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Optimal Selection of Blocked Robust Parameter Designs and their Applications

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Abstract

Blocking is a useful technique to control systematic variation in experiments. Robust parameter design is widely used as an effective tool to reduce process variability by appropriate selection of control factors to make the process insensitive to noise. In this paper, we propose and study a method for selecting the optimal blocked robust parameter designs when some of the control-by-noise interactions are included in the model. We then discuss how to search for the best designs according to this method and present some results for designs of 8 and 16 runs.

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1. Introduction

Robust parameter design, originally proposed by Taguchi [15], is an important method for variation reduction in industry processes and products. It has been widely used as an effective tool to reduce process variability by appropriate selection of control factors to make the process insensitive to noise. The practical and theoretical importance of this design has been discussed by Phadke [11] and Nair [10]. An outstanding issue in robust parameter design is the choice of experimental plan. Most robust design experiments have used product arrays for joint study of control and noise factors. In these designs, separate arrays are chosen for the control and the noise factors and then each combination in the control factor array is paired with each combination in the noise factor array, producing a matrix of data. When the number of factors is large, a large experimental run size is needed for product arrays, making experiments costly and impractical. Recent statistical research on robust design experiments has favored single arrays which include effects for both control factors and noise factors. This idea has been discussed by many authors including Lorenzen and Villalobos [9], Vining and Myers [18], Welch, Yu, Kang and Sacks [19], Shoemaker, Tsui and Wu [12], Freeny and Nair [5], Steinberg and Bursztyn [13, 14], Tsui [17], and Berube and Nair [1]. Recently the problem of selecting optimal single arrays for robust parameter design has been addressed in the literature. Frey, Engelhardt, and Greitzer [6] proposed an adaptive one-factor-at-a-time technique that provides large improvements with an economy of run size under some conditions. Bingham and Sitter [2] proposed a criterion and constructed the tables of small arrays applicable in split-plot parameter design experiments. Wu and Zhu [21] developed a general framework for selecting optimal single arrays based on their effect ordering principle which includes main effects and all the interactive effects in their model. In practical applications of robust parameter designs it is often the case that only some of the control-by-noise interactions are important and need to be estimated in addition to the control and noise effects.

In some experiments, in order to reduce extraneous error and increase the precision of inferences, it is often desirable to group the experimental units into blocks such as different fields in a plant study. Hence blocked designs are often used in order to make data collection more efficient.

In this paper we propose and study a method to select optimal blocked robust parameter designs for the models containing some of the control-by-noise interactions. Section 2 of the paper introduces robust parameter design, two-level fractional factorial design, and blocking. Section 3 describes the proposed method for selecting optimal blocked robust parameter designs. Section 4 examines how to search for best designs based on this method and presents some results for designs of 8 and 16 runs. Section 5 concludes the paper with an illustrative example.

2. Robust parameter design and fraction factorial design

2.1 Robust parameter design

Taguchi [15] postulated that there are two types of factor which operate on a process: control factors and noise factors. Control factors are variables whose levels remain unchanged in the process once they are set, whereas the levels of the noise factors are hard or impossible to control within the process and cause unwanted variation in the response. The goal of robust parameter design is to determine levels of the control factors which cause the response to be robust to changes in the levels of the noise variables, thereby to find settings of the control factors that will minimize variability. When a robust parameter design experiment is conducted, both the control factors and noise factors are varied systematically. The basic idea is to choose the control factors such that they are insensitive to the noise factors. This will reduce the effect of uncontrollable variations on the response. To achieve this goal, information on two-factor interactions between control and noise factors is particularly useful. The blocked robust parameter designs selected from two-level fractional factorial designs would enjoy the nice properties of this class of designs such as balance, orthogonality, and reduced run size of the experiment. To help understanding our method, a brief review of two-level fractional factorial design and blocking are given in Section 2.2 and Section 2.3 respectively.

2.2 Two-level fractional factorial design

Two-level fractional factorial designs have been proven useful for efficient data collection and are widely used in many areas of scientific investigation. They allow us to study many factors with relatively small run size. The importance of this class of designs has long been established by Box, Hunter and Hunter [4]. A regular two-level fractional factorial design is commonly referred to as a 2^{m-p} design. It has m two-level factors with 2^{m-p} runs, and is completely determined by p independent *defining relations*, also called *defining words*. When $p = 0$, it is reduced to a full factorial 2^m design. A *defining word* is given by a word of letters which are labels of factors denoted by 1, 2, …, m. The number of letters in a word is called its *word-length*. The group of *defining words* generated by the p independent *defining words* is called the *defining contrast subgroup* and the length of the shortest is called the *resolution* of the design. The vector $W(d) = (A_1(d), A_2(d), \ldots, A_{\mathrm{m}}(d))$ is called the *word-length pattern* of the design d, where $A_i(d)$ is the number of words of length i in the *defining contrast subgroup*. A key concern in fractional factorial design is how to choose a fraction of the full factorial design for a given run size and number of factors. The *resolution* criterion proposed by Box and Hunter [3] selects 2^{m-p} design that has higher *resolution*. Since two designs having the same *resolution* may have a different *word-length pattern* and may not be equally good, Fries and Hunter [7] proposed the *minimum aberration* criterion to further discriminate 2^{m-p} designs. For two designs d_1 and d_2, suppose r is the smallest value such that $A_r(d_1) \neq A_r(d_2)$. We say that d_1 has less aberration than d_2 if $A_r(d_1) < A_r(d_2)$. If no design has less aberration than d_1, then d_1 is said to have *minimum aberration*. The *minimum aberration* criterion is the most commonly used criterion for 2^{m-p} design selection.

In what follows, an example is used to introduce 2^{m-p} design. Suppose we wish to perform an experiment with eight runs and several factors at two levels, labeled +1 and −1. Table 1 gives an 8-run saturated design with its columns arranged in Yates's order. The Yates's order of the columns of an 8-run saturated design can be written as ($\boldsymbol{a}_1$, $\boldsymbol{a}_2$, a_1a_2, $\boldsymbol{a}_4$, a_1a_4, a_2a_4, $a_1a_2a_4$) where other columns can be generated from the three independent columns $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_4$.

Table 1. Columns of the 8-run saturated design in Yates's order

Run	**1**	**2**	3 (3=12)	**4**	5 (5=14)	6 (6=24)	7 (7=124)	Response
1	−1	−1	+1	−1	+1	+1	−1	y_1
2	−1	−1	+1	+1	−1	−1	+1	y_2
3	−1	+1	−1	−1	+1	−1	+1	y_3
4	−1	+1	−1	+1	−1	+1	−1	y_4
5	+1	−1	−1	−1	−1	+1	+1	y_5
6	+1	−1	−1	+1	+1	−1	−1	y_6
7	+1	+1	+1	−1	−1	−1	−1	y_7
8	+1	+1	+1	+1	+1	+1	+1	y_8

If we use columns 1, 2, and 4 of Table 1 to set the levels of three factors A, B, and C, respectively, then y_1 through y_8 in Table 1 represent the responses at 2^3 possible combinations of factor settings. This gives a 2^3 = 8 run, two-level, three-factor, full factorial design. By using this design, the main effects of *A*, *B*, and *C*, as well as their interactions *AB*, *AC*, *BC*, and *ABC* can be estimated. A significant disadvantage of the full factorial design is that the run size of the design rises geometrically with the number of factors. In order to conduct experiments more efficiently, fractional factorial designs may be employed to reduce the size of the experiment. For example, for estimating the effects of seven factors each having two levels, a full factorial design will need 2^7=128 runs, a fractional factorial design may need only 2^{7-4} =8 runs. The economy of run size afforded by the fractional factorial design comes at a cost. In a fractional factorial design, each main effect is clear of other main effects, but may be confounded with two-factor interactions or higher order interactions. For example, if we want to study one more factor *D* in addition to *A*, *B*, and *C* using the 8-run design, we have different choices. For design d_1, we assign the levels of factor *D* to the column 7. This gives a 2^{4-1} fraction factorial design. Since 7=124, the estimate for the main effect *D* could not be separated from the effect of the interaction between *A*, *B*, and *C*. That is *D*=*ABC*. The resolution of the design is 4 and the *word-length pattern* $W(d_1)$ = (0, 0, 0, 1). For another design d_2, we assign factor *D* to column 6=24. The resolution is 3 and $W(d_2)$ = (0, 0, 1, 0). Obviously, d_1 is better than d_2 because it has higher resolution and minimum aberration. Both definition of resolution and minimum aberration are based on the hierarchical principle: (i) lower order interactions are more important than higher order interactions, (ii) effects of the same order are equally important. The advantage of d_1 is obvious based on the principle because the main effects in d_1 are confounded with three-factor interactions and the main effects in d_2 are confounded with two-factor interactions.

2.3 Blocking and its applications

Blocking is a commonly used technique to control systematic variation in experiments. Such variation might occur from field-to-field, day-to-day, or batch-to batch. Without blocking, the systematic variation can influence the accuracy and efficiency of effect estimation. Blocking can effectively eliminate the systematic variance by grouping the runs of an experiment into blocks. In a blocked design, the variance due to blocks is modeled. Hence it is removed from the residual variance; thereby effectively reducing the magnitude of the estimated experimental error. Criteria for the choice of blocks are most frequently different settings or environments for the conduct of the experiment. In any case, blocks should be chosen so that the units within blocks are as homogeneous as possible. Blocking can be accomplished through the use of blocking factors in a design. For 2^{m-p} designs, since there are many choices to assign blocking factors to the unused columns of a saturated design, blocking schemes and methods are needed to select columns that reduce the bias (as one did in the selection of unblocked designs). In blocked 2^{m-p} designs, interactions between treatment and blocking factors are assumed to be non-existent, a necessary condition for the effectiveness of blocking. See Box, Hunter, & Hunter [4] for the principles and assumptions in the construction of block designs. For the models containing main effects and blocking effects, many blocking schemes for 2^{m-p} designs have been discussed in the literature. In this article, we discuss the blocking schemes for robust parameter designs for the models containing some control-by-noise interactions in addition to main effects and blocking effects.

3. A method for selecting optimal blocked robust parameter design

3.1 The optimal design selection criterion

Suppose that we wish to estimate the control effects, noise effects, and some control-by-noise interactions using a blocked robust parameter design designs. Then the fitted model should include all the control factors, noise factors, blocking factors, and the important control-by-noise interactions.

If the effects not in the postulated model cannot be completely ignored, they will bias the estimates of the effects in the model. To solve this problem, the key issues are to permit estimation of the control effects, noise effects, blocking effects, and the important control-by-noise interactions in the postulated model and to minimize the bias caused by the effects not in the model. The optimal design should be selected to minimize the contamination of these non-negligible effects on the model. The proposed design selection criterion is given below:

Optimal design selection criterion: Let N_j, j = 2, 3, …, m be the number of j-factor interactions not in the model that are confounded with the effects in the postulated model including control effects, noise effects, blocking effects, and the important control-by-noise interactions. The optimal blocked robust parameter design is selected by sequentially minimizing $N_2,\ldots, N_m$.

To gain further insight, we now examine the criterion in detail. The postulated model consists of all the control factors, noise factors, blocking effects, and control-by-noise interactions. Those 2-factor interactions not in the model and higher-order interactions generally cause a bias on the estimation of the effects in the model. The measure of this bias, as given by N_j, is the number of the j-factor interactions outside the model that are confounded with the effects in the model. Under the hierarchical principle that lower-order effects are more important than higher-order effects (Wu and Hamada [20]), to minimize the bias, we should sequentially be minimizing N_2, N_3, …, N_m. The vector (N_2, N_3, …, N_m) is called the confounding pattern of a design. Therefore this criterion selects the design that minimizes N_2. If several designs have the same number of N_2, it selects the design that minimizes N_3 among the designs that have minimum N_2, and so on.

We now look at an example. Suppose that we want to study one noise factor A, three control factors B, C, and D, and two control-by-noise interactions AB and AC by using a blocked design of 8 runs. We consider two designs d_1 and d_2. Based on Table 1, for d_1, we assign factors A, B, C, and D to columns 1, 4, 7, and 2 and the blocking factor to column 3_b (We use 'b' to indicate blocking factor). The interactions to be estimated should be 14 and 17. Since 7 = 124 and 3_b =12, the *defining contrast subgroup* of the design is given by I=1247=123$_b$=3$_b$47. Hence we have 14=27, 17=24, 3_b=12, 3_b=47, 1=247, 2=147, 4=127, and 7=124. Therefore N_2 = 4 and N_3 = 4. Note that the interactions between blocking factors and treatment factors are assumed not existent and are not counted here. The confounding pattern of d_1 is (4, 4, 0) meaning that four 2-factor interactions and four 3-factor interactions not in the model are confounded with the effects in the model. For design d_2, we assign factors A, B, C, and D to columns 4, 2, 3, and 1 and the blocking factor to column 5_b. The interactions to be estimated should be 42 and 43. Since 3=12 and 5_b=47, the *defining contrast subgroup* of design d_2 is given by I=123=145$_b$=2345$_b$. Hence we have 1=23, 2=13, 3=12, 5_b=14, 5_b=234, 24=134, 34=124 and 4=1234. Hence the confounding pattern of d_2 is (4, 3, 1) meaning that four 2-factor interactions, three 3-factor interactions, and one 4-factor interaction not in the model are confounded with the effects in the model. Based on our design selection criterion, design d_2 is better than d_1 because $N_2(d_1) = N_2(d_2)$ and $N_3(d_1) > N_3(d_2)$ where $N_2(d_1)$, $N_2(d_2)$ $N_3(d_1)$, and $N_3(d_2)$ denote N_2 and N_3 for d_1 and d_2 respectively.

3.2 Theoretical justification of the criterion

Tang and Deng [16] showed that the minimum aberration criterion is equivalent to a criterion that sequentially minimizes the number of j-factor interactions aliased with the main effects. Their result was extended by Ke and Tang [8] to the cases that include some important two-factor interactions. Here we extend their results to the blocked robust parameter design cases. Suppose that we are interested in estimating all the block factors, control factors, noise factors, and some of the control-by-noise interactions by using a 2^{m-p} design. Then the fitted model is given by

$$Y = \alpha_0 I + W_1\gamma_1 + \varepsilon \tag{1}$$

where Y denotes the vector of n observations, α_0 is the grand mean, I denotes the vector of n ones, γ_1 is the vector of parameters containing all the control factors, noise factors, and important control-by-noise interactions, W_1 is the corresponding design matrix, and ε is the vector of uncorrelated random errors, assumed to have mean 0 and a constant variance. Since other interactions may not be negligible, the true model can be written as

$$Y = \alpha_0 I + W_1\gamma_1 + X_2\alpha_2 + X_3\alpha_3 + \cdots + X_m\alpha_m + \varepsilon, \tag{2}$$

where α_2 is the vector of remaining two-factor interactions and X_2 is the corresponding design matrix, α_j is the vector of j factors interactions and X_j is the corresponding matrix. The least square estimator $\hat{\gamma}_1 = (W_1^T W_1)^{-1} W_1^T Y = n^{-1} W_1^T Y$ from the fitted model in (1) has expectation, taken under the true model in (2), of $E(\hat{\gamma}_1) = \gamma_1 + P\alpha_2 + P_3\alpha_3 + \cdots + P_m\alpha_m$, where $P_2 = n^{-1}W_1^T X_2$ and $P_j = n^{-1}W_1^T X_j$. So the bias of $\hat{\gamma}_1$ for estimating γ_1 is given by

$$\text{Bias}(\hat{\gamma}_1, \gamma_1) = P_2\alpha_2 + P_3\alpha_3 + \cdots + P_m\alpha_m. \tag{3}$$

Note $P_2\alpha_2$ is the contribution of α_2 to the bias, and $P_j\ \alpha_j$ is the contribution of α_j to the bias. Because α_j is unknown, we have to work with P_j. One size measure for a matrix $P = (p_{ij})$ is given by $\|P\|^2 \overset{def}{=} \text{trace}(P^{\text{T}}P) = \sum_{i,j} p_{ij}^2$. Under the hierarchical assumption that lower order effects are more important than higher order effects, to minimize the bias of $\hat{\gamma}_1$ we should sequentially minimize $\|P_2\|^2, \ldots, \|P_m\|^2$. For regular 2^{m-p} designs, the entries of P_j are 0 or 1, and thus $\|P_j\|^2$ is simply the number of j-factor interactions aliased with the effects in the postulated model in (1). Now let $N_j = \|P_j\|^2$. Based on the above results, we can select optimal robust parameter designs by sequentially minimizing $N_2, \ldots, N_m$ where N_j is the number of j-factor interactions not in the model confounded with the effects in the postulated model.

4. Searching for best designs

4.1 Searching method

In this paper, we consider 2^{m-p} designs of 8 runs and 16 runs. For designs of 8 runs there are only 7 columns in a saturated design. The choice of the blocked robust parameter design is limited and the optimal design is not hard to select according to our criterion. For designs of 16 runs, there are 15 columns in a saturated design. There are a lot of choices for a robust parameter design. The optimal blocked robust parameter design of 16 runs is not easy to select according to this criterion. We use a computer program to calculate the confounding pattern for each choice of the designs for the given number of blocking factors, control factors, noise factors, and important control-by-noise interactions. Then select the best one that has minimum N_2. If several designs have the same N_2, we select the one that minimizes N_3, and so on. Because noise factors are hard to control, the number of noise factors included in robust parameter design experiments is often small. In this paper, we only consider up to 2 noise factors. Through our search effort we have found optimal robust parameter designs of 8 and 16 runs for up to 2 noise factors and 2 control-by-noise interactions. In our search effort we have used (N_2, N_3, N_4) instead of the entire vector (N_2, …, N_m) to reduce the computing burden. Actually five-factor and higher order interactions are very small and usually negligible in practice.

4.2 Optimal blocked robust parameter designs of 8 and 16 runs

Table 2 and 3 present the optimal robust parameter designs of 8 runs for the models containing up to 2 noise factors and 2 control-by-noise interactions, Tables 4−6 present the optimal designs of 16 runs for the models containing up to 2 noise factors and 2 control-by-noise interactions. In these tables, the entries under "$k_c + k_n$" give the number of control factors plus the number of noise factors. The entries under "model" indicate which model is under consideration for the models containing two control-by-noise interactions. Let C_1 and C_2 be control factors and N_1 and N_2 noise factors. An entry of 2(a) denotes the model with two control-by-noise interactions of C_1–N_1 and C_2–N_2, 2(b) denotes the model of C_1–N_1 and C_2–N_1, and 2(c) denotes the model of C_1–N_1 and C_1–N_2. The entries under "block factor" give the design columns for the block factors in the fitted model. The entries under "control factor" give the design columns for the control factors. The entries under "noise factor" give the design columns for the noise factors. Column j in these tables denotes the j-th column in the saturated design with its columns arranged in Yates's order. The Yates's order of the columns of an 8-run saturated design was given in Section 2.2.The Yates's order of the columns of a 16-run saturated design can be written as ($\boldsymbol{a_1}$, $\boldsymbol{a_2}$, a_1a_2, $\boldsymbol{a_4}$, a_1a_4, a_2a_4, $a_1a_2a_4$, $\boldsymbol{a_8}$, a_1a_8, a_2a_8, $a_1a_2a_8$, a_4a_8, $a_1a_4a_8$, $a_2a_4a_8$, $a_1a_2a_4a_8$) where other columns can be generated from the four independent columns $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_4}$, and $\boldsymbol{a_8}$. The entries under "c-n interaction" show how to assign the factors involved in the control-by-noise interactions. The last column in these tables gives (N_2, N_3, N_4).

For unblocked robust parameter designs, Wu and Zhu [21] provided the optimal design tables based on their effect ordering principle which treated all the control-by-noise interactions and the main effects as equally important. Their tables provided generally good designs for estimating main effects and all the control-by-noise interactions. For blocked robust parameter designs, if only some of the control-by-noise interactions are important and need to be estimated, which is the case in many practical applications, our blocked optimal design tables can be directly applied.

Table 2. Optimal blocked robust parameter designs of 8 runs for the models containing one c-n interaction

$k_c + k_n$	block factor	control factor	noise factor	c-n interaction	(N_2, N_3, N_4)
3 + 1	3	1 2 7	4	(1, 4)	(3, 4, 0)
2 + 2	3	1 2	4 7	(1, 4)	(3, 4, 0)
4 + 1	6	1 2 3 4	5	(2, 5)	(9, 8, 4)
3 + 2	6	1 2 3	4 5	(2, 5)	(9, 8, 4)

Table 3. Optimal blocked robust parameter designs of 8 runs for the models containing two c-n interactions

$k_c + k_n$	model	block factor	control factor	noise factor	c-n interaction	(N_2, N_3, N_4)
3 + 1	2(b)	5	1 2 3	4	(2, 4)(3, 4)	(4, 3, 1)
2 + 2	2(b)	5	2 3	1 4	(2, 4)(3, 4)	(4, 3, 1)
2 + 2	2(c)	5	1 4	2 3	(4, 2)(4, 3)	(4, 3, 1)

Table 4. Optimal blocked robust parameter designs of 16 runs for the models containing one noise factor and one c-n interaction

$k_c + k_n$	block factor	control factor	noise factor	c-n interaction	(N_2, N_3, N_4)
4 + 1	11	1 2 4 7	8	(1, 8)	(0, 6, 1)
5 + 1	13	1 2 7 8 11	4	(1, 4)	(1, 16, 2)
6 + 1	14	1 4 7 8 11 13	2	(1, 2)	(2, 37, 4)
7 + 1	5	1 4 7 8 11 13 14	2	(1, 2)	(7, 56, 16)
8 + 1	7	1 2 3 5 8 9 14 15	4	(2, 4)	(19, 64, 80)
9 + 1	11	1 2 3 4 5 6 9 14 15	8	(2, 8)	(31, 88, 160)
10 + 1	15	1 2 3 4 5 8 9 10 13 14	6	(1, 14)	(44, 129, 272)
11 + 1	11	1 2 3 4 5 8 9 10 13 14 15	6	(1, 6)	(59, 188, 432)
12 + 1	12	1 2 3 4 5 8 9 10 11 13 14 15	6	(2, 12)	(77, 264, 660)

Table 5. Optimal blocked robust parameter designs of 16 runs for the models containing one noise factor and two c-n interactions

$k_c + k_n$	model	block factor	control factor	noise factor	c-n interaction	($N2$, $N3$, $N4$)
4 + 1	2(b)	11	1 2 4 7	8	(1, 8)(2, 8)	(0, 6, 2)
5 + 1	2(b)	13	1 2 7 8 11	4	(1, 4)(2, 4)	(2, 16, 4)
6 + 1	2(b)	14	2 4 7 8 11 13	1	(2, 1)(4, 1)	(4, 35, 8)
7 + 1	2(b)	6	2 4 7 8 11 13 14	1	(2, 1)(4, 1)	(10, 56, 24)
8 + 1	2(b)	10	1 2 3 5 8 9 14 15	4	(2, 4)(3, 4)	(22, 68, 88)
9 + 1	2(b)	12	1 2 3 4 5 6 9 14 15	8	(2, 8)(3, 8)	(34, 96, 172)
10 + 1	2(b)	7	1 2 3 4 5 6 8 9 10 14	13	(1, 13)(2, 13)	(48, 141, 288)

Table 6. Optimal blocked robust parameter designs of 16 runs for the models containing two noise factors and two c-n interactions

$k_c + k_n$	model	block factor	control factor	noise factor	c-n interaction	($N2$, $N3$, $N4$)
3 + 2	2(a)	5	1 4 15	2 8	(1, 2)(4, 8)	(1, 3, 5)
3 + 2	2(b)	11	1 2 15	4 8	(1, 8)(2, 8)	(0, 6, 2)
3 + 2	2(c)	11	4 8 15	1 2	(8, 1)(8, 2)	(0, 6, 2)
4 + 2	2(a)	13	1 2 7 11	4 8	(1, 4)(2, 8)	(2, 16, 4)
4 + 2	2(b)	13	1 2 8 11	4 7	(1, 4)(2, 4)	(2, 16, 4)
4 + 2	2(c)	13	4 7 8 11	1 2	(4, 1)(4, 2)	(2, 16, 4)
5 + 2	2(a)	14	1 4 7 11 13	2 8	(1, 2)(4, 8)	(4, 35, 8)
5 + 2	2(b)	14	2 4 8 11 13	1 7	(2, 1)(4, 1)	(4, 35, 8)
5 + 2	2(c)	14	1 7 8 11 13	2 4	(1, 2)(1, 4)	(4, 35, 8)
6 + 2	2(a)	5	1 4 7 11 13 14	2 8	(1, 2)(4, 8)	(10, 56, 24)
6 + 2	2(b)	6	2 4 8 11 13 14	1 7	(2, 1)(4, 1)	(10, 56, 24)
6 + 2	2(c)	6	1 7 8 11 13 14	2 4	(1, 2)(1, 4)	(10, 56, 24)
7 + 2	2(a)	7	1 2 3 5 9 14 15	4 8	(2, 4)(3, 8)	(22, 68, 88)
7 + 2	2(b)	10	2 3 5 8 9 14 15	1 4	(2, 4)(3, 4)	(22, 68, 88)
7 + 2	2(c)	10	1 4 5 8 9 14 15	2 3	(4, 2)(4, 3)	(22, 68, 88)
8 + 2	2(a)	11	1 2 3 4 5 6 9 15	8 14	(2, 8)(3, 14)	(34, 96, 172)
8 + 2	2(b)	12	2 3 4 5 6 9 14 15	1 8	(2, 8)(3, 8)	(34, 96, 172)
8 + 2	2(c)	12	1 4 5 6 8 9 14 15	2 3	(8, 2)(8, 3)	(34, 96, 172)
9 + 2	2(a)	15	1 2 3 4 6 8 9 13 14	10 5	(1, 10)(2, 5)	(48, 141, 288)
9 + 2	2(b)	7	1 2 4 5 6 8 9 10 14	3 13	(1, 13)(2, 13)	(48, 141, 288)
9 + 2	2(c)	7	3 4 5 6 8 9 10 13 14	1 2	(13, 1)(13, 2)	(48, 141, 288)

5. An illustrative example

The optimal blocked robust parameter designs of 8 runs and 16 runs are listed in Tables 2−6. When we plan to study several control factors, noise factors, and some important control-by-noise interactions by using a blocked 2^{m-p} design of 8 or 16 runs, we can choose an optimal design directly from these tables to satisfy our needs. Now an example is employed to illustrate how to use these optimal design tables.

Suppose that in an experiment, the experimenter want to study four control factors - *Nitrogen*, *Phosphorus*, *Potassium*, and *moisture* and two noise factors - *temperature* and *light*. She would like to use a blocked 2^{m-p} design of 16 runs with one blocking factor that is the subdivision of the field. In addition to the block factor, control factors and noise factors, she also wants to estimate the control-by-noise interaction between *Nitrogen* and *temperature* and the control-by-noise interaction between *Nitrogen* and *light*. The model for this experiment is 2(c) with interaction pattern of C_1–N_1 and C_1–N_2. The optimal design for this model can be found in Table 6. Now let us look at the row for $k_c + k_n = 4 + 2$ and model 2(c) in Table 6. We see that the column for blocking factor is 13, the columns for control factors are 4, 7, 8 and 11, and the columns for noise factors are 1 and 2. To complete the specification of the optimal design, we need to appropriately assign the control and noise factors to the 6 columns 1, 2, 4, 7, 8, and 11. The "c-n interaction" column in Table 6 says that we should assign *Nitrogen* to column 4, assign *temperature* and *light* to columns 1 and 2, and assign other control factors to columns 7, 8, and 11 arbitrarily. This design has $N_2 = 2$, meaning that only two 2-factor interactions not in the model are confounded with the effects in the model. This design is the best in the sense that no other design has smaller N_2 than this one for the given model.

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