T.ERTL, H.HEROLD, B.FINKBEINER, H.RUDER,
Lehrstuhl für Theoretische Astrophysik der Universität Tübingen

SELF-CONSISTENT MODELLING OF PULSAR MAGNETOSPHERES

The problem - fundamental to the physics of pulsars - of determining the global structure of the magnetospheres of rapidly rotating, strongly magnetized neutron stars has not yet been solved self-consistently (for a review cf. [1]). We report on some progress that we have achieved by numerically modelling the magnetosphere of an aligned rotator where the rotation axis is parallel to the magnetic axis of the neutron star. Here, the unipolar induction, which should be responsible for populating the magnetosphere with charged particles pulled out from the neutron star surface via field emission can be studied in purity, whereas electromagnetic wave effects are neglected. In the stationary, axially symmetric case the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ in the pulsar magnetosphere can be described (in cylindrical coordinates ρ, φ, z) by the electrostatic potential $\Phi(\rho, z)$, the magnetic flux function $\Psi(\rho, z)$ and the poloidal magnetic field $B_\varphi(\rho, z)$: $\mathbf{E} = -\nabla\Phi$ and $\mathbf{B} = (1/\rho)\nabla\Psi \times \mathbf{e}_\varphi + B_\varphi \mathbf{e}_\varphi$. The charge density ρ_e and the current density $\mathbf{j}$ determine the electric potential via the Poisson equation and the magnetic field via Ampere's law which here read in suitable units such that all quantities are dimensionless:

$$\Delta\Phi = -\rho_e; \qquad \Big(\Delta - \frac{2}{\rho}\frac{\partial}{\partial\rho}\Big)\Psi = -\rho\, j_\varphi \tag{1a}$$

$$\frac{\partial}{\partial z}(\rho B_\varphi) = -\rho\, j_\rho; \qquad \frac{\partial}{\partial z}(\rho B_\varphi) = -\rho\, j_z \tag{1b}$$

The magnetosphere is formed by a collisionless plasma, in which the particles are expected to be extremely relativistic due to the huge electric and magnetic fields (the unipolar induction voltage between pole and equator of the neutron star is typically of the order 10^{17}–10^{18}V). The charge and current densities are therefore derived from the zeroth and first momentum of the particle distribution function $f(\mathbf{r}, \mathbf{p})$ which is determined by the Vlasov equation

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{p}}[(\mathbf{E} + \mathbf{v}\times\mathbf{B} + {}^{\text{radiation}}_{\text{damping}})f] = 0 \tag{2}$$

The velocity $\mathbf{v}$ is given by $\mathbf{v} = \mathbf{p}/\sqrt{(\varepsilon^2 + \mathbf{p}^2)}$, where the dimensionless parameter ε is defined by $\varepsilon = 2mc^2/(eB_0 a^2\Omega)$, i.e. by the ratio between rest mass energy and unipolar induction energy (B_0 is the polar magnetic field strength, a is the radius and Ω is the angular velocity of the neutron star). Because typical values for ε are extremely small ($\varepsilon \sim -10^{-12}$ for electrons, $\varepsilon \sim 10^{-9}$ for protons), the quantity $\Gamma = \varepsilon\gamma$ should be of order unity, at least if the radiation reaction during phases of acceleration can be neglected. This, however, is not the case as can be seen by studying the trajectories of particles in realistic pulsar vacuum fields. The Lorentz-Dirac equation of motion in the Landau approximation , can be written as ($|\varepsilon| \ll 1$)

$$\dot{\mathbf{v}} = \frac{1}{\Gamma}\mathbf{F}; \quad \dot{\Gamma} = \mathbf{E}\cdot\mathbf{v} - D_0\Gamma^2\mathbf{F}^2 \quad \text{with} \quad \mathbf{F} = \mathbf{E} + \mathbf{v}\times\mathbf{B} - \mathbf{v}(\mathbf{E}\cdot\mathbf{v}) \tag{3}$$

For typical pulsar parameters the value of the dimensionless damping constant $D_0 = e^2/(6\pi\epsilon_0)\,\Omega/(mc^3\varepsilon^3)$ is in the order of $D_0 \sim -10^{14}$ for electrons, and $D_0 \sim 10$ for protons. Thus, at least for the electrons, the radiation reaction force dominates the particle motion. Large values of $|D_0|$ imply that the factor of D_0 in (3) always remains very small; this leads us to the assumption, which we confirmed by numerical integration of eqs. (3) (cf. [2]), that during the motion the condition $\mathbf{F} \approx 0$ is fulfilled, which means that the radiation reaction is locally minimized along the trajectory. This is a condition for the velocity and yields, for given $\mathbf{E}$ and $\mathbf{B}$ fields

$$\mathbf{v} = \frac{1}{\mathbf{B}^2 + P^2}\Big[\mathbf{E}\times\mathbf{B} + \frac{1}{P}(\mathbf{E}\cdot\mathbf{B})\mathbf{B} + P\mathbf{E}\Big] \tag{4a}$$

with

$$P^2 = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{1}{2}\Big[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{E}\cdot\mathbf{B})^2\Big]^{\frac{1}{2}} \tag{4b}$$

i.e. we obtain a local velocity field $\mathbf{v} = \mathbf{v}(\mathbf{E}, \mathbf{B})$ and thus a fluid-like picture for the particle motions in the magnetosphere. The main characteristics of this damping–free motion is, that the particles come to rest in surfaces where $\mathbf{E}\cdot\mathbf{B} = 0$.

Based on these results, the task to determine a self-consistent solution is simpler than before, but still difficult due to the great non-linearity of the problem. Our approach is based on the idea to start from the vacuum solution and to fill up the magnetosphere with the particles which are pulled out from the neutron star surface. This is not a real time-dependent calculation, since we assume that the electric field is always described by an electrostatic potential, but for transporting the charge with the velocity (4) we need to solve the time-dependent continuity equation $\dot{\rho} + \mathrm{div}(\rho\mathbf{v}) = 0$ and therefore $\dot{\mathbf{E}}$ cannot be omitted in Ampere's law. Thus eqs. (1b) have to be replaced by

$$\Big(\Delta - \frac{2}{\rho}\frac{\partial}{\partial\rho}\Big)(\rho B_\varphi) = \rho\Big(\frac{\partial j_z}{\partial\rho} - \frac{\partial j_\rho}{\partial z}\Big). \tag{5}$$

In summary, we solve at each time step the elliptic equations (1a) with Dirichlet boundary conditions (the change of Φ at infinity has also to be derived from Ampere's law) and equation (5) with the von Neumann boundary condition $\partial(\rho B_\varphi)/\partial r = \rho j_\vartheta$ on the star's surface, where the simple emission law $j = \sigma E_\|$ is assumed. The three elliptic equations are solved by successive over-relaxation (SOR) in a vectorizable checkerboard scheme. For the continuity equation an explicit discretization in time with 2-dimensional Flux Corrected Transport is used in order to preserve steep gradients in the charge density.

We start with the vacuum fields of an uncharged aligned rotator, i.e. an electric quadrupole and a magnetic dipole resulting in $\mathbf{E}\cdot\mathbf{B} < 0$ everywhere. Therefore, only negative particles can be emitted and then transported along the magnetic field lines towards the equator where they accumulate. This causes a change in the electric field such that the $\mathbf{E}\cdot\mathbf{B} = 0$ surface rises from the equator towards the polar field line forcing the negative particles to follow it. As soon as $\mathbf{E}\cdot\mathbf{B} > 0$ at some part of the star's surface positive particles can enter the magnetosphere and stream out along the equator. After about one revolution (T=6.25) two regions of charged particles (the negative ones around the pole, the positive ones around the equator) separated by a vacuum gap have formed which then develop into a charge separated pulsar wind. The charge densities after 30 revolutions which are shown in Figs. 1 and 2 represent an almost stationary configuration. The electric field (Fig. 3) evolves as to achieve $\mathbf{E}\cdot\mathbf{B} \sim 0$ in the wind zone (Fig. 4). The poloidal magnetic field essentially does

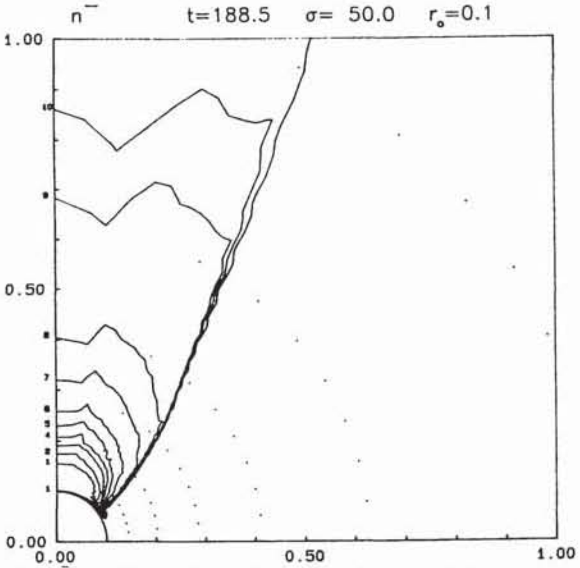

Fig.1: Density of the negative particles

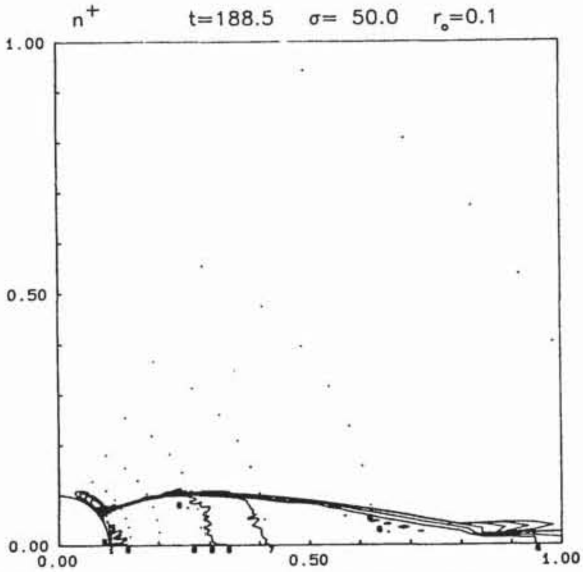

Fig.2: Density of the positive particles

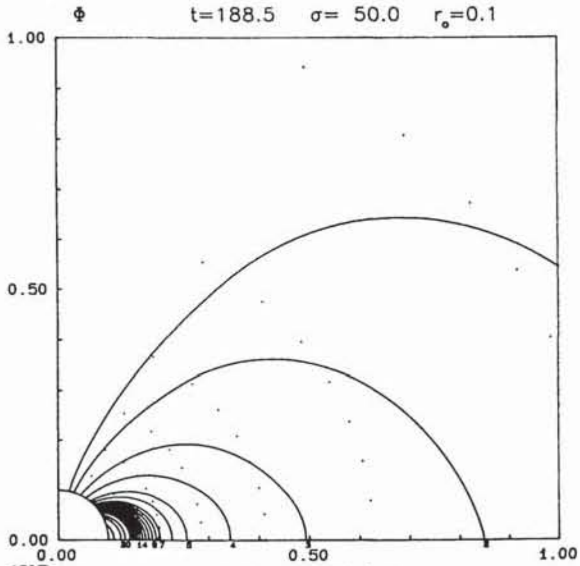

Fig.3: The electric potential Φ

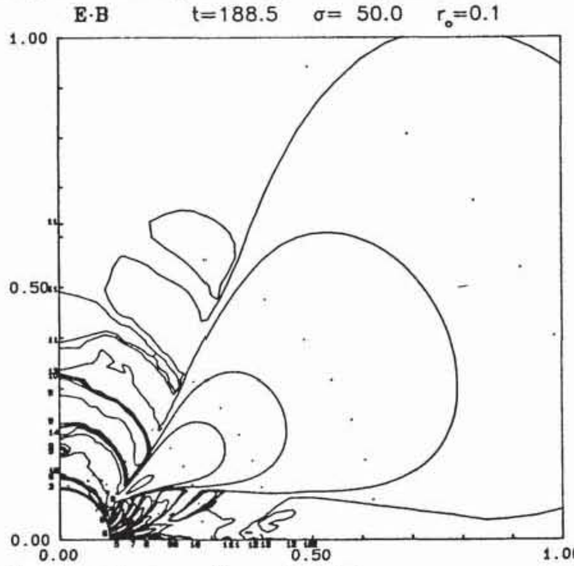

Fig.4: Contour lines of $\mathbf{E} \cdot \mathbf{B}$

not change, but the toroidal field shows huge variations along the surfaces limiting the populated regions. The neutron star becomes positivly charged because of the negative particles leaving the star at the beginning. The positive particles soon catch up and after about one revolution the currents of positive and negative particles out of the star exactly match each other. Lateron the amount of charge for each particle species as well as the total currents into and out of the magnetosphere approach a constant value.

For the first time in pulsar magnetospheric theory it seems that a stationary self-consistent solution for the aligned rotator has been found by quasi-time-dependent simulation. Further physical implications like the dependence of the solution on the emissivity σ of the neutron star and on the rotation frequency (r_0) still have to be studied before the model can be extended for the slightly oblique rotator.

[1] Michel, F.C., 1982 Rev. Mod. Phys. **54**, 1.
[2] Herold, H., Ertl, T., Ruder, H., 1985, Mitt. Astron. Ges. **63**, 174.