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X-4. Energy Relaxation and Intervalley Relaxation of Hot Electrons in *n*-type Germanium*

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The average current density in *n*-type germanium subjected to superimposed strong *ac* and weak *dc* electric fields was both measured and calculated for frequencies of 9 and 35 GHz. Repopulation relaxation of the conduction band valleys and energy relaxation were taken into account. Experimental and theoretical results agree qualitatively.

§ 1. Relaxation Times

The *ac* conductivity of *n*-type germanium in strong electric fields of a ⟨001⟩ direction depends on the product of the *ac* field frequency and a quantity τ_e which essentially is the energy relaxation time as shown by Gibson *et al.*[1]. However, for another field direction, e.g. ⟨111⟩, there are repopulations of the conduction band valleys which in addition influence the *ac* conductivity with another time constant τ_i.[2]

Assuming a Maxwellian-type energy distribution with an electron temperature T_e the reciprocal energy relaxation time is calculated as

$$\tau_e^{-1}=\frac{16m^*s^2W_0}{3\sqrt{\pi}\,kT\sqrt{T/T_e}}+0.17\frac{\sqrt{\pi}\,W_0x_0^2(T/T_e)^{3/2}\sinh\left\{\frac{x_0}{2}(1-T/T_e)\right\}}{3(1-T/T_e)\sinh(x_0/2)}\times\left\{-H_1^{(1)}\left(i\frac{x_0}{2}T/T_e\right)\right\}, \qquad (1)$$

where $x_0T=400°K$ is the optical phonon temperature, $s=5.4\times10^5$ cm/sec is the sound velocity, $m^*=0.12m_0$ is the effective mass and $W_0=4.09\times10^9(T/°K)^{1.5}$ sec^{-1} is the probability for scattering of thermal electrons by acoustic phonons as taken from zero-field mobility data by Koenig *et al.*[3]; $-H_1^{(1)}$ is the Hankel function and the factor 0.17 approximates the theoretical zero-field mobility μ_0 to the observed $T^{-1.67}$-dependence. τ_e is plotted vs. T_e/T for lattice temperatures T of 85 and 273°K in Fig. 1 (dashed curves). The order of magnitude is 10^{-11} sec in agreement with observations by Gibson *et al.*[1] The solid curves in Fig. 1 were obtained from observed conductivity vs. *dc* field intensity data $\sigma(E)$ in ⟨001⟩ field direction[4] by means of the energy balance equation

$$0=e\mu(E)E^2-\frac{3}{2}k(T_e-T)/\tau_e(E), \qquad (2)$$

and $\mu=\mu_0\sigma(E)/\sigma_0$. A relation between field intensity E and electron temperature T_e was established by the assumption

$$\sigma(E)/\sigma_0=(T_e/T)^{-1/p}, \qquad (3)$$

where the exponent $-1/p$ depends on the scattering mechanism. A value of -0.67 was chosen which is acceptable for both $\mu_0\propto T^{-1.67}$ and predominant optical phonon scattering at large field intensities. The agreement between the semiempirical and the theoretical values of τ_e is considered as good, especially at large field intensities where the Maxwellian distribution is a good approximation even for the small carrier densities of the samples.[5]

The intervalley scattering time τ_i is calculated as

$$\tau_i^{-1}=\tau_{iH}^{-1}+3\tau_{iC}^{-1}, \qquad (4)$$

where

$$\tau_{i\rho}^{-1}=\frac{1}{2}\cdot10^{11}\,\text{sec}^{-1}\sqrt{\pi\theta_i/T_\rho}\cdot\times\cosh\left\{\frac{\theta_i}{2}(T^{-1}-T_\rho^{-1})\right\}\cdot\left\{-H_1^{(1)}\left(\frac{i}{2}\theta_i/T_\rho\right)\right\}\Big/\sinh\left(\frac{\theta_i}{2T}\right). \qquad (5)$$

$\rho=H$ or C and $\theta_i=315°K$ is the intervalley phonon temperature. With τ_i as defined by eq. (4) the population N_C of the cool valley after applying a field in the direction of the valley axis will change with time t according to

$$dN_C/dt=(N_{Cs}-N_C)/\tau_i, \qquad (6)$$

where N_{Cs} is the stationary population. This

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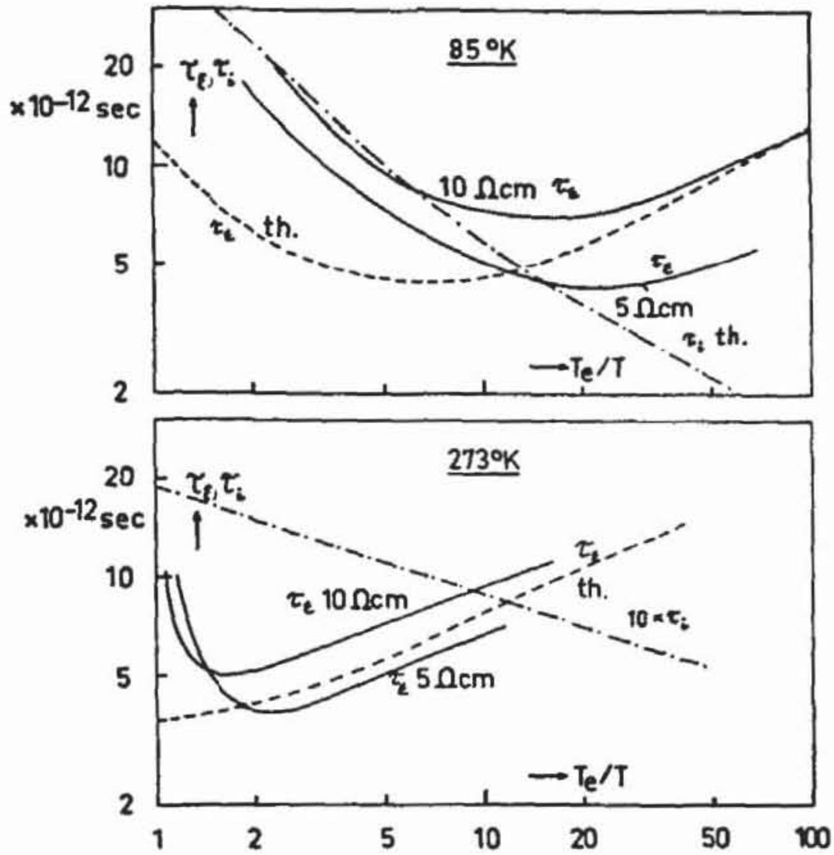

Fig. 1. Energy relaxation time τ_ε and repopulation relaxation time τ_i vs. hot valley temperature to lattice temperature ratio.

equation has been established by Schmidt-Tiedemann[5] for the range of warm electrons. Equations (4)—(6) are valid under the assumption that the energy transfer between valleys by intervalley scattering is negligible. In Fig. 1 τ_i is plotted vs. T_H/T where $(T_C-T)/T$ is approximately $0.032((T_H-T)/T)^{2/3}$ at 85°K and 1/2 of this at 273°K and has been calculated from the "effective" field intensities[4] and the energy balance equation for a ⟨111⟩ field direction. While at $T=85$°K τ_i and τ_ε have the same order of magnitude, $\omega^2\tau_i^2 \ll \omega^2\tau_\varepsilon^2$ at 273°K.

§ 2. Relaxation Processes at Microwave Frequencies

Gibson *et al.*[1] determined τ_ε at 300°K by applying a strong *dc* field and a colinear microwave field to a sample and determining its microwave conductivity σ_{ac} and dielectric constant κ from microwave absorption and phase shift. As pointed out by Gunn[6] the evaluation of σ_{ac} and κ neglects that the sample is of finite length and not in *dc* contact with the top and bottom walls of the waveguide. Therefore we think it more reliable to determine the *dc* conductivity of the sample by applying a weak *dc* field E_0 while the carriers are heated by a colinear pulsed strong microwave electric field of amplitude E_1. Frequencies were 9.375 and 35.12 GHz, pulse times 82 and 52 nsec, respectively, the rep. rate 30 pulses/sec and the sample temperatures 79 and 273°K. The apparatus was essentially as previously described[7] except that the sample was subjected to a travelling rather than a standing wave. The X-band guide was tapered to a clearance of 5 mm at the position of the sample. The filamentary samples of dimensions $1\times1\times10\,\text{mm}^3$ were cut from the same single crystal in either a ⟨001⟩ or ⟨111⟩ direction.

We measured the pulse height ΔU and the *dc* voltage U_0 between the ends of the sample of length l and calculated the relative increase in *dc* field in the sample due to the resistance increase by the strong *ac* field,

$$\Delta E/E_0 = l\cdot\Delta U/(b\cdot U_0)\,, \qquad (7)$$

where b is the clearance of the waveguide. Since the *dc* current is kept constant by a large series resistor, the time average of the current density relative to $(E_0+\Delta E)\sigma_0$ for $E_0 \ll E_1$ is given by

$$1/(1+\Delta E/E_0) = \int_0^{2\pi}(E_0+E_1\cos\omega t)\sigma d(\omega t)/(2\pi E_0\sigma_0)\,. \qquad (8)$$

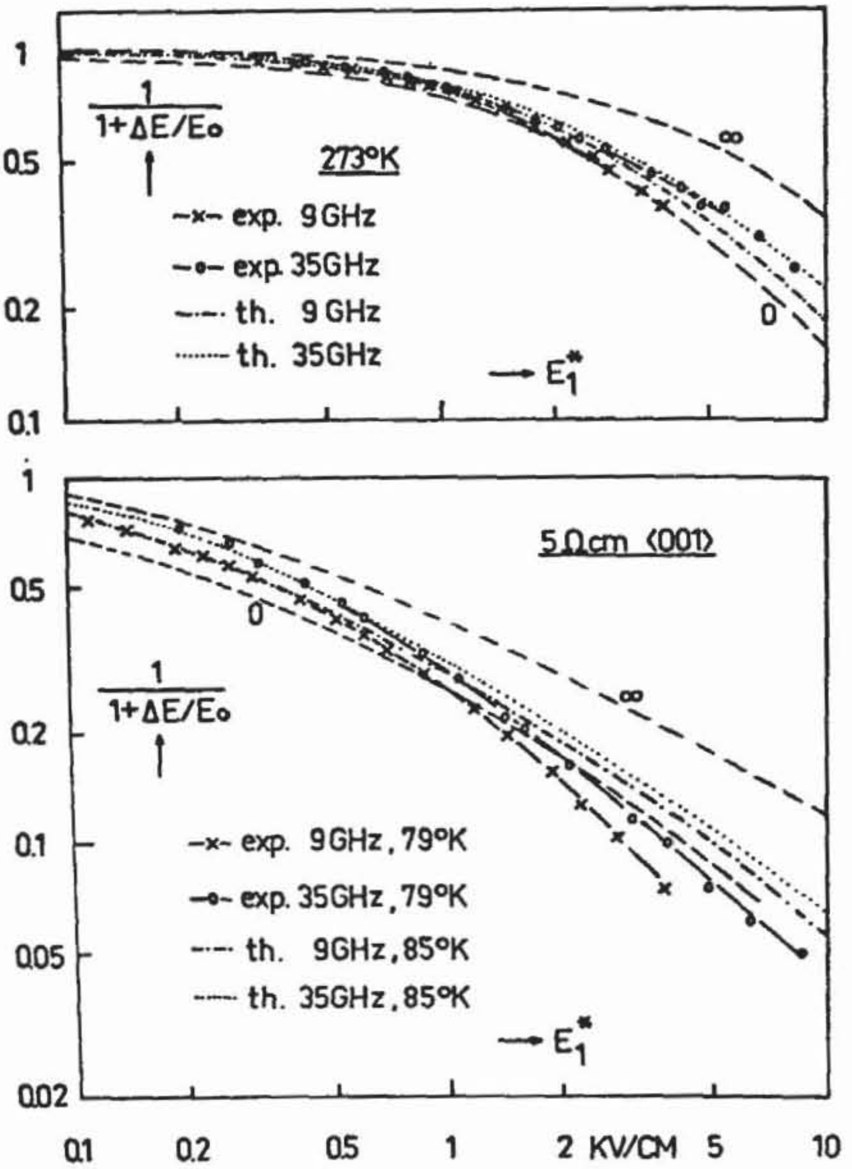

Fig. 2. Average sample current with both *dc* and *ac* fields normarized by current with *dc* field alone vs. amplitude of *ac* field, exp=experimental, *th*=calculated from observed *dc* characteristic (the difference between 79K° and 85°K is not significant).

The field amplitude E_1 in the sample is calculated from the average microwave power P absorbed by the sample, the volume V and an average conductivity $\bar{\sigma}$ of the part of the sample inside the waveguide:

$$E_1=(2P/(\bar{\sigma}V))^{1/2}\,, \qquad (9)$$

where $\bar{\sigma}$ is given by

$$\bar{\sigma}=\sigma_0\cdot(1+\Delta E/E_0)^{-1}\cdot(1+f_1)/(1-f_2)\,. \qquad (10)$$

The quantities f_1 and f_2 are $\ll 1$ and will be calculated in the next section. We introduce a field amplitude

$$E_1{}^*=(2P(1+\Delta E/E_0)/(\sigma_0 V))^{1/2}\,, \qquad (11)$$

that depends on observed quantities only. There is no need for introducing an *ac* conductivity instead of σ_0 in eq. (10) since according to observations by Conwell and Fowler[8] both conductivities are equal for frequencies of less than 35 GHz, temperatures above 77°K and field intensities above 0.5 kV/cm.

In Fig. 2 $1/(1+\Delta E/E_0)$ is plotted vs. $E_1{}^*$ for a ⟨001⟩ orientation and both frequencies. For 9 GHz it is smaller than for the higher frequency. This is true for all the orientations, temperatures and dopings (characterized by the room temperature resistivity 5 and 10 ohm cm) we used.

The ⟨111⟩ data are plotted in Fig. 3 in the form of the ratio $(1+\Delta E/E_0)^{-1}_{\langle 001\rangle}/(1+\Delta E/E_0)^{-1}_{\langle 111\rangle}$. We note that at both temperatures of 79°K and 273°K the 35 GHz data are above the 9 GHz data, and this is true also for the 10 ohm cm

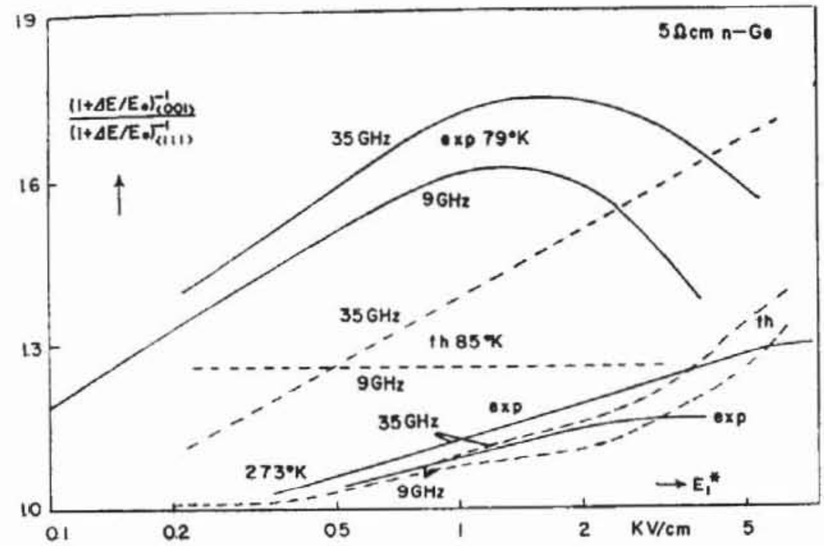

Fig. 3 Ratio of average currents for ⟨001⟩ and ⟨111⟩ field directions.

samples. This behaviour is in contrast to the direct expectation that at the higher frequency the anisotropy of the current should be smaller due to the additional influence of the repopulation relaxation of the conduction band valleys.

§ 3. Analysis of Microwave Measurements

For an *ac* field in ⟨111⟩ direction the left-hand side of eq. (2) is $d(\frac{3}{2}kT_e)/dt \neq 0$ with $T_e = T_\rho$ depending on the type of valley ($\rho = H$ or C). For $E_0 \ll E_1$ the following result is obtained from eq. (3):

$$1/(1+\Delta E/E_0) = f_3 \cdot (1 + f_4 - f_2)/2\pi \,, \qquad (12)$$

where

$$f_3 = \int_0^{2\pi} (N_C y_C^{-1/2} + 3N_H y_H^{-1/2}) d(\omega t)/N \qquad (13)$$

$$f_4 = E_1 \int_0^{2\pi} M_C (y_C^{-1/2} - y_H^{-1/2}) \cos \omega t \, d(\omega t)/(N \cdot f_3) \qquad (14)$$

$$f_2 = E_1 \int_0^{2\pi} (N_C z_C y_C^{-3/2} + 3N_H z_H y_H^{-3/2}) \cos \omega t \times d(\omega t)/(2N \cdot f_3) \,, \qquad (15)$$

and f_1 as defined by eq. (10) is given by

$$f_1 = \int_0^{2\pi} (N_C y_C^{-1/2} + 3N_H y_H^{-1/2}) \cos 2\omega t \, d(\omega t)/(N \cdot f_3) \,. \qquad (16)$$

The time—independent total carrier density is $N = N_C + 3N_H$. The quantities N_C and $M_C = -3M_H$ are solutions of eq. (6) and

$$dM_C/d(\omega t) = (dN_{C0}/dE - M_C - (N_{C0} - N_C) d \ln \tau_i/dE) /(\omega \tau_i) \,, \qquad (17)$$

and the quantities y_ρ and z_ρ ($\rho = C$ or H) are solutions of

$$dy_\rho/d(\omega t) = \alpha \cdot y_\rho^{(1-p)/2} \cdot (E_1^2 \cos^2 \omega t - h_\rho)/p \qquad (18)$$

$$dz_\rho/d(\omega t) = \alpha \cdot y_\rho^{(1-p)/2} \cdot (2E_1 \cos \omega t - z_\rho h_\rho' - z_\rho (p-1)(E_1^2 \cos^2 \omega t - h_\rho)/(2y_\rho))/p \,, \qquad (19)$$

where $\alpha = 4\mu_0 e/(3kT\omega)$ and

$$h_\rho = y_\rho^{1/2} (y_\rho^{p/2} - 1) 3kT/(2\mu_0 e \tau_\varepsilon) \,, \qquad (20)$$

and $h_\rho' = dh_\rho/dy_\rho$. In these equations N_{C0} and τ_ε are the carrier density in the cool valley and the energy relaxation time as determined from *dc* field measurements but with E replaced by $E_1 \cos \omega t$. The repopulation relaxation time was calculated from eqs. (4) and (5). h_ρ was replaced by that value of E^2 for which in the *dc* experiment a mobility ratio μ/μ_0 equal to μ_ρ/μ_0 was found where μ_ρ is given by eq. (4) of ref. 4). Initial conditions for eqs. (18)—(21) were chosen arbitrarily and the machine calculation was done over so many periods of length 2π that $1/(1+\Delta E/E_0)$ changed from one period to the next by less than 1%.

For a ⟨001⟩ field direction all 4 valleys are equivalent and thus $N_C = N_H = N/4$, $y_C = y_H$ and $z_C = z_H$ which makes $f_4 = dN_C/dt = 0$. The results of eq. (12) are plotted vs. E_1^* in Figs. 2 and 3. The limiting cases $\omega\tau_\varepsilon$ and $\omega\tau_i \to 0$ and ∞ have been included in Fig. 2. In the limit ∞ the quantity $1/(1+\Delta E/E_0)$ is identical with the *dc* characteristic σ/σ_0 vs. E for *dc* fields $E = E_1/\sqrt{2}$ observed at 85°K with the same samples.[4)]

A comparison between the experimental and the calculated data shows qualitative agreement. In strong fields larger deviations from Ohm's law are observed than calculated. Carrier scattering into the silicon-like valleys where the carriers become less mobile would qualitatively explain the discrepancy. However, it remains open if the discrepancies would be solved by a more refined theory of scattering within and between the low valleys of the conduction band. Figure 3 shows that at $E_1^*>0.5$ kV/cm the calculated anisotropy is also larger at the higher frequency although there is no quantitative agreement between experimental and theoretical data. Apparently the hot valleys that carry the essential part of the longitudinal current component,[4] are much hotter at a $\langle 111\rangle$ than a $\langle 001\rangle$ field direction and τ_e so much larger according to Fig. 1 that at a higher frequency the average $\langle 111\rangle$ current is more reduced than the average $\langle 001\rangle$ current, the ratio of the currents increased therefore. The repopulation relaxation seems to be less important in this respect.

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DISCUSSION

Paige, E. G. S.: At low temperatures, low fields and high frequency, $\omega\tau>1$ (τ, momentum relaxation time). Was this incorporated in your calculations?

Seeger, K.: The influence of momentum relaxation on the high-field *ac* conductivity may be neglected even at 77°K at field intensities above 1/2 kV/cm as shown by experiments of Conwell and Fowler presented at the Prague Conference in 1960.