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# Explicit $CP$ Violation of the Higgs Sector in the Next-to-Minimal Supersymmetric Standard Model

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## ABSTRACT

We study the explicit  $CP$  violation of the Higgs sector in the minimal supersymmetric standard model with a gauge singlet Higgs field. The magnitude of  $CP$  violation is discussed in the limiting cases of  $x \gg v_1, v_2$  and  $x \ll v_1, v_2$ , where  $x$  and  $v_{1,2}$  denote VEV of singlet and doublet Higgs scalars, respectively. Our numerical predictions of the electric dipole moments of electron and neutron lie around the present experimental upper limits. It is found that the large  $CP$  violation effect reduces the magnitude of the lightest Higgs boson mass in the order of a few ten GeV.

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## 1. Introduction

The physics of  $CP$  violation has attracted much recent attention in the light that the  $B$ -factory will go on line in the near future at KEK and SLAC. The central subject of the  $B$ -factory is the test of the standard model(SM), in which the origin of  $CP$  violation is reduced to the phase in the Kobayashi-Maskawa matrix[1]. However, there has been a general interest in considering other approaches to  $CP$  violation since many alternate sources exist. The attractive extension of the standard Higgs sector is the two Higgs doublet model(THDM)[2], yielding both charged and neutral Higgs bosons as physical states. The THDM with the soft breaking term of the discrete symmetry demonstrates explicit or spontaneous  $CP$  violation[3,4,5]. On the other hand, the recent measurements of the gauge couplings at  $M_Z$  scale suggest the minimal supersymmetric standard model(MSSM) is a good candidate beyond the standard model in the standpoint of the unification[6]. In this model,  $CP$  violation has been investigated with the soft symmetry breaking terms[7] since there is no  $CP$  violating source in the Higgs sector at the tree level although two Higgs doublets exist[2]. However, the spontaneous  $CP$  violation could occur in the neutral Higgs exchange through a one-loop potential suggested by Maekawa[8] in the MSSM. Unfortunately, the scenario to violate  $CP$  spontaneously by radiative correction requires a lighter Higgs boson mass[9,10] than its lower limit obtained at LEP[11].

The spontaneous  $CP$  violation in the extended supersymmetric model was discussed by some authors[12,13,14]. The most challenging approach is to add a gauge singlet Higgs field  $N$  to the MSSM. This next-to-MSSM(NMSSM) was studied by many authors especially in the interests of mass spectra of Higgs sectors[15,16]. The detailed analysis of the mass spectra in this model was studied by Ellis et al.[16], in which  $CP$

violation in the Higgs sector was neglected. Although there is a "NO-GO" theorem of the spontaneous  $CP$  violation in the NMSSM[13], the radiative correction may open the way to cause the spontaneous  $CP$  violation as shown by Babu and Barr[14]. In this scenario, the light Higgs boson is also unavoidable. On the other hand, additional singlet  $N$  could cause explicit  $CP$  violation in the Higgs sector even at tree level. In this paper, we study the explicit  $CP$  violation of the Higgs sector in the NMSSM phenomenologically. The lightest Higgs boson could be heavier than the  $Z^0$  boson by including radiative corrections.

In section 2, the explicit  $CP$  violation is studied with the Higgs potential in general. In section 3, the magnitude of the explicit  $CP$  violation is discussed in the special limiting cases of the vacuum expectation values(VEV) of the singlet Higgs boson  $N$ . In section 4, the numerical results are discussed by using the recent experimental bounds such as masses of Higgs scalars and the electric dipole moments of neutron and electron. Section 5 is devoted to summary.

## 2. Explicit $CP$ violation in Higgs Potential

In order to give masses to all the quarks and leptons, and to cancel gauge anomalies, at least two Higgs doublets  $H_1, H_2$  are required in a supersymmetric version of the standard model[7]. Our discussing model is the MSSM to which a gauge singlet Higgs scalar  $N$  has been added with the requirement that the superpotential contains only cubic terms[15,16] as follows:

$$W = h_U Q u^c H_2 + h_D Q d^c H_1 + h_E L e^c H_1 + \lambda H_1 H_2 N - \frac{1}{3} k N^3 + \dots , \quad (1)$$

where  $Q, L, u^c, d^c$  and  $e^c$  are usual notations of quarks and leptons, and the ellipsis

stands for possible nonrenormalizable terms. The effective scalar potential is given as

$$V_{\text{Higgs}} = V_F + V_D + V_{\text{soft}} , \quad (2)$$

$$V_F = |\lambda|^2[ (|H_1|^2 + |H_2|^2)|N|^2 + |H_1 H_2|^2 ] + |k|^2|N|^4 \\ - (\lambda k^* H_1 H_2 N^{*2} + H.c.) - |\lambda|^2 (H_1^0 H_2^0 H^{+*} H^{-*} + H.c.) , \quad (3)$$

$$V_D = \frac{g^2}{8} (H_2^\dagger \hat{\sigma} H_2 + H_1^\dagger \hat{\sigma} H_1)^2 + \frac{g'^2}{8} (|H_2|^2 - |H_1|^2)^2 , \quad (4)$$

$$V_{\text{soft}} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_N^2 |N|^2 - (\lambda A_\lambda H_1 H_2 N + H.c.) \\ - \left( \frac{1}{3} k A_k N^3 + H.c. \right) , \quad (5)$$

where  $H_1 \equiv (H_1^0, H^-)$ ,  $H_2 \equiv (H^+, H_2^0)$ ,  $H_1 H_2 \equiv H_1^0 H_2^0 - H^- H^+$  and  $\hat{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$ .

The radiative effect of the top-quark and top-squark is significant for the mass spectra of the Higgs bosons as pointed out by some authors in the MSSM[17]. This leading-log radiatively induced potential is given as follows:

$$V_{\text{top}} = \frac{3}{16\pi^2} \left[ (h_t^2 |H_2|^2 + M_{sq}^2)^2 \ln \frac{(h_t^2 |H_2|^2 + M_{sq}^2)}{Q^2} - h_t^4 |H_2|^4 \ln \frac{h_t^2 |H_2|^2}{Q^2} \right] , \quad (6)$$

where we have assumed degenerate squarks:  $M_{\tilde{t}_L} = M_{\tilde{t}_R} = M_{sq} \gg m_t$ . The potential  $V_{\text{top}}$  should be added to  $V_{\text{Higgs}}$  in eq.(2).

In general,  $\lambda$ ,  $k$ ,  $A_\lambda$  and  $A_k$  are complex, however, by redefining the global phase of the fields  $H_2$  and  $N$ , we can take

$$\lambda A_\lambda \geq 0 , \quad k A_k \geq 0 , \quad (7)$$

without loss of any generality. If we allow  $CP$  violation explicitly in the Higgs scalar sector,  $\lambda k^*$  is a complex.

The VEV of the Higgs potential  $V_{\text{Higgs}}$  is composed of the neutral sector and the charged sector written as

$$\langle V_{\text{Higgs}} \rangle = \langle V_{\text{neutral}} \rangle + \langle V_{\text{charged}} \rangle . \quad (8)$$

Our discussion is concentrated on the neutral Higgs sector because there is no  $CP$  violation in the charged Higgs sector. Since the contribution of  $V_{\text{top}}$  is not important for qualitative studies of the explicit  $CP$  violation, we discuss the magnitude of  $CP$  violation without  $V_{\text{top}}$  in sections 2 and 3. However,  $V_{\text{top}}$  contributes significantly to the mass spectra of the Higgs bosons, so we include this effect in the numerical analyses in section 4. Neglecting  $V_{\text{top}}$  for simplicity, we can write

$$\begin{aligned}
\langle V_{\text{neutral}} \rangle &= \lambda^2(|x|^2|v_1|^2 + |x|^2v_2^2 + |v_1|^2v_2^2) + k^2|x^4| - v_2(\lambda k^*v_1x^{*2} + \lambda^*kv_1^*x^2) \\
&\quad + \frac{g^2 + g'^2}{8}(|v_1|^2 - v_2^2)^2 + m_{H_1}^2|v_1|^2 + m_{H_2}^2v_2^2 + m_N^2|x^2| \\
&\quad - \lambda A_\lambda v_2(v_1x + v_1^*x^*) - \frac{kA_k}{3}(x^3 + x^{*3}) ,
\end{aligned} \tag{9}$$

where VEV's of the neutral Higgs scalar fields are defined as follows:

$$v_1 \equiv \langle H_1^0 \rangle , \quad v_2 \equiv \langle H_2^0 \rangle , \quad x \equiv \langle N \rangle . \tag{10}$$

The VEV's  $v_1$  and  $x$  are taken to be complex, and  $v_2$  is taken to be a real positive number without loss of generality. Therefore,  $v_1$  and  $x$  are replaced with

$$v_1 \implies v_1 e^{i\alpha} , \quad x \implies x e^{i\omega} , \tag{11}$$

where  $v_1$  and  $x$  in RHS are redefined to be real positive numbers, and we give familiar definitions such as

$$\tan \beta \equiv \frac{v_2}{v_1} , \quad v^2 \equiv v_1^2 + v_2^2 . \tag{12}$$

We also introduce a phase for  $\lambda k^*$  as follows:

$$\lambda k^* = \lambda k e^{i\varphi} , \tag{13}$$

where  $\lambda$  and  $k$  in RHS are redefined as positive real numbers. One can use the minimization conditions of  $V_{\text{neutral}}$  to re-express the soft supersymmetric breaking masses

$m_{H_1}^2, m_{H_2}^2, m_N^2$  in terms of the three VEV's and of the remaining parameters  $\lambda, k, A_\lambda, A_k$ :

$$\begin{aligned}
m_{H_1}^2 &= \lambda A_\lambda \frac{v_2 x}{v_1} \cos(\alpha + \omega) - \lambda^2(x^2 + v_2^2) + \lambda k \frac{v_2 x^2}{v_1} \cos(\varphi + \alpha - 2\omega) \\
&\quad + \frac{g^2 + g'^2}{4}(v_2^2 - v_1^2) \quad , \\
m_{H_2}^2 &= \lambda A_\lambda \frac{v_1 x}{v_2} \cos(\alpha + \omega) - \lambda^2(x^2 + v_1^2) + \lambda k \frac{v_1 x^2}{v_2} \cos(\varphi + \alpha - 2\omega) \\
&\quad + \frac{g^2 + g'^2}{4}(v_1^2 - v_2^2) \quad , \\
m_N^2 &= \lambda A_\lambda \frac{v_1 v_2}{x} \cos(\alpha + \omega) + k A_k x \cos 3\beta - \lambda^2(v_1^2 + v_2^2) - 2k^2 x^2 \\
&\quad + 2\lambda k v_1 v_2 \cos(\varphi + \alpha - 2\omega) \quad .
\end{aligned} \tag{14}$$

The presence of phases  $\alpha$  and  $\omega$  allows in principle for the spontaneous  $CP$  violation. This case was discussed numerically by Babu and Barr[14]. We work in the vacuum of  $\alpha = 0$  and  $\omega = 0$  since we consider the case of the explicit  $CP$  violation in this paper.

Let us study the masses of the Higgs scalars. The physical charged Higgs fields is given by

$$C^+ \equiv \cos \beta H^+ + \sin \beta H^{-*} \quad , \tag{15}$$

while the orthogonal combination corresponds to an unphysical Goldstone boson. The physical charged Higgs boson mass is given as follows:

$$m_C^2 = m_W^2 - \lambda^2 v^2 + \lambda(A_\lambda + kx \cos \varphi) \frac{2x}{\sin 2\beta} \quad , \tag{16}$$

where  $m_W^2 = g^2 v^2/2$ . On the other hand, the neutral Higgs scalar masses are given by  $5 \times 5$  mass matrix. Decomposing the neutral Higgs fields into their real imaginary components

$$H_1^0 \equiv \frac{S_1 + iP_1}{\sqrt{2}} \quad , \quad H_2^0 \equiv \frac{S_2 + iP_2}{\sqrt{2}} \quad , \quad N \equiv \frac{X + iY}{\sqrt{2}} \quad , \tag{17}$$

shifting  $H_1^0$ ,  $H_2^0$ ,  $N$  by their expectation values, and expanding the neutral Higgs scalar part of  $V_{\text{Higgs}}$ , we get the mass matrix of the neutral Higgs scalars. After expressing  $P_1$  and  $P_2$  in terms of the neutral Goldstone boson  $G^0 \equiv \cos \beta P_1 - \sin \beta P_2$  and its orthogonal state  $A \equiv \sin \beta P_1 + \cos \beta P_2$ , we get  $5 \times 5$  mass matrix for the Higgs bosons  $A$ ,  $Y$ ,  $S_1$ ,  $S_2$  and  $X$  as follows:

$$M_{\text{Higgs}}^2 = \begin{bmatrix} M_{AY}^{AY} & M_{S_1 S_2 X}^{AY} \\ (M_{S_1 S_2 X}^{AY})^T & M_{S_1 S_2 X}^{S_1 S_2 X} \end{bmatrix}, \quad (18)$$

where  $M_{AY}^{AY}$ ,  $M_{S_1 S_2 X}^{AY}$  and  $M_{S_1 S_2 X}^{S_1 S_2 X}$  are  $2 \times 2$ ,  $2 \times 3$  and  $3 \times 3$  submatrices, respectively. The matrix  $M_{AY}^{AY}$  is the one for the Higgs pseudoscalars  $A$  and  $Y$  as follows:

$$M_{AY}^{AY} = \begin{pmatrix} \frac{\lambda x v^2}{v_1 v_2} (A_\lambda + kx \cos \varphi) & \lambda v (A_\lambda - 2kx \cos \varphi) \\ \lambda v (A_\lambda - 2kx \cos \varphi) & \frac{\lambda v_1 v_2}{x} A_\lambda + 3A_k kx + 4\lambda k v_1 v_2 \cos \varphi \end{pmatrix}. \quad (19)$$

The matrix  $M_{S_1 S_2 X}^{S_1 S_2 X}$  is the one for the Higgs scalars  $S_1$ ,  $S_2$  and  $X$  as follows:

$$M_{S_1 S_2 X}^{S_1 S_2 X} = \begin{pmatrix} \bar{g}^2 v_1^2 + \frac{\lambda v_2 x}{v_1} & v_1 v_2 (2\lambda^2 - \bar{g}^2) & 2\lambda^2 v_1 x \\ + \frac{\lambda v_2 x}{v_1} (A_\lambda + kx \cos \varphi) & -\lambda x (A_\lambda + kx \cos \varphi) & -\lambda v_2 (A_\lambda + 2kx \cos \varphi) \\ v_1 v_2 (2\lambda^2 - \bar{g}^2) & \bar{g}^2 v_2^2 & 2\lambda^2 v_2 x \\ -\lambda x (A_\lambda + kx \cos \varphi) & + \frac{\lambda v_1 x}{v_2} (A_\lambda + kx \cos \varphi) & -\lambda v_1 (A_\lambda + 2kx \cos \varphi) \\ 2\lambda^2 v_1 x & 2\lambda^2 v_2 x & \frac{\lambda v_1 v_2}{x} A_\lambda \\ -\lambda v_2 (A_\lambda + 2kx \cos \varphi) & -\lambda v_1 (A_\lambda + 2kx \cos \varphi) & -A_k kx + 4k^2 x^2 \end{pmatrix}, \quad (20)$$

where  $\bar{g}^2 \equiv (g^2 + g'^2)/2$ . The matrix  $M_{S_1 S_2 X}^{AY}$  is the mixing terms of the scalar and pseudoscalar components as follows:

$$M_{S_1 S_2 X}^{AY} = \begin{pmatrix} \frac{k\lambda v_1 x^2}{v} \sin \varphi & \frac{k\lambda v_2 x^2}{v} \sin \varphi & 2k\lambda v x \sin \varphi \\ -2k\lambda v_2 x \sin \varphi & -2k\lambda v_1 x \sin \varphi & -2k\lambda v_1 v_2 \sin \varphi \end{pmatrix}. \quad (21)$$

This submatrix is zero if  $CP$  is conserved, that is to say,  $\varphi = 0$ . Then, the matrix  $M_{\text{Higgs}}^2$  in eqs.(18)  $\sim$  (21) is reduced to the one given by Ellis et al.[16].

### 3. $CP$ Violation in Special Limiting Cases

In general,  $CP$  symmetry is violated due to the scalar and pseudoscalar mixing of eq.(21). Its magnitude depends on the values of the Higgs potential parameters, especially,  $x$ . Following analyses of the Higgs mass spectra by Ellis et al.[16], we study the magnitude of  $CP$  violation in the three special limiting cases: (A)  $x \gg v_1, v_2$  with  $\lambda$  and  $k$  fixed, (B)  $x \gg v_1, v_2$  with  $\lambda x$  and  $kx$  fixed and (C)  $x \ll v_1, v_2$ . These limits are discussed in the phenomenological standpoint.

#### (A) Limits of $x \gg v_1, v_2$ ( $\lambda, k$ fixed)

In this limit with  $A_\lambda, A_k \simeq O(x)$ , the matrix  $M_{\text{Higgs}}^2$  in eqs.(18)~(21) becomes very simple. Remaining only the terms of order  $O(x^2)$ , the Higgs scalar  $X$  and the Higgs pseudoscalar  $Y$  almost decouple from other Higgs bosons since these mixing terms are at most order  $O(x)$ . The masses squares of  $X$  and  $Y$  bosons are an order of  $O(x^2)$  and then, those mixing is negligible small. The effect of  $X$  and  $Y$  contributes to our result in the order of  $v_1/x$  and  $v_2/x$  through the mixings. Therefore, it is enough for  $CP$  violation to consider  $3 \times 3$  submatrix as to  $A, S_1$  and  $S_2$ . Then, the mass matrix is given in the  $A - S_1 - S_2$  system as follows:

$$M_{\text{Higgs}}^2 = \begin{bmatrix} 2\lambda x A_\sigma / \sin 2\beta & \lambda k x^2 \cos \beta \sin \varphi & \lambda k x^2 \sin \beta \sin \varphi \\ \lambda k x^2 \cos \beta \sin \varphi & \bar{g}^2 v^2 \cos^2 \beta + \lambda x A_\sigma \tan \beta & (\lambda^2 - \frac{\bar{g}^2}{2}) v^2 \sin 2\beta - \lambda x A_\sigma \\ \lambda k x^2 \sin \beta \sin \varphi & (\lambda^2 - \frac{\bar{g}^2}{2}) v^2 \sin 2\beta - \lambda x A_\sigma & \bar{g}^2 v^2 \sin^2 \beta + \lambda x A_\sigma \cot \beta \end{bmatrix} \quad (22)$$

where  $A_\sigma \equiv A_\lambda + kx \cos \varphi$  is defined conveniently and  $A_\sigma$  is taken to be of  $O(x)$ . By rotating this matrix using  $U_0$  with

$$U_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{bmatrix}, \quad (23)$$



we get simple form of the matrix  $M_{\text{Higgs}}^{\prime 2} = U_0^T M_{\text{Higgs}}^2 U_0$  in the new basis of  $A - S'_1 - S'_2$  as follows:

$$M_{\text{Higgs}}^{\prime 2} = \begin{bmatrix} \frac{2\lambda x A_\sigma}{\sin 2\beta} & \lambda k x^2 \sin \varphi & 0 \\ \lambda k x^2 \sin \varphi & (\bar{g}^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta)v^2 & (\lambda^2 - \bar{g}^2)v^2 \sin 2\beta \cos 2\beta \\ 0 & (\lambda^2 - \bar{g}^2)v^2 \sin 2\beta \cos 2\beta & (\bar{g}^2 - \lambda^2)v^2 \sin^2 2\beta + \frac{2\lambda x A_\sigma}{\sin 2\beta} \end{bmatrix}. \quad (24)$$

In this matrix, the (2-2), (2-3), (3-2) components are very small because these are order of  $O(v^2)$  but others are  $O(x^2)$ . Therefore, the submatrix of  $S'_1 - S'_2$  system is almost diagonal one. Now, we consider only  $A - S'_1$  submatrix, which leads to  $CP$  violation, as follows:

$$M_{\text{Higgs}}^{\prime 2} = \begin{bmatrix} \frac{2\lambda x A_\sigma}{\sin 2\beta} & \lambda k x^2 \sin \varphi \\ \lambda k x^2 \sin \varphi & (\bar{g}^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta)v^2 \end{bmatrix}. \quad (25)$$

Since this matrix has a hierarchical structure, one should investigate these mass eigenvalues carefully. While the pseudoscalar mass is very large as  $O(x)$ , the scalar mass is very small as  $O(v)$ . In order to get the condition of positive eigenvalues, we take the determinant of this matrix:

$$\text{Det}[M_{\text{Higgs}}^{\prime 2}] \geq 0, \quad (26)$$

which gives a constraint  $\lambda k x^2 \sin \varphi \leq O(xv)$ . Since  $\lambda$  and  $k$  are constants, we get

$$\sin \varphi \leq O(v/x), \quad (27)$$

which means the scalar-pseudoscalar mixing vanishes in the  $x \rightarrow \infty$  limit. Therefore, it is concluded that  $CP$  violation is minor in this limit.

### (B) Limits of $x \gg v_1, v_2(\lambda x, kx \text{ fixed})$

This limit leads the NMSSM to the MSSM without the Higgs singlet field as discussed Ellis et al.[16]. In this limit with  $A_\lambda, A_k \simeq O(v)$ , the  $X$  and  $Y$  boson decouple

from other bosons, and then the matrix  $M_{\text{Higgs}}^2$  in eqs.(18)~(21) reduces to the same  $3 \times 3$  matrix in eq.(22). However, masses of  $X$  and  $Y$  are same order of other Higgs bosons in contrast with the case (A). Using the same orthogonal matrix in eq.(23), we get also the similar matrix as the one in eq.(24) for the  $A - S'_1 - S'_2$  system as follows:

$$M_{\text{Higgs}}'^2 = \begin{bmatrix} \frac{2\bar{\lambda}A_\sigma}{\sin 2\beta} & \bar{\lambda} \bar{k} \sin \varphi & 0 \\ \bar{\lambda} \bar{k} \sin \varphi & (\bar{g}^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta)v^2 & (\lambda^2 - \bar{g}^2)v^2 \sin 2\beta \cos 2\beta \\ 0 & (\lambda^2 - \bar{g}^2)v^2 \sin 2\beta \cos 2\beta & (\bar{g}^2 - \lambda^2)v^2 \sin^2 2\beta + \frac{2\bar{\lambda}A_\sigma}{\sin 2\beta} \end{bmatrix}, \quad (28)$$

where the definitions  $\bar{\lambda} \equiv \lambda x$  and  $\bar{k} \equiv kx$  are fixed to be constants, while  $\lambda$  and  $k$  are order of  $O(1/x)$ . In contrast with the matrix of eq.(22), this matrix has not a hierarchical structure in the considering limit since  $\bar{\lambda}$  and  $\bar{k}$  are finite numbers. Therefore, the submatrix of  $S'_1 - S'_2$  in eq.(28) are far from the diagonal matrix in general. Now, let us discuss the magnitude of  $CP$  violation for the special case of  $\tan \beta$ .

The first case is the one with  $\tan \beta = 0$  and  $\infty$ . Since  $\sin 2\beta = 0$ , the submatrix of the  $S'_1 - S'_2$  system is exactly diagonal. The scalar-pseudoscalar mixing is occurred only in the  $A - S'_1$  submatrix. The mixing angle is given as follows:

$$\tan 2\theta_{AS'_1} = \frac{2\bar{\lambda} \bar{k} \sin \varphi}{(\bar{g}^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta)v^2 - \frac{2\bar{\lambda}A_\sigma}{\sin 2\beta}} \simeq -\frac{\bar{k}}{A_\sigma} \sin \varphi \sin 2\beta. \quad (29)$$

Thus, the scalar-pseudoscalar mixing vanishes in  $\tan \beta = 0$  or  $\infty$  limit since it is proportional to  $\sin 2\beta$  even if  $\sin \varphi \simeq 1$ . Then, the  $CP$  violation effect is expected generally to vanish. However, we should pay attention to an exceptional case that the  $CP$  violating effect depends on  $\tan \beta$  significantly. We will discuss this case in analyses of the electric dipole moments of the section 4.

The second case is the one of  $\tan \beta = 1$ , which gives  $\cos 2\beta = 0$ . In this case, the

scalar-pseudoscalar mixing is also occurred only in the  $A-S'_1$  submatrix since the  $S'_1-S'_2$  submatrix is exactly diagonal. Then, the  $S'_1-S'_1$  component is  $\lambda^2 v^2$  which is order of  $O(v^4/x^2)$ . This hierarchical structure of the mass matrix gives strong constraint for the mixing angle as discussed in the limiting case (A). Applying the positivity condition of the Higgs scalar mass in eq.(26) leads

$$\sin \varphi \leq O\left(\frac{v}{x}\right). \quad (30)$$

Thus,  $CP$  violation also vanishes in the case of  $\tan \beta = 1$ .

In order to get the finite  $CP$  violation, we should choose the region of  $\tan \beta \neq 0, 1$  and  $\infty$ . If we could adjust the parameter such as

$$2\bar{\lambda}A_\sigma \simeq \bar{g}^2 v^2 \cos^2 2\beta \sin 2\beta, \quad (31)$$

by choosing the suitable  $\tan \beta$ , the large scalar-pseudoscalar mixing is expected. However, since the radiative correction  $V_{\text{top}}$  becomes significant in this situation, we shall give the numerical analyses in section 4.

### (C) Limits of $x \ll v_1, v_2$

In the  $x = 0$  limit with  $A_\lambda, A_k \simeq O(v)$ , the submatrix  $M_{S'_1 S'_2 X}^{AY}$  is described as

$$M_{S'_1 S'_2 X}^{AY} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2k\lambda v_1 v_2 \sin \varphi \end{pmatrix}, \quad (32)$$

where only the (2-3) component remains to be finite, that is, the scalar-pseudoscalar mixing exists only in the  $X$  and  $Y$  mixing. Since squares of  $X$  and  $Y$  boson masses are order of  $O(v^3/x)$  and other matrix elements are at most  $O(v^2)$ , these bosons decouple from other Higgs bosons except for the case of  $\sin 2\beta = 0$ . Ellis et al.[16] have found that the large  $\tan \beta$  or  $\cot \beta$  ( $\sin 2\beta = 0$ ) is not allowed by studying the constraint that

the symmetry-breaking vacuum is a deeper minimum than the symmetry vacuum in the case of  $x \ll v_1, v_2$ . So, we do not need to consider the case of  $\sin 2\beta = 0$ .

The submatrix of the  $X - Y$  system is given as follows:

$$M_{\text{Higgs}}^2 = \begin{pmatrix} \frac{\lambda A_\lambda v^2}{2x} \sin 2\beta & -k\lambda v^2 \sin 2\beta \sin \varphi \\ -k\lambda v^2 \sin 2\beta \sin \varphi & \frac{\lambda A_\lambda v^2}{2x} \sin 2\beta + 2\lambda k v^2 \sin 2\beta \cos \varphi \end{pmatrix}. \quad (33)$$

Then, the the mixing angle is given as follows:

$$\tan 2\theta_{XY} = -\frac{2k\lambda v^2 \sin 2\beta \sin \varphi}{2k\lambda v^2 \sin 2\beta \cos \varphi} = -\tan \varphi, \quad (34)$$

so the maximal mixing of the scalar-pseudoscalar is realized in the case of  $\varphi = \pm\pi/2$ . The phenomena induced by  $X$  and  $Y$  Higgs bosons may show the large  $CP$  violation. However, the mass eigenvalues are infinite in the  $x = 0$  limit, so the  $CP$  violation effect on the low energy phenomena would vanish.

#### 4. Numerical Discussion of Explicit $CP$ violation

In this section, we show the numerical examples to realize the large  $CP$  violation. Generally, the large  $CP$  violation could be caused by choosing  $x$  to be  $O(v)$ . However, then, the Higgs boson spectroscopy is very different from the MSSM because  $S_1, S_2$  and  $A$  bosons mix significantly with  $X$  and  $Y$  bosons. In our interest, we present numerical study of the similar case to the MSSM spectroscopy, but the case with  $CP$  violation. This is just the limit in case (B).

In the previous section, we have neglected the radiatively induced potential  $V_{\text{top}}$  for simplicity because the qualitative result is not changed even if we include it. Now, we should include the  $V_{\text{top}}$  term in our numerical analyses. In the leading-log approximation, this potential contributes only to the mass matrix element  $M_{S_2}^{S_2}$  in eq.(20) as

follows:

$$M_{S_2}^{S_2} = (\bar{g}^2 + \Delta)v_2^2 + \frac{\lambda v_1 x}{v_2}(A_\lambda + kx \cos \varphi) , \quad (35)$$

where

$$\Delta = \frac{3h_t^4}{4\pi^2} \left[ \ln \left( \frac{M_{sq}^2}{m_t^2} \right) + p \right] , \quad (36)$$

where  $p$  denotes non-logarithmic terms. In the following calculations, we fix  $\Delta = 0.5$ , which corresponds to  $M_{sq} = 3\text{TeV}$  and  $m_t = 175\text{GeV}$  with  $p = 1$ [14].

**Fig.1**

In Fig.1, we display a plot of the experimentally allowed region in the  $\cos \varphi - \lambda$  plane for fixed values of the other parameters, which are

$$x = 10v, \quad k = 0.1, \quad A_\lambda = v, \quad A_k = v, \quad \tan \beta = 10 . \quad (37)$$

One experimental constraint is that the two Higgs bosons have not been produced in the decay of a real  $Z^0$ [11]. The lower boundary (small  $\lambda$ ) in Fig.1 corresponds to  $m_{h_1} + m_{h_2} = m_{Z^0}$ , where  $m_{h_1}$  and  $m_{h_2}$  are two lightest Higgs boson masses. The other constraint is that a light Higgs boson has not been produced in the  $Z^0 \rightarrow Z^0 h$  process, where  $h$  is a physical Higgs boson. If  $h = \sum_{i=1}^5 \alpha_i \Phi_i$ , where  $\alpha_i$  and  $\Phi_i$  denote mixing factors and neutral Higgs boson fields  $S_1, S_2, A, X, Y$ , respectively, the cross section for this process is approximately proportional to  $|\alpha_1 \cos \beta + \alpha_2 \sin \beta|^2 m_h^{-1}$ . The non-observation of this process gives the upper boundary (large  $\lambda$ ) in Fig.1 by  $m_h \geq (60\text{GeV})|\alpha_1 \cos \beta + \alpha_2 \sin \beta|^2$ [11]. In addition, the pseudoscalar and scalar bosons should be heavier than 24GeV and 44GeV, respectively[11]. This constraints are satisfied in the allowed region of Fig.1.

In Fig.2, the allowed region of  $\lambda$  is shown in the case of  $\tan\beta = 1 \sim 100$  at  $\cos\varphi = 0$ . Other parameters are fixed as given in eq.(37). It is remarked that the allowed region vanishes below  $\tan\beta \simeq 1.5$ . This result is consistent with the qualitative discussion of (B) in section 3, in which  $\varphi$  is constrained to be very small at  $\tan\beta \simeq 1$ , and  $\varphi \simeq \pi/2$  is allowed at  $\tan\beta = \infty$ . In both results of Figs. 1 and 2, we fix  $k = 0.1$ , which gives the most wide allowed area of  $\lambda$ . As far as we take  $k = 0.03 \sim 0.2$ , the allowed region is obtained.

**Fig.2**

The electric dipole moment(EDM) of electron or neutron is very important quantities to constrain the phase  $\varphi$ . In our scheme, the EDM of electron is calculated in the two-loop level as shown by Barr and Zee[18]. The neutron EDM is also predicted in two-loop level. Both three gluon operator proposed by Weinberg[19] and quark-gluon operator by Gunion and Wyler[20] are taken into account in our calculation. Since the estimation of the hadronic matrix elements is model-dependent, the ambiguity with a few factors should be taken into consideration in the prediction of the neutron EDM. Here, we use the model proposed by Chemtob[21,22]. The recent experimental upper limit of the electron EDM is  $4 \times 10^{-27} e \cdot \text{cm}$ [23] and that of the neutron EDM is  $11 \times 10^{-26} e \cdot \text{cm}$ [11]. It should be remarked that the Barr-Zee operator and the quark-gluon operator are exceptional  $CP$  violating operators as discussed in (B) of section 3. Since these operators have a term which is proportional to  $\tan^2\beta$ , this term contributes to the EDM significantly at  $\tan\beta \gg 1$  even if the scalar-pseudoscalar mixing is very small. In fact, we find the large predicted EDM at  $\tan\beta = 10$  in Figs. 3 and

4. In these figures, we give the numerical predictions of the electron EDM and the neutron EDM in the allowed region of  $\lambda$  in Fig.1. The upper(lower) boundary of the predictions corresponds to the upper(lower) one of  $\lambda$  in Fig.1. Those predictions lie around experimental upper limits except for the region of  $\cos \varphi \simeq \pm 1$ . If the small  $\lambda$ ,  $O(0.01)$ , is taken, our predictions are below the experimental limits even if the phase  $\varphi$  is a maximal one  $\pi/2$ . We expect both electron EDM and neutron EDM will be observed around  $10^{-27} \sim 10^{-26} e \cdot \text{cm}$  in the near future.

**Figs.3 and 4**

Let us discuss about the mass of the lightest Higgs boson in our scheme. What is the  $CP$  violating effect on it ? The top-loop effect seems to be significant in the lightest Higgs boson mass in the case without  $CP$  violation[17]. We show the mass of the lightest Higgs boson versus  $\lambda$  for fixed  $\cos \varphi = 1, 0.5, 0, -0.5$  in Fig.5. In the region of  $\lambda = 0.01 \sim 0.1$ , the  $CP$  violating effect reduces the magnitude of the mass in the order of  $10 \sim 20\text{GeV}$ . The qualitative result is not so changed if we take  $M_{\text{sq}} \ll 3\text{TeV}$ .

**Fig.5**

## 5. Summary

We have studied the explicit  $CP$  violation of the Higgs sector in the MSSM with a gauge singlet Higgs field. The magnitude of  $CP$  violation is discussed in the limiting cases of  $x \gg v_1, v_2$  and  $x \ll v_1, v_2$ . We have shown that the large  $CP$  violation is realized in the region of  $\tan\beta \geq 1.5$  for the case of  $x \gg v_1, v_2$  with the fixed values of  $\lambda x$  and  $kx$ . In other cases, the explicit  $CP$  violation is minor for the Higgs sector. Since  $CP$  violation in the Higgs sector does not occur in the MSSM without a gauge singlet Higgs field,  $CP$  violation is an important signal of the existence of the gauge singlet Higgs field. In the present case of the Higgs sector, the predictions of the electron EDM and the neutron EDM lie around the experimental upper limits. Our results suggest that these EDM's will be observed in the near future if  $CP$  is explicitly violated through the Higgs sector in the NMSSM. Furthermore, we have found that the large  $CP$  violation effect reduces the magnitude of the lightest Higgs boson mass in the order of a few ten GeV. Thus, the explicit  $CP$  violation due to the gauge singlet Higgs boson will give us interesting phenomena in the forthcoming experiments.

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## Figure Captions

### Fig.1

The allowed region in the  $\cos \varphi - \lambda$  plane for  $x = 10v$ ,  $k = 0.1$ ,  $A_\lambda = v$ ,  $A_k = v$  and  $\tan \beta = 10$ . The lower boundary corresponds to  $m_{h_1} + m_{h_2} = m_{Z^0}$  and the upper boundary is given by the nonobservation of  $Z^0 \rightarrow Z^{0*}h$ .

### Fig.2

The allowed region in the  $\tan \beta - \lambda$  plane at  $\cos \varphi = 0$ . The notations are same as in Fig.1

### Fig.3

The predicted electron EDM in the allowed region in Fig.1. The lower(upper) boundary corresponds to the lower(upper) one in Fig.1. The dotted-line denotes the experimental upper-limit.

### Fig.4

The predicted neutron EDM in the allowed region in Fig.1. The lower(upper) boundary corresponds to the lower(upper) one in Fig.1. The dotted-line denotes the experimental upper-limit.

### Fig.5

The predicted lightest Higgs boson mass versus  $\lambda$  for  $\cos \varphi = 1, 0.5, 0, -0.5$  in the case of  $\Delta = 0.5$ .

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