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Design consideration of a dual-beam ESPI optical system for contouring

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Design consideration of a dual-beam ESPI optical system for contouring. A discussion of the essential technical advantages, limitations, various kinds of errors and required characteristics of dual-beam illumination ESPI optical system by tilting illuminations for contouring is attached. Attractive features are the simplification of the optical system, the stationary reference contour planes and the possibility of adjusting various parameters for contouring.

Konstruktionsüberlegungen für optische Zweistrahl-ESPI-Systeme für die Konturierung. Eine Diskussion der wichtigen technischen Vorteile, Begrenzungen, verschiedene Fehlerquellen und notwendige Charakteristiken des optischen Zweistrahl-ESPI-Systems bei schräger Beleuchtung wird vorgestellt. Attraktive Eigenschaften sind die Vereinfachung der optischen Systeme, die stationären Kontur-Referenzebenen, und die Möglichkeit der Justierung verschiedener Parameter an die Konturierung.

1. Introduction

Electronic speckle pattern phase-shifting interferometry is a kind of technique for nondestructively testing optically rough components [1] and has been known for a long time. Contouring using dual-beam illumination ESPI by tilting the illumination beams [2] which has been recently reported is a good technique for testing various kinds of optically rough surfaces because of its variable sensitivity and measuring range. The interferometer has the another advantage that the imaging system does not have to resolve the fine grating structure and can therefore be easily adopted to ESPI. By altering the illuminating angles, this technique can conveniently change the measurement sensitivity like two-wavelength techniques and allows the measurement of profiles of deeper surfaces than single-wavelength techniques with the smooth reference beam do [3]. It uses only one wavelength to measure objects so that the better contrast of correlation fringes can be easily obtained than that in the two-wavelength techniques [4–5].

This paper discusses the essential technical advantages, limitations, various kinds of errors and required characteristics of ESPI optical system by tilting dual-beam illuminations for contouring. The stationary reference contour planes, the formulae of various kinds of errors and the possibility of adjusting various parameters for contouring are presented. Several available optical arrangements are given out.

2. Optical arrangement of a dual-beam phase-shifting speckle pattern interferometer

Fig. 1(a) shows the schematic of the experimental arrangement of a dual beam interferometer. The laser beam is collimated and expanded. Then it is split into two illuminating beams *A* and *B* of equal amplitudes via a beam splitter. To introduce a phase shift one of the mirrors is attached to a piezoelectric translator (PZT). The two beams with an included angle illuminate the object to

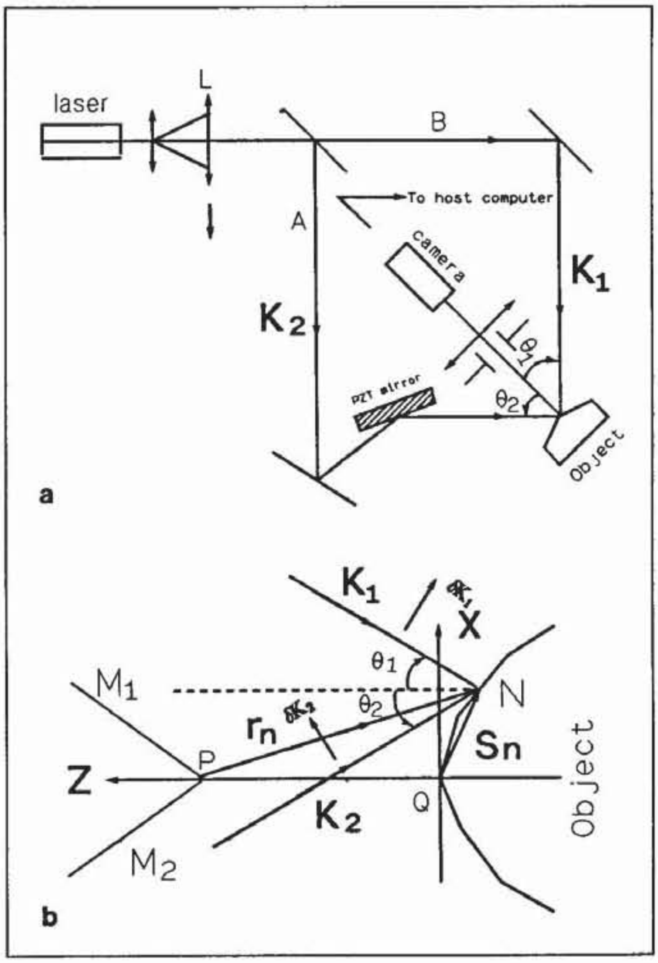

Fig. 1. (a) The schematic optical arrangement of a dual-beam interferometer. (b) The vector geometry of the interferometer.

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be contoured. A TV camera images the object which is digitized by a host computer and displayed on a monitor. Two exposures are made with the object when the two illuminating beams are tilted, and the subtracted speckle pattern is displayed on the monitor.

3. Theory and the formation of the stationary reference contour planes

Referring to fig. 1(b), let $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ be the unit direction vectors of two illuminating beams, respectively. One of them, $\boldsymbol{K}_2$, is reflected by PZT mirror. The correlation fringes are obtained by subtracting the intensity records before and after tilting anti-symmetrically two illuminating beams. The phase difference determined the correlation fringes can be derived as follows [2]

$$\begin{aligned}\Delta\Psi_{nq} &= \frac{2\pi}{\lambda}\boldsymbol{S}_n\cdot(\delta\boldsymbol{K}_2-\delta\boldsymbol{K}_1)\\ &= \frac{2\pi}{\lambda}M(\boldsymbol{S}_n\cdot\boldsymbol{d}),\end{aligned} \tag{1}$$

where $\boldsymbol{S}_n$ is the relatively positive vector between the test points and $\delta\boldsymbol{K}_1$ and $\delta\boldsymbol{K}_2$ are the differential vectors of the illuminating beams $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ respectively when the illumination beams are tilted; $\delta\beta$ is the tilting angle of the illuminating beams; θ_1 and θ_2 are the illuminating angles of beams A and B, respectively; $\boldsymbol{d}$ and M are the unit director vector of the contour planes and the measurement sensitivity of this interferometer, respectively, and

$$\boldsymbol{d} = (\delta\boldsymbol{K}_2-\delta\boldsymbol{K}_1)\Big/\left[2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\right], \tag{2}$$

$$M = 2\delta\beta\sin\left(\frac{\theta_1+\theta_2}{2}\right). \tag{3}$$

Eq. 2 and 3 show that the direction and the interval of the contour planes depend on only the parameters of the ESPI optical system and are not related to the test object. Therefore, for an optical system with certain parameters the contour planes are stationary reference planes not related to the test object. According to our habits observing the object, the nearer the test point is apart from the viewing system, the higher the test point is. Through eq. 1, we can derive that the positive direction of $\boldsymbol{d}$ should be identical to Z axis in order that the measuring results are completely the same as the shape of the test object. Here the positive direction of the vector $\boldsymbol{d}$ is defined as the direction along which the highness of the test object increases.

4. Determination of the measurement sensitivity and the measuring range

Under the standard measurement, let $\theta_1=\theta_2=\theta$, and the measurement sensitivity can be written

$$M = 2\delta\beta\sin\theta. \tag{4}$$

Eq. 3 and 4 show the possibility of changing the measurement sensitivity by altering the illuminating angles when the wavelength λ and the tilting angle are determined.

In order to avoid 2π ambiguity of the speckle interferometer, the only constraint on the spacing of detector elements is that from one point detector to the next, the change in phase due to the tilt of the illuminating beams can not be larger than π. This constraint can be expressed as follows when the normal contour planes are obtained

$$\sigma/P\max\left(\frac{dz}{dx}\right) < \frac{\lambda}{M}, \tag{5}$$

where σ is the spacing of the adjacent two detector elements; P is the magnification of the viewing system; $\frac{dz}{dx}$ is the grade along X axis of the test object. During measuring, the illuminating angles must be appropriate to the grade $\frac{dz}{dx}$ in order to keep the even illumination. The larger the grade is, the smaller the illuminating angles should be. This is just appropriate to the requirement of the sensitivity, i.e. the larger the grade is, the lower the sensitivity should be. In general, the proportion of two illumination intensities I_1 and I_2 should be within the range from 1:5 to 5:1. In practical engineering applications, we first choose the suitable illumination angles, then adjust the tilting angle of the illuminating beams so that the suitable sensitivity can be obtained.

The measuring range of the test object depends on the magnification P of the imaging objective and the size of the detector arrays. The area of the measuring range is P times as large as the area of the detector arrays. In general, we can change the magnification P to meet the requirement of the measuring range.

5. Phase shifting dual-beam speckle pattern interferometer

To obtain quantitative data from the interferograms, the phase of one of the illuminating beams in the speckle interferometer is shifted with respect to the other in steps. The phase maps are calculated from the phase shifted intensity data taken before and after the tilt of the illuminating beams. If the illuminating beams $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ produce two different speckle patterns Γ_1 and Γ_2, the difference phases between the speckle patterns are evaluated by taking three frames of intensity data in our experiment, while shifting the phase of one of the beams. If the illuminating beam $\boldsymbol{K}_2$ is reflected by mirror attached with PZT, three frames of intensity data are recorded as follows:

$$I_1 = I_0[1+\gamma\cos(\Delta-\delta)] \tag{6a}$$

$$I_2 = I_0[1+\gamma\cos(\Delta)] \tag{6b}$$

$$I_3 = I_0[1+\gamma\cos(\Delta+\delta)], \tag{6c}$$

where I_0 is the average intensity; γ is the modulation factor; δ is the constant phase shift step; and Δ is the

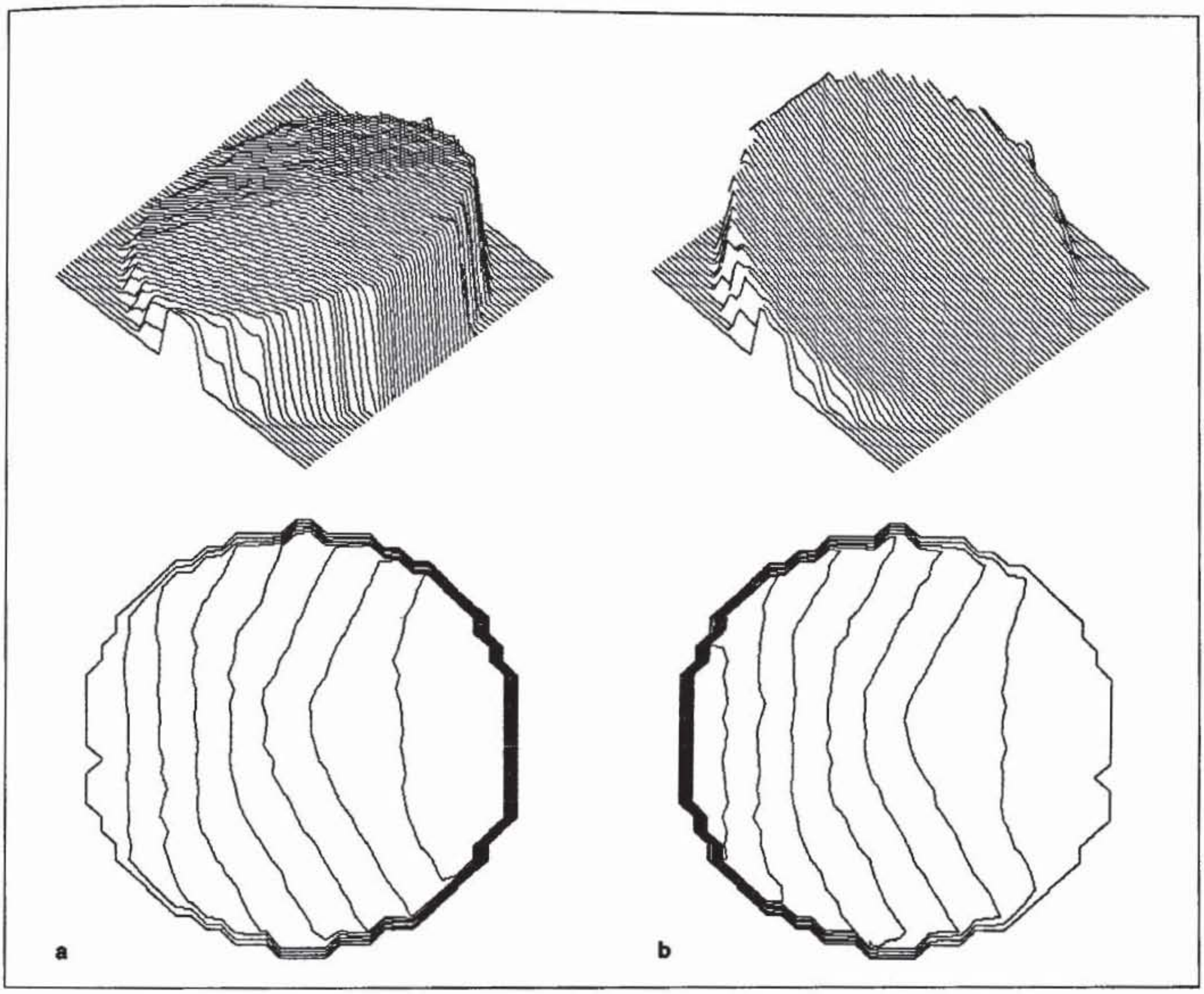

Fig. 2. 3-D plots and phase maps of pyramid as the test object when the reference beam is stationary, i.e. $\delta \boldsymbol{K}_2 = 0$ and (a) $\boldsymbol{K}_1$ is rotated anti-clockwise; (b) $\boldsymbol{K}_1$ is rotated clockwise.

difference phase to be determined between two illumination beams. The phase difference calculated at each point in the interferogram is

$$\Delta = \arctan\left[\frac{I_3 - I_1}{I_3 + I_1 - 2I_2}\right]\tan(\delta/2).$$

$\tan(\delta/2)$ is unity as the phase shift in our case is 90°. Eq. 6 shows that the measurement result Δ is the advance phase of the beam $\boldsymbol{K}_2$ reflected by PZT mirror (usually called the reference beam and expressed as $\boldsymbol{K}_r$) with respect to the beam $\boldsymbol{K}_1$ (usually called the measuring beam and expressed as $\boldsymbol{K}_m$). Thus the direction vector $\boldsymbol{d}$ of the contour planes can usually be written for all of the speckle pattern interferometer

$$\boldsymbol{d} \propto f_1(\boldsymbol{K}_r) - f_2(\boldsymbol{K}_m), \tag{7}$$

where $f_i(\boldsymbol{K})$ is a function expression related to the vector $\boldsymbol{K}$.

This formula makes the possibility of determining the director of the vector $\boldsymbol{d}$ in eq. 1 and 2 when the interferometer with completely symmetric dual-beam illuminations is adopted. This is very significant for choosing the direction of the contour planes and the tilting director of the illuminating beams, calibrating the inclined contour planes and constructing the reasonable optical arrangements[6].

From eq. 7 and 1, we find that 3-D plot of the test object is similar to itself when $\delta \boldsymbol{K}_2 = 0$ and $\boldsymbol{K}_1$ is rotated anti-clockwise; and the 3-D plot of the test object is inversion of the test object when $\delta \boldsymbol{K}_2 = 0$ and $\boldsymbol{K}_1$ is rotated clockwise. Fig. 2(a) and (b) show that the conclusions are right.

6. Formation and compensation of system error

This contouring technique needs the eikonals [7] of two illumination beams from the source S to each test point

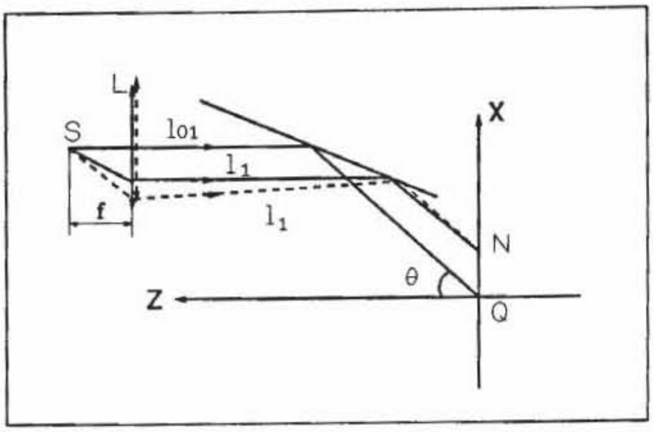

Fig. 3. Geometry for analysis of the change of the optical paths.

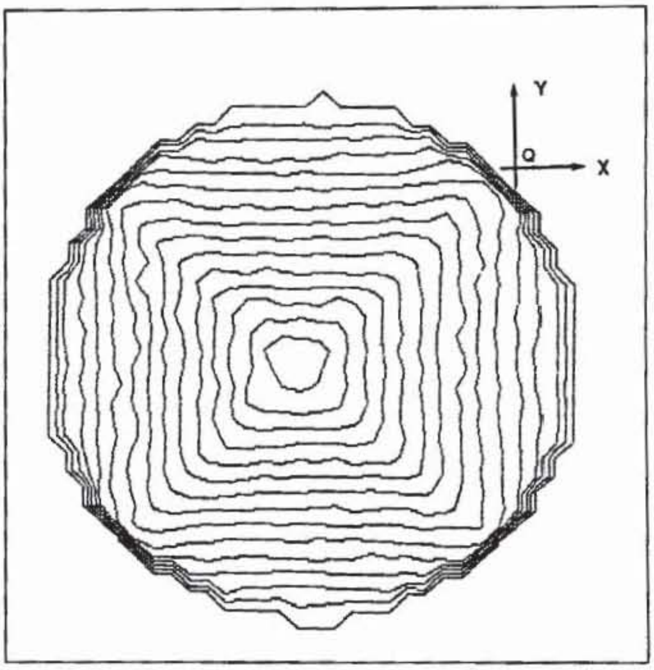

Fig. 4. The distorted contour map due to the change of the optical paths.

of the object constant before and after the tilt [2–3]. This requirement is difficult to be met especially for the large test objects. The most simple and convenient method of tilting the illumination beams is to shift the collimated lens. The measuring error Δ due to change of the eikonals caused by shifting the collimating lens has been discussed in the Appendix A when the plane illuminating wavefront is used. The conclusion is as follows (referring to fig. 3)

$$\Delta \cong \frac{x^2 \delta\beta \sin 2\theta}{f} + \frac{2x\, l_{01}}{f} \sin\theta\, \delta\beta^2 . \tag{8}$$

The contour map with this kind of error of pyramid is presented in fig. 4 and the distortion along X axis can be observed from fig. 4 and from the "live" fringes on the monitor during the experiment. The change Δ of the optical paths will directly be added in the measuring values, so eq. 8 also shows the possibility of calibrating this kind of error by host computer.

7. The measuring error caused by the inclined contour plane due to tilting the illuminating beams

Eq. 1 can be derived out when we approximately consider that the change $\delta \boldsymbol{K}$ of the illuminating vector $\boldsymbol{K}$ is perpendicular to the vector $\boldsymbol{K}$. In fact, this approximate will cause the direction error of the vector $\boldsymbol{d}$, thus the measuring error is caused. When we consider the effect of the tilting angle, the vector geometry of the difference vectors $\delta \boldsymbol{K}_1$ and $\delta \boldsymbol{K}_2$ is drawn in fig. 5. The practical director angle α of the contour planes can be expressed as

$$\alpha = \frac{1}{2}(\theta_1 - \theta_2) - \frac{\delta\beta}{2} . \tag{9}$$

When $\theta_1 = \theta_2$, the error $\Delta\alpha$ of the angle α due to the tilt of the illuminating beams is $-(\delta\beta/2)$. The relationship between the measuring values η and the real shape z can be expressed as [6]

$$z = (\eta - x \sin\alpha)/\cos\alpha , \tag{10}$$

where x is X-coordinate value of the test point. The error of the measuring values can be written in our symmetrically experimental arrangement [6]

$$\Delta z = \rho + x(\delta\beta/2) , \tag{11}$$

where ρ is the stochastic error of the measuring values. $\delta\beta$ is about 2 mrad in our experiment. If not calibrated, the largest systematic error will be about 20 µm when the diameter of the measuring range is 40 millimeters. Therefore, it is very necessary to calibrate the measuring values according to eq. 11 in our optical arrangement in order to get more precious results.

8. Decorrelation effect on the dual-beam interferometer

When phase-shifting techniques are applied to speckle pattern interferometry, the randomness of the speckle will

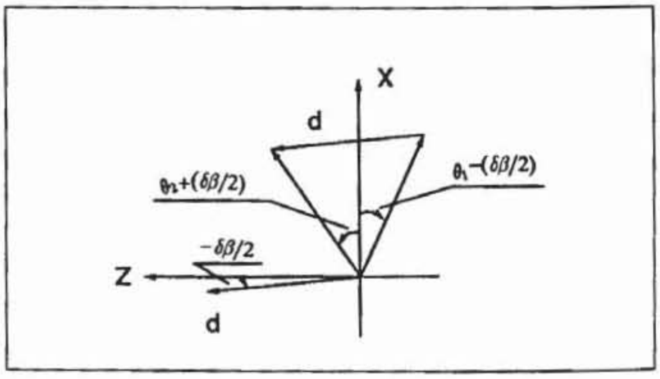

Fig. 5. The vector geometry when considering the effect of the tilting angle.

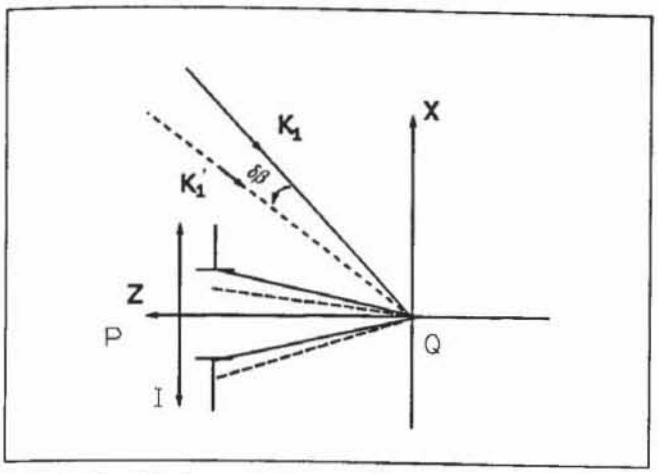

Fig. 6. Speckle decorrelation due to the collection of different scattering contributions.

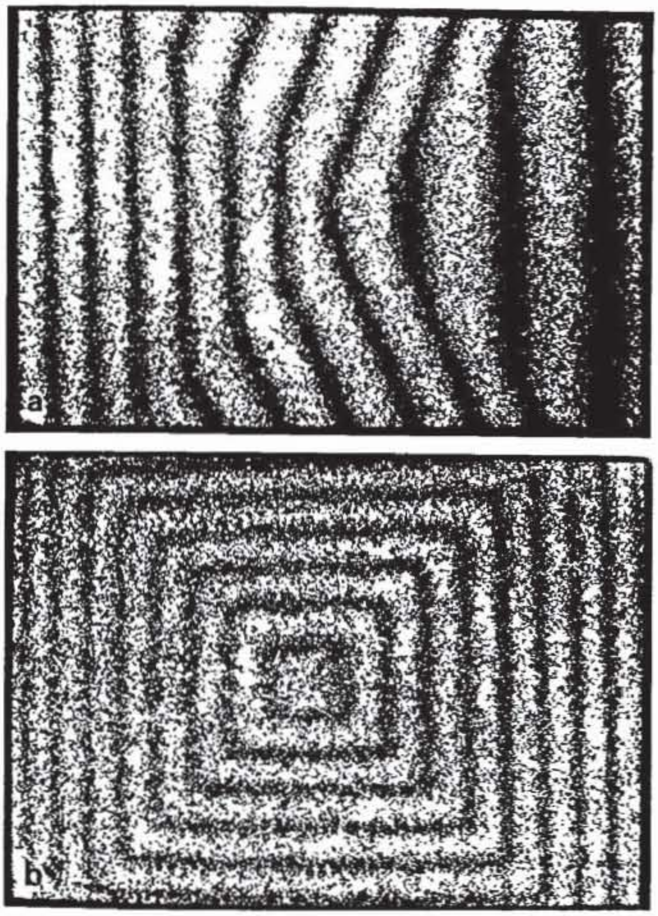

Fig. 7. (a) The contour fringes when tilting one of two illumination beams. (b) The correlation fringes when tilting two illumination beams.

cause many noisy data points. Other than points where the intensity saturates the CCD element, there are two main contributors to these noisy points which are fundamental limitations to this technique. They are (1) decorrelation of the speckle between before and after the tilt of the illuminating beams, and (2) low modulation of the measured intensity at a given pixel as the phase is shifted [8].

Speckle decorrelation in this case is caused by changes in the collected scattering angles from the test object which contribute to a single image point. Each point on the object imaged onto CCD element scatters light into a large solid angle. In general, the imaging system will only collect a small portion of this light. As the illuminating beams are tilted, the incident angles of two illuminating beams will be changed, and different scattering contributions will be collected by the optical imaging system than before the tilt. The imaging system will have a displaced collecting cone compared to the situation before the tilt as shown in fig. 6. Thus the intensity and phase of the imaged speckle will change slightly. According to this theory, if the correlation function is $\mathbb{R}_1$ when one of the illuminating beams is tilted, we can get the correlation function $\mathbb{R}_2$ with dual-beam tilt as follows

$$\mathbb{R}_2 = \mathbb{R}_1 * \mathbb{R}_1 . \tag{12}$$

Obviously, if we tilt one of the illuminating beams, the decorrelation will decrease compared to tilting dual beams with the same angle. This effect can be demonstrated in fig. 7 which shows two groups of pictures with the correlation fringes when the tilting angles is same.

Low modulation is a foundational problem in all phase-shifting techniques [8]. It can be improved by decreasing the size of the detector element and suitably increasing the intensity of the illuminations.

9. Tilting one of the illuminating beams

When tilting one of the illuminating beams, the better contrast of the correlation fringes can be obtained and the change Δ of the optical paths will increase compared to the tilting dual-beam illuminations. Although this technique is not related to static illuminating beams, the illumination angle of the beam still is limited by the requirement of the illuminating evenness. At the same time, the sensitivity is limited by $M = \delta\beta$ so that the adjustable range of M is very small. Usually we can use the anti-symmetric optical arrangements tilting dual-beams instead of tilting one of two illuminations in the practical engineering applications so that the measuring sensitivity can be adjusted.

10. Several available optical arrangements

The optical arrangement in the fig. 1(a) can work well only for simultaneously tilting two illumination beams and the optical arrangement in the fig. 8(a) is suitable to tilting one or two illumination beams. Fig. 8(b) shows the anti-symmetric optical arrangement with two beams tilted. Fig. 8(c) shows the optical set-ups with one illumination tilted. At this time, the reflectors should be controlled by the same computer interface so that the same amount of rotation can be obtained.

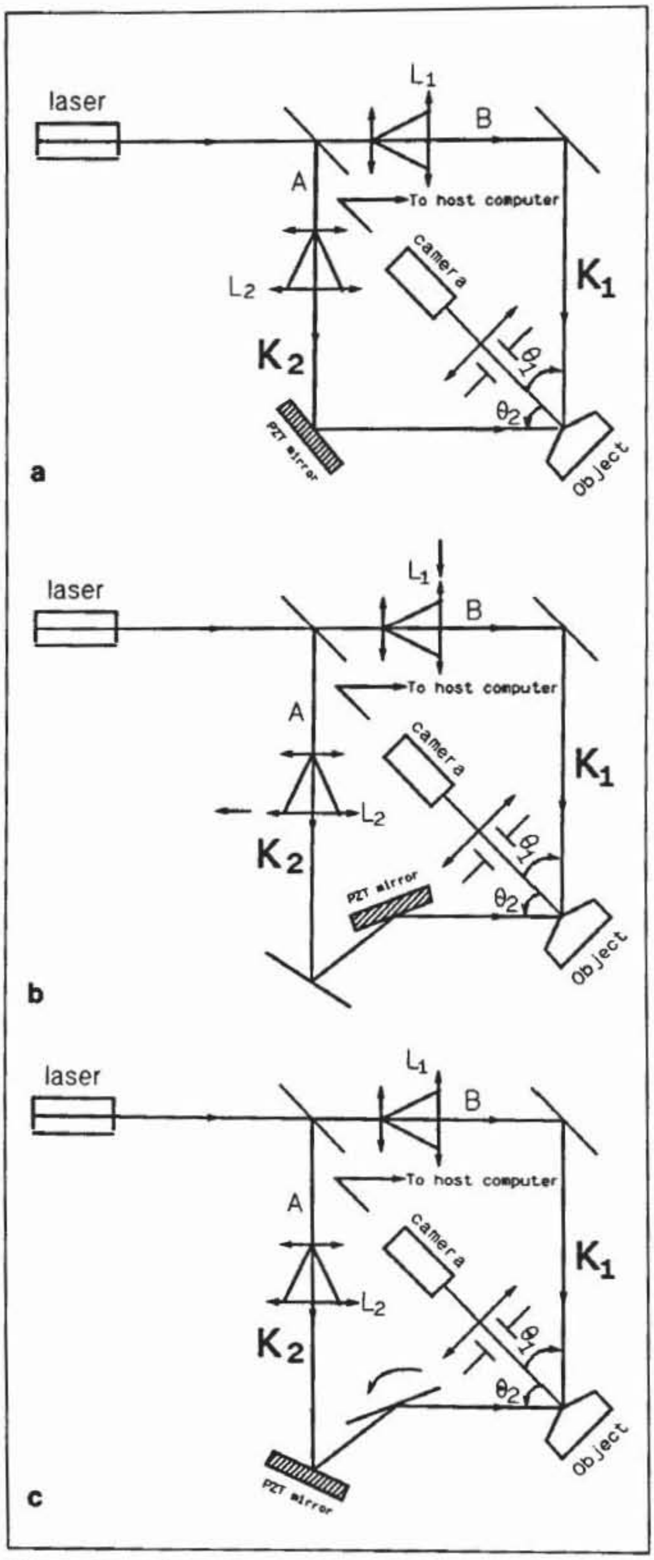

Fig. 8. Several practical optical arrangements.

The choice of the experimental arrangement should meet the requirements of the illumination evenness. When the range of (dz/dx) values is not symmetric about zero, we adopt the optical arrangements in fig. 8(b) or fig. 8(c) so that the more even illuminations can be obtained. At this time, the arrangement in fig. 8(b) is of the advantage of adjustable sensitivity.

Appendix A: Influence of the change of the optical paths

In our set-up the scanning of the illumination beams was introduced by shifting the collimation lens L (see fig. 1(a)). The optical paths before and after shifting L will be different from each other. We assume that the eikonals of the beam illuminating the same test point N in the illumination K_1 are l_1 and l'_1, respectively, before and after shifting the lens and l_{01} and l'_{01} are the eikonals of the illumination K_1 on the test central point Q before and after tilting. If the shift is very small, we assume that the thickness D and D' of the collimation lens passed by the light l_1 and l'_1 are constant; i.e. $D = D'$. The change Δl_1 of the eikonal l_1 due to the tilt can be calculated as follows (see fig. 3)

$$l_1 = (l_{01} - f - x\sin\theta) + \sqrt{f^2 + (x/\cos\theta)^2} + nD,$$

and

$$\begin{aligned} l'_1 &= (l'_{01} - f - x\sin\theta)/\cos\delta\beta \\ &\quad + \sqrt{f^2 + [x/\cos\theta + (l'_{01} - x\sin\theta)\,\delta\beta]^2} + nD \\ &\cong (l'_{01} - f - x\sin\theta) \\ &\quad + \sqrt{f^2 + [x/\cos\theta + (l'_{01} - x\sin\theta)\,\delta\beta]^2} + nD', \\ &\qquad (\delta\beta \ll 1) \end{aligned}$$

we can derive out

$$\begin{aligned} \Delta l_1 = l'_1 - l_1 &= (l'_{01} - l_{01}) \\ &\quad + \sqrt{f^2 + [x/\cos\theta + (l_{01\cdot} - x\sin\theta)\,\delta\beta]^2} \\ &\quad - \sqrt{f^2 + (x/\cos\theta)^2}, \end{aligned} \tag{13}$$

where n is the refractivity of the collimation lens. According to the same method, we can get

$$\begin{aligned} \Delta l_2 = l'_2 - l_2 &= (l'_{02} - l_{02}) \\ &\quad + \sqrt{f^2 + [x/\cos\theta + (l_{02\cdot} + x\sin\theta)\,\delta\beta]^2} \\ &\quad - \sqrt{f^2 + (x/\cos\theta)^2}. \end{aligned} \tag{14}$$

In the completely symmetric optical arrangement as shown in fig. 1(a), we can get

$$(l'_{01} - l_{01}) = (l'_{02} - l_{02}).$$

Therefore, referring to eq. 7, the measuring error Δ due to the change of the eikonals caused by shifting the collimating lens can be written as

$$\begin{aligned} \Delta = \Delta l_2 - \Delta l_1 &\cong \frac{x(l'_{02} - l'_{01})\,\delta\beta\cos\theta}{f} + \frac{x^2\,\delta\beta\sin 2\theta}{f} \\ &\quad + \frac{(l'^2_{02} - l'^2_{01})}{2f}\,\delta\beta^2 + \frac{x(l'_{01} + l'_{02})\sin\theta}{f}\,\delta\beta^2. \end{aligned} \tag{15}$$

In our experimental arrangement, keep $l_{01} = l_{02}$, i.e. $l'_{01} = l'_{02}$ and eq. 15 can be written

$$\Delta = \frac{x^2\,\delta\beta\sin 2\theta}{f} + \frac{2xl'_{01}}{f}\sin\theta\,\delta\beta^2. \tag{16}$$

Seeing reference [2], $l'_{01} = l_{01} + (l_{01}\,\delta\beta)^2/2f \cong l_{01}$, so the eq. 16 can be changed into

$$\Delta \cong \frac{x^2\,\delta\beta\,\sin 2\theta}{f} + \frac{2x\,l_{01}}{f}\sin\theta\,\delta\beta^2 .$$

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