H. HEROLD, T. ERTL, H. RUDER, Theoretische Astrophysik, Tübingen

GENERATION OF RELATIVISTIC PARTICLES IN PULSAR MAGNETOSPHERES

The problem - fundamental for the physics of pulsars - of determining the global structure of the magnetosphere in a self-consistent way has not yet been solved satisfactorily. We report on some progress in this direction, which we have achieved by studying the trajectories of individual charged particles in the electromagnetic vacuum fields of an aligned rotator. For typical pulsar parameters (period $P \approx 0.1$ s, radius $a \approx 10$ km, polar magnetic field strength $B_0 \approx 10^8 T$) the unipolar induction voltage between pole and equator is in the order of $10^{17} - 10^{18}$V, which, if completely transformed into kinetic energy of the particles, would lead to Lorentz factors γ of about 10^{12} for electrons. It is convenient to introduce the ratio of rest mass energy and unipolar induction energy $\varepsilon = 2mc^2/(eB_0 a^2 \Omega)$ as a dimensionless parameter ($\Omega = 2\pi/P$), which for electrons has the value ε(electron) = $-1.6\ 10^{-12}(P/0.1s)(B_0/10^8T)^{-1}(a/10km)^{-2}$. Investigating at first the acceleration of the particles from the neutron star surface we find that the particles gain relativistic energies - and thus velocities approaching the speed of light - within distances of about µm for electrons and mm for protons due to the electric field component parallel to the magnetic field. In addition, it turns out that after this phase the directions of the velocities are independent of the initial non-relativistic velocities and only depend on the electric and magnetic fields at the starting point. The further motion of the particles is described by the Lorentz-Dirac equation, which in Landau approximation and for $|\varepsilon|<<1$ reads

$$\dot{\underline{v}} = [\underline{E} + \underline{v}\times\underline{B} - (\underline{E}\cdot\underline{v})\underline{v}]/\Gamma \qquad (1)$$

$$\dot{\Gamma} = \underline{E}\cdot\underline{v} - D_0\Gamma^2[(\underline{E}+\underline{v}\times\underline{B})^2-(\underline{E}\cdot\underline{v})^2] \qquad (2)$$

with the damping constant $D_0 = e^2/(6\pi\varepsilon_0)\,\Omega\,/(mc^3)\varepsilon^{-3}$ and $\Gamma = \varepsilon\gamma$. The units of the dimensionless quantities used are: c/Ω (= light-cylinder radius) for the position vector, $1/\Omega$ for t, c for $\underline{v}$, $cB_0(\Omega a/c)^2/2$ for $\underline{E}$, and $B_0(\Omega a/c)^2/2$ for $\underline{B}$. Equation (1) implies that the relation $|\underline{v}| = c = 1$ holds during the whole motion.

To gain some insight, in a next step we have numerically integrated the equations of motion (1) and (2) neglecting the radiation reaction term ($D_0 = 0$). For a magnetic dipole field and an electric monopole plus quadrupole field the resulting trajectories are shown in Fig. 1. Depending on the starting point on the surface there are particles which escape to infinity and other ones which remain in relatively limited regions near the star because they do not deviate very far from their magnetic field lines. For distances comparable with the light cylinder the particles depart from their corresponding magnetic field lines which leads to complicated trajectories certainly not describable by a pressureless hydrodynamical picture.

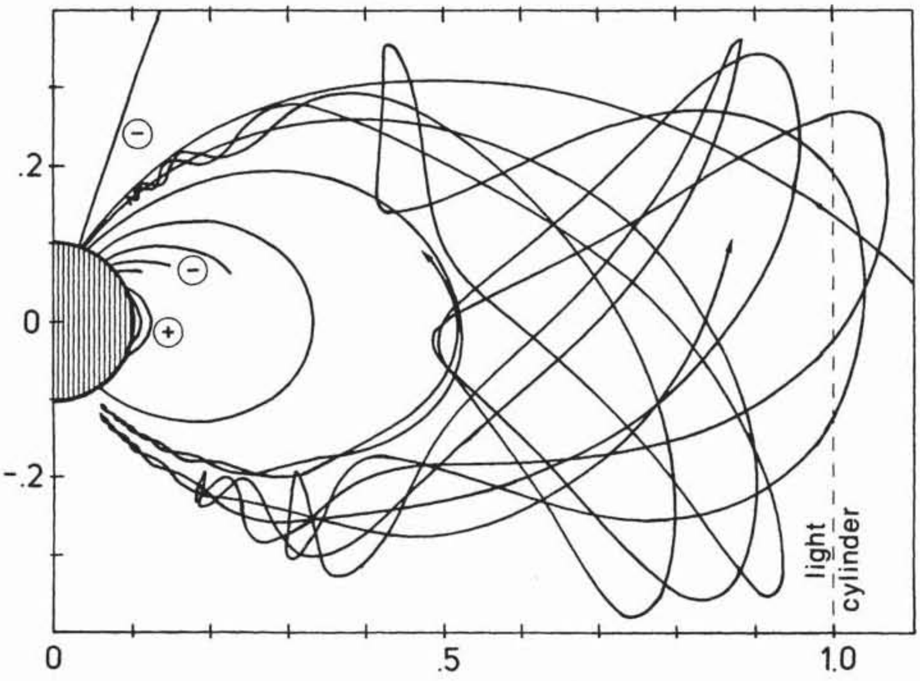

Fig. 1 : Trajectories of charged particles in a magnetic dipole and an electric monopole (positively charged neutron star) plus quadrupole field without radiation reaction.

The occurring regions with strong curvature give rise to the question of how much the trajectories are modified by the radiation reaction force. That the influence of radiation damping can be strong is also suggested by the value of the damping constant D_0 for typical pulsar parameters

$$|D_0| = 9 \cdot 10^{13} (P/0.1s)^{-4} (B_0/10^8 T)^3 (a/10km)^6 (m/m_e)^{-4} \qquad (3)$$

To demonstrate this explicitly, in Fig. 2 we compare the most complicated

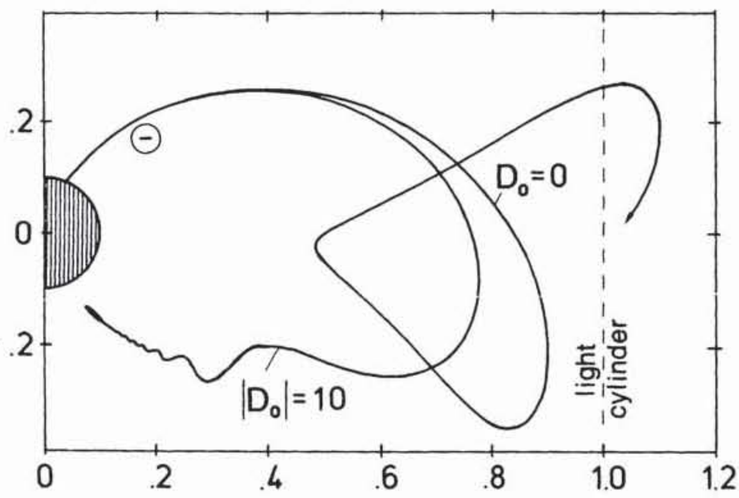

Fig.2 : Comparison of an undamped (D_0=0) and a radiatively damped (D_0=10) particle trajectory. The radiation reaction leads to the capture of the particle in the region $\underline{E} \cdot \underline{B} \simeq 0$.

undamped trajectory of Fig. 1 which is shown here only partially ($D_0 = 0$) with the corresponding radiatively damped trajectory ($|D_0| = 10$). Even with this for electrons unrealistically small radiation damping, the effect is tremendous, the particle does no longer gyrate, but is trapped in a region with $\underline{E} \cdot \underline{B} \approx 0$. As it can be concluded from eqs. (1) and (2) and has been confirmed by numerical integration, with increasing damping constant the trajectories approach more and more the limiting case characterized by the condition

$$\underline{F} = \underline{E} + \underline{v} \times \underline{B} - (\underline{E} \cdot \underline{v})\underline{v} \approx 0 \qquad (4)$$

Thus the damping term $D_0 \Gamma^2 \underline{F}^2$ in eq. (2) remains small, and the radiation reaction is locally minimized during the whole motion. Equation (4) can be solved for $\underline{v}$ yielding a local velocity field, which is a unique function of $\underline{E}$ and $\underline{B}$:

$$\underline{v} = [\underline{E} \times \underline{B} + (\underline{E} \cdot \underline{B})\underline{B}/W + W\underline{E}]/(\underline{B}^2 + W^2) \qquad (5)$$

with

$$W^2 = \tfrac{1}{2}(\underline{E}^2 - \underline{B}^2) + \tfrac{1}{2}[(\underline{E}^2 - \underline{B}^2)^2 + 4(\underline{E} \cdot \underline{B})^2]^{\frac{1}{2}} \qquad (6)$$

Therefore, the inclusion of the radiation reaction opens again the possibility of describing the pulsar magnetosphere by a fluid-like picture. Within the frame of these approximations we are going to compute self-consistent solutions for the aligned rotator. Independently from such solutions, an estimate for the maximum energies up to which the particles are accelerated by unipolar induction can be derived. Combining eqs. (1) and (2) one obtains

$$\dot{\Gamma} = \underline{E} \cdot \underline{v} - D_0 \Gamma^4 \, \dot{\underline{v}}^2 \qquad (7)$$

Assuming that a wind solution exists and using eqs. (5) and (6) it turns out from eq. (7) that for $r \to \infty$ Γ asymptotically approaches a value in the order of $|D_0|^{-1/8}$, which means a maximum Lorentz factor of

$$\gamma_{max} \approx 10^{10} (P/0.1\mathrm{s})^{-\frac{1}{2}} (B/10^8\,\mathrm{T})^{\frac{5}{8}} \; (a/10\mathrm{km})^{\frac{5}{4}} \; (m/m_e)^{-\frac{1}{2}} \qquad (8)$$

For typical pulsars this estimate yields energies of $5 \cdot 10^{15}$ eV for electrons and $2 \cdot 10^{17}$ eV for protons.