_A$. This type of GUT model has been first discussed by Panagiotakopoulos [9], who studied $SU(6) \times U(1)$ models constructed using the Tian-Yau manifold divided by Z_3. However, in Ref[9] down-type quarks, lepton-doublet and right-handed neutrinos are denoted as g^c, H_d and S, respectively. This is due to the flipped type of assignment of matter fields, in which assignment the $SU(6)$ contains the flipped $SU(5)$.

This paper is organized as follows. In section 2 we briefly review the relation between flux breaking and gauge symmetry at the Planck scale and then carry out the classification of the gauge groups. It is shown that through the abelian flux breaking there possibly appear three kinds of $SU(5) \times U(1)^2$ gauge symmetry as mentioned above. Among them the aligned $SU(5) \times U(1)^2$ model can be consistent with proton stability, when $\langle S \rangle \gtrsim 10^{16}$GeV. In section 3 we find gauge hierarchies for four types of models and clarify the processes of symmetry breaking to G_{st}. By introducing an appropriate discrete symmetry suggested from Gepner model in section 4, we construct a simple three-generation model with the aligned $SU(5) \times U(1)^2$ gauge symmetry and discuss its phenomenological implication. In the model the generation and the anti-generation numbers are 4 and 1, respectively. It is pointed out that there is an interesting solution for the triplet-doublet splitting problem and for solar neutrino problem. At low energies this model is in line with the minimal supersymmetric standard model except for the existence of G_{st}-singlet superfields. Section 5 is devoted to summary and discussion. We also show that there is a realistic solution for the μ-problem.

2 Flux breaking mechanism

In Calabi-Yau compactification on multiply-connected manifold K there generally exists a nontrivial Wilson loop U on K and then the available gauge group G at the Planck scale is reduced to a subgroup of E_6. The nontrivial U gives rise to the discrete symmetry $\overline{G}_d$, which is an embedding of $G_d = \Pi_1(K)$ in the E_6. Then G consists of the generators of E_6 which commute with all elements of $\overline{G}_d$. This mechanism is called flux breaking or Hosotani mechanism [10]. Phenomenologically it is required that the group G contains G_{st}. The generators of E_6 are denoted as $\{\ H_i,\ E_\xi\ \}$ in Cartan-Weyl basis, where H_i's are diagonal generators and E_ξ's are ladder operators associated with root vectors ξ. In the abelian flux breaking the Wilson loop U is expressed as

$$U = \exp(2\pi i \sum_i z_i H_i), \tag{1}$$

where z_i's are real parameters. In this case we obtain

$$U E_\xi U^{-1} = \exp\left\{2\pi i (Z, \xi)\right\}\ E_\xi, \tag{2}$$

where

$$(Z, \xi) = \sum_i z_i \xi_i, \qquad [H_i, E_\xi] = \xi_i E_\xi. \tag{3}$$

Under the condition $G_{st} \subset G$ the vector Z is described in terms of three real parameters α, β, γ as [5]

$$Z = \alpha\ \Theta_1 + \beta\ \Theta_2 + \gamma\ \Theta_3, \tag{4}$$

where $\Theta_i (i = 1, 2, 3)$ stand for three linearly independent $SU(3)_c \times SU(2)_L$-neutral weights in **27** representation and coincide with weights of E^c, S, N^c, respectively. The relations $(\Theta_i, \Theta_j) = \delta_{ij} + 1/3$ hold. When $U(1)_Y$ is embedded into G, $U(1)_Y$-generator is

$$Y = \frac{1}{3}(5\Theta_1 - \Theta_2 - \Theta_3), \tag{5}$$

which is orthogonal to $\Theta_{2,3}$.

Among gauge bosons in the **78** of E_6, there are three sets of $(\mathbf{3}, \mathbf{2})$ gauge bosons with respect to $SU(3)_c \times SU(2)_L$ [5]. We denote three representatives of root vectors corresponding to these three sets of gauge bosons as $\xi^{(\mathrm{A})}, \xi^{(\mathrm{B})}, \xi^{(\mathrm{C})}$. The quantum numbers of these root vectors are

$$\begin{aligned} \xi^{(\mathrm{A})} \quad &: \quad (\mathbf{3}, \mathbf{2}, -5/3, 0, 0), \\ \xi^{(\mathrm{B})} \quad &: \quad (\mathbf{3}, \mathbf{2}, 1/3, -1, -1), \\ \xi^{(\mathrm{C})} \quad &: \quad (\mathbf{3}, \mathbf{2}, 1/3, 1, -1) \end{aligned} \tag{6}$$

under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_I \times U(1)_\eta$ symmetry, where $U(1)_I$ and $U(1)_\eta$ correspond to $\Theta_2 \mp \Theta_3$ axes, respectively. Referring to $U(1)_Y$-charges we can discriminate $\xi^{(A)}$ from $\xi^{(B)}$ and $\xi^{(C)}$. On the other hand, $\xi^{(B)}$ and $\xi^{(C)}$ are indistinguishable with respect to G_{st}. Inner products of these root vectors with Z become

$$(Z, \xi^{(\mathrm{A})}) = -\alpha, \quad (Z, \xi^{(\mathrm{B})}) = -\beta, \quad (Z, \xi^{(\mathrm{C})}) = -\gamma. \tag{7}$$

The gauge group G at the Planck scale is determined depending on values of these parameters α, β, γ. From Eq.(2), if we obtain

$$(Z, \xi) \equiv 0 \qquad \text{mod } 1, \tag{8}$$

then the E_ξ becomes a generator of G. To the contrary, if we get

$$(Z, \xi) \not\equiv 0 \qquad \text{mod } 1, \tag{9}$$

the E_ξ is not a generator of G. In the case $\alpha, \beta, \gamma \not\equiv 0$ we do not have any kinds of $SU(5)$ symmetry and all three kinds of $(\mathbf{3}, \mathbf{2})$ gauge boson become massive at $O(M_{\rm Pl})$ [5]. On the other hand, when one of α, β, γ becomes zero (mod 1), there appear three kinds of $SU(5)$ symmetry as

$$\text{(A)} \quad \alpha \equiv 0, \quad \beta, \ \gamma \not\equiv 0 \qquad : \qquad G \supset SU(5)_S \times U(1)^2, \tag{10}$$

$$\text{(B)} \quad \beta \equiv 0, \quad \gamma, \ \alpha \not\equiv 0 \qquad : \qquad G \supset SU(5)_A \times U(1)^2, \tag{11}$$

$$\text{(C)} \quad \gamma \equiv 0, \quad \alpha, \ \beta \not\equiv 0 \qquad : \qquad G \supset SU(5)_F \times U(1)^2. \tag{12}$$

Here $U(1)^2$-axes in these cases correspond to $\Theta_2 \mp \Theta_3$, $\Theta_3 \mp \Theta_1$ and $\Theta_1 \mp \Theta_2$, respectively. As will be discussed in the next section, depending on the relations between two nonzero parameters the gauge group G varies from $SU(5) \times U(1)^2$ to $SU(6) \times SU(2)$ and is classified into four cases. For a moment, we concentrate on the case $G = SU(5) \times U(1)^2$. In a $\mathbf{27}$ representation of E_6, there are two sets of (D_1, L_1, S_1) and (D_2, L_2, S_2). $D_{1,2}$ reside in the representation of $(\mathbf{3}^*, \mathbf{1}, \frac{2}{3})$ as to $(SU(3)_c, SU(2)_L, U(1)_Y)$. Similarly, $L_{1,2}$ and $S_{1,2}$ reside in $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ and $(\mathbf{1}, \mathbf{1}, 0)$, respectively. (D_1, L_1, S_1) have positive $U(1)_I$-charges and (D_2, L_2, S_2) have negative $U(1)_I$-charges, respectively. A phenomenologically viable model implies that three generations of down-type quarks $(\mathbf{3}^*, \mathbf{1}, \frac{2}{3})$ (denoted as D^c) remain massless at TeV scale. Thus the other $(\mathbf{3}^*, \mathbf{1}, \frac{2}{3})$ field in $\mathbf{27}$ is considered as the leptoquark (denoted as

g^c). We can assign D_1 to down-type quarks D^c without the loss of generality. Then the sign of $U(1)_I$-charges is fixed. One can write down eleven E_6-invariant Yukawa couplings by using **27** fields as

$$\begin{array}{llll} gS_1D_2, & gS_2D_1, & QD_1L_2, & QD_2L_1, \\ H_uL_1S_2, & H_uL_2S_1, & E^cL_1L_2, & U^cD_1D_2, \\ QQg, & QU^cH_u, & U^cE^cg. & \end{array}$$

Now it is phenomenologically plausible for us to require the existence of the Yukawa couplings $QD_1L_2(= QD^cH_d)$ and $E^cL_1L_2(= E^cLH_d)$ to obtain available Dirac masses of quarks and leptons at weak scale. Since we take D_1 as down-quark D^c, we should assign L_2 to H_d and L_1 to L, respectively. Moreover, the coupling $H_uL_1S_2$ can give the neutrino Dirac masses through $\langle H_u \rangle$ so that we should assign S_2 to right-handed neutrino N^c. Consequently, under the assignment of $D_1 = D^c$ we must take (D_1, L_1, S_1) as (D^c, L, S), which have positive $U(1)_I$-charges, and also (D_2, L_2, S_2) as (g^c, H_d, N^c), which have negative $U(1)_I$-charges. We denote here S_1 as S. For three cases (A), (B) and (C) irreducible decompositions of the **27** matter fields under $SU(5) \times U(1)^2$ are shown in Table I. In the case (A) both S and N^c reside in **1** of $SU(5)$. While, in the case (B) S resides in **10** of $SU(5)$ but N^c in **1**. In the case (C) N^c resides in **10** of $SU(5)$ but S in **1**.

Table I

The $SU(5)_S$ in the case (A) is just the standard $SU(5)$ [3]. In order to break down $SU(5)_S$ into G_{st} at an intermediate energy scale, $(\mathbf{3}, \mathbf{2}, -5/3)$ gauge superfields under $SU(3)_c \times SU(2)_L \times U(1)_Y$ should become massive via Higgs mechanism. However, there are no $(\mathbf{3}, \mathbf{2}, -5/3)$ chiral superfields in the **27** which would be absorbed to give masses to $(\mathbf{3}, \mathbf{2}, -5/3)$ gauge superfields. Thus in the case (A) we can not construct a realistic model. The standard $SU(5)$-GUT model is excluded in the Calabi-Yau string theory.

The $SU(5)_A$ in the case (B) can be broken into $SU(3)_c \times SU(2)_L$, when S develops a nonzero VEV. This is due to the fact that S belongs to the **10** of $SU(5)_A$. When we decompose $SU(5)_A$ into $SU(3)_c \times SU(2)_L \times U(1)_{Y'}$, we obtain

$$Y' = \frac{1}{3}(5\Theta_2 - \Theta_3 - \Theta_1). \tag{13}$$

The $U(1)_Y$ is given by a linear combination of $U(1)_{Y'}$ and $U(1)^2$ aside from $SU(5)_A$. In this case a quark-doublet superfield Q is absorbed by $(\mathbf{3}, \mathbf{2}, 1/3)$ gauge superfields via Higgs mechanism. The $(\mathbf{3}, \mathbf{2}, 1/3)$ gauge superfields gain masses of order $\langle S \rangle$. Proton decay is caused not only by the interactions of $(\mathbf{3}, \mathbf{2}, 1/3)$ gauge superfields but also by the interactions of leptoquark superfields g and g^c. In the $SU(5)_A \times U(1)^2$ model we have four independent Yukawa coupling constants $\lambda^{(r)}$ $(r = 1 \sim 4)$ which appear in the superpotential

$$\begin{aligned} W_Y &= \lambda^{(1)} \left(QQg + Qg^cL + g^cSg \right) \\ &\quad + \lambda^{(2)} \left(QU^cH_u + QH_dD^c + g^cU^cD^c + SH_dH_u \right) \\ &\quad + \lambda^{(3)} \left(LH_dE^c + gU^cE^c \right) + \lambda^{(4)} \left(LH_uN^c + gD^cN^c \right), \end{aligned} \tag{14}$$

where the generation indices are omitted and λ's are all expected to be $O(1)$. From Eq.(14) leptoquark superfields g and g^c also gain masses of the order $\langle S \rangle$ through the Yukawa interactions $\lambda^{(1)} g^cSg$. Thus, at energies below $\langle S \rangle$, g and g^c decouple from the effective theory. Therefore, if $\langle S \rangle$ is equal to or larger than $O(10^{16})$GeV, this model is consistent with proton stability.

The flipped type of $SU(5)$ in the case (C) can be decomposed into $SU(3)_c \times SU(2)_L \times U(1)_{Y''}$. The generator Y'' is given by

$$Y'' = \frac{1}{3}(5\Theta_3 - \Theta_1 - \Theta_2). \tag{15}$$

When N^c develops a nonzero VEV, a quark-doublet superfield Q is absorbed to give masses of $O(\langle N^c \rangle)$ to $(\mathbf{3}, \mathbf{2}, 1/3)$ gauge superfields. In the $SU(5)_F \times U(1)^2$ model Yukawa interactions are given by

$$\begin{aligned} W_Y &= \lambda^{(1)} \left(QQg + QD^cH_d + D^cN^cg \right) \\ &\quad + \lambda^{(2)} \left(QH_uU^c + Qg^cL + D^cg^cU^c + N^cH_uL \right) \\ &\quad + \lambda^{(3)} \left(H_uH_dS + g^cgS \right) + \lambda^{(4)} \left(LH_dE^c + U^cgE^c \right). \end{aligned} \tag{16}$$

In the symmetry breaking due to a nonzero $\langle N^c \rangle$, leptoquark superfields g and g^c can not gain masses of $O(\langle N^c \rangle)$. Unless S develops a large VEV, we are led to the fast proton decay. To avoid this difficulty, S also should develop its VEV of $O(\gtrsim 10^{16})$GeV. Thus in the case (C) it is required that both $\langle S \rangle$ and $\langle N^c \rangle$ are $O(\gtrsim 10^{16})$GeV. In this scheme of symmetry breaking it is impossible for us to get a large Majorana-mass of right-handed neutrino [11]. Furthermore, since the Yukawa couplings of g^cgS and H_uH_dS take a common value $\lambda^{(3)}$, we can not solve the triplet-doublet splitting problem. Thus the $SU(5)_F \times U(1)^2$ model is not realistic. It should be noted that the present $SU(5)_F \times U(1)^2$ model is quite different from the flipped $SU(5) \times U(1)$ model. The $U(1)$ factor group in the flipped $SU(5) \times U(1)$ model corresponds to $(4\Theta_1 - \Theta_2)/2$, while an extra $U(1)$ symmetry in the $SU(5)_F \times U(1)^2$ model relative to the flipped $SU(5) \times U(1)$ model corresponds to Θ_2-axis, i.e. $U(1)_\psi$. Due to the extra $U(1)_\psi$ symmetry, many of the Yukawa interactions such as $(\mathbf{10}^* \cdot \mathbf{10}^* \cdot \mathbf{5}^*)$, $(\mathbf{10} \cdot \mathbf{10}^* \cdot \mathbf{1})$ which appear in the flipped $SU(5) \times U(1)$ model are forbidden. As a consequence, unlike in the flipped $SU(5) \times U(1)$ model the triplet-doublet splitting mechanism and see-saw mechanism [12] [13] are not at work in the $SU(5)_F \times U(1)^2$ model. Furthermore, there is a sharp distinction between the present $SU(5)_F \times U(1)^2$ model and the flipped $SU(5)_F \times U(1)$ model with respect to their generation structure.

When all but one of α, β, γ are zero (mod 1), there appears an $SO(10) \times U(1)$ gauge group. In this case we also have three kinds of model. For instance, in the case $\alpha, \gamma \equiv 0$ and $\beta \not\equiv 0$ the $SO(10)$ referred here is the same as the usual one. As mentioned above, not only the case $\alpha \equiv 0$ but also the case $\gamma \equiv 0$ are unfavorable. Thus we have no possibilities of the $SO(10) \times U(1)$ gauge symmetry.

Next we consider the case of non-abelian flux breaking. Root vectors of the E_6 perpendicular to those of G_{st} are restricted only to $\pm(\Theta_2 - \Theta_3)$. Since these root vectors compose $SU(2)$ group, the remaining gauge symmetry is at most $SU(6)$. This $SU(6)$ involves $SU(5)_S$ but neither $SU(5)_A$ nor $SU(5)_F$. Since we have no realistic solutions for the $SU(5)_S$-GUT, $SU(3)_c$ and $SU(2)_L$ should be already separated in the non-abelian flux breaking at the Planck scale.

3 Gauge hierarchies

As discussed in the previous section, the realistic scenarios with $SU(5) \times U(1)^2$ gauge symmetry are limited only to the case (B) of flux breaking

$$\beta \equiv 0, \quad \gamma,\ \alpha \not\equiv 0. \tag{17}$$

Then we proceed to study the case (B) including the aligned $SU(5) \times U(1)^2$ model. To explain large Majorana-masses of right-handed neutrinos, we now consider the hierarchical symmetry breaking with $\langle S \rangle \gtrsim 10^{16}\mathrm{GeV} \gg \langle N^c \rangle \gg m_{\mathrm{susy}}$ [15] [11], where m_{susy} represents the susy breaking scale of $O(1)$TeV. Some of Gepner models [14] potentially implement this hierarchical type of symmetry breaking.

To maintain supersymmetry down to m_{susy}, the D-terms should vanish at large scales $\langle S \rangle$ and $\langle N^c \rangle$. It is realized by assuming the existence of mirror chiral superfields of S and N^c and by setting $\langle S \rangle = \langle \overline{S} \rangle$ and $\langle N^c \rangle = \langle \overline{N}^c \rangle$. In what follows we take up this scheme of symmetry breaking. The gauge group G in the region ranging from the Planck scale to the scale $\langle S \rangle$ is classified into four cases depending on the relations between α and γ. For example, when $\alpha - \gamma \equiv 0$, there appears $SU(2)_R$

symmetry associated with root vectors $\pm(\Theta_3 - \Theta_1)$. Chiral superfields in the **27** of E_6 are decomposed into the irreducible representations of G. These situations are summarized as follows;

$$
\begin{array}{lll}
\text{(B1)} & \alpha+\gamma \equiv \alpha-\gamma \equiv 0 & : \quad G = SU(6)\times SU(2)_R \\
& & (\mathbf{15},\mathbf{1}) \ : \ Q, g, g^c, L, S \\
& & (\mathbf{6}^*,\mathbf{2}) \ : \ U^c, D^c, H_u, H_d, E^c, N^c \\
\text{(B2)} & \alpha+\gamma \equiv 0,\ \alpha-\gamma \not\equiv 0 & : \quad G = SU(6)\times U(1)_R \\
& & (\mathbf{15},0) \ : \ Q, g, g^c, L, S \\
& & (\mathbf{6}^*,1) \ : \ U^c, H_d, N^c \\
& & (\mathbf{6}^*,-1) \ : \ D^c, H_u, E^c \\
\text{(B3)} & \alpha+\gamma \not\equiv 0,\ \alpha-\gamma \equiv 0 & : \quad G = SU(5)\times SU(2)_R\times U(1) \\
& & (\mathbf{10},\mathbf{1},\frac{2}{3}) \ : \ Q, g^c, S \\
& & (\mathbf{5},\mathbf{1},-\frac{4}{3}) \ : \ L, g \\
& & (\mathbf{5}^*,\mathbf{2},-\frac{1}{3}) \ : \ U^c, D^c, H_u, H_d \\
& & (\mathbf{1},\mathbf{2},\frac{5}{3}) \ : \ E^c, N^c \\
\text{(B4)} & \alpha+\gamma \not\equiv 0,\ \alpha-\gamma \not\equiv 0 & : \quad G = SU(5)\times U(1)_R\times U(1) \\
& & (\mathbf{10},0,\frac{2}{3}) \ : \ Q, g^c, S \\
& & (\mathbf{5},0,-\frac{4}{3}) \ : \ L, g \\
& & (\mathbf{5}^*,1,-\frac{1}{3}) \ : \ U^c, H_d \\
& & (\mathbf{5}^*,-1,-\frac{1}{3}) \ : \ D^c, H_u \\
& & (\mathbf{1},-1,\frac{5}{3}) \ : \ E^c \\
& & (\mathbf{1},1,\frac{5}{3}) \ : \ N^c
\end{array}
$$

When S develops a nonzero VEV, the gauge group G is spontaneously broken into a smaller group G'. For each case we have

$$\begin{aligned}
&\text{(B1)} \qquad G' = SU(4) \times SU(2)_L \times SU(2)_R, \\
&\text{(B2)} \qquad G' = SU(4) \times SU(2)_L \times U(1)_R, \\
&\text{(B3)} \qquad G' = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1), \\
&\text{(B4)} \qquad G' = SU(3)_c \times SU(2)_L \times U(1)_R \times U(1).
\end{aligned}$$

The $SU(4)$ in the cases (B1) and (B2) is the Pati-Salam $SU(4)$ [16]. In the cases (B1) and (B2), $(\mathbf{4}, \mathbf{2})$, $(\mathbf{4}^*, \mathbf{2})$ and $(\mathbf{1}, \mathbf{1})$ gauge superfields under $SU(4) \times SU(2)_L$ absorb a pair of Q, L and $\overline{Q}, \overline{L}$ and $(S - \overline{S})/\sqrt{2}$ via Higgs mechanism. In the cases (B3) and (B4), $(\mathbf{3}, \mathbf{2})$, $(\mathbf{3}^*, \mathbf{2})$ and $(\mathbf{1}, \mathbf{1})$ gauge superfields under $SU(3)_c \times SU(2)_L$ absorb a pair of Q and $\overline{Q}$ and $(S - \overline{S})/\sqrt{2}$.

In the subsequent symmetry breaking due to nonzero $\langle N^c \rangle$ the gauge group G' is broken to G_{st}. In the case (B1) a pair of U^c, E^c and $\overline{U}^c, \overline{E}^c$ and $(N^c - \overline{N}^c)/\sqrt{2}$ are absorbed by gauge superfields. In the case (B2) a pair of U^c and $\overline{U}^c$ and $(N^c - \overline{N}^c)/\sqrt{2}$ and in the case (B3) a pair of E^c and $\overline{E}^c$ and $(N^c - \overline{N}^c)/\sqrt{2}$ are absorbed. In the case (B4) only $(N^c - \overline{N}^c)/\sqrt{2}$ is absorbed.

4 A simple model

In this section we construct a simple three-generation model for the case (B4) $G = SU(5)_A \times U(1)^2$. From the observation of this example we will see that the discrete symmetry of the compactified manifold controls many parameters of the low-energy effective theory. To obtain three-generation models at low energies, the difference between the generation number and the anti-generation number should be three at the Planck scale. Concretely, here the generation number and the anti-generation number are taken as 4 and 1, respectively. This generation structure is illustrated in Table II.

Table II

When the effective theory has Gepner type of discrete symmetry $Z_{2k+1} \times Z_2$ coming from the symmetry of the compactified manifold, nonrenormalizable terms of the superpotential have peculiar structure. Especially, if Z_{2k+1}-charges of $S_0, \overline{S}$ and $N_0^c, \overline{N}^c$ are 1 and k, respectively, the nonrenormalizable terms incorporated only by these fields are of special form [11]

$$W_{NR} \sim \lambda M_C^3 \left[\left(\frac{S_0 \overline{S}}{M_C^2} \right)^{2k} + k \left(\frac{N_0^c \overline{N}^c}{b^2 M_C^2} \right)^2 - 2c \left(\frac{S_0 \overline{S}}{M_C^2} \right)^k \left(\frac{N_0^c \overline{N}^c}{b^2 M_C^2} \right) \right], \qquad (18)$$

where M_C represents the compactification scale and λ, b and c are real constants of $O(1)$. Here we assume that the soft susy-breaking mass parameter $m_{S_0}^2 + m_{\overline{S}}^2$, whose running behavior is controlled by the renormalization group equation, becomes negative in the energy region $O(10^{17})$GeV. As investigated in Ref.[11], carrying out the minimization of the scalar potential under the conditions $k = 3, 4, \cdots$ and $0 < c < \sqrt{2k}$ $(c \neq \sqrt{k})$, we obtain

$$\langle S_0 \rangle = \langle \overline{S} \rangle \sim M_C x, \qquad (19)$$

$$\langle N_0^c \rangle = \langle \overline{N}^c \rangle \sim M_C x^k \qquad (20)$$

with

$$x = \left(\frac{m_{\rm susy}}{M_C} \right)^{1/(4k-2)}. \qquad (21)$$

Through Higgs mechanism $(\mathbf{3}, \mathbf{2})$ and $(\mathbf{3}^*, \mathbf{2})$ gauge superfields become massive at the scale $\langle S_0 \rangle = \langle \overline{S} \rangle$. It is Q_0 and $\overline{Q}$ that are absorbed by $(\mathbf{3}, \mathbf{2})$ and $(\mathbf{3}^*, \mathbf{2})$ gauge superfields. Since gauge interactions are diagonal with respect to the generation degree of freedom, the superfields absorbed here have the same generation indices as S_0 and $\overline{S}$. Thus at energies below $\langle S_0\rangle = \langle \overline{S}\rangle$ only three generations of quark $Q_i (i = 1, 2, 3)$ remain massless. Through the symmetry breaking $(S_0 - \overline{S})/\sqrt{2}$ is also absorbed by a gauge superfield associated with a diagonal generator. Remaining massless S fields become $S_i (i = 1, 2, 3)$ and $(S_0 + \overline{S})/\sqrt{2}$.

Leptoquark superfields $g,\ g^c\ (\overline{g},\ \overline{g}^c)$ gain their masses of order $\langle S_0\rangle = \langle \overline{S}\rangle$ through the Yukawa interactions $\sum_{i,j=0\sim 3} \lambda^{(1)}_{i0j}\, g^c_i S_0 g_j\ (\overline{\lambda}^{(1)}\, \overline{g}^c \overline{S}\,\overline{g})$. Here $\lambda^{(1)}_{i0j}$ can be considered as a matrix with respect to the indices i, j. If this matrix is rank four, all g and g^c are massive. On the other hand, doublet Higgs get their masses through the Yukawa interactions $\sum_{i,j=0\sim 3} \lambda^{(2)}_{0ij}\, S_0 H_{di} H_{uj}\ (\overline{\lambda}^{(2)}\, \overline{S}\,\overline{H}_d \overline{H}_u)$. If the matrix $\lambda^{(2)}_{0ij}$ is rank three, a pair of H_u and H_d remains massless and the other three pairs of them become massive at the scale $\langle S_0\rangle = \langle \overline{S}\rangle$. As seen in Table-II, since g, g^c, H_u and H_d belong to different irreducible representations of $SU(5)_A$ with each other, it is likely that the Yukawa couplings $\lambda^{(1)}_{i0j}$ and $\lambda^{(2)}_{0ij}$ have distinct structure with respect to their ranks. In the present model triplet-doublet splitting is attributable to the disparity of ranks of $\lambda^{(1)}_{i0j}$ and $\lambda^{(2)}_{0ij}$. In Eq.(14) we have the Yukawa interactions $\sum \lambda^{(4)}_{ijk} (L_i H_{uj} + g_i D^c_j) N^c_k$. In the subsequent symmetry breaking due to a nonzero $\langle N^c_0\rangle$ there possibly appear the mixings between H_{uj} and $L^\dagger_i$ and between D^c and $g^\dagger$. If $\lambda^{(4)}_{ij0}$'s vanish for all i and j, these mixings are avoidable. The condition $\lambda^{(4)}_{ij0} = 0$ can be explained under appropriate charge assignments for the discrete symmetry $Z_{2k+1} \times Z_2$ to L_i, H_{uj} and N^c_0.

In the present model U^c, D^c, L and E^c also have four generations and an anti-generation at the Planck scale. If these superfields have appropriate charges of the discrete symmetry $Z_{2k+1} \times Z_2$, we get the nonrenormalizable terms

$$\frac{1}{M_C^{2l_1-1}} (S_0\overline{S})^{l_1} (U^c_0 \overline{U}^c) + \frac{1}{M_C^{2l_2-1}} (S_0\overline{S})^{l_2} (D^c_0 \overline{D}^c) + \frac{1}{M_C^{2l_3-1}} (S_0\overline{S})^{l_3} (L_0 \overline{L}) + \frac{1}{M_C^{2l_4-1}} (S_0\overline{S})^{l_4} (E^c_0 \overline{E}^c), \qquad (22)$$

where $l_i < 2k-1$ $(i = 1 \sim 4)$. These terms induce masses for U_0^c, D_0^c, L_0, E_0^c and $\overline{U}^c, \overline{D}^c, \overline{L}, \overline{E}^c$. In fact, by substituting $S_0, \overline{S}$ by their nonzero VEVs, we get

$$\begin{aligned} M_{U_0^c,\overline{U}^c} &\sim M_C x^{2l_1}, \qquad M_{D_0^c,\overline{D}^c} \sim M_C x^{2l_2}, \\ M_{L_0,\overline{L}} &\sim M_C x^{2l_3}, \qquad M_{E_0^c,\overline{E}^c} \sim M_C x^{2l_4}. \end{aligned} \tag{23}$$

Consequently, at energies below $M_{U_0^c,\overline{U}^c}$, $M_{D_0^c,\overline{D}^c}$, $M_{L_0,\overline{L}}$ and $M_{E_0^c,\overline{E}^c}$ there remain only three generations of U^c, D^c, L and E^c.

At the scale $\langle N_0^c \rangle$, $(N_0^c - \overline{N}^c)/\sqrt{2}$ is absorbed by a gauge superfield. By assigning appropriate charges to N_i^c, we can obtain large Majorana-masses of N^c, which lead to sufficiently small neutrino masses by see-saw mechanism. As shown in Ref.[11], the superpotential Eq.(18) leads to Majorana-masses

$$M_M \sim M_C x^{2k} = \sqrt{m_{\rm susy} M_C}\, x \tag{24}$$

for $N_i^c (i = 1, 2, 3)$ and for

$$N' = \cos\theta \frac{1}{\sqrt{2}}(N_0^c + \overline{N}^c) + \sin\theta \frac{1}{\sqrt{2}}(S_0 + \overline{S}) \tag{25}$$

with

$$\theta \sim x^{k-1}. \tag{26}$$

Thus at energies below M_M available G_{st}-singlet superfields are limited only to $S_i (i = 1, 2, 3)$ and to

$$S' = -\sin\theta \frac{1}{\sqrt{2}}(N_0^c + \overline{N}^c) + \cos\theta \frac{1}{\sqrt{2}}(S_0 + \overline{S}), \tag{27}$$

whose masses are $O(m_{\rm susy})$.

As an example, let us consider the case $k = 5$. In this case the discrete symmetry becomes $Z_{11} \times Z_2$. The Z_{11}-charges of $S_0, \overline{S}$ and $N_0^c, \overline{N}^c$ are 1 and 5, respectively. Here we take $l_1 = l_2 = l_3 = l_4 = 3$ and take numerical values of M_C and $m_{\rm susy}$ as

$$M_C \cong \frac{M_{\rm Pl}}{\sqrt{8\pi}} \cong 10^{18.4}{\rm GeV}, \qquad m_{\rm susy} = 10^3{\rm GeV}. \tag{28}$$

In this case we get $x = 10^{-0.86}$ and mass hierarchies become

$$\langle S_0 \rangle = \langle \overline{S} \rangle \cong 10^{17.5}\mathrm{GeV}, \tag{29}$$

$$\langle N_0^c \rangle = \langle \overline{N}^c \rangle \cong 10^{14.1}\mathrm{GeV}, \tag{30}$$

$$M_{U_0^c, \overline{U}^c},\ M_{D_0^c, \overline{D}^c},\ M_{L_0, \overline{L}},\ M_{E_0^c, \overline{E}^c} \cong 10^{13.3}\mathrm{GeV}, \tag{31}$$

$$M_M \cong 10^{9.8}\mathrm{GeV}. \tag{32}$$

Large Majorana-masses M_M obtained here solve the solar neutrino problem [11]. At energies below M_M this model is in accord with the minimal supersymmetric standard model except for the existence of singlet fields $S_i (i = 1, 2, 3)$ and S'.

Now it is interesting to study the unification of gauge coupling constants. In the aligned $SU(5) \times U(1)^2$ model, $SU(3)_c$ and $SU(2)_L$ gauge couplings should be unified at the scale $\langle S \rangle$ but not at the Planck scale. On the other hand, due to possible existence of gauge kinetic mixing terms unification of abelian gauge couplings is not straightforward [17]. Here we confine ourselves to non-abelian gauge couplings. The one-loop renormalization group equation for gauge couplings reads

$$\frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2 \qquad (i = 3, 2) \tag{33}$$

with $t = \ln(\mu/\mu_0)$. In the model explored above the coefficients of β-functions for $SU(3)_c$ and $SU(2)_L$ gauge couplings are given by

$$b_3 = -1, \qquad b_2 = 2 \tag{34}$$

in the energy region from $\langle S \rangle$ to $M_{U_0^c, \overline{U}^c}$ and

$$b_3 = -3, \qquad b_2 = 1 \tag{35}$$

in the energy region from $M_{U_0^c, \overline{U}^c}$ to m_{susy}. Therefore, for each region the difference $b_2 - b_3$ becomes 3 and 4, respectively. To the contrary, in the minimal supersymmetric standard model the difference is equal to 4 over the range from $M_{\rm GUT}$ to $m_{\rm susy}$. The present model leads to the relation

$$\alpha_2(M_Z)^{-1} - \alpha_3(M_Z)^{-1} = \frac{1}{4\pi}\left[8\,\ln\left(\frac{M_C}{M_Z}\right) - \frac{2l+3}{2k-1}\,\ln\left(\frac{M_C}{m_{\rm susy}}\right)\right] \tag{36}$$

in the one-loop renormalization group calculation, where $l = l_1 = l_2 = l_3 = l_4$. We use the unification condition $\alpha_3 = \alpha_2$ at the scale $\langle S\rangle$ and Eqs.(19) and (21). As far as the difference $\alpha_2(\mu)^{-1} - \alpha_3(\mu)^{-1}$ is concerned, the two-loop effect gives only a small correction to the one-loop effect. After numerical calculations we find that when $l = k-2$, the unification of $SU(3)_c$ and $SU(2)_L$ gauge couplings at the scale $\langle S\rangle$ is consistent with experimental data. Detailed renormalization group analysis of gauge couplings including abelian ones will be presented elsewhere.

5 Summary and Discussion

In Calabi-Yau string compactification, there possibly exist three kinds of $SU(5) \times U(1)^2$ gauge symmetry which contain $SU(3)_c \times SU(2)_L \times U(1)_Y$. Among them realistic models can be constructed only in the case of the aligned $SU(5) \times U(1)^2$ gauge symmetry, in which the $SU(5)$ differs from the standard $SU(5)$ and also from the flipped $SU(5)$. In this model the gauge group $G = SU(5)_A \times U(1)^2$ at the Planck scale is spontaneously broken into G_{st} by two stages when G_{st}-neutral fields in the **27** of E_6 develop nonzero VEVs. At the first stage, when the field S in **10** of $SU(5)_A$ evolves its VEV of $O(\gtrsim 10^{16})$GeV, G is broken into

$$G' = SU(3)_c \times SU(2)_L \times U(1)^2. \tag{37}$$

Subsequent symmetry breaking from G' to G_{st} is attributed to a nonzero VEV of N^c. Although the unification scale of all fundamental interactions is the Planck scale, $SU(3)_c$ and $SU(2)_L$ gauge couplings come together at the scale $\langle S\rangle = O(10^{17.5})$GeV.

Therefore, the string threshold effect takes part in the unification of $SU(5)_A$ and $U(1)^2$ gauge couplings but does not in the unification of $SU(3)_c$ and $SU(2)_L$ gauge couplings.

In this paper we constructed a simple three-generation model with the aligned $SU(5) \times U(1)^2$. Under appropriate charge assignments of Gepner type of discrete symmetry mass spectra of the model comes down to as follows. Through Higgs mechanism at the scale $\langle S_0 \rangle$, chiral superfields $Q_0, \overline{Q}$ and $(S_0 - \overline{S})/\sqrt{2}$ are absorbed by gauge superfields. At the same scale all of g, g^c and $\overline{g}, \overline{g}^c$ become massive. All but one set of H_u and H_d also gain their masses. At the next stage of symmetry breaking due to $\langle N_0^c \rangle$, $(N_0^c - \overline{N}^c)/\sqrt{2}$ is absorbed. Chiral superfields U_0^c, D_0^c, L_0, E_0^c and $\overline{U}^c, \overline{D}^c, \overline{L}, \overline{E}^c$ get masses of order $M_C\, x^{2l_i}$ through the nonrenormalizable interactions. Chiral superfields $N_i^c (i = 1, 2, 3)$ and $N' (\sim N_0^c + \overline{N}^c)$ get large Majorana-masses $M_M \sim M_C\, x^{2k} = \sqrt{m_{\rm susy} M_C}\, x$ also via nonrenormalizable interactions. Thus, the triplet-doublet splitting problem and solar neutrino problem can be solved with the aid of the discrete symmetry. Consequently, at energies below M_M, $M_{U_0^c, \overline{U}^c}$, $M_{D_0^c, \overline{D}^c}$, $M_{L_0, \overline{L}}$ and $M_{E_0^c, \overline{E}^c}$ available superfields are reduced to three generations of $Q_i, U_i^c, D_i^c, L_i, E_i^c (i = 1, 2, 3)$, a pair of Higgs superfield H_u, H_d and singlet superfields $S_i (i = 1, 2, 3), S' (\sim S_0 + \overline{S})$. The model obtained here is in accord with the minimal supersymmetric standard model except for the existence of singlet fields $S_i (i = 1, 2, 3)$ and S' with masses of $O(m_{\rm susy})$.

In the present model we can find a realistic solution also for the μ-problem. Since there is a nonrenormalizable term

$$\frac{1}{M_C^{2n}} (S_0 \overline{S})^n S_0 H_u H_d \tag{38}$$

for a pair of light Higgs fields H_u and H_d, we obtain the induced μ-term with

$$\mu \sim M_C\, x^{2n+1}. \tag{39}$$

If the sum of Z_{2k+1}-charges of H_u and H_d is 1, then we get $n = 2k - 1$ and

$$\mu \sim m_{\rm susy}\, x. \tag{40}$$

By taking $m_{\rm susy} \sim 1\text{TeV}$ and $k = 5$, one finds

$$\mu \sim 100\text{GeV}. \tag{41}$$

This is a plausible solution for the μ-problem. Moreover, there is a possibility that the present model gives a plausible interpretation of quark/lepton mass hierarchy. The problem will be studied in detail elsewhere [18]. The discrete symmetry of the compactified manifold as well as the supersymmetry breaking and the gauge hierarchy plays an important role in connecting the superstring theory with the standard model and in determining the parameters of the standard model.

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Table Captions

Table I Irreducible decompositions of the **27** matter superfields under three kinds of $SU(5) \times U(1)^2$. $U(1)^2$-axes in three cases correspond to $\Theta_2 \mp \Theta_3$, $\Theta_3 \mp \Theta_1$ and $\Theta_1 \mp \Theta_2$, respectively. The numbers in parentheses are the dimensions of the $SU(5)$ representations and the quantum numbers of $U(1)^2$.

Table II The generation and anti-generation structure of matter superfields in a simple three-generation model with the aligned $SU(5)\times U(1)^2$. $U(1)^2$-axes correspond to $\Theta_3 - \Theta_1$ and $\Theta_3 + \Theta_1$.

Table I

$SU(5) \times U(1)^2$	(A) $SU(5)_S$	(B) $SU(5)_A$	(C) $SU(5)_F$
$(\mathbf{10}, \ 0, \ \frac{2}{3})$	$Q, \ U^c, E^c$	$Q, \ g^c, S$	$Q, \ D^c, N^c$
$(\mathbf{5}^*, \ 1, -\frac{1}{3})$	D^c, L	U^c, H_d	g^c, H_u
$(\mathbf{5}^*, -1, -\frac{1}{3})$	g^c, H_d	D^c, H_u	U^c, L
$(\ \mathbf{5}, \ 0, -\frac{4}{3})$	$g, \ H_u$	$L, \ g$	$g, \ H_d$
$(\ \mathbf{1}, -1, \ \frac{5}{3})$	N^c	E^c	S
$(\ \mathbf{1}, \ 1, \ \frac{5}{3})$	S	N^c	E^c

Table II

$SU(5)_A \times U(1)^2$	generation		anti-generation
$(\mathbf{10}, \ 0, \ \frac{2}{3})$	$(Q, \ g^c, S)_i$	$(i = 0, 1, 2, 3)$	$(\overline{Q}, \ \overline{g}^c, \overline{S})$
$(\mathbf{5}^*, \ 1, -\frac{1}{3})$	$(U^c, H_d)_i$	$(i = 0, 1, 2, 3)$	$(\overline{U}^c, \overline{H}_d)$
$(\mathbf{5}^*, -1, -\frac{1}{3})$	$(D^c, H_u)_i$	$(i = 0, 1, 2, 3)$	$(\overline{D}^c, \overline{H}_u)$
$(\ \mathbf{5}, \ 0, -\frac{4}{3})$	$(L, \ g)_i$	$(i = 0, 1, 2, 3)$	$(\overline{L}, \ \overline{g})$
$(\ \mathbf{1}, -1, \ \frac{5}{3})$	$(E^c)_i$	$(i = 0, 1, 2, 3)$	$(\overline{E}^c)$
$(\ \mathbf{1}, \ 1, \ \frac{5}{3})$	$(N^c)_i$	$(i = 0, 1, 2, 3)$	$(\overline{N}^c)$