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A Distributed Solution to the Adjustable Robust Economic Dispatch Problem

(final draft)

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Abstract: The problem of maintaining balance between consumption and production of electric energy in the presence of a high share of intermittent power sources in a transmission grid is addressed. A distributed, asynchronous optimization algorithm, based on the ideas of cutting-plane approximations and adjustable robust counterparts, is presented to compute economically optimal adjustable dispatch strategies. These strategies guarantee satisfaction of the power balancing constraint as well as of the operational constraints for all possible realizations of the uncertain power generation or demand. The communication and computational effort of the proposed distributed algorithm increases for each computational unit only slowly with the number of participants, making it well suited for large scale networks. A distributed implementation of the algorithm and a numerical study are presented, which show the performance in asynchronous networks and its robustness against packet loss.

Keywords: Distributed Robust Optimization, Economic Dispatch, Asynchronous Algorithms

1. INTRODUCTION

It is a declared goal of many industrial nations to increase the share of "green energy' in the domestic energy mix. Due to the uncertain nature of renewable power sources, one major control challenge is to maintain balance between the consumption and generation of electrical power.

Current practice is to optimize a nominal dispatch plan with regard to some predicted consumption and production trajectories. To compensate prediction errors, a hierarchy of control mechanisms is implemented to maintain the stability, cf. Bergen and Vittal [1999]. In order to not violate device and transmission constraints even in the presence of (unavoidable) prediction errors, the nominal dispatch plans need to be fairly conservative. To overcome this conservatism one can optimize, for example, over adjustable plans instead of nominal ones. One idea proposed in this direction is to use stochastic programming tools, e.g. Dvijotham et al. [2012]. Another approach recently proposed in Warrington et al. [2012] and Bienstock et al. [2012] is based on the theory of *affinely adjustable robust counterparts*. The idea, also known as *linear decision rules* (eg. Garstka and Wets [1974]), is well known in operations research. For an introduction, examples and theoretical treatment see the book Ben-Tal et al. [2009]. It is known that linear decision rules are, apart from the one-dimensional case cf. Bertsimas et al. [2010], in general not optimal compared to arbitrary decision rules. However, finding the latter is in general computationally intractable, which justifies to search for affine rules.

The contributions of this paper are as follows. We consider a simple power network model, involving controllable and uncontrollable devices affected by external, unknown disturbances. The robust economic dispatch problem is to design a control strategy over a finite horizon that ensures a balancing of the power supply and demand at all times. We focus on control strategies that are affine in the disturbance signal and formulate the robust counterpart of the resulting optimization problem.

The main contribution of this paper is the proposal and implementation of a distributed, asynchronous algorithm for solving the robust economic dispatch problem. First, we show that the robust counterpart has a very characteristic structure, involving fairly complex local constraint sets and few coupling constraints. Then, we show that the problem decomposes as the dual problem formulation is considered. Based on these considerations, we illustrate the applicability of a fully distributed, asynchronous algorithm, the cutting-plane consensus algorithm, recently proposed in Bürger et al. [2012b]. We provide a novel interpretation of the method for this problem class. Namely, we show that the algorithm results to be a trajectory exchange method, where processors exchange predicted trajectories to make cooperative decisions. Finally, we present the results of a distributed, asynchronous implementation of the optimization algorithm in a spatially distributed computation network. The implementation shows that the algorithm performs well in asynchronous networks with extremely different computation speeds and has a high robustness against failures in the communication network, such as packet losses. The imple-

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mentation proves the suitability of the cutting-plane consensus algorithm for solving management and decision problems over wide-area communication networks.

The remainder of this paper is organized as follows. Section 2 introduces the economic dispatch problem subject to uncertainties and the affinely adjustable robust counterpart method. In Section 3 the problem structure is exploited, and an algorithm is presented to solve it in a completely distributed fashion. Finally, Section 4 provides the implementation results and shows the robustness of the presented solution.

Notation: The notation employed is standard. Upper case letters are used for matrices and lower case for vectors. $[A]_k$, $[b]_k$ denotes row k of the matrix A and element k of the vector k, respectively. Calligraphic letters, e.g., $\mathcal{C}$, denote index sets.

2. ROBUST ECONOMIC DISPATCH PROBLEM

The *robust economic dispatch problem* is to compute a power generation schedule to satisfy an uncertain demand while minimizing the economic cost. We consider here an adjustable planning approach and design *affine design decision* rules over a finite prediction horizon which ensure that the uncertain demand is satisfied and all operational constraints are met.

2.1 Network Model and Problem Statement

We consider a simplified model of a power distribution network, consisting of ideal transmission lines which connect different devices such as power generating plants, distribution systems or industrial consumers. We assume that the operational cost critically depends on the generated active power p and only marginally on the voltage which we will neglect in our model, cf. Bergen and Vittal [1999].

Device Models A device might be controllable (e.g. generator) or uncontrollable (e.g. general load) and it might be affected by uncertainties (e.g. wind farm). Each device has an associated *power schedule* $p_i = [p_i(1) \ \dots \ p_i(T)] \in \mathbb{R}^T$ over a given time horizon T. We stick to the convention, that $p(\tau) \geq 0$ means a positive power flow from the device into the power grid. Let in the following $\mathcal{C}$ be the index set of all controllable devices, $\mathcal{V}$ the index set of uncontrollable devices. Some of the devices are influenced by an external, unknown disturbance δ_i. We denote the index set of all devices affected by uncertainties with $\mathcal{D}$.

Uncontrollable Devices: The time evolution of an uncontrollable device is modeled as

$$p_i = r_i + G_i \delta_i, \tag{1}$$

where $r_i \in \mathbb{R}^T$ is a forecast reference trajectory and $G_i \in \mathbb{R}^{T\times T}$ a lower triangular matrix. Furthermore, $\delta_i \in \mathbb{R}^T$ is a possible forecast error which belongs to some given bounded set

$$\Delta_i = \{\delta \in \mathbb{R}^T \mid Q_i\delta + q_i \in \mathbf{K_i}\} \tag{2}$$

with $\mathbf{K}_i$ a closed, convex, pointed cone in $\mathbb{R}^{N_i}$, and $Q_i \in \mathbb{R}^{N_i \times T}$, $q_i \in \mathbb{R}^{N_i}$ given.

Controllable Devices: A controllable device is represented by a linear dynamical system, with the control input $u_i(\tau)$ and the associated power schedule being the systems output, i.e.,

$$\begin{aligned} x_i(\tau+1) &= \tilde{A}_i x_i(\tau) + \tilde{b}_{i,u} u_i(\tau) + \tilde{b}_{i,\delta}\delta_i(\tau) \\ p_i(\tau) &= \tilde{c}_i^T x_i(\tau), \ x_i(0) = x_{i,0}. \end{aligned} \tag{3}$$

with $x_i(\tau)$, $\tilde{b}_{i,u}$, $\tilde{b}_{i,\delta}$, $\tilde{c}_i \in \mathbb{R}^{n_i}$, $\tilde{A}_i \in \mathbb{R}^{n_i \times n_i}$. We incorporate here explicitly the possibility, that a controllable device is affected by an uncertainty $\delta_i \in \Delta_i$, as it is, e.g., the case with curtailable wind farms. It is a standard result that the linear dynamical system (3) can be solved explicitly for the state and output trajectories

$$\begin{aligned} x_i &= A_i x_{i,0} + B_{i,u} u + B_{i,\delta}\delta \\ p_i &= C_i^T x_i \end{aligned} \tag{4}$$

with $x_i \in \mathbb{R}^{n_i \cdot T}$, $B_{i,u}$, $B_{i,\delta}$, $C_i \in \mathbb{R}^{n_i \cdot T \times T}$, $A_i \in \mathbb{R}^{n_i \cdot T \times n_i \cdot T}$. Note that in this form p_i corresponds directly to the power schedule.

Each controllable device has a set of operational constraints, which might depend explicitly on the uncertainties affecting the device and must be satisfied for all possible realizations of the disturbances. The set of admissible state and input trajectories for device i is given by

$$\mathcal{X}_i \times \mathcal{U}_i = \{(x,u) \mid T_i x + U_i u + V_i \delta \leq v_i \quad \forall \delta \in \Delta_i\}. \tag{5}$$

Economic Dispatch Problem Using this general model, we are ready to formalize the economic dispatch problem.

Power Balance Constraint: We assume an ideal transmission grid with neither loss nor limits on the line currents. Under this assumption the only remaining constraint is the net power balance: At each time instant the overall power production has to match the overall power consumption

$$\sum_{i\in\mathcal{C}} p_i(\tau) + \sum_{i\in\mathcal{V}} p_i(\tau) = 0, \quad \tau \in \{1,\dots,T\}. \tag{6}$$

The power balance constraint introduces T equality constraints.

Optimization Objective: Each controllable device has associated an economic convex cost function $J_i(x_i,u_i) : \mathbb{R}^{n_i \cdot T} \times \mathbb{R}^T \to \mathbb{R}$. We assume here that the cost functions are piecewise linear, i.e.,

$$\begin{aligned} J_i(x_i,u_i) = \max\{ & c_{1,x}^T x_i + c_{1,u}^T u_i + d_1, \ \dots, \\ & c_{m_i,x}^T x_i + c_{m_i,u} u_i + d_{m_i}\} \end{aligned} \tag{7}$$

with $c_{j,x} \in \mathbb{R}^{n_i \cdot T}$, $c_{j,u} \in \mathbb{R}^T$, $d_j \in \mathbb{R}$. The overall objective is the sum of all device objectives.

Taking these components together, we can formalize the optimal dispatch problem with uncertainties:

$$\begin{aligned} \min_{u_i} \ & J = \sum_{i\in\mathcal{C}} J_i(x_i,u_i) \\ \text{s.t. } & \text{Power Balance Constraint (6)} \\ & \text{Device Constraints (5).} \end{aligned} \tag{8}$$

Due to the uncertainties in the constraints and objective, problem (8) is not directly feasible in the present form. To overcome this issue, we will use in the following the idea of affinely adjustable robust counterparts, see Ben-Tal et al. [2009].

2.2 Affinely Adjustable Robust Counterpart

To handle the uncertainties, we seek here for feedback rules that depend explicitly on the measured disturbance sequence, i.e., $u_i = u_i(\delta_1, \ \dots, \ \delta_{|\mathcal{D}|})$. We restrict ourselves here to *affine decision rules* of the form

$$u_i = \rho_i + \sum_{j\in\mathcal{D}} \mathrm{P}_{ij}\delta_j \tag{9}$$

with $\mathrm{P}_{ij} \in \mathbb{R}^{T\times T}$. In order to maintain causality, the disturbance feedback gains P_{ij} are required to be lower triangular matrices. Diagonal elements in the matrix P_{ij} represent the ability of a system to react on the current disturbance. That is, if the diagonal elements exist, the disturbance $\delta_j(\tau)$ is known at the time $u_i(\tau)$ is applied. This assumption is necessary for at least some devices to maintain exact power balancing.

After restricting to the affine decision rules (9), the resulting optimization problem still depends on the uncertainties δ_i. However, one can now formulate the robust counterpart , i.e., a nominal optimization problem with the same solution as the uncertain problem, see Ben-Tal et al. [2009].

Controllable Devices: The set of admissible decision rules for a controllable device i has to be designed such that the operational constraints (5) are satisfied. This leads to an infinite dimensional problem description with an equivalent nominal robust counterpart.

Proposition 1. The set of feasible feedback rules, respecting (5) is given by

$$\Pi_i(x_0) = \left\{ (\rho_i, \{P_{ij}\}_{j\in\mathcal{D}}) \;\middle|\; \begin{array}{r} \rho_i \in \mathbb{R}^T, P_{ij} \in \mathbb{R}^{T\times T}, \text{lower triang.} \\ \exists\, y_{jk} \in \mathbf{K}_j^* \\ [F]_k\rho_i + \sum_{j\in\mathcal{D}} q_j^T y_{jk} \le [h]_k \\ y_{jk}^T Q_j = -[F P_{ij} + G_j]_k \\ \forall \text{ rows } k,\ \forall j \in \mathcal{D} \end{array} \right\} \tag{10}$$

with $F = T_i B_{i,u} + U_i$, $G_j = T_i B_{i,\delta_j} + V_i$, $h = v_i - T_i A_i x_{i,0}$ and $\mathbf{K}^*$ the cone dual to $\mathbf{K}$.

The proof follows from Ben-Tal et al. [2009] and a similar result has been presented in Warrington et al. [2012].

Power Balance Constraint: As a next step we consider the net balancing constraint (6). Under the proposed affine control law the balancing constraint becomes

$$\sum_{i\in C} C_i(A_i x_{0,i} + B_{i,u}(\rho_i + \sum_{j\in\mathcal{D}} P_{ij}\delta_j) + B_{i,\delta}\delta_i) = - \sum_{i\in\mathcal{V}} (r_i + G_i\delta_i) \quad \forall \delta_j \in \Delta_j,\ \forall j \in \mathcal{D}. \tag{11}$$

Proposition 2. Constraint (11) is satisfied for all disturbance realisations $\delta_j \in \Delta_j$, $\forall j \in \mathcal{D}$ if and only if the following equations hold:

$$\sum_{i\in C} C_i(A_i x_{0,i} + B_{i,u}\rho_i) = -\sum_{i\in\mathcal{V}} r_i \tag{12a}$$

$$\sum_{i\in C} C_i(B_{i,u}P_{ij} + \mathbf{1}_{ij}B_{i,\delta}) = -G_j \qquad \forall j \in \mathcal{D}, \tag{12b}$$

where $\mathbf{1}_{ij}$ denotes the indicator function which is 1 if $i = j$ and 0 otherwise.

We again omit the proof since a similar result has been published in Warrington et al. [2012].

Optimization Objective: It remains to define the objective (7) in the presence of disturbances. We focus on a min-max formulation and aim to minimize the worst case loss

$$J_i(\rho_i, \{P_{ij}\}_j) = \max_{\delta_j\in\Delta_j} J_i(x_i, \rho_i + \sum_{j\in\mathcal{D}} P_{ij}\delta_j).$$

The main advantage of the min-max problem formulation is that, besides the absolute bounds, no further properties on the underlying disturbance distribution have to be known.

We transform the optimization problem into epigraph form by introducing one new variable $\bar{J}_i$ together with the constraint $\bar{J}_i \ge \max_{\delta_j\in\Delta_j} J_i(x_i, \rho_i + \sum_{j\in\mathcal{D}} P_{ij}\delta_j)$ for each controllable device. Introducing the robust counterpart, similar as before, leads to the set of feasible upper bounds on the cost:

$$\Gamma_i(x_0) = \left\{ \bar{J} \;\middle|\; \begin{array}{r} \bar{J} \in \mathbb{R} \\ \exists\, y_{jk} \in \mathbf{K}_j^* \\ \sum_{j\in\mathcal{D}} q_j^T y_{jk} + c_{k,u}^T\rho_i + c_{k,x}^T(Ax_0 + B_u\rho_i) + d_k \le \bar{J} \\ y_{jk}^T Q_j = -[(c_{k,x}^T B_u + c_{k,u}^T)P_{ij} + \mathbf{1}_{ij} c_{k,x}^T B_\delta] \\ \text{for } k = 1 \ldots m_i,\ \forall j \in \mathcal{D} \end{array} \right\} \tag{13}$$

Summarizing, we obtain the *robust economic dispatch problem*

$$\begin{aligned} \min_{\rho_i, P_{ij}, \bar{J}_i} \quad & J = \sum_{i\in C} \bar{J}_i \\ \text{s.t.} \quad & \text{Pwr Balance Constraints (12a), (12b)} \\ & (\rho_i, \{P_{ij}\}_{j\in\mathcal{D}}, \bar{J}_i) \in \Pi_i \times \Gamma_i \quad \forall i \in C. \end{aligned} \tag{14}$$

The robust economic dispatch problem (14) has a very characteristic structure. It has a linear objective function, some linear coupling constraints, and $|C|$ *local* constraint sets. Each local constraint set belongs to exactly one controllable device and contains the local operational constraints, restricting the power generation ρ_i and feedback gains P_{ij}, as well as the upper bound for the cost function $\bar{J}_i$.

3. DISTRIBUTED SOLUTION

We are now seeking an algorithm to solve the robust economic dispatch problem in a fully distributed way. That is, we want a processor to be placed at the controllable devices, with the local operational constraints of the device only known to this processor. The processors should cooperatively solve the large scale robust economic dispatch problem, by exchanging messages over an asynchronous communication network.

3.1 Lagrange Dual and Separability

The first step for deriving the distributed algorithm is to consider the dual problem of (14). We formulate a partial Lagrangian of the problem by dualizing the coupling constraints (12). Let $\lambda \in \mathbb{R}^T$ and $\Lambda_j \in \mathbb{R}^{T\times T}$ $\forall j \in \mathcal{D}$ be the dual variables corresponding to the constraints (12a), (12b), respectively. Now, the dual optimization problem of (14) becomes

$$\begin{aligned} \max_{\lambda,\Lambda_j} \ & \lambda^T \sum_{i\in\mathcal{V}} r_i + \sum_{j\in\mathcal{D}} < \Lambda_j, G_j > + \\ & \sum_{i\in C} \min_{\rho_i, P_{ij}} \Big\{ \bar{J}_i + \lambda^T C_i(A_i x_{0,i} + B_{i,u}\rho_i) + \\ & \qquad \sum_{j\in\mathcal{D}} < \Lambda_j, C_i(B_{i,u}P_{ij} + \mathbf{1}_{ij}B_{i,\delta}) > \Big\} \\ \text{s.t.} \ & (\rho_i, \{P_{ij}\}_j, \bar{J}_i) \in \Pi_i \times \Gamma_i \quad \forall i \in C. \end{aligned} \tag{15}$$

Next, for each controllable device i we introduce a new variable u_i and reformulate the max-min problem (15) as a separable *semi-infinite program*:

$$\begin{aligned} \max_{\lambda,\Lambda_j,u_i} \ & \lambda^T \sum_{i\in\mathcal{V}} r_i + \sum_{j\in\mathcal{D}} < \Lambda_j, G_j > + \sum_{i\in C} u_i \\ \text{s.t.} \ & u_i \le \bar{J}_i + \lambda^T C_i(A_i x_{0,i} + B_{i,u}\rho_i) + \\ & \qquad \sum_{j\in\mathcal{D}} < \Lambda_j, C_i(B_{i,u}P_{ij} + \mathbf{1}_{ij}B_{i,\delta}) > \\ & \forall (\rho_i, \{P_{ij}\}_j, \bar{J}_i) \in \Pi_i \times \Gamma_i \quad \forall i \in C. \end{aligned} \tag{16}$$

This problem has a nice structure. It has one linear objective function and each processor (i.e., controllable device) is assigned one convex constraint set, represented here by a linear semi-infinite constraint. This form is advantageous, as it allows now to apply the *cutting-plane consensus algorithm*, recently presented in Bürger et al. [2012a] and Bürger et al. [2012b].

3.2 The Cutting-Plane Consensus Algorithm

The conceptual idea of the algorithm can informally be described as follows. Instead of solving the semi-infinite program (16) directly, each processor solves a linear program with a polyhedral outer-approximation of the semi-infinite constraint set. Each processor $i \in C$ stores and updates its own set of constraints, denoted with $\mathcal{B}^{[i]}$. The processors exchange their set of constraints, generate new constraints and then update their set $\mathcal{B}^{[i]}$. Under some additional technical modifications, the processors will eventually all compute the optimal solution. As a main contribution of this section, we show that the cutting-plane consensus algorithm applied to the economic dispatch problem turns out to be a trajectory exchange method, similar to the methods intensively studied in distributed model predictive control, see e.g., Richards and How [2007]. We first introduce the approximate program and then describe the generation of new constraints.

We replace the semi-infinite constraints in (16) with a set of linear constraints by selecting suitable sampling points from the sets $\Pi_i \times \Gamma_i$. Let

$$\mathcal{B} = \left\{(\rho_i^k, \{\mathrm{P}_{ij}^k\}_j, \bar{J}_i^k) \in \Pi_i \times \Gamma_i, i \in C, k = \{1, \ldots, K_i\}\right\},$$

be such a set of sampling points. With this we formulate the approximate program

$$\begin{aligned} \max_{\lambda,\Lambda_j,u_i}\ & \lambda^T \sum_{i\in\mathcal{V}} r_i + \sum_{j\in\mathcal{D}} < \Lambda_j, G_j > + \sum_{i\in C} u_i \\ \text{s.t. }\ & u_i \leq \bar{J}_i^k + \lambda^T C_i(A_i x_{0,i} + B_{i,u}\rho_i^k) + \\ & \quad \sum_{j\in\mathcal{D}} < \Lambda_j, C_i(B_{i,u}\mathrm{P}_{ij}^k + \mathbf{1}_{ij}B_{i,\delta}) > \\ & \forall\ (\rho_i^k, \{\mathrm{P}_{ij}^k\}_j, \bar{J}_i^k) \in \mathcal{B}. \end{aligned} \tag{17}$$

The polyhedral constraint set of the linear program (17) is an outer-approximation of the semi-infinite constraint set in (16). That is, any feasible point of (16) is also feasible for (17). An additional (technical) challenge arises from the non-uniqueness of the optimal solution of (17). For the algorithm to work correctly, it is necessary to employ a *unique solution* rule, when solving the linear program, see Bürger et al. [2012b] for a detailed explanation. We consider here, as in Bürger et al. [2012b] the unique minimal 2-norm solution to (17).

The constraints of (17) are affine in the decision variables u_i, λ, Λ_j. The constant term is simply $\bar{J}_i^k$, i.e., the upper bound on the cost function for the corresponding processor. The constraint data corresponding to the multiplier λ is

$$p_{i,nom}^k := C_i(A_i x_{0,i} + B_{i,u}\rho_i^k).$$

That is, the multiplier λ is associated to the predicted nominal power generation schedule of the controllable device, corresponding to the sampled input trajectory ρ_i^k. Similarly, the multipliers Λ_j are associated to the disturbance dependent adjustable control components, i.e.,

$$P_{ij}^k := C_i(B_{i,u}\mathrm{P}_{ij}^k + \mathbf{1}_{ij}B_{i,\delta}).$$

In summary the constraint data of device i, i.e., $\mathcal{B}^{[i]}$ contains several *plans*. A plan consists of (i) a "predicted" nominal power schedule, i.e., $p_{l,nom}^k, l \in C$, (ii) the corresponding feedback gains $\{P_{lj}^k\}_j$ and (iii) the corresponding upper bound of the cost functions $\bar{J}_l^k$. The power generation corresponding to this sampling point can be simply computed as

$$p_i^k := p_{i,nom}^k + \sum_{j\in\mathcal{D}} P_{ij}^k \delta_j.$$

The processors store and exchange such power generation plans between each other. Note that only the generation plan needs to be exchanged, while the corresponding input sequence, i.e., $\rho_i^k, \{\mathrm{P}_{ij}^k\}_j$, remains private information of the device.

To define the optimal solution of (16) it is not sufficient to store only one plan for each device. It is well known in linear programming (and also in semi-infinite optimization, see Hettich and Kortanek [1993]) that at most as many constraints as decision variables are necessary to determine a feasible solution. Therefore, for the robust economic dispatch problem, each device needs to store at most

$$d = T + |\mathcal{D}|\frac{T(T+1)}{2} + |C|$$

plans (i.e., constraints of the approximate program). In fact, once the approximate linear program (17) is solved, one can drop all except of d active constraints, without changing the optimal solution. We will say in the following that these d active constraints are the *relevant plans*, i.e., they are the plans determining the optimal solution to the approximate program.

The next decisive question is how the given approximation (17) can be improved further. Let $z_q = (u_q, \lambda_q, \{\Lambda_{q,j}\}_j)$ denote the argument of the optimal solution to the approximate program (17). Consider the following optimization problem corresponding to device $i \in C$:

$$\begin{aligned} u_i^*(\lambda_q, \{\Lambda_{j,q}\}_{j\in\mathcal{D}}) = \min_{\rho_i, P_{ij}, \bar{J}_i}\ & \bar{J}_i + \lambda_q^T C_i(A_i x_{0,i} + B_{i,u} p_i) + \\ & \sum_{j\in\mathcal{D}} < \Lambda_{j,q}, C_i(B_{i,u}P_{ij} + \mathbf{1}_{ij}B_{i,\delta}) > \\ \text{s.t. }\ & (p_i, \{P_{ij}\}_j, \bar{J}_i) \in \Pi_i \times \Gamma_i. \end{aligned} \tag{18}$$

Now, if $u_{q,i} \leq u_i^*(\lambda_q, \{\Lambda_{q,j}\}_{j\in\mathcal{D}})$, then the optimal solution (u_q, $\lambda_q, \{\Lambda_{q,j}\}_j$), computed with the approximate linear program, is also feasible for the corresponding semi-infinite constraint. Otherwise, the approximation is not exact enough, and processor i generates a new linear constraint of the form

$$\begin{aligned} u_i \leq & \bar{J}_i^* + \lambda^T C_i(A_i x_{0,i} + B_{i,u}\rho_i^*) \\ & + \sum_{j\in\mathcal{D}} < \Lambda_j, C_i(B_{i,u}\mathrm{P}_{ij}^* + \mathbf{1}_{ij}B_{i,\delta}) >, \end{aligned} \tag{19}$$

where $\bar{J}_i^*, \rho_i^*, \{\mathrm{P}_{ij}^*\}_j$ is the optimal solution of (18). The problem (18) is basically a robust optimal planning problem for a controllable device. The local constraints $\Pi_i \times \Gamma_i$ represent the local operational constraints of the device and the robust counterpart of the cost function. Additionally, the nominal generation schedule is "penalized" by the dual variables λ, and the adjustable component is penalized by Λ_{ij}.

Finally, the generated plans need to be exchanged. After having solved the approximate program and having created a new constraint, the processor sends *all relevant plans* to its neighbors.

We formally summarize the computational procedure each processor performs in Algorithm 1. For an in-depth discussion of the algorithm and a convergence proof we refer to Bürger et al. [2012b]. We want to point out that the convergence of the algorithm does not depend on the linearity of the underlying optimization problem. The distributed algorithm, as presented here is applied to the dual of the economic dispatch problem,

Algorithm 1 Cutting-Plane Consensus Algorithm

Require: Initial set of plans $\mathcal{B}_0^{[i]}$

loop

(S1) Transmit the set of plans $\mathcal{B}_t^{[i]}$ to all out-neighbors and receive the plans $\mathcal{B}_t^{[j]}$ from all in-neighbors.

(S2) Form the approximate linear program (17) and compute the 2-norm solution $\lambda(t)$, $\Lambda_j(t)$, $u_i(t)$; determine the set of relevant plans $\mathcal{B}_{tmp}^{[i]}$.

(S3) Compute a new plan, i.e., a nominal power trajectory p_i^k, the feedback components $\{P_{ij}^k\}_j$, and the cost bound $\bar{J}_i^k$ by solving the local planning problem (18).

(S4) Add the new plan $(p_i^k, \{P_{ij}^k\}_j, \bar{J}_i^k)$ to the set, i.e., $\mathcal{B}_{t+1}^{[i]} = \mathcal{B}_{tmp}^{[i]} \cup (p_i^k, \{P_{ij}^k\}_j, \bar{J}_i^k)$.

end loop

and will solve the dual problem. However, it is shown in Bürger et al. [2012b] that the algorithm allows to reconstruct the optimal primal solution. This is due to the fact, that the processors store and exchange predicted plans. These plans allow to reconstruct the primal solution from the computed dual solution. Finally we want to point out that the algorithm iterates through a series of feasible solutions, i.e., when it is stopped too early the solution might not be optimal yet, but it will be feasible.

We will show in the next section, that the algorithm performs well in practice in asynchronous communication networks. We show in particular, that the algorithm is extremely robust against packet losses in the communication.

4. DISTRIBUTED IMPLEMENTATION

The described model and algorithm was implemented in Matlab in two different versions. First, an asynchronous version of the algorithm was implemented, running on the spatially distributed computers attached to the institutes network. A second implementation was performing the distributed algorithm in a synchronous fashion on one computer. We used ROME, a modeling language for robust optimization, cf. Goh and Sim [2011], for the local subproblems and the IBM CPLEX solver for all optimization programs. In the following we first describe the devices we used, followed by details on the distributed implementation and finally a short computational study on the convergence and message loss.

For comparison we also implemented the complete centralized optimization problem using ROME and IBM CPLEX. For smaller size problems, e.g., two and four device cases, we were able to solve the robust economic dispatch problem centrally, and we could verify in this way that the proposed distributed algorithm computed the correct solution. However for the larger model considered in this section, the centralized approach ran into memory problems and could not compute the solution.

4.1 Network Model and Devices

In our study we consider a grid of 16 controllable devices [2] with an all-to-all communication. We implemented three small, five medium and three large generators plus two small, two medium size and one large storage. A small generator, e.g. gas turbine, and all storages, e.g. pumped hydro-power, are able to react on current disturbances, i.e., have nonzero diagonal entries in the gain matrices P_{ij}. Medium size and large generators have only subdiagonal entries, that is, they can react on past disturbances only.

A generator is modeled without relevant dynamics but with constraints on the maximum and minimum power output as well as with rate constraints. A storage has a direct feed-through from input to output, as well. Beyond it has an integral behaviour to simulate the storage level. The model incorporates constraints on maximum and minimum power output, rate constraints and minimum and maximum storage level. In general, the smaller a device the less restrictive are the respective rate constraints, but the more expensive is the produced power. The larger a device the larger is the maximum power output and storage capacity.

The considered uncontrollable devices are an uncertain demand and intermittent wind energy production, both without dynamics, i.e., the matrix Q contains only diagonal elements. The demand $p_D(\tau) = q_{D,pred}(\tau) + \delta_D(\tau)$ was modeled with a sinusoidal reference trajectory and infinity norm bounds on the disturbance, i.e.,

$$\Delta_D = \{\delta \in \mathbb{R}^T \mid \|\delta(\tau)\| \le d_{\max}(\tau)\}.$$

We used error bounds of 10% as reported in Vilar et al. [2012]. The wind energy production $p_W(\tau) = q_{W,pred}(\tau) + \delta_W(\tau)$ was modelled with a sinusoidal reference trajectory $q_{W,pred}(\tau)$, as well. The considered disturbance bounds were

$$\Delta_W = \{\delta \in \mathbb{R}^T \mid \|\delta(\tau)\| \le d_{\max}(\tau),\ \|\delta(\tau+1) - \delta(\tau)\| \le r_{\max}\}.$$

We chose values increasing from ±5% to ±20% of the installed capacity for the absolute bounds and a rate constraint of 20% according to Hodge and Milligan [2011] and Wan [2011].

4.2 Distributed Implementation

For the distributed implementation we used 16 personal computers, ranging from rather old dual core with 2.2 Ghz to eight core machines with 2.83 Ghz and available memory from two GB to eight GB. During the simulation they where partially used for daily tasks. Each computer was running the algorithm of one controllable device and communicating with all other devices. For the communication we used UDP sender and receiver objects of the DSP System Toolbox in Matlab. UDP is a very fast, low overhead transmission protocol without guarantee of delivery or flow control. It only provides checksums for data integrity and therefore seemed very suitable for the algorithm and our our simulation.

In Figure 1 the results of one simulation are shown. We stopped the optimization after the slowest processor completed 300 iterations. In the same time the fastest computer completed over 1200 iterations. It was more than 4 times faster, showing the asynchrony of the algorithm in practice. Furthermore the message loss is depicted with a color code from white meaning 0% loss over yellow and red to black depicting 100% message loss. Despite the heavy loss ranging from 40.7% up to 71.4% due to the simple UDP communication, the obtained solution agreed with the solution from the synchronous implementation with perfect communication up to 0.01%.

4.3 Message Loss and Convergence

The results of the asynchronous implementation motivated a computational study on the effect of message loss on the convergence which was performed with a synchronous implementation on a single computer. We simulated message loss

[2] 16 was the maximum number of personal computers available for the study.

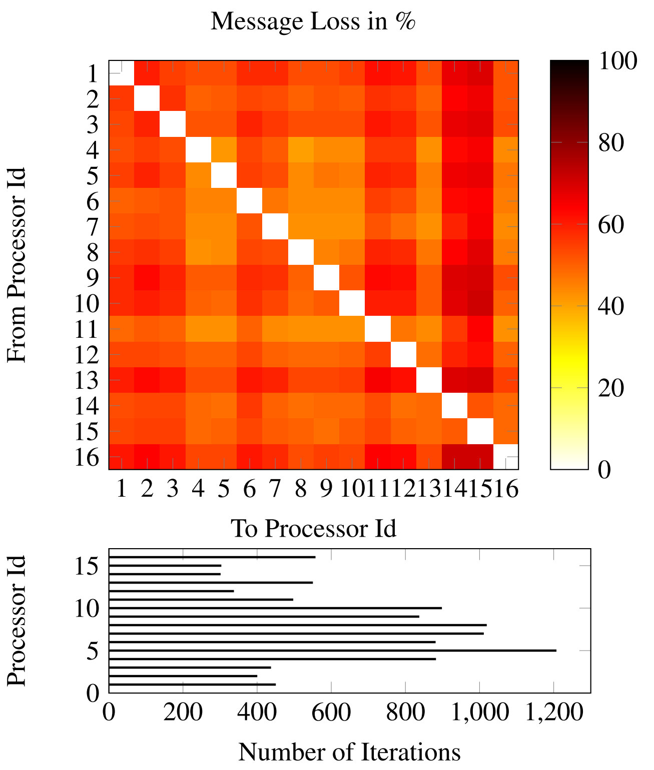

Fig. 1. Results of a distributed implementation using UDP communication between 16 personal computers with different hardware. Despite the heavy message loss and asynchrony the correct solution was found.

from 0% to 90% by dropping the desired percentage of messages chosen randomly through a uniform distribution. Figure 2 shows the convergence results of the 16 controllable devices case. We were not able to compute the optimal value J^* using the centralized approach. Therefore, the minimum considered here, i.e., J^*, is the best value achieved in a serial implementation of the novel algorithm. The result shows, that even with 90% loss the convergence is quite fast and about 40 to 80 iterations depending on the desired accuracy were enough.

5. CONCLUSION

We proposed a novel distributed, asynchronous algorithm to efficiently solve the economic dispatch problem under uncertainty. In particular we applied the theory of affinely adjustable robust counterparts and showed how the resulting optimization program can be solved in a distributed way by the recently proposed cutting-plane consensus algorithm. We provided a novel interpretation of the cutting-plane consensus as a trajectory exchange method, highlighting the connection to approaches in MPC literature. A computational study was performed that demonstrates the applicability of the proposed algorithm in highly heterogeneous, asynchronous networks with a high rate of message loss.

Ongoing work includes a more detailed modeling of the power network with transmission constraints as well as methods to decrease the problem dimension or the number of exchanged messages. Future work will focus on the implementation of the proposed algorithm in an receding horizon fashion.

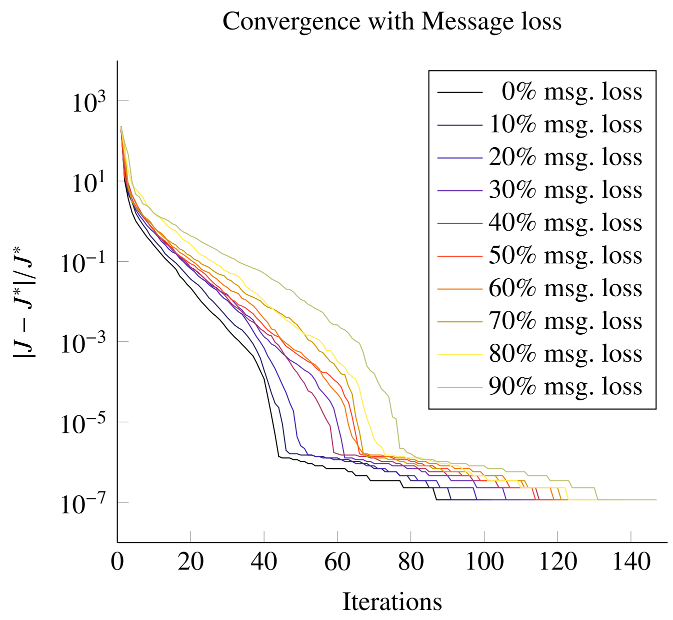

Fig. 2. Convergence of the algorithm in a synchronous implementation with different percentages of message loss.

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