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Visualization in Astrophysics

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This paper reports on progress we have made in modelling cosmic X-ray sources on supercomputers. The results we present are meant to serve as an example for the fact that sophisticated visualization techniques play a crucial role in scientific computing. Among the graphical methods we demonstrate, raytracing in curved space-time and a physically motivated 3D-volume rendering algorithm might be of interest to the graphics community in general.

1. Introduction

In a clear night we can observe with the naked eye several thousands of objects in the sky. A modern telescope like the one at Mount Palomar Observatory reveals millions of stars, nebulae and galaxies. Together with observations in the radio, UV, X-ray, and γ-ray band of the electromagnetic spectrum, astronomers have collected over the years a huge amount of data which serves as a basis for the more theoretically oriented astrophysicists like us to develop theories in order to explain the physics that is going on up there. However, the closer the predictions of the models are to match the observations, the more complex the models become, which for many astronomical objects leads to the situation that one can only solve their underlying equations with the help of modern supercomputers. Fortunately, our group has fairly good access to the Cray-2 of the nearby University of Stuttgart, and we use this facility extensively to simulate various aspects of certain astrophysical objects. Without really having been aware of the activities already going on in the field of Visualisation in Scientific Computing, the huge amount of data produced by the supercomputers, the problems in steering such long-run simulations efficiently, and the need to communicate our results to other scientists automatically forced us into developing graphical methods to deal with these issues. By now, having learned that ViSC is becoming a research area of its own right, we find our needs as well as our problems so accurately described in the SIGRAPH Report on ViSC [1], that we were encouraged to present some of our results to the graphics community.

2. X-Ray pulsars

In order for the non-astrophysics reader to be able to appreciate our color pictures we also have to describe the physical scenario (Fig. 1) to some detail. The cosmic objects we concentrate on in this article are the *X-ray pulsars* which are the strongest X-ray sources in our milky way. An X-ray pulsar actually is a close binary system, which means that it consists of two stars which circle around each other at a relatively small distance. One of the two components is a normal star (e.g. like our sun), the other one is a fast rotating neutron

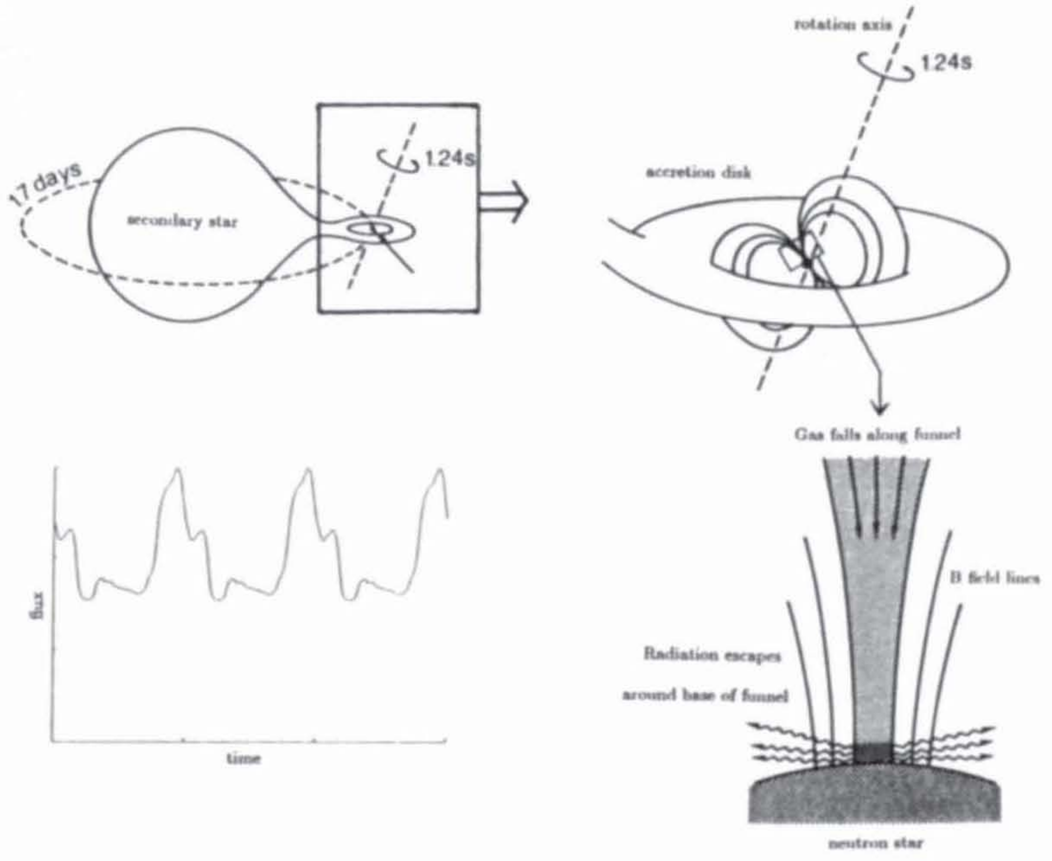

Fig. 1: The physical components of an X-ray pulsar: a binary system, an accretion disk, an accretion column together with an observed light curve

star. A neutron star is a very compact object with a radius of only about 10 km, but a mass of about one solar mass. Like the earth, a neutron star exhibits a magnetic dipole field, this field, however, is 10^{12} times stronger than that of the earth. As we will understand later on, the neutron star emits intensive X-rays from an area close to its magnetic poles. Since the magnetic axis of the neutron star is in general not aligned with its rotational axis, the observer on earth sees the X-ray emission pulsed at the rotational frequency. From the light curve which is the temporal resolution of the observed radiation intensity one can also derive other details of the system like the distance between the objects and the various angles of inclination.

3. Accretion disks

The enormous mass density of about 500 million tons per cm^3 inside the neutron star produces a very strong gravitational field around it. This field not only deforms the companion star, but even pulls matter from it which then falls towards the neutron star. However, because of the conservation of angular momentum this matter will not directly hit the neutron star surface but will form a thin *accretion disk* around it. It is easily conceivable from the schematic scenario in Fig. 1 that for certain geometrical situations not only the companion star but also the accretion disk will hide the X-ray emission area from the terrestrial observer and will influence the X-ray light curve of the object. Furthermore, due to the

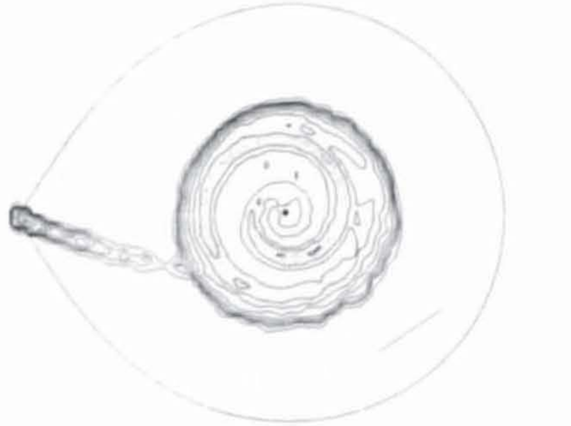

Fig. 2: Contour plot of the particle density of a model accretion disk

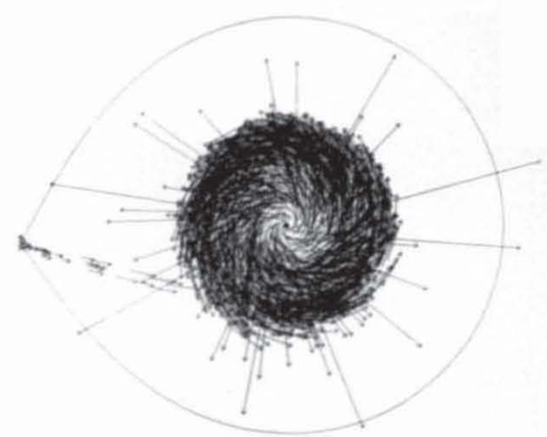

Fig. 3: Some velocity vectors corresponding to the particles in Fig. 2

viscous friction in the accretion disk, the disk itself will be hot and will contribute to the radiation of the system in the visible part of the spectrum. Therefore, in order to be able to model the observational data, a detailed knowledge of the accretion disk is required and we try to gain a deeper understanding of the related phenomena by simulating the formation of accretion disks with the help of a computer.

Physically, the matter of the accretion disk is well described by hydrodynamic equations where the only external forces are the gravitational attraction of the two stars and the internal forces are the pressure gradient and a viscous interaction. The equations are solved by means of a numerical method called particle simulation in which many real particles are grouped together to form a massive pseudo-particle. We then follow the trajectory of each pseudo-particle by integrating its properly scaled equation of motion. The viscosity term is modelled by assuming that at each time step the velocities of the pseudo-particles within a certain neighbourhood are redistributed in such a way that the deviations from the local mean velocity are reduced. Although this method allows us to simulate the system with much less particles than there are in reality (on the order of 10^{25}) we still need more than 10^4 pseudo-particles to achieve the desired accuracy, which leaves us with a problem which is tractable only on current supercomputers.

For numerical reasons the simulation was carried out with a white dwarf as the accreting object instead of a 1000 times smaller and much faster rotating neutron star. Binary systems of this type are called cataclysmic variables. We start our computation without a disk and let particles stream through the interior Lagrange point at a constant rate. After a few orbital periods of the stars (typically several hours in real time as well as in Cray-2 CPU time) a stationary disk has formed which means that the same amount of matter which feeds the accretion disk from the companion star leaves it at its inner edge to fall onto the neutron star. Traditionally, the results of such a simulation would be presented as contour plots of the computed particle density, possibly enhanced by some representative velocity vectors as shown in Figs. 2 and 3. However, this form of presentation does not give a realistic impression of the disk because, even if one could zoom in on it with a telescope, the variations in density are not a directly observable quantity. Furthermore a pile of contour plots does not appear to be an adequate means of monitoring the state of a very expensive computation because in practice it is usually only available long after the simulation run has finished.

A much more suggestive way of visualizing the computed accretion disk is shown in Figs. 4* and 5*. We place the two-dimensional disk in relation to the other objects in the scene letting the colour coding represent the local temperature which is calculated from the viscous

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* See page 546 for Figures 4 and 5.

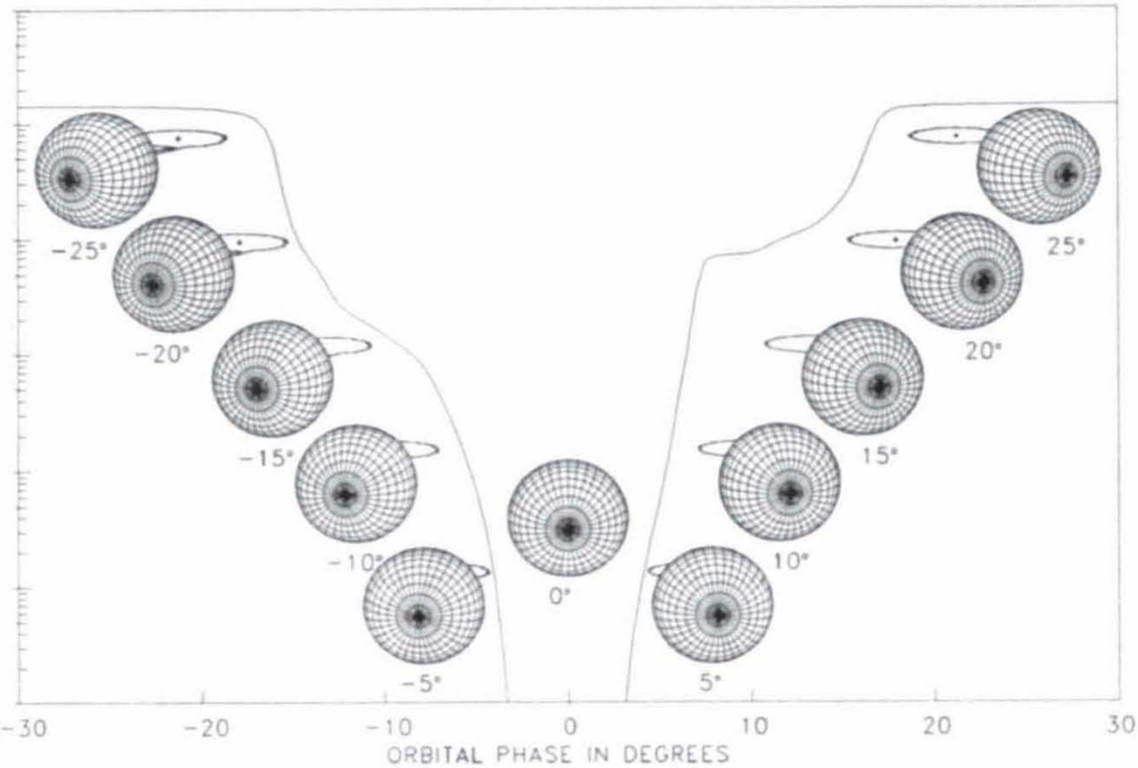

Fig. 6: The light curve resulting from the accretion disk in Fig. 5

heating in the disk. We also try to get as many of those pictures and as soon as possible on to our workstations in order to understand the complex dynamics involved in the formation process and in order to be able to react promptly on unexpected simulation results. This procedure is still a long way from the desirable realtime visualization of supercomputer simulations, but we are currently involved in a networking research project to develop a remote procedure call mechanism which is adequate to close this gap. Since the presented pictures actually display what one would see (if one would be able to see more than just the visible part of the electromagnetic spectrum), a simple but correct way to compute the light curve of the total radiation coming from the disk itself is to sum up over its visible pixels on the screen (Fig. 6).

4. Radiation hydrodynamics

A detailed analysis of Fig. 5 reveals a hot spot where the particle stream coming from the companion star hits the accretion disk as well as a hot region at the inner border of the disk. In the case of an X-ray pulsar the inner edge of the accretion disk is coupled to the pulsar magnetosphere which forces the accreting particles to follow the magnetic field lines. When they finally hit the surface of the neutron star at the magnetic poles the strong gravitational field has accelerated them to velocities of up to one half of the speed of light and the strong magnetic field has channeled them in such a way that about 100 billion tons per second fall onto a region of only a few km^2. During the sudden deceleration shortly above the neutron star surface the huge amount of kinetic energy is converted to radiation and results in a hot spot at the bottom of an *accretion column* where temperatures of about 100 million degrees produce an X-ray emission of about 10^{30} Watt which is more than 1000 times the energy that our sun radiates in the whole spectrum. Since both magnetic poles circle around the rotational axis the X-ray beams are observed as pulses each time they wipe across the earth.

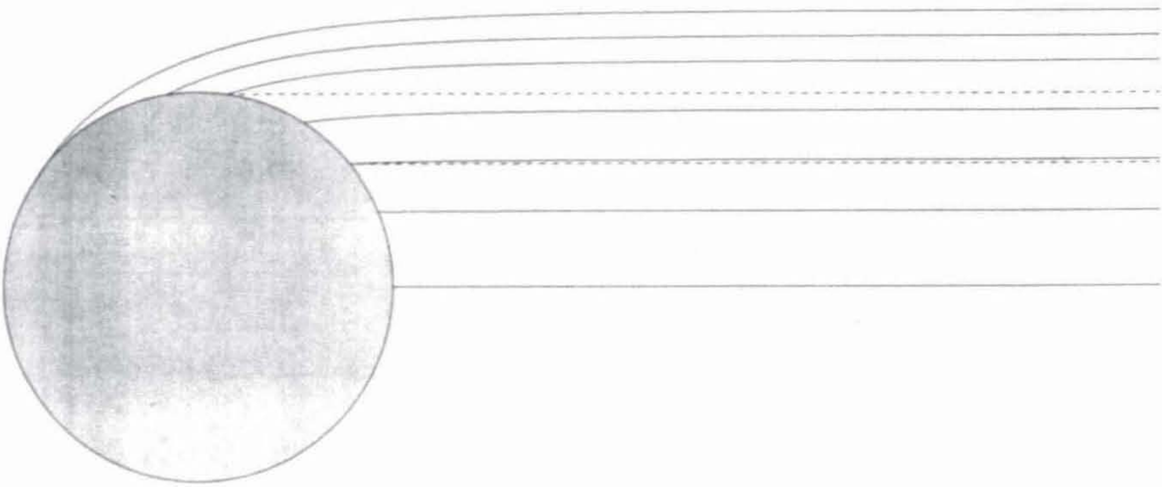

Fig. 9: Photon trajectories from the neutron star's surface to a distant observer

In order to explain the resulting light curve like the one in Fig. 1 one has to model how the radiation is produced in the accretion column and how it can pass through the hot and magnetized plasma which moves with relativistic speed. On the one hand the infalling plasma drags along the generated X-ray quantas forcing them to leave the column primarily in direction towards the neutron star surface. On the other hand the plasma itself is decelerated by the enormous radiation pressure. Thus, one has to solve the complicated problem of spectral radiative transfer in fast moving media selfconsistently with the hydrodynamics of the infalling plasma. Numerically this leads to an iterative procedure with the solution of an elliptic partial differential equation in each step, which again requires the computational power of a modern vector computer.

The result of these calculations (details can be found in [2]-[5]) is, for each point on a grid covering the surface of the accretion cylinder, the intensity of the radiation of a certain energy which leaves the column there in a specific direction (which is also discretized in two angles). This three-dimensional array is again visualized in such a way that it shows what an observer would actually see if he could zoom in on the accretion column with a high resolution X-ray telescope (Fig. 7*). In this special case our approach, which appeals to the scientific intuition, led us to gain additional physical insight. Only by being puzzled by a black screen, which we got when looking at the accretion column from steep angles, the question arose what happens to X-ray photons which do not reach the observer because they hit the neutron star surface. The answer is that they get reprocessed there and reemitted isotropically which leads to a halo-component around the accretion disk (Fig. 8*). Again, we compute model light curves by summing up the visible pixels for the various rotational phases and it turns out that the halo-component plays an important role here.

5. Relativistic light deflection

A further complication in modelling the observed light curves results from the fact that there are phases where we can see both emission regions at the same time although the magnetic poles are located exactly in opposite direction on the neutron star surface. This is an implication of General Relativity: if a massive object becomes as compact as a neutron star, the structure of space-time in its vicinity is described by a curved, rather than Euclidean geometry. According to General Relativity, photons follow the so-called null geodesics in the given metric of space-time. In a curved space-time these paths differ significantly from the straight lines in Euclidean flat space. As a result, light rays which are emitted near

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* See page 546 for Figures 7 and 8.

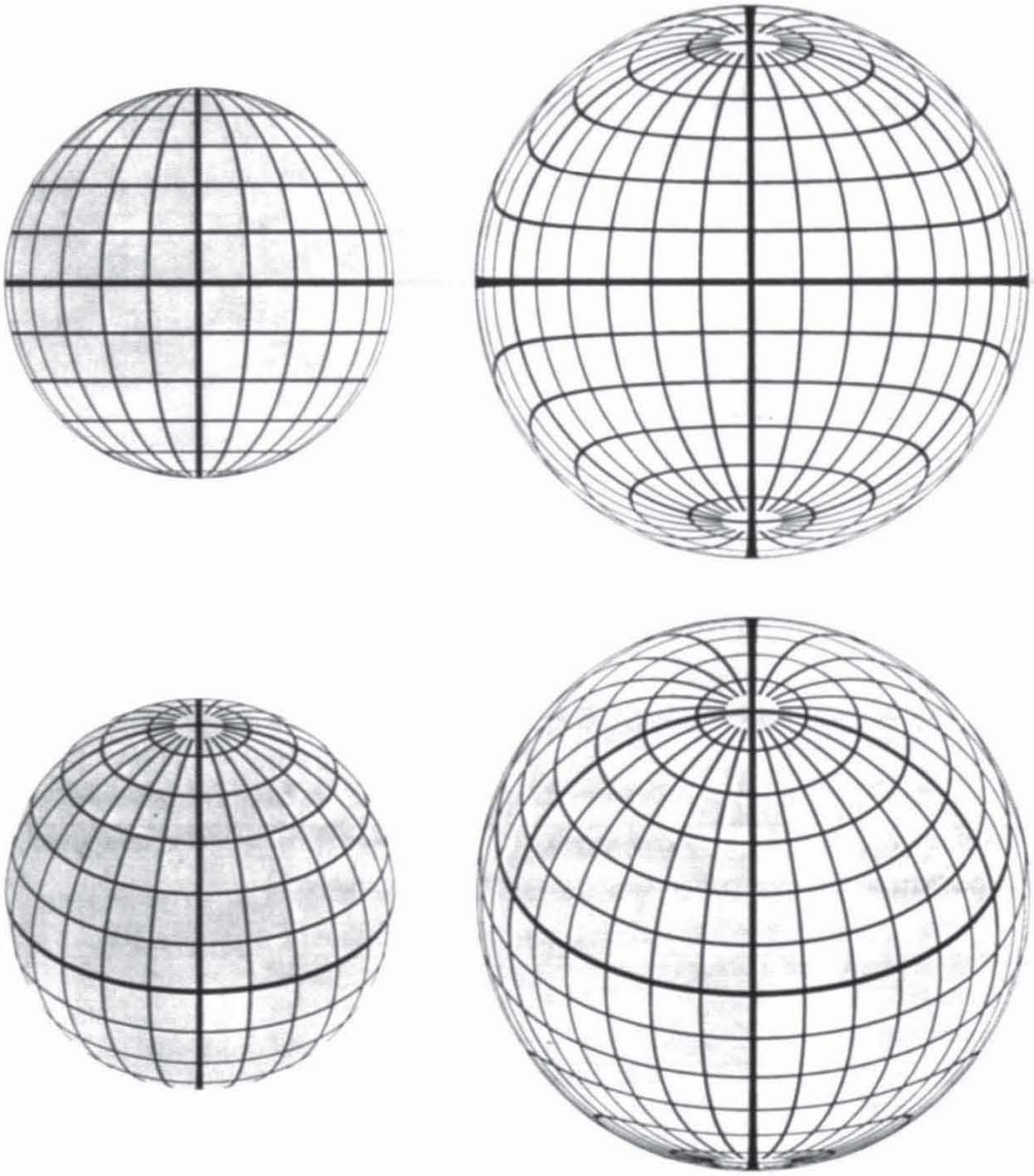

Fig. 10: Neutron star with spherical grid in Euclidean flat space (left) and in curved space-time (right) for different angles of inclination

the surface of a neutron star will not follow a straight line but will be deflected by more than 45°. The amount of deflection is determined by the ratio $\rho = r_*/r_S$ of the neutron star's radius r_* to its Schwarzschild radius $r_S = 2GM_*/c^2$, where G, c, and M_* denote the gravitational constant, the speed of light in vacuum, and the star's mass. Smaller values of ρ mean a more compact star and consequently a stronger light deflection. In the following, we will always use $\rho = 2$, which corresponds to a neutron star with a mass of about $1.5\mathrm{M}_\odot$ (solar masses) and a radius of about 9 km.

Fig. 9 shows the paths of photons as they travel from the neutron star's surface to an asymptotically distant observer, compared to the straight rays of flat space. As a consequence of the light deflection the field of view is stretched and photons emitted on the back side of the star can reach the observer. In order to compute the displayed trajectories we use a raytracing technique for non-perspective views. For each pixel on a hypothetical screen at the distant observer we determine the location on the surface of the star where

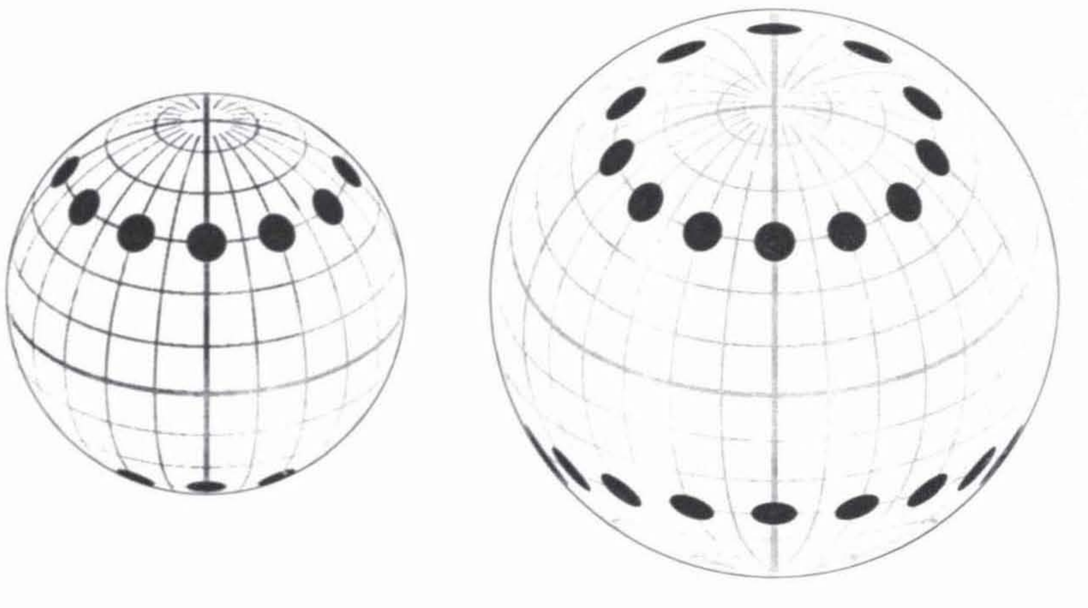

Fig. 11: Rotating neutron star with antipodal hot spots

the corresponding light ray has been emitted. Unfortunately, the equation describing the light path cannot be written in closed form as in flat space. Rather, we have to numerically compute a (one-dimensional) integral along the path which is a problem that is somewhat analogous to tracing a light ray in a medium with a continuously varying index of refraction.

The effects of the relativistic light deflection are visualized in Fig. 10. They show a star that has a spherical coordinate grid mapped onto its surface as it appears in flat space on the left side and in curved space-time on the right side for various angles of inclination of the polar axis with respect to the line-of-sight. The grid lines have a width of 3° and are evenly spaced at $\Delta\vartheta = \Delta\varphi = 15^\circ$, lines of $\vartheta = 90^\circ$ or $\varphi = n \cdot 90^\circ$ are shown with double width for ease of visualization. The pictures in Fig. 10 have been computed in two steps: First, any pair of neighbouring points was checked for whether a line of the coordinate grid passes between the points. If this is the case, the point closest to the line was set. In the second step, the lines were given the width mentioned above, and the points on the screen that correspond to a point on the star lying on such a broadened line were set. These broadened lines give the picture a much more 'realistic' appearance, and they also contain some information about the physics of the situation (cf. [6]). However, the second step alone would not be sufficient to obtain a satisfactory picture, since it might miss lines which appear very narrow due to geometric reasons. Therefore, the first step is necessary to ensure that all visible lines will be shown completely in the picture. This procedure requires around 500 seconds of CPU time on a CRAY-2 to compute one picture with a resolution of 2400 × 2400 points for output on a laser printer.

Fig 11. shows a pair of antipodal hot spots rotating together with the neutron star at 12 different phases of the rotation. The angle between the center of the hot spot and the axis of the star's rotation is chosen to be 45°. The visibility of the hot spots in curved space-time is significantly different from the flat space case: obviously both hot spots will be visible at the same time for certain angles of inclination, a fact which dramatically changes the shape of the observed light curves.

6. Atoms in strong magnetic fields

In addition to the various aspects already mentioned, we also need to know the behaviour of matter in strong magnetic fields in order to be able to understand the cosmic X-ray sources in detail. For the most simple physical atomic system, the hydrogen atom, the effect of a magnetic field of small strength is well known as the usual Zeeman splitting of the emission lines into three components. For large values of the field strength, however, the Lorentz force of the external magnetic field dominates the internal Coulomb force, changing the atomic structure drastically (for details see [7]).

Quantum mechanically the problem can be described by Schrödinger's equation with a Hamilton-operator $\hat{H}$, which includes the kinetic energy of the electron, the Coulomb interaction between the proton and the electron, and the influence of the magnetic field. Mathematically we are dealing with an eigenvalue problem. The eigenvalues are the energies of the eigenstates Ψ, which determine the probability distribution $|\Psi|^2$ of the electron. A standard method for solving such an eigenvalue problem is the expansion of Ψ in a complete basis where the set of coefficients of this expansion is called the eigenvector. These coefficients can be determined approximately by diagonalizing the truncated matrix. For the case where the strength of the Lorentz force and the Coulomb force becomes comparable, these matrices can have dimensions up to 225.000 with about 10^8 nonvanishing elements. The CPU time on the CRAY-2 needed to diagonalize such a matrix and to compute 100 eigenvalues and eigenvectors is only about 100 minutes if one can use the full 2 Gigabyte of main memory. From the computed eigenvectors the eigenstates can be rebuilt and displayed for deeper insights.

Fig. 12 shows a traditional way of displaying the probability distribution Ψ as a function plot of a two-dimensional array. However, since Ψ is a three-dimensional function, it takes several projections and a lot of imagination to regain the full information. In order to obtain a correct impression of the probability distribution of the eigenstates in the real three dimensional space with only one picture, we use the method of line-of-sight integration (cf. [8]): The three dimensional scalar field $|\Psi|^2$ emits "light" with an intensity proportional to the numeric values of the scalar field, which it penetrates without absorption The scalar field is therefore equivalent to an "optically thin self-radiating gaseous nebula". The intensity which a distant observer receives from such an object out of a specific direction is just the integral along the line-of-sight of the intensities emitted towards the observer.

We used this algorithm to visualize the $5d_0$ state of a hydrogen atom for different magnetic field strengths in Fig. 13. Obviously, the shape of the hydrogen atom becomes longer and thinner with the increase of the magnetic field. In Figs. 14* and 15* the hydrogen atom — still in a homogeneous magnetic field — is shown in a highly excited state. Such *Rydberg atoms* are studied because they allow to test the correspondence principle which states that for high quantum numbers all observable quantities in classical mechanics must have their counterparts in quantum mechanics The classical system of an electron and a proton is known to show chaotic behaviour under certain circumstances, which, if found also in quantum systems, could help to describe the symptoms of quantum chaos.

7. Conclusions

The cosmic objects we told you about in this paper are so incredibly far away (millions of light years) that for the near (and probably for the far) future there is no chance to get pictures of them which show more than just a point of light. The same is true for objects on the opposite side of the physical length scale: No imaging technique is known that can reveal the internal structure of even the simplest atom. However, with the modelling capabilities

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* See page 546 for Figures 14 and 15.

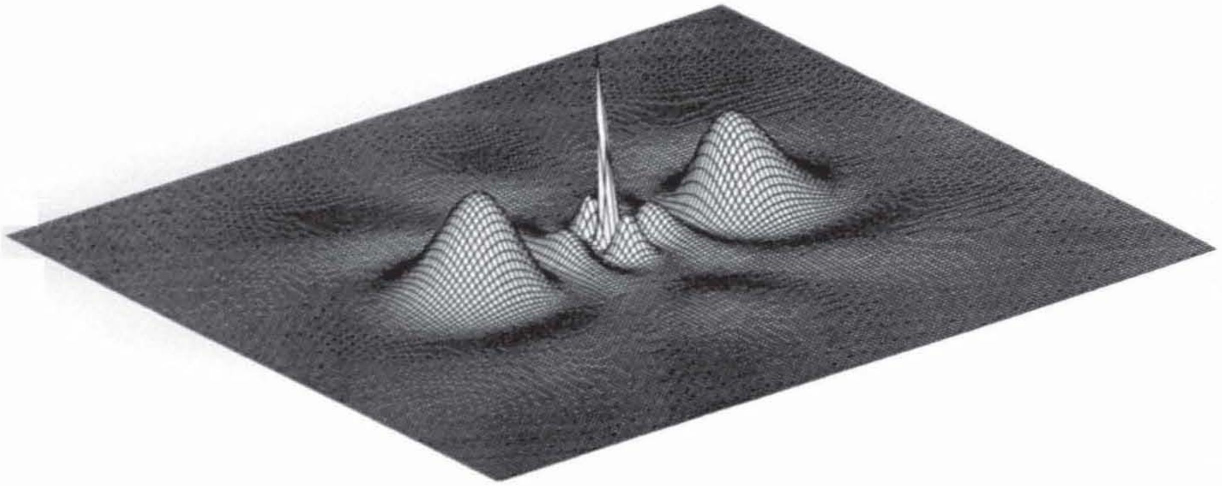

Fig. 12: Three-dimensional plot of the probability of presence distribution of an H-atom in an excited state

of supercomputers and the high-resolution color graphics of modern workstations, we are able to produce synthetic but physically correct pictures, zoom in on them — and see the unseen.

Acknowledgement

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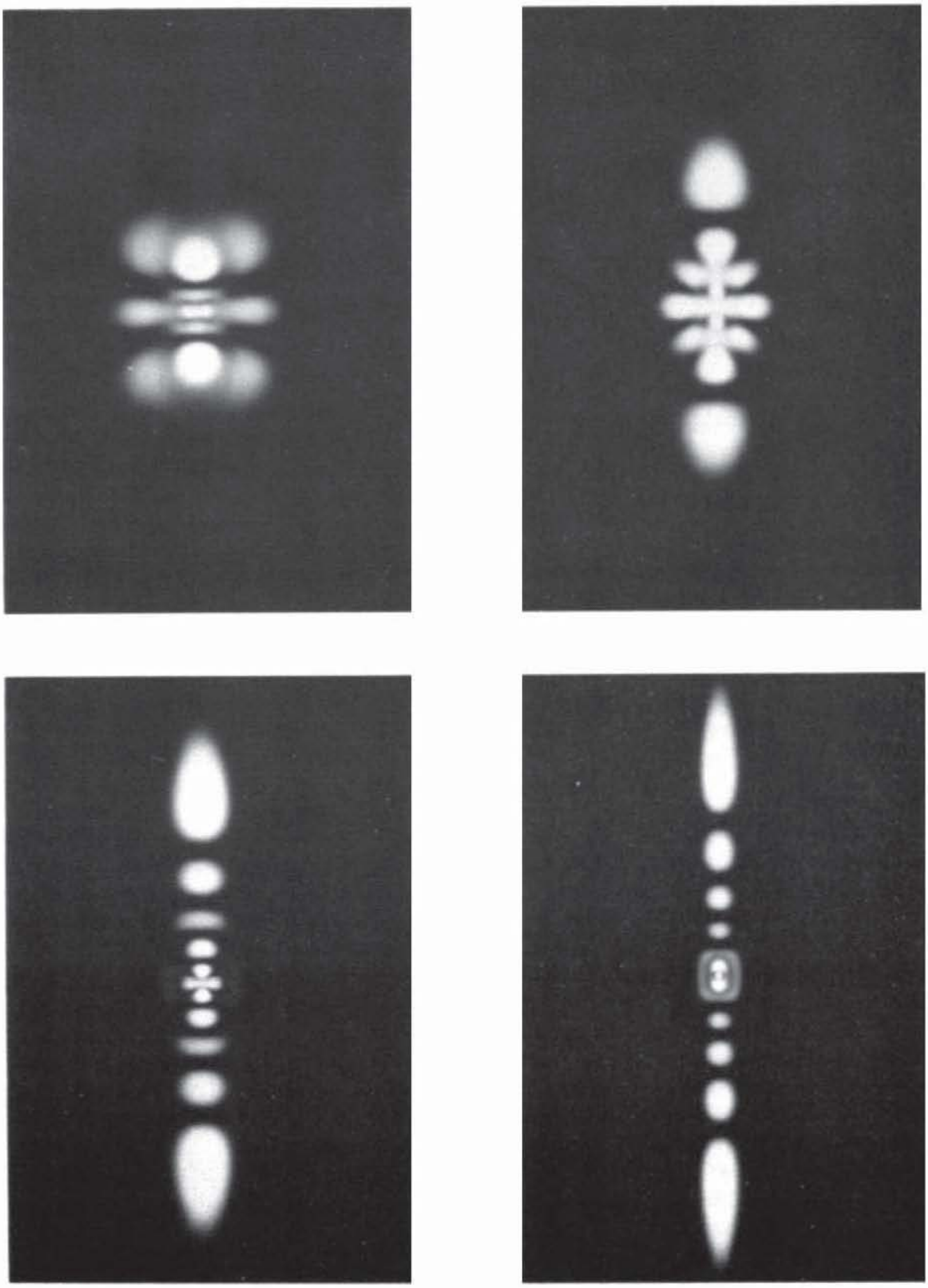

Fig. 13: Probability of presence of the electron in an excited state of the H-atom for increasing values of the magnetic field strength