Multibody Systems and Robot Dynamics

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Abstract

The method of multibody systems has been developed during the last two decades with application to various engineering topics, including robotics and walking machines. On the other hand, special algorithms for robot dynamics are available featuring the high computational efficiency required for control purposes. This paper shows the close relation between both approaches. Essential criteria for the efficiency of dynamics software are the numbers of coordinates used, which should be minimal. For illustration a two-body system is considered, including open and closed loop configurations.

1. INTRODUCTION

Many engineering systems, like vehicles, spacecrafts or robots, can be modeled as multibody systems. The complexity of the dynamical equations called for the development of computer-aided formalisms starting a quarter of a century ago. The state-of-the-art was presented at IUTAM symposia and other meetings, documented in the corresponding proceedings, see Magnus [1], Haug [2], Bianchi and Schiehlen [3]. More specialized formalisms have been developed for robot control featuring high computational efficiency, see Hollerbach [4], Walker and Orin [5], Vukobratovic and Kircanski [6], Brandl, Johanni and Otter [7], Valasek [8]. It is quite clear that a more specialized formalism can be better trimmed to fit the requirements. On the other hand, the equations of motion are invariant with respect to the generalized coordinates chosen. Thus, general purpose formalisms must generate the same results. Differences in computational complexity cannot be due to mechanical modeling but only to numerical evaluation. Special properties of a mechanical system may remain hidden in a general purpose programme, resulting in additional computational costs.

In this paper both the multibody system and the robot dynamic approach are presented. Both approaches are compared with respect to their computational efficiency. A two-body system with open and closed loop topology is used for illustration.

2. MULTIBODY SYSTEM APPROACH

Multibody systems consist of rigid bodies, constraint elements like joints, bearings and supports, and force elements realized by springs, dampers or servomotors, respectively. Using the free body principle for body i, the position is given by the 3x1-translation vector r_{Ii} of an arbitrary body-fixed point O_i and by the 3x3-rotation tensor S_{Ii} relating the body-fixed frame i to the inertial frame I, Fig. 1. Then, by differentiating, the absolute 3x1-translational and rotational acceleration vectors a_{Ii} and α_{Ii}, respectively, are obtained.

For the formulation of the dynamical Newton's and Euler's equations the body-fixed frame i is more adequate. Therefore, the acceleration vectors will be resolved in frame i and represented by an overall 6x1-acceleration vector

$$a_i = [a_{Ii}^T S_{Ii} \quad \alpha_{Ii}^T S_{Ii}]^T. \tag{1}$$

Then, Newton's and Euler's equations read as

$$M_i a_i + k_i = q_i \quad , \quad i = 1,\dots,p, \tag{2}$$

where M_i is a time-invariant 6x6-inertia matrix composed of the masses m_i, the 3x1-vector c_i between the reference point O_i and the centre of mass C_i and the 3x3-tensor of moments of inertia I_{Oi}. Further, the 6x1-vector k_i represents the gyroscopic forces and $q_i = [f_{Oi}^T \ l_{Oi}^T]^T$ is the 6x1-vector summarizing the 3x1-force vector f_{Oi} and the 3x1-torque vector l_{Oi} also resolved in the body-fixed frame i. Introducing q constraints, the free system of p bodies is assembled as a holonomic system. Then, all the kinematical quantities depend on the $f = 6p-q$ generalized coordinates of the system represented by the fx1-position vector y. In particular, it yields for scleronomic systems

$$r_{Ii} = r_{Ii}(y) \quad , \quad S_{Ii} = S_{Ii}(y). \tag{3}$$

By differentiating with respect to the inertial frame considering the generalized coordinates and resolving in the body-fixed frame, one finally arrives at the 6x1-acceleration vector

$$a_i = \overline{J}_i \ddot{y} + \overline{a}_i \quad , \quad i = 1,\dots,p \tag{4}$$

where the 6x1-Jacobian matrix $\overline{J}_i$ is introduced, see e.g. Ref. [9].

Further, the reaction forces and torques $\overline{q}_i^r = [f_i^{rT} \ l_i^{rT}]^T$ have to be added

$$q_i = \overline{q}_i^e + \overline{q}_i^r = \overline{q}_i^e + \overline{Q}_i g \tag{5}$$

where $\overline{q}_i^a$ are the remaining applied forces, $\overline{Q}_i$ is a 6xq-distribution matrix and g represents the qx1-vector of the generalized reaction forces. Then, the Newton-Euler equations of the total system read as

$$\overline{\overline{M}}\ \overline{J}\ \ddot{y} + \overline{k} = \overline{q} + \overline{Q}g \tag{6}$$

where $\overline{\overline{M}} = \text{diag}\ (M_i) = \text{const},\ \overline{J}^T = [\overline{J}_1^T\ \overline{J}_2^T\ \ldots\ \overline{J}_p^T]$ and $\overline{Q}^T = [\overline{Q}_1^T\ \overline{Q}_2^T\ \ldots\ \overline{Q}_p^T]$ represent global matrices of the system. Due to the orthogonality of free motions and reaction forces in constrained systems it yields according to the principle of virtual work

$$\overline{J}^T\ \overline{Q} = 0. \tag{7}$$

Using (7) the differential algebraical system of equations (6) can be resolved in the pure differential equations of motion

$$M\ \ddot{y} + k = q \tag{8}$$

and the pure algebraical equations of reaction

$$N\ g + \hat{q} = \hat{k} \tag{9}$$

where $M = \overline{J}^T\ \overline{\overline{M}}\ \overline{J}$ is the fxf inertia matrix and $N = \overline{Q}^T\ \overline{\overline{M}}^{-1}\ \overline{Q}$ represents the gxg-reaction matrix of the constrained system. The solution of both equations (8) and (9) requires first of all the inversion of the inertia or reaction matrix, respectively, which is expensive and time-consuming for larger systems.

Even if the principle of virtual power is used, sometimes referred to as Kane's equations, see e.g. Quirt and Anderson [10], the inversion of the inertia matrix cannot be avoided.

3. THE ROBOT DYNAMICS APPROACH

Robots are multibody systems with chain topology. This special property can be used to achieve recursive formalisms less expensive and time-consuming.

According to Fig. 2, it yields for the absolute acceleration of body 2 in its body-fixed frame

$$a_2 = C_2 a_1 + J_2 \ddot{y}_2 + \xi_2 \tag{10}$$

and for its the reaction

$$\overline{q}_2^r = Q_2 g_2 - C_3^T Q_3 g_3, \tag{11}$$

where the local matrices C_i, J_i, Q_i are related to joints 2 and 3. Note that both the f_ix1-vector y_i of generalized coordinates and the q_ix1-

vectors g_i of generalized reaction forces are related to the corresponding joints.
Summarizing these results for the total system, it follows from (10) and (11)

$$a = C\,a + J\,\ddot{y} + \xi, \tag{12}$$

$$\bar{q}^r = (E-C)^T\,Q\,g, \tag{13}$$

where C is a 6px6p-sparse matrix with the matrices C_i on the lower sub-diagnonal and $J = \mathrm{diag}(J_i)$, $Q = \mathrm{diag}(Q_i)$. Then, comparing with (6), the following global matrices are obtained

$$\bar{J} = (E-C)^{-1}\,J, \tag{14}$$

$$\bar{Q} = (E-C)^T\,Q. \tag{15}$$

While $\bar{Q}$ remains a 6pxq-sparse matrix, $\bar{J}$ is a 6pxf-lower block-triangular matrix.
By definition, from (8) and (9) it is found that even for a chain system the fxf-inertia matrix is a full matrix; only the qxq-reaction matrix is a band matrix. However, the fxf-inertia matrix M has a special structure shown in Ref. [11]. The application of the Gaussian algorithm starting with the last row and the introduction of proper abbreviations results in a lower triangular fxf-inertia matrix $\hat{M}$ which means a completely recursive formalism, see Ref. [11]. Then, the corresponding differential equations read as

$$\hat{M}\,\ddot{y} + \hat{k} = \hat{q} \tag{16}$$

with properly changed fx1-vectors $\hat{k}$ and $\hat{q}$. Thus, it turns out that the essential point of all the recursive robot dynamic formalisms is the choice of a minimal number of relative coordinates related to the joints of the chain.

4. COMPUTATIONAL EFFICIENCY

In a detailed study Valasek [8] has shown that the multibody system approach with a full inertia matrix requires $O(n^3)$ operations while the robot dynamics approach needs only $O(n)$ operations. Up to $p = 8$ bodies the multibody system approach is still competitive with the robot dynamics approach due to the rather complicated expression of matrices $\tilde{M}_i$, shown in Ref. [11]. However, there remains a principal difference between both approaches. The robot dynamics approach is restricted to chain topology while the multibody system approach is quite general. It will be shown, however, that the recursively computed equations of motion can be used to analyse the closed loop topology of robots often found in technological applications.

The equations of motion of the open loop robot read from (8) or (16), respectively, as

$$\ddot{y} = M^{-1}(q - k). \tag{17}$$

The q_c closing conditions constrain the robot's motion explicitly or implicitly,

$$y = \begin{bmatrix} z \\ \hline y_c(z) \end{bmatrix} \quad or \quad \phi(y) = 0\ , \tag{18}$$

respectively, and can be found from robot kinematics, see Eppinger and Kreuzer [12]. Here, z means the vector of the $f_c = f - q_c$ conserved coordinates of the robot, and $y_c(z)$ as well as $\phi(y)$ are q_cx1 - vector functions. Then, introducing (18) in (17), the reactions have to be added again,

$$\begin{bmatrix} \ddot{z} \\ \hline I_c(z)\ddot{z} + \dot{I}_c(z)\dot{z} \end{bmatrix} = M^{-1}\{(q-k) + Q_c\, g_c\}. \tag{19}$$

According to the virtual work principle it yields $Q_c^T[E \mid I_c^T]^T = 0$ and from (19) the q_cx1-vector of the generalized closing reactions is easily obtained

$$N_c\, g_c = Q_c^T\{M^{-1}(q-k) - [0 \mid (\dot{I}_c\dot{z})^T]^T\} \tag{20}$$

where $N_c = Q_c^T M^{-1} Q_c$ is the q_cxq_c-reaction matrix. Then, the first f_c equations of (19) represent the equations of motion of the closed loop robot. This approach is computationally efficient since the open loop equations (17) have to be supplemented only by the $q_c < 6$ closing reactions available from (20).

5. ILLUSTRATION BY TWO-BODY ROBOT

An example that simply illustrates the method proposed is a two-body robot, Fig. 3. The first body is attached by a pin joint to the ground; the second body is attached at its center of mass C_2 by a pin joint to the first body, resulting in an open chain. Using the angles α and δ as relative generalized coordinates, the equations of motion (8) read as

$$\begin{bmatrix} I_1+m_2R^2+I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} M_1+F_2(R\sin\alpha+L\sin(\alpha+\delta)) \\ F_2\, L\sin(\alpha+\delta) \end{bmatrix} \tag{21}$$

with a full 2x2-inertia matrix M. Further, by application of the Gaussian algorithm or a recursive formalism according to (16) the equations of motion read as

$$\begin{bmatrix} I_1+m_2R^2 & 0 \\ I_2 & I_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} M_1+F_2R\sin\alpha \\ F_2\,L\sin(\alpha+\delta) \end{bmatrix} \tag{22}$$

showing the triangular form of the 2x2 inertia matrix M. In this special case, a further simplification is possible by choosing absolute generalized coordinates α, β resulting in a diagonal inertia matrix,

$$\begin{bmatrix} I_1+m_2R^2 & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} M_1+F_2R\sin\alpha \\ F_2\,L\sin\beta \end{bmatrix}. \tag{23}$$

For closing the loop, eqs.(23) are rewritten according to (19) as

$$\begin{bmatrix} \ddot{\alpha} \\ \hline \beta'\ddot{\alpha} + \beta''\dot{\alpha}^2 \end{bmatrix} = \begin{bmatrix} (I_1+m_2R^2)^{-1} & 0 \\ 0 & I_2^{-1} \end{bmatrix} \left\{ \begin{bmatrix} M_1+F_2\,R\sin\alpha \\ \hline F_2\,L\sin\beta \end{bmatrix} + \begin{bmatrix} R\cos\alpha \\ \hline -\,L\cos\beta \end{bmatrix} F \right\} \tag{24}$$

where $L\sin\beta(\alpha) = R\cos\alpha$ represents the kinematical relation. Following (20) the generalized closing relation is obtained as

$$\begin{aligned} F = {} & \frac{1}{I_2\,R^2\cos^2\alpha + I\,L^2\cos^2\beta}\,[(-\,I_2\,R\cos\alpha)M_1 \\ & + (-I_2\,R^2\sin\alpha\cos\alpha + I\,L^2\sin\beta\cos\beta)F_2 \\ & + (-\,I\,L\,\beta\cos\beta)\;I_2\dot{\alpha}^2], \\ & I = I_1+m_2R^2. \end{aligned} \tag{25}$$

Finally, (25) is put to the first equation of (24) and the closed loop equation is at hand, making full use of the recursively inverted inertia matrix of the open chain system.

6. CONCLUSION

It has been shown that the multibody system approach and the robot dynamics approach are closely related. The essential point is the choice of a minimal number of relative coordinates featuring sparse matrices. Further, it is possible to make full use of the recursively inverted inertia matrix for closed lpop systems applying the concept of generalized reaction forces in the closing joint. Then, high computational efficiency is achieved.

ACKNOWLEDGEMENTS

The author acknowledges the generous support of this work during a sabbatical spent at the Department of Mechanical Engineering, Queen's University, Kingston, Canada.

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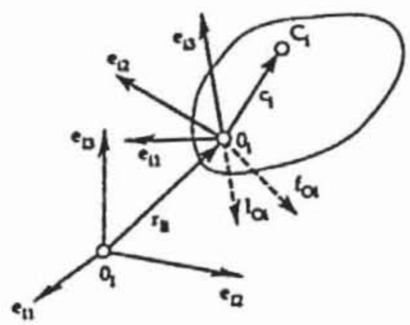

Fig. 1. Free body

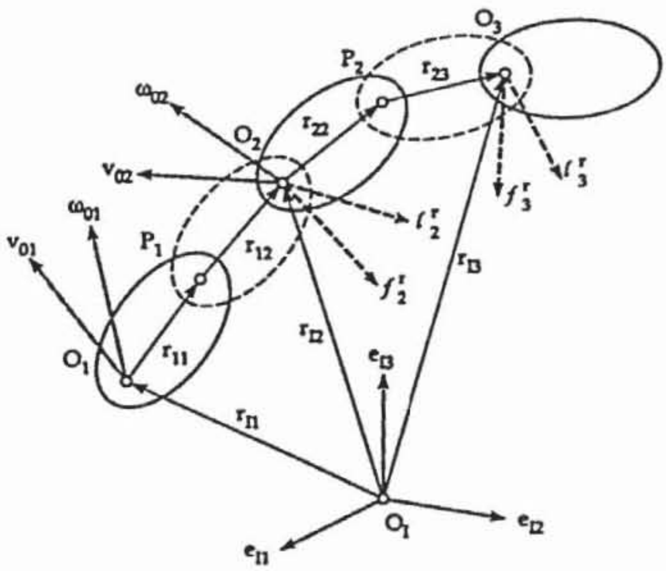

Fig. 2 Chain of 3 bodies and 2 joints

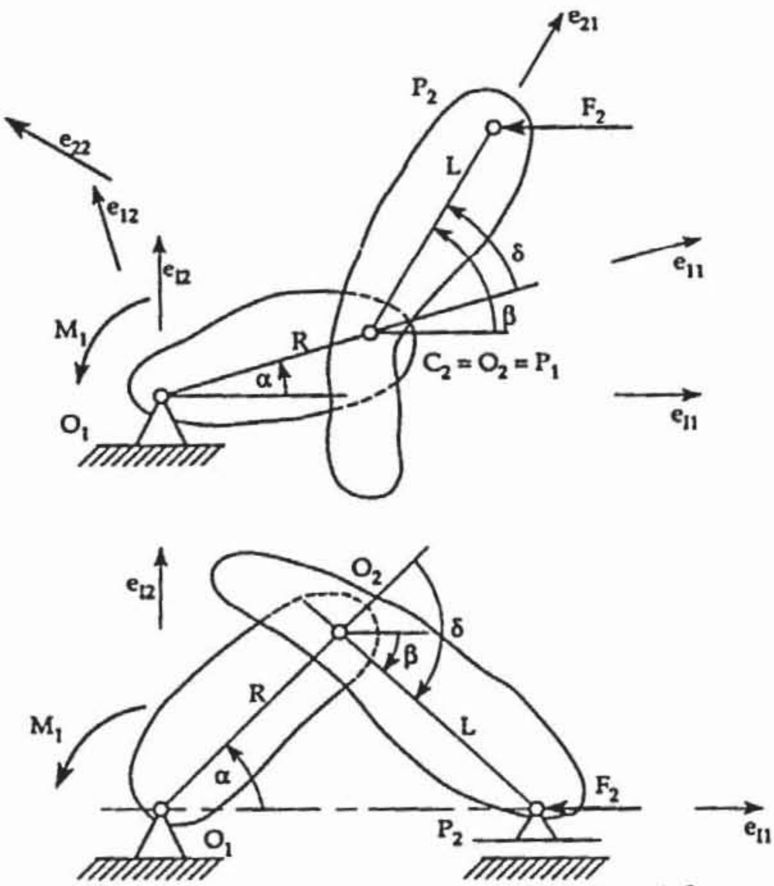

Fig. 3 Two-body robot, open and closed loop