HEURISTIC FOR ASYMMETRIC CAPACITATED VEHICLE ROUTING PROBLEM

Tjutju Tarliah Dimyati

Industrial Engineering Department, Pasundan University
tjutjutarliah@ymail.com; adimyati@bdg.centrin.net.id

ABSTRACT

The vehicle routing problem (VRP) is commonly defined as the problem of designing optimal delivery or collection routes from one or several depots to a set of geographically scattered customers, under a variety of side conditions. This problem is generally described through a graph, whose arcs represent the road sections and vertices correspond to the depot and customer locations. The arcs (and consequently the corresponding graph) can be directed or undirected, depending on whether they can be traversed in only one direction or in both directions. Since each arc is associated with a cost then if the graph is directed, the cost matrix is asymmetric and the corresponding problem is called asymmetric vehicle routing problem (AVRP). Although the symmetric problems are special cases of the asymmetric ones, the latter were much less studied in the literature. In this paper, a type of problem, called the Asymmetric Capacitated Vehicle Routing Problem (ACVRP) is discussed and a heuristic algorithm is proposed to solve the problem.

***Keywords**: Vehicle Routing, Asymmetric, Capacitated, Heuristic*

1. INTRODUCTION

The Vehicle Routing Problem (VRP) has been thoroughly studied in a variety of areas, such as Operations Research, Artificial Intelligence, etc. It can be defined as a problem of finding the optimal routes of delivery or collection from one or several depots to a number of cities or customers, while satisfying some constraints. Collection of household waste, gasoline delivery trucks, goods distribution, and mail delivery are the most used applications of the VRP. Since the VRP plays a vital role in distribution and logistics, thousands of papers have been written on several VRP variants such as Capacitated Vehicle Routing Problem (CVRP), VRP with Time Windows (VRPTW), VRP with Backhauls (VRPB), VRP with Pick-Up and Delivery (VRPPD) and Periodic VRP (PVRP) (Toth, 2002).

In the CVRP, all the customers correspond to deliveries and the demand are deterministic and may not be split. The vehicles are identical and based at a single central depot, and only the capacity restrictions for the vehicles are imposed. Typically, the problem objective is to find delivery routes starting and ending at the depot that minimize a total travel cost or distance without violating the capacity constraints of the vehicles. This type of problem has been studied extensively in the literature. Many exact algorithms, which were dominated by branch-and-cut methods, have been proposed for solving the problem (see (Ralphs, 2003), (Toth, 2002), (Baldacci, 2004), (Lysgaard, 2004), (Fukasawa, 2006)). However, the efficient solution of CVRP is still challenging problem, which requires the use of heuristics to large problems. Fortunately, several families of heuristics have been proposed. They can be broadly classified into two main classes, i.e. classical heuristics and meta-heuristics. The quality of solutions produced by meta-heuristics is much higher than that obtained by classical heuristics, but the price to pay is increased computing time. Some classical heuristics for the problem have been surveyed by (Laporte, 2002), while meta-heuristics have been surveyed by (Gendreau, 2002).

The road network where the movements of the delivery vehicles are performed is represented by a graph with arcs and vertices. Arcs represent roads and vertices correspond to the road intersections, junctions, customer locations, and the depot. The arcs and its corresponding graph can be directed or undirected, depending on whether they can be traversed in only one direction or in both directions. If the graph is directed, the distance from one point to another in a given space can be different from the inverse distance. Since each arc is associated with a cost then if the graph is directed, the cost matrix is asymmetric, and the corresponding problem is called Asymmetric Capacitated Vehicle Routing Problem (ACVRP). The scheduling problems with sequence dependent setup time and resource constraints, as well as the problem of distribution in urban areas where distances depend on one-way directions impose on roads, are practical applications of ACVRP in which the distance matrix associated to customers' set is asymmetric.

Although the symmetric problems are special cases of the asymmetric ones, the latter were much less studied in the literature. Most of the exact and heuristic algorithms from the literature considered only the symmetric and in many cases, only the Euclidean version of the problem. Algorithms for the non-Euclidean CVRP, and especially the Asymmetric CVRP, have been neglected in the literature by comparison with the extensive effort devoted to studying Euclidean VRP. So far, there is only one heuristic (Vigo, 1996) and two exact methods (Fischetti, 1994), (Laporte, 1986) for ACVRP found in the literature. This state of affairs is at odds with the fact that asymmetric models are relevant to a wider range of applications, and more general than symmetric models. Hence it is important to study heuristic methods for ACVRP.

The purpose of this paper is to propose a heuristic algorithm developed for ACVRP, although it may also be used for the symmetric instances. The rest of this paper is organized as follows: In section 2 a problem formulation of ACVRP is discussed. Section 3 is devoted to a discussion of the developed heuristic. Section 4 gives a numerical example for the proposed heuristic. The conclusion is put in section 5.

2. FORMULATION OF THE ACVRP

2.1. Problem Identification

The Asymmetric Capacitated Vehicle Routing Problem (ACVRP) is defined on a complete directed graph G = (V, A) where $V=\{0,1,2, \ldots, n\}$ is the set of nodes (vertices), 0 is the *depot* (*origin*), and the remaining nodes are *customers*. The set $A = \{(i, j): i, j \in V, i \neq j\}$ is the set of arcs. Each customer $i \in V \setminus \{0\}$ is associated with a positive integer demand q_i and each arc (i, j) is associated with asymmetric deterministic travel cost c_{ij} (i.e. $c_{ij} \neq c_{ji}$ in general) which might violate the triangular inequality (i.e. $c_{ij} \leq c_{ik} + c_{kj} \;\forall\; i,j,k$). In this case, the cost of traveling from node i to node j can be approximated by the distance d_{ij} with $d_{ij} \neq d_{ji}$.

There are m vehicles with identical capacity Q. The ACVRP consists of determining a set of m vehicle routes satisfying the following conditions:

- Each route starts and ends at the depot,
- Each customer is served exactly by one vehicle,
- The total demand of each route does not exceed the vehicle capacity Q,
- The total cost of all routes is minimized.

Because of the problem objective and in order to ensure feasibility, it is assumed that the number of available vehicles m is equal to the minimum number of vehicles sufficient to serve all the customers. Mathematically, the minimum number of vehicles needed to serve all customers can be stated as:

$$m = \left\lceil \frac{\sum_{i \in V} q_i}{Q} \right\rceil$$

2.2. Mathematical Model

Consider integer variables x_{ij} which indicate whether a vehicle goes from node i to node j or not, and additional continuous variables u_i for $i \in V \setminus \{0\}$ that represent the load of the vehicle after visiting customer i. The mathematical model of the problem is:

$$\text{Min} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \qquad (1)$$

Subject to

$$\sum_{i \in V} x_{ij} = 1 \qquad j \in V \setminus \{0\} \qquad (2)$$

$$\sum_{j \in V} x_{ij} = 1 \qquad i \in V \setminus \{0\} \qquad (3)$$

$$\sum_{i \in V} x_{i0} = m \qquad (4)$$

$$\sum_{j \in V} x_{0j} = m \qquad (5)$$

$$u_i - u_j + Qx_{ij} \leq Q - q_j \quad i,j \in V \setminus \{0\},\ i \neq j,\ q_i + q_j \leq Q \qquad (6)$$

$$q_i \leq u_i \leq Q \qquad i \in V \setminus \{0\} \qquad (7)$$

$$x_{ij} \in \{0,1\} \qquad i,j \in V \qquad (8)$$

The objective function given in (1) states that costs should be minimized. Constraint (2) states that for each node, excluding node 0, there can only be a single outgoing arc to any other node, while constraint (3) states that there can only be a single incoming arc to a node from any other node, excluding node 0. Therefore, the first two constraints (2) and (3), force that a single vehicle goes into and out of every node. Constraint (4) and (5) ensure that a total of *m* vehicles leave the depot and then return to it. Constraints (6) and (7) are sub-tour elimination constraints imposing both the capacity and connectivity requirements of ACVRP.

Since the ACVRP is known to be NP-hard combinatorial optimization problem, the time it takes to solve the problem is grow exponentially as the number of customers increases. In other words, the exact algorithms can only solve relatively small problems. On the contrary, a heuristic is a solution strategy that produces an answer without any formal guarantee as to the quality. Heuristics clearly are methods which produce good solutions in practice but do not guarantee optimality. Therefore, in the next section a relatively simple heuristic algorithm is developed to obtain a quick solution for a large problem.

3. HEURISTIC FOR ACVRP

Classical VRP heuristics can be broadly classified into three categories: constructive heuristics, the two-phase heuristics, and the improvement methods. In the two-phase heuristics, the problem is decomposed into its two natural components i.e., clustering of nodes into feasible routes and actual route construction, with possible feedback loops between the two stages. The two-phase heuristics are divided into two classes: cluster first route second methods and route first cluster second methods. In the first case, nodes are first organized into feasible clusters, and a vehicle route is constructed for each of them. In the second case, a tour is first built on all nodes and is then segmented into feasible vehicle routes.

When finding an initial solution to a routing problem, the initialization criteria refers to the process of finding the first node to insert into a route, which is referred to as the seed node. Because of the strong influence of the seeds in the assignment of nodes to routes, and therefore on the final solution obtained, the choice of the seeds is a very critical step in applying cluster first route second methods. To avoid unpredictable results led by the seed selection, the heuristic developed in this paper is based on the idea of route first cluster second methods instead of cluster first route second methods, which are more extensively used.

The proposed heuristic for ACVRP consists of three steps: creating initial route, grouping customers, and building final feasible route for each group of customers obtained in the second step.

3.1. Creating Initial Route

Generate a *traveling salesman tour* on all customers without considering the capacity constraint of the vehicles. In this paper, the initial route will be done using two simple procedures. The first procedure is the well-known *Nearest Neighbor Algorithm* (NNA), which can be used both for the asymmetric and the symmetric instances.

The procedure of *NNA* is as follows:

1. Select the depot as a starting node and insert the node closest to it
2. Iteratively insert the node closest to the last node inserted until no more node is available

3. Connect the last node inserted with the depot

The second procedure is specially proposed for the asymmetric problem. In this paper, the initial route will be created using a modified procedure of the well-known *Johnson's Algorithm* for *n-jobs two machines problem* (Johnson, 1974). In this proposed procedure, all the nodes on the graph, except the depot, are sequenced as follows:

1. Let d_{0i} be the distance from depot to node i and d_{i0} be the distance from node i to depot ($i \in V \setminus \{0\}$)
2. List the d_{0i} and d_{i0} on a separate two columns
3. For every node $i \in V$ find the minimum distance of the two columns. If this minimum distance found on the d_{0i} column, assign the corresponding node to the first available place in the sequence. Otherwise, assign the node to the last available place in the sequence. Remove this node from the list of nodes to be sequenced. Ties are resolved by randomly choosing either position in the sequence for the assignment.
4. Repeat the process until all nodes have been sequenced.
5. Connect the first and the last nodes of the sequence with the depot.

3.1. Grouping Customers

The feasibility of the route will not depend on the sequence the customers are visited in, but rather depends on the set of customers assigned to the vehicle, i.e. the delivery of the route does not exceed the vehicle's capacity. Therefore, the initial route obtained from the previous step is then divided into a number of routes that satisfies capacity constraint. It is assumed that the number of available vehicles is at least the same as the number of routes obtained.

3.2. Building Final Routes

For each group of customers obtained in the second step, find a tour of minimal distance that visits all nodes exactly once, starting and ending at the depot. Since this step is actually solving the *Traveling Salesman Problem* for each group of customer, it can be performed by any heuristic or exact method. Nevertheless, in this paper it will performed by the *Nearest Neighbor Algorithm*.

4. NUMERICAL EXAMPLE

Consider an ACVRP where a depot has to serve nine customers. The distances between points are shown in Table 1 and the demand at each customer is shown in Table 2. Let the capacity of the available vehicles is equal to 23.

Table 1. Distance matrix

From \ To	Depot	1	2	3	4	5	6	7	8	9
Depot	0	10	3	6	9	5	7	8	2	4
1	5	0	5	4	2	3	6	10	7	8
2	4	9	0	7	8	6	5	2	4	3
3	7	1	3	0	4	8	9	10	5	2
4	3	2	6	5	0	7	4	8	9	3
5	2	7	2	7	5	0	5	6	4	10
6	5	6	9	6	6	3	0	7	5	7
7	6	4	8	4	7	4	8	0	3	8
8	4	3	4	7	8	5	5	4	0	5
9	3	5	5	3	4	6	6	5	7	0

Table 2. Demand at each customer

Customer	1	2	3	4	5	6	7	8	9
Demand	4	6	5	4	7	3	5	4	4

If the *NNA* is assigned to create the initial route, the result will be such as follows:

Depot – 8 – 1 – 4 – 9 – 3 – 2 – 7 – 5 – 6 – Depot

Since the total demand of all customers is 42 and the vehicle capacity is 23, there will be two routes to determine in the next step. The first route will involves the group of customers 8, 1, 4, 9, and 3 with the total delivery of 21 and the second route will involves the group of customers 2, 7, 5, and 6 with the total delivery of 21 also.

The last step is to reassign the *NNA* to build the final tour for each of the customers group. The matrix distance for the first and the second group of customers is shown in Table 3 and Table 4, respectively.

Table 3. Distance matrix for group 1

From \ To	Depot	8	1	4	9	3
Depot	0	2	10	9	4	6
8	4	0	3	8	5	7
1	5	7	0	2	8	4
4	3	9	2	0	3	5
9	3	7	5	4	0	3
3	7	5	1	4	2	0

Table 4. Distance matrix for group 2

From \ To	Depot	2	7	5	6
Depot	0	3	8	5	7
2	4	0	2	6	5
7	6	8	0	4	8
5	2	2	6	0	5
6	5	9	7	3	0

The final route obtained from the third step for the first group is:

Depot – 8 – 1 – 4 – 9 – 3 – Depot

with the total travel distance of 20, while for the second group is:

Depot – 2 – 7 – 5 – 6 – Depot

with the total travel distance of 19.

To give more illustration of how the proposed algorithm works, suppose the vehicles capacity is 16 instead of 23. The solution will consists of three final routes, they are:

- *Depot – 8 – 1 – 4 – 9 – Depot*, with the total delivery of 16 and the total distance of 13 in the first group,
- *Depot – 2 – 7 – 3 – Depot*, with the total delivery of 16 and the total distance of 16 in the second group, and
- *Depot – 5 – 6 – Depot*, with the total delivery of 10 and the total distance of 15 in the third group.

In what follows, the same problem will be solved using the modified *Johnson's Algorithm* in the step of creating the initial route. The distance from depot to each node and from each node to depot is listed in Table 5 below.

Table 5. List of the distance

i	d_{0i}	d_{i0}	i	d_{0i}	d_{i0}
1	10	5	6	7	5
2	3	4	7	8	6
3	6	7	8	2	4
4	9	3	9	4	3
5	5	2			

The initial route is then *Depot – 8 – 2 – 3 – 7 – 1 – 6 – 4 – 9 – 5 – Depot*.
If the vehicle capacity is 23 then the first route will involves customers 8, 2, 3 and 7, with the total delivery of 20, and the second route will involves customers 1, 6, 4, 9, and 5, with the total delivery of 22.

Using the *NNA* in the last step, the resulted final routes are:

Depot – 8 – 2 – 7 – 3 – Depot

with the total travel distance of 19 in the first group, and

Depot – 9 – 4 – 1 – 5 – 6 – Depot

with the total travel distance of 23 in the second group.

If the vehicle capacity is 16 then the three final routes are:

- *Depot – 8 – 2 – 3 – Depot*, with the total delivery of 15 and the total distance of 20 in the first group,
- *Depot – 6 – 1 – 7 – Depot*, with the total delivery of 12 and the total distance of 29 in the second group, and
- *Depot – 9 – 4 – 5 – Depot*, with the total delivery of 15 and the total distance of 17 in the third group.

The results for this example show that the final routes obtained are better when the *NNA* applied in the first step, than those when the modified *Johnson's Algorithm* applied. It can be easily accepted, since the quality of the final routes is strongly depends on the quality of the initial route. All the distance from one customer point to another are considered in building the initial route by *NNA*, while the modified *Johnson's Algorithm* only considering the distance from depot to each customer point and the inverse distance.

4. CONCLUSION

In this paper, a variant of *Capacitated Vehicle Routing Problem* in which the distance matrix is asymmetric is considered. Despite the fact that this type of problem were much less studied in the literature, asymmetric models are relevant

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to a wider range of applications, and more general then symmetric models. In this study, the problem was formulated as an *Integer Linear Program* and a heuristic algorithm based on the *route first cluster second* method is developed.

The proposed heuristic consists of three steps, i.e. creating the initial route, grouping the customers, and building the final routes. The Nearest Neighbor Algorithm and the modified Johnson's Algorithm are then proposed to apply in the first step. While the Nearest Neighbor Algorithm can be applied for both asymmetric and symmetric instances, the proposed modified Johnson's Algorithm can only be applied for asymmetric instances.

This study can be extended in several directions. For instance, it might need to consider cases when the vehicle's capacity is not the only restriction imposed, as found in many variants of VRP.

5. REFERENCES

(a) Baldacci, R., Hadjiconstantinou E. and A. Mingozzi, (2004), "An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation," *Operations Research* 52, pp. 723-738.

(b) Fischetti M., Toth P, and D. Vigo, (1994), "A Branch-And-Bound Algorithm for the Capacitated Vehicle Routing Problem on Directed Graphs," *Operations Research*, 42 (5), pp. 846-859.

(c) Fukasawa, R., Longo, H., Lysgaard, J., Poggi, D.M., Reis, M., Uchoa, E. and R. F. Werneck, (2006). "Robust branch-and-cut-and- price for the capacitated vehicle routing problem," *Mathematical Programming Series*, 106, 2006, pp. 491–511.

(d) Gendreau, M, Laporte, G and Jean-Yves Potvin, (2002), "Metaheuristics for the capacitated vrp," In: P. Toth and D. Vigo, editors, The Vehicle Routing Problem, *SIAM Monographs on Discrete Mathematics and Applications*, SIAM, pp. 129-154.

(e) Johnson, L.A. and D.C. Montgomery, (1974), "Operations research in production planning, scheduling, and inventory control," John Wiley & Sons Inc., New York.

(f) Laporte, G., Mercure, H. and Y. Nobert, (1986), "An exact algoritm for the asymmetrical capacitated vehicle routing problem," *Networks*, 16, 1986, pp. 33-46.

(g) Laporte, G. and Frédéric Semet, (2002), "Classical heuristics for the capacitated vrp," In: P. Toth and D. Vigo, editors, The Vehicle Routing Problem, *SIAM Monographs on Discrete Mathematics and Applications*, SIAM, pp. 109-128.

(h) Lysgaard, J, Letchford, A.N. and Richard W. Eglese, (2004), "A new branch-and-cut algorithm for the capacitated vehicle routing problem," *Mathematical Programming, Series A*, 100, pp. 423–445.

(i) Ralphs, T.K., Kopman, L., Pulleyblank, W.R. and L. E. Trotter, (2003), "On the capacitated vehicle routing problem," *Mathematical Programming Series* B, 94, pp. 343-359.

(j) Toth, P. and D. Vigo, (2002), "An overview of vehicle routing problems," In: P. Toth and D. Vigo, editors, The Vehicle Routing Problem, *SIAM Monographs on Discrete Mathematics and Applications*, SIAM, pp. 1-26.

(k) Toth, P. and D. Vigo, (2002), "Models, relaxations and exact approaches for the capacitated vehicle routing problem," *Discrete Applied Mathematics* 123, pp. 487-512.

(l) Vigo, D., (1996). "A heuristic algorithm for the asymmetric capacitated vehicle routing problem," *European Journal of Operational Research*, 89, pp. 108-126.

AUTHOR BIOGRAPHIES

Tjutju T. Dimyati is a lecturer in Industrial Engineering Department of Pasundan University, Bandung. She received her

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degree of Doctoral in Industrial Engineering from The Institute Technology Bandung in 2004. Her research interests are in the area of Operations Research, Logistics Planning, and Production Planning and Inventory Control. Currently she is doing research in the area of Organizational Culture as well.