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A Large Majorana-Mass From Calabi-Yau Superstring Models

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Abstract

In Calabi-Yau superstring models it is found that two large intermediate energy scales of symmetry breaking can be induced for special types of the nonrenormalizable interactions. In the models one set of $SO(10)$-singlet, right-handed neutrino and their mirror chiral superfields is needed. Through the study of the minimization of the scalarpotential, the conditions for the presence of two large intermediate scales are obtained. In this scheme a Majorana-mass possibly amounts to $O(10^{9\sim10}\mathrm{GeV})$. This large Majorana-mass solves the solar neutrino problem and also is compatible with the cosmological bound for stable light neutrinos.

Superstring theory is the only known candidate of consistent unification of all fundamental interactions. Lacking a means of addressing the non-perturbative problems, at present we are unable to select a true string vacuum theoretically. However, we can make use of phenomenological requirements on superstring-derived models as a valuable clue to classify the string vacua corresponding to a huge number of distinct classical solutions. From this point of view it is important for us to understand how to connect superstring theory with the standard model.

In Calabi-Yau superstring models the gauge symmetry at the unification scale is rank-6 or rank-5 and is larger than the standard gauge symmmetry $G_{st} = SU(3)_c \times SU(2)_L \times U(1)_Y$ with rank-4 [1]. Consequently, there exist intermediate energy scales of symmetry breaking between the unification scale and the electroweak scale. Furthermore, in Calabi-Yau superstring models there appear extra matter fields which are not contained in the minimal supersymmetric standard model. In fact, we generally have G_{st}-neutral but E_6-charged chiral superfields and their mirror chiral superfields. In order that Calabi-Yau superstring theory is brought into contact with the standard model, some of these G_{st}-neutral matter fields have to develop non-vanishing vacuum expectation values(VEVs) at the intermediate energy scales.

In the following we specialize in the case that the gauge symmetry at the unification scale is rank-6. In this case there should exist two intermediate energy scales of the symmetry breaking, which are represented by the VEVs $\langle S \rangle$ and $\langle N \rangle$. Here we denote an $SO(10)$-singlet chiral superfield and a right-handed neutrino chiral superfield (ν_R^c) as S and N, respectively, which belong to **27**-representation of E_6. If these VEVs are sufficiently large compared with the soft supersymmetry(susy) breaking scale $m_{SUSY} = O(10^3\text{GeV})$, we have to make the D-terms vanish at such large scales. This is realized by setting $\langle S \rangle = \langle \overline{S} \rangle$ and $\langle N \rangle = \langle \overline{N} \rangle$, where $\overline{S}$ and $\overline{N}$ stand for mirror chiral superfields of S and N, respectively. Since the superfield S participates in a Yukawa interaction with leptoquark chiral superfields, the order of magnitude of $\langle S \rangle (\langle \overline{S} \rangle)$ determines the lifetime of proton. To be consistent with the proton stability, it is required that $\langle S \rangle \geq O(10^{16}\text{GeV})$. On the other hand, a non-vanishing $\langle N \rangle$

implies the lepton number violation. Therefore, the magnitude of $\langle N \rangle (\langle \overline{N} \rangle)$ seems to be closely linked to a Majorana-mass(M-mass) of the right-handed neutrino. Experimentally neutrino masses are so small compared with quark masses and charged lepton masses. Seesaw mechanism provides an interesting solution for the neutrino mass problem by introducing large M-masses for right-handed neutrinos [2]. If we take the solar neutrino problem seriously, the M-mass of the right-handed neutrino should be of order $10^{9\sim 12}$GeV[3]. Also this large M-mass is compatible with the cosmological bound for stable light neutrinos [4].

As mentioned above, from the proton stability the condition

$$\langle S \rangle \geq O(10^{16}\mathrm{GeV}) \tag{1}$$

should be satisfied. How can we derive such large intermediate scales in Calabi-Yau superstring models? The discrete symmetry of the compactified manifold possibly accomplishes this desired situation [5]. In superstring models there exist effective non-renormalizable(NR) terms in the superpotential. The order of magnitudes of $\langle S \rangle$ and $\langle N \rangle$ are governed by these NR terms. Along this fascinating line the problems of two large intermediate scales of symmetry breaking and mass matrices have been studied first by Masip [6]. In the analysis general structures of the scalarpotential are not sufficiently clarified and conditions on the NR terms for the presence of two large intermediate scales are obscure.

In this paper we find the constraints on the NR terms for the presence of two large intermediate scales and of a large M-mass. Furthermore, we show that two intermediate scales of symmetry breaking are

$$\langle S \rangle \geq O(10^{16}\mathrm{GeV}), \qquad O(10^{15}\mathrm{GeV}) \geq \langle N \rangle \geq O(10^{13}\mathrm{GeV}) \tag{2}$$

for special types of the NR terms and that a M-mass of right-handed neutrino becomes $M_M \sim m_{SUSY}(\langle S \rangle / \langle N \rangle)^2$. Its numerical value of the M-mass possibly amounts to $O(10^{9\sim 10}\mathrm{GeV})$.

First we take up the NR interactions in the superpotential coming from a pair of

S and $\overline{S}$ chiral superfields. The NR terms are of the form

$$W_{NR} = \sum_{p=2}^{\infty} \lambda_p {M_C}^{3-2p} (S\overline{S})^p, \tag{3}$$

where M_C is the unification scale. Dimensionless coupling λ_p's are of order one. However, if the compactified manifold has a specific type of discrete symmetry, some of λ_p's become vanishing. For instance, in the four-generation model obtained from the Calabi-Yau manifold with the high discrete symmetry $S_5 \times {Z_5}^5$, this symmetry requires that $\lambda_p = 0$ for $p \neq 4$ (mod 5) [5]. When we denote the lowest number of p as n, the NR terms are approximately written as $W_{NR} \cong \lambda_n {M_C}^{3-2n} (S\overline{S})^n$.

To maintain susy down to a TeV scale, the scalarpotential should satisfy F-flatness and D-flatness conditions at the large intermediate scale. Then we have to set $\langle S \rangle = \langle \overline{S} \rangle$. As far as D-terms are concerned, the VEV can be taken as large as we want. Incorporating the soft susy breaking terms, we have the scalarpotential

$$\begin{aligned} V &= n^2 {\lambda_n}^2 {M_C}^{6-4n} \left(|S|^{2(n-1)} |\overline{S}|^{2n} + |S|^{2n} |\overline{S}|^{2(n-1)} \right) \\ &\quad + \frac{1}{2} \sum_\alpha {g_\alpha}^2 \left(S^\dagger T_\alpha S - \overline{S}^\dagger T_\alpha \overline{S} \right)^2 + V_{soft}, \end{aligned} \tag{4}$$

$$V_{soft} = {m_S}^2 \, |S|^2 + {m_{\overline{S}}}^2 \, |\overline{S}|^2, \tag{5}$$

where the T_α are Lie algebra generators and ${m_S}^2$ and ${m_{\overline{S}}}^2$ are the running scalar masses squared from the soft susy breaking. S and $\overline{S}$ develop nonzero VEVs when ${m_S}^2 + {m_{\overline{S}}}^2 < 0$. In the renormalization group analysis for the four-generation model, it has been proven that ${m_S}^2 + {m_{\overline{S}}}^2$ possibly becomes negative at the large intermediate scale $O(10^{16}\text{GeV})$ [7]. By minimizing V, we obtain the VEVs as

$$\langle S \rangle \simeq \langle \overline{S} \rangle \sim M_C \left(\frac{\sqrt{-m_S^2}}{M_C} \right)^{1/2(n-1)} . \tag{6}$$

The difference $\langle S \rangle - \langle \overline{S} \rangle$ is negligibly small and we put ${m_S}^2 = {m_{\overline{S}}}^2$ approximately. In the case $n = 4$ as in the four-generation model, the intermediate energy scale becomes

$$\langle S \rangle \simeq \langle \overline{S} \rangle \sim O(10^{16}\text{GeV}) \tag{7}$$

for $M_C = 10^{18\sim19}$GeV. If $n = 2$, then we have $\langle S \rangle \sim 10^{11}$GeV, which leads to the fast proton decay. Through the Higgs mechanism, the $(S - \overline{S})/\sqrt{2}$ are absorbed into a massive vector superfield with its mass of $O(g_\alpha \langle S \rangle)$. The component $(S + \overline{S})/\sqrt{2}$ have masses of order $O(10^3\text{GeV})$ irrespectively of n. In the case of only a pair of S and $\overline{S}$ it is impossible for us to get sufficiently large M-masses compared with the soft susy breaking scale.

Next we proceed to study the NR terms which consist of pairs of S, N and $\overline{S}$, $\overline{N}$ chiral superfields, provided that there appear S, N and $\overline{S}$, $\overline{N}$ superfields in adequate Calabi-Yau models. In this case we potentially derive two intermediate energy scales of symmetry breaking. Here we assume the NR interactions

$$W_{NR} = {M_C}^3 \left[\lambda_1 \frac{(S\overline{S})^n}{{M_C}^{2n}} + \lambda_2 \frac{(N\overline{N})^m}{{M_C}^{2m}} + \lambda_3 \frac{(S\overline{S})^i (N\overline{N})^j}{{M_C}^{2(i+j)}} \right], \tag{8}$$

where n, m, i and j are generally integers with

$$n,\ m \geq 2, \qquad i,\ j \geq 1 \tag{9}$$

and λ_i's are constants of $O(1)$. In certain types of Calabi-Yau models we suppose that the exponents n, m, i and j are settled on appropriate values due to the discrete symmetry of the Calabi-Yau manifold. By minimizing the scalarpotential including the soft susy breaking terms

$$V_{soft} = {m_S}^2\, |S|^2 + {m_{\overline{S}}}^2\, |\overline{S}|^2 + {m_N}^2\, |N|^2 + {m_{\overline{N}}}^2\, |\overline{N}|^2, \tag{10}$$

we can determine the energy scales of symmetry breaking, that is, $\langle S \rangle$ and $\langle N \rangle$. The scalar mass parameters ${m_S}^2$ and ${m_N}^2$ evolve according to the renormalization group equations. As in the four-generation model, we expect that ${m_S}^2$ becomes negative at the large intermediate scale(M_I). On the other hand, it is natural to expect that ${m_N}^2$ remains still positive at M_I scale, because N and $\overline{N}$ have different gauge quantum numbers from S and $\overline{S}$ and also have different Yukawa interactions. Hereafter we take ${m_S}^2 < 0$ and ${m_N}^2 > 0$ at M_I scale. However, the sign of ${m_N}^2$ is not essential in the following discussions. From the D-flatness condition we get $\langle S \rangle = \langle \overline{S} \rangle$ and

$\langle N\rangle = \langle \overline{N}\rangle$ in the approximation ${m_S}^2 = {m_{\overline{S}}}^2$ and ${m_N}^2 = {m_{\overline{N}}}^2$. To find solutions in which the VEVs are real, here we parametrize as

$$\langle S\rangle = \langle \overline{S}\rangle = M_C\, x, \qquad \langle N\rangle = \langle \overline{N}\rangle = M_C\, y. \tag{11}$$

For convenience' sake, instead of λ_i's we use the parameters a, b and c defined as

$$\lambda_1 = \frac{a}{n}, \qquad \lambda_2 = \frac{a}{m}\, b^{-2m}, \qquad \lambda_3 = -\frac{ac}{ij}\, b^{-2j}, \tag{12}$$

where b and c are put as positive. For negative c we have no solutions in which x and y are real. Let us introduce two dimensionless functions f and g :

$$f(x,y) \;\equiv\; {M_C}^{-2}\, \frac{\partial W}{\partial S}\bigg| = a\left(x^{2n-1} - \frac{c}{j} x^{2i-1}\left(\frac{y}{b}\right)^{2j}\right), \tag{13}$$

$$g(x,y) \;\equiv\; {M_C}^{-2}\, \frac{\partial W}{\partial N}\bigg| = \frac{a}{b}\left(\left(\frac{y}{b}\right)^{2m-1} - \frac{c}{i} x^{2i}\left(\frac{y}{b}\right)^{2j-1}\right), \tag{14}$$

where $\ldots|$ means the values at $S = \overline{S} = \langle S\rangle$ and $N = \overline{N} = \langle N\rangle$. By using the D-flatness condition we have the scalarpotential

$$ {M_C}^{-4} V| = 2\, f(x,y)^2 + 2\, g(x,y)^2 - 2\, {\rho_x}^2 x^2 + 2\, {\rho_y}^2 y^2 \tag{15}$$

with

$$ {\rho_x}^2 = -\frac{{m_S}^2}{{M_C}^2}\ (>0), \qquad {\rho_y}^2 = \frac{{m_N}^2}{{M_C}^2}\ (>0). \tag{16}$$

We now turn to study the absolute minimum of the scalarpotential V. At the minimum point the conditions

$$\frac{\partial V}{\partial S} = \frac{\partial V}{\partial \overline{S}} = \frac{\partial V}{\partial N} = \frac{\partial V}{\partial \overline{N}} = 0 \tag{17}$$

have to be satisfied. In the present notation the conditions are expressed as

$$f\, f_x + g\, g_x - {\rho_x}^2\, x = 0, \tag{18}$$

$$f\, f_y + g\, g_y + {\rho_y}^2\, y = 0, \tag{19}$$

where $f_x = \partial f/\partial x$ and so forth. Solving the above equations and calculating the second derivatives such as $\partial^2 V/\partial S^2$, we find local minima and saddle points. Since the scalarpotential is symmetric under the reflection $x \to -x$ and/or $y \to -y$, it is sufficient for us to consider only the first quadrant in the $x-y$ plane.

Let us consider the case $n/m = i/(m-j)$. In this case at the region $x^n \sim y^m$ the first terms of Eqs. (13) and (14) become the same order with the second terms coincidentally. This situation is of critical importance in the minimization of the scalarpotential. Furthermore, we take $n > m$ so that $\langle S \rangle > \langle N \rangle$. Thus we study the case

$$\frac{n}{m} = \frac{i}{m-j} > 1. \tag{20}$$

In this case it can be proven that there are the following two (three) local minimum points for $j = 1$ ($j \geq 2$). The values of the scalarpotential at these points are calculable.

Point A: $(x, y) = (x_0, y_0)$.

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)} \rho_x{}^2 x_0{}^2, \tag{21}$$

where

$$\begin{aligned} x_0 &= \left(\frac{\rho_x}{\sqrt{2n-1}\, a\, \xi} \right)^{1/2(n-1)}, \\ y_0 &= b \left(\frac{c}{i} \right)^{1/2(m-j)} x_0{}^{i/(m-j)} \quad (\ll x_0), \\ \xi &= \left| 1 - \frac{i}{j} \left(\frac{c}{i} \right)^{n/i} \right|. \end{aligned} \tag{22}$$

Point B: $(x, y) = (x'_0, 0)$.

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)} \rho_x{}^2 x'_0{}^2, \tag{23}$$

where

$$x'_0 = \left(\frac{\rho_x}{\sqrt{2n-1}\, a} \right)^{1/2(n-1)}. \tag{24}$$

Point C: $(x, y) = (x'_0, y'_0) \quad (j \geq 2)$.

$$M_C{}^{-4} V \cong -\frac{4(n-1)}{(2n-1)} \rho_x{}^2 x'_0{}^2, \tag{25}$$

where

$$\begin{aligned} y_0' &= b\left(\frac{i^2b^2}{(2j-1)c}\left(1+\sqrt{1+R}\right)\right)^{1/2(j-1)} {x_0'}^{(n-i-1)/(j-1)} \quad (\ll y_0), \\ R &= -\frac{(2n-1)(2j-1){\rho_y}^2}{i^2{\rho_x}^2} \quad (<0). \end{aligned} \tag{26}$$

Point A is a solution which was found by Masip[6]. At this point not only two terms in $g(x,y)$ cancel out with each other in their leading order but also the leading term in $f\,f_y$ of Eq.(19) cancels out $g\,g_y$. In the expansion the ratio of the next-to-leading terms to the leading ones is $O((y_0/x_0)^2)$. In the case $j \geq 2$ Point C becomes a local minimum only for $1+R \geq 0$ and is not a solution in the case $j=1$. When $0<\xi<1$ Point A is the absolute minimum. This condition on ξ is translated as

$$0 < c < i\left(\frac{2j}{i}\right)^{i/n} \text{ and } \; c \neq i\left(\frac{j}{i}\right)^{i/n}. \tag{27}$$

It is worth noting that under this condition the Point A is the absolute minimum independent of the sign of ${m_N}^2$. For illustration we show the behavior of the scalar-potential for the case $(n,i,m,j)=(6,3,2,1)$ in Fig.1. In Fig.1 the vertical axis is taken as

$$v = \left(2{M_C}^4{\rho_x}^2{x_0}^2\right)^{-1} V + 1 \tag{28}$$

and instead of x and y the horizontal axes are taken as $\overline{x} = (x/x_0)^3$ and $\overline{y} = y/y_0$ so that the point $(\overline{x},\overline{y}) = (1,1)$ becomes the absolute minimum (Point A). In the case $(n,i,m,j)=(6,3,2,1)$ the condition (27) leads to $0<c<\sqrt{6}$ and $c \neq \sqrt{3}$. In Fig.1 we put $a=b=c=1$. As seen in Fig.1, local minima (Points A and B) are located at bottoms of very deep valleys.

Fig.1

We are now in a position to evaluate the mass matrix for S, N and $\overline{S}, \overline{N}$ at the absolute minimum (Point A). The components $(S-\overline{S})/\sqrt{2}$ and $(N-\overline{N})/\sqrt{2}$ are absorbed into massive vector superfields due to the Higgs mechanism. For the remaining components the mass matrix is of the form

$$\begin{pmatrix} O(1) & O(x_0/y_0) \\ O(x_0/y_0) & O((x_0/y_0)^2) \end{pmatrix} m_{SUSY} \tag{29}$$

with the basis $(S+\overline{S})/\sqrt{2}$ and $(N+\overline{N})/\sqrt{2}$. Thus we obtain a large M-mass

$$M_M = \frac{2(m-j)}{\sqrt{2n-1}\,\xi}\,(c/i)^{n/i}\sqrt{-m_S{}^2}\,(x_0/y_0)^2\,, \tag{30}$$

which is associated with the eigenstate

$$\frac{1}{\sqrt{2}}(N+\overline{N}) - \frac{i}{\sqrt{2}\,(m-j)}\,(y_0/x_0)\,(S+\overline{S}). \tag{31}$$

The enhancement factor $(x_0/y_0)^2$ depends on n and m as

$$(x_0/y_0)^2 \sim (1/\rho_x)^{(n-m)/(n-1)m} \tag{32}$$

with ${\rho_x}^{-1} = 10^{15\sim16}$. Since the exponent $(n-m)/(n-1)m$ decreases with increasing m, we take $m=2$ so as to get a sufficiently large M-mass M_M. Then we have $j=1, n=2i$ and obtain

$$(x_0/y_0)^2 = 10^{7\sim8} \qquad \text{for} \;\; n \geq 6. \tag{33}$$

This means that the M-mass becomes

$$M_M = O\left(10^{9\sim10}\mathrm{GeV}\right) \tag{34}$$

by taking $\sqrt{-m_S{}^2} = O(10^3\mathrm{GeV})$. Consequently, a large M-mass can be induced from the NR interactions of S, N and $\overline{S}, \overline{N}$ which are of the form

$$W_{NR} = {M_C}^3\lambda_1\left[\left(\frac{S\overline{S}}{{M_C}^2}\right)^n + \frac{n}{2}\left(\frac{N\overline{N}}{b^2\,{M_C}^2}\right)^2 - 2c\left(\frac{S\overline{S}}{{M_C}^2}\right)^{n/2}\left(\frac{N\overline{N}}{b^2\,{M_C}^2}\right)\right] \tag{35}$$

with $0 < c < \sqrt{n}$ and $c \neq \sqrt{n/2}$. For comparison we tabulate the orders of $\langle S\rangle, \langle N\rangle$ and M_M for several cases of the set (n,i,m,j) in Table I. As seen in this Table, unless $m=2$ and $j=1$, M_M attains to only at most $O(10^7\mathrm{GeV})$. The case $m=2$ and $j=1$ is indispensable for solving the solar neutrino problem.

Table I

Untill now we consider the case (20). In the other cases, for example,

$$
\begin{aligned}
\frac{n-i}{j} > \frac{n}{m} > \frac{i}{m-j} > 1, \\
1 < \frac{n-i}{j} < \frac{n}{m} < \frac{i}{m-j},
\end{aligned}
$$

we have no interesting solutions in which a large M-mass is derived from the minimization of the scalarpotential.

In conclusion, we found that a large M-masss can be induced from the NR interactions of S, N and $\overline{S}, \overline{N}$ in Calabi-Yau superstring models. A pair of S, N and $\overline{S}, \overline{N}$ chiral superfields is needed to get this amazing result. It is essential that two large intermediate energy scales of symmetry breaking given by $\langle S\rangle(\langle\overline{S}\rangle)$ and $\langle N\rangle(\langle\overline{N}\rangle)$ emerge as a consequence of the minimization of the scalarpotential. This implies that the gauge symmetry should be rank-6 at the unification scale. Furthermore, the special sets $m = 2, j = 1, n = 2i \geq 6$ in the NR interactions Eq.(8) are necessary for realistic scenarios. In fact, the M-mass becomes $O(10^{9\sim10}\mathrm{GeV})$ for these sets. This large M-mass solves the solar neutrino promlem and also is compatible with the cosmological bound for stable light neutrinos. Special form of the NR terms suggests that the superstring model possesses an appropriate discrete symmetry coming from distinctive structure of the compactified manifold. The detailed study of the present models will be presented elsewhere [8].

References

[1] S.Cecotti, J.P.Derendinger, S.Ferrara, L.Girardello and M.Roncadelli, Phys. Lett. **156B** (1985) 318.
J.D.Breit, B.A.Ovrut and G.C.Segre, Phys. Lett. **158B** (1985) 33.
M.Dine, V.Kaplunovsky, M.Mangano, C.Nappi and N.Seiberg, Nucl. Phys. **B259** (1985) 549.
T.Matsuoka and D.Suematsu, Nucl. Phys. **B274** (1986) 106; Prog. Theor. Phys. **76** (1986) 886.

[2] M.Gell-Mann, P.Ramond and S.Slansky, in Supergravity, eds. D.Freedman *et al.* (North-Holland, Amsterdam, 1979).
T.Yanagida, KEK lectures, eds. O.Sawada *et al.* (1980)912; Phys. Rev. **D23** (1981) 196.
R.Mohapatra and S.Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[3] S.P.Mikheyev and A.Y.Smirnov, Yad. Fiz. **42** (1985) 1441; Nuov. Cim. **9C** (1986) 17.
L.Wolfenstein, Phys. Rev. **D17** (1978) 2369; Phys. Rev. **D20** (1979) 2634.
S.A.Bludman, D.Kennedy and P.Langacker, Phys. Rev. **D45** (1992) 1810; Nucl. Phys. **B374** (1992) 373.

[4] R.Cowsik and J.McClelland, Phys. Rev. Lett. **29** (1972) 669.

[5] C.A.Lutkin and G.G.Ross, Phys. Lett. **214B** (1988) 357.
C.Hattori, M.Matsuda, T.Matsuoka and H.Mino, Prog. Theor. Phys. **82** (1989) 599.

[6] M.Masip, Phys. Rev. **D46** (1992) 3601.

[7] P.Zoglin, Phys. Lett. **228B** (1989) 47.

[8] N.Haba, C.Hattori, M.Matsuda T.Matsuoka and D.Mochinaga, in preparation.

Table Captions

Table I

The energy scales of symmetry breaking $\langle S\rangle$ and $\langle N\rangle$ and a large Majorana-mass M_M in GeV unit for various cases of (n,i,m,j). Here we take $M_C = 10^{18.5}$GeV and $\sqrt{-m_S{}^2} = 10^3$GeV.

Figure Captions

Fig. 1

The structure of the scalarpotential in the case $(n,i,m,j) = (6,3,2,1)$ with $a = b = c = 1$. The vertical axis is taken as the normalized scalarpotential v (see text). The horizontal axes are $\overline{x} = (x/x_0)^3$ and $\overline{y} = y/y_0$, where $x = \langle S\rangle/M_C$ and $y = \langle N\rangle/M_C$.
(a) The overview of the scalarpotential v. The Point A (the absolute minimum) is located at $(\overline{x},\overline{y}) = (1,1)$ and the Point B is a local minimum.
(b) The comparison of values of the scalarpotential v between Point A and Point B. A solid (dashed) curve represents the calculation of v vs. $\overline{x}$ along the line $\overline{x} = \overline{y}$ ($\overline{y} = 0$).

Table I

n	i	m	j	$\langle S\rangle$ (GeV)	$\langle N\rangle$ (GeV)	M_M (GeV)
4	2	2	1	$10^{15.9}$	$10^{13.1}$	$10^{8.1}$
6	3	2	1	$10^{16.9}$	$10^{13.5}$	$10^{8.8}$
8	4	2	1	$10^{17.4}$	$10^{13.6}$	$10^{9.1}$
10	5	2	1	$10^{17.6}$	$10^{13.7}$	$10^{9.2}$
12	6	2	1	$10^{17.8}$	$10^{13.7}$	$10^{9.2}$
20	10	2	1	$10^{18.1}$	$10^{13.7}$	$10^{9.2}$
6	4	3	1	$10^{16.7}$	$10^{14.7}$	$10^{6.6}$
9	6	3	1	$10^{17.5}$	$10^{15.3}$	$10^{6.4}$
12	8	3	1	$10^{17.8}$	$10^{15.4}$	$10^{6.6}$
6	2	3	2	$10^{16.9}$	$10^{15.2}$	$10^{5.4}$
9	3	3	2	$10^{17.5}$	$10^{15.2}$	$10^{5.8}$
12	4	3	2	$10^{17.8}$	$10^{15.3}$	$10^{5.9}$