AN INNOVATIVE ALGORITHM TO ACCURATELY SOLVE THE EULER EQUATIONS FOR ROTARY WING FLOW

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1. INTRODUCTION

Due to the ability of Euler methods to treat rotational, nonisentropic flows and also to correctly transport on the rotation embedded in the flow field it is possible to correctly represent the inflow conditions on the blade in the stationary hovering flight of a helicopter, which are significantly influenced by the tip vortices (blade–vortex interaction) of all blades.

In the following it is mainly reported on the development of a robust Euler method (ref. 1) for computing the transonic flow around a rotor blade by using the Wake–Capturing method. One of the central points of this work was to prove that the radial change of the transport equation at the rotor blade had to particularly be taken into account in contrast to the method used in the calculation of fixed wings, where the transport velocity q over the examined control surface of a discrete control volume is assumed to be constant. This resulted in far-reaching consequences, when the flows are calculated, and completely new algorithms. Furthermore, it is shown that also the very complex starting procedure of a helicopter rotor can be very well described by a simple Euler method that is to say without a wake-model.

The algorithm based on the procedure is part of category of upwind schemes, in which the difference formation orientates to the actual, local flow state that is to say to the typical disturbance expansion direction. Hence, the artificial dissipation required for the numerical stability is included in a nat ural way adapted to the real flow state over the break-up error of the difference equation and has not to be included from outside. This makes the procedure robust. An implicit solution algorithm is used, where the invertation of the coefficient matrix is carried out by means of a Point–Gauß–Seidel relaxation.

After a short presentation of the basic equations and the numerical solution method the prinicipal results of the dissertation (ref. 1) are presented.

2. DYNAMICAL EQUATION AND SOLUTION METHOD

2.1 Euler equations in the rotating cylindrical reference system

According to the flow field of a rotor, the compressible Euler equations are formulated in a blade–fixed cylindrical coordinate system which rotates together with the rotation velocity ω of the rotor. This demands the additional examination of the centrifugal force and coriolis force. The Euler equations are (ref. 1) :

$$\frac{\partial \phi}{\partial t} + \frac{1}{r}\frac{\partial E}{\partial \theta} + \frac{\partial F}{\partial r} + \frac{\partial G}{\partial z} = K^{(z)} \qquad (2.1)$$

with
$$\phi = (\rho, \rho u, \rho v, \rho w, e)^T \qquad (2.2)$$

as solution factor and the fluxes:

$$E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho vu \\ \rho wu \\ \rho hu \end{pmatrix} \quad F = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho wv \\ \rho hv \end{pmatrix} \quad G = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ \rho hw \end{pmatrix} \qquad (2.3)$$

as well as the term

$$K^{(z)} = -\frac{\rho}{r}\begin{pmatrix} v \\ 2v(u-\omega r) \\ v^2-(u-\omega r)^2 \\ vw \\ v(h-(\omega r)^2) \end{pmatrix} \qquad (2.4)$$

where

$$e = \frac{p}{\kappa - 1} + \rho\frac{q^2}{2} \qquad (2.5)$$

represent the specific total energy per unit volume and

$$h = \frac{e + p}{\rho} \qquad (2.6)$$

the specific total enthalpy per unit mass.

The vector of the right–hand side is composed of terms from the differentiation in the cylindrical coordinate system as well as the centrifugal and coriolis forces influencing the control volume as a result of the induced velocities.

2.2 Separation of the relative contributions from the dependant variables

Krämer (ref. 1) recognized that one of the elementary demands on a solution algorithm, namely the exact conservation of the original uniform flow in an undisturbed flow field, is not fulfilled when the Euler equations represented above are used. The reason for this difficulty is the use of relative terms that is to say the representation of the velocity and energy referred to the rotating reference system.

Krämer (ref. 1) solves the problem by eliminating the contributions of the velocity of the free flow from initial direction from the conservative flow contributions, and therefore, Krämer significantly improves the quality of the solution.

2.2.1 Determination of flux balance through a control volume

The transport of mass, momentum and energy through the surface of a blade-fixed and thus stationary control volume is carried out by the velocity q, which consists of an induced contribution, the so-called disturbance velocity, and the rotation velocity. The equation for a rotor blade rotating with the velocity ω at a body-fixed coordinate system is:

$$\bar{q} = q + (\omega \times r) \quad , \tag{2.7}$$

where q describes the pure contribution of the disturbance velocity.
For a cylindrical coordinate system the equation is

$$\bar{q} = \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} u-\omega r \\ v \\ w \end{bmatrix} \quad . \tag{2.8}$$

The application of the integral form of the Euler equation requires the determination of the mass, momentum and energy fluxes through the cell surfaces of the single finite volumes:

$$\iint_S \phi \cdot q \; dS = \iint_S \phi(q \cdot n) dS \quad , \tag{2.9}$$

where ϕ stands in for any element of the solution vector, and n represents the surface normal unit vector.

In contrast to the fixed wing calculation, where the transport velocity (q·n) over the examined surface is assumed to be constant, there will be radial variability due to the ($\omega \cdot r$)-contribution at the rotorblade flow.

A discrete approximation of the surface integral

$$\iint_S \phi(q \cdot n) dS \cong \sum_{i=1}^{m} \phi_i \; q_i n_i S_i \quad , \tag{2.10}$$

where the index i is the ith cell surface of an examined volume element, is only possible if all participating terms over the integrating surface are constant. This is not the case because of equation (2.7). Nevertheless, to maintain a simple formulation according to equation (2.10), Krämer (ref. 1) divided the flow through cell surface in two triangles (see fig. 1) and examined each of the two triangles separately.

This method allows the mathematically exact calculation of the unknown contributing integral in a simple way.

$$\iint_S \phi(\omega \times r) n dS \cong -\omega \cdot \sum_{i=1}^{m} \left[\phi \cdot \int_r r \cdot z(r) dr \right]_i =$$
$$-\omega \cdot \sum_{i=1}^{m} \phi_i \cdot \left(r_{i,1} S_{i,1}^{(\vartheta)} + r_{i,2} S_{i,2}^{(\vartheta)} \right) \tag{2.11}$$

with $r_{i,1}, r_{i,2}$ as radial coordinates of the surface center of the two triangles and $S_{i,1}^{(\vartheta)}, S_{i,2}^{(\vartheta)}$ as projection surfaces of the two triangles in flow direction.

According to Krämer (ref. 1) the approximation of equation (2.9) for a cylindrical coordinate system is now

$$\iint_S \phi(q \cdot n) dS = \iint_S \phi(\bar{q} - (\omega \times r)) n dS \cong$$
$$\sum_{i=1}^{m} \phi_i \bar{q}_i n_i S_i + \omega \cdot \sum_{i=1}^{m} \phi_i \cdot \left(r_{i,1} S_{i,1}^{(\vartheta)} + r_{i,2} S_{i,2}^{(\vartheta)} \right) . \tag{2.12}$$

2.2.2 Transition to absolute flow terms

However, the division of the transport velocity and the separated approximation of both flux contributions are not enough, as long as ϕ is a function of r. If the Euler equations are formulated in relative flow terms, which are referred to a blade-fixed rotating coordinate system, it is not possible any longer to assume that the elements of the solution vector over the examined cell surface are constant. This can be pointed out by considering the specific total energy per volume unit

$$e = \frac{p}{\kappa-1} + \frac{\rho}{2} q^2 \quad ,$$
$$= \frac{p}{\kappa-1} + \frac{\rho}{2} (\bar{q}^2 - 2\bar{q}(\omega \times r) + (\omega r)^2) \qquad (2.13)$$

and the specific total enthalpie per unit mass

$$h = \frac{\kappa}{\kappa-1} \frac{p}{\rho} + \frac{q^2}{2} \quad ,$$
$$= \frac{\kappa}{\kappa-1} \frac{p}{\rho} + \frac{1}{2} (\bar{q}^2 - 2\bar{q}(\omega \times r) + (\omega r)^2) \quad . \qquad (2.14)$$

Krämer (ref. 1) shows the difficulty by examining the circumferential velocity and the energy in the cylindrical system. If in equation (2.12)

$$\rho u = \rho(\bar{u}+\omega r)$$
$$e = \frac{p}{\kappa-1} + \frac{\rho}{2} ((\bar{u}+\omega r)^2+v^2+w^2) \quad ,$$

is substituted for ϕ, one obtains

$$\iint_S \rho u(q \cdot n) dS = \iint_S \rho u(\bar{q} \cdot n) dS + \iint_S \rho u(\omega r) dS^{(\vartheta)}$$

$$\iint_S e(q \cdot n) dS = \iint_S e(\bar{q} \cdot n) dS + \iint_S e(\omega r) dS^{(\vartheta)} \qquad (2.15)$$

respectively.

If only the last term is examined that is to say the contribution which corresponds to a transport of the flow terms with the free stream velocity and which is about orders higher than each of the first contributions in some distance from the rotor blade, one obtains for the ith surface (the index i is not taken into account for the sake of simplicity):

$$\iint_S \rho u(\omega r) dS^{(\vartheta)} \cong \omega \cdot \rho\bar{u} \cdot \int_r r \cdot z(r) dr + \omega^2 \cdot \rho \cdot \int_r r^2 \cdot z(r) dr$$

$$\text{and} \iint_S e(\omega r) dS^{(\vartheta)} \cong \omega \cdot \bar{e} \cdot \int_r r \cdot z(r) dr + \omega^2 \cdot \rho\bar{u} \cdot \int_r r^2 \cdot z(r) dr + \frac{\omega^3}{2} \cdot \rho \cdot \int_r r^3 \cdot z(r) dr \qquad (2.16)$$

respectively, with the new term $\bar{e}$ according to equation (2.17).

In contrast to the first integral terms according to equation (2.12) which can be simply replaced by a summation form, this is not possible any longer if r appears in the powers r^3 and r^4, respectively. Here, the only possibility to limit the calculation cost and yet to obtain a mathematically exact representation of the velocity and energy balance is to use the relative flow variables instead of the absolute ones, and to eliminate all $(\omega \cdot r)$-contributions from the terms of the solution vector. Hence, the integral terms of higher order can be dropped in r.

Krämer (ref. 1) introduces for this purpose besides the disturbance velocities within the solution vector a new pair of terms which is totally independant of the contribution of the free flow, namely

$$\bar{\bar{e}} = \frac{p}{\kappa-1} + \frac{\rho}{2} \bar{q}^2 \quad , \qquad (2.17)$$

$$\bar{\bar{h}} = \frac{\kappa}{\kappa-1} \frac{p}{\rho} + \frac{\bar{q}^2}{2} \quad . \qquad (2.18)$$

The terms $\bar{\bar{e}}$ and $\bar{\bar{h}}$ are defined as absolute specific total energy per volume unit and as absolute specific total enthalpy per unit mass, respectively. Krämer (ref. 1) uses equations (2.17) and (2.18) and shows that, if the algorithm is used, this is advantageous compared with a formulation with energy and rothalpy according to

$$\bar{h} = h - \frac{1}{2} (\omega r)^2 \quad ,$$
$$\bar{e} = e - \frac{\rho}{2} (\omega r)^2 \qquad (2.19)$$

The new energy equation formed with $\bar{\bar{e}}$ is derived from the original energy equations by exact mathematical transformations. Thus, the physical content of this energy conservation law is maintained, and the numerical difficulties due to discretization which occur when e and $\bar{e}$, respectively, are used can be avoided.

Due to this transformation there are now new relations for the Euler equations. Since the vectorial writing has not changed, equation (2.1) is maintained formally. The new elements of the vectors ϕ, E, F, G and K, however, are different :

$$\begin{pmatrix} \rho \\ \rho\bar{u} \\ \rho v \\ \rho w \\ \bar{\bar{e}} \end{pmatrix}_t + \frac{1}{r} \begin{pmatrix} \rho u \\ \rho\bar{u}u+p \\ \rho vu \\ \rho wu \\ \rho\bar{\bar{h}}u-p\omega r \end{pmatrix}_\vartheta + \begin{pmatrix} \rho v \\ \rho\bar{u}v \\ \rho v^2+p \\ \rho wv \\ \rho\bar{\bar{h}}v \end{pmatrix}_r + \begin{pmatrix} \rho w \\ \rho\bar{u}w \\ \rho vw \\ \rho w^2+p \\ \rho\bar{\bar{h}}w \end{pmatrix}_z = -\frac{\rho}{r} \begin{pmatrix} v \\ 2v\bar{u} \\ v^2-\bar{u}^2 \\ vw \\ v\bar{\bar{h}} \end{pmatrix} \qquad (2.20)$$

with

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \bar{u}+\omega r \\ \bar{v} \\ \bar{w} \end{bmatrix} .$$

For the numerical solution the equations of motion are transformed in any curvilinear coordinate system according to the common methods of the literature (see (refs. 2 and 3)).

3. RESULTS

In order to validate the developed method measurements carried out by Caradonna & Tung (ref. 4) for a two-bladed model rotor in the hovering flight were used. As a first test case a blade-tip Mach number of 0.815 and a collective angle of attack of 5° are chosen.

The comparison between the calculated pressure distribution and the measured data shows that there is a very good agreement (fig. 2) at the first three radial stations.

Only at the outer radial station the used method calculates a low reduced pressure that is to say a too strong blade-tip loss. It seemed reasonable to assume that, for example, the influence of the local grid refinement, the grid size or the geometric design of the blade-tip etc. are responsible for this low pressure, but they did not prove to be applicable for this test case according to (ref. 1). Otherwise, the results obtained by the Wake-Capturing method are very satisfying.

Figure 3 shows the distribution of the circulation density in a reference plane which cuts the rotor disk 30° behind the blade in vertical direction. The contraction of the tip vortex can be seen very exactly. Below the tip vortex of the actual blade the tip vortex of the blade rotating ahead can be seen fairly discrete. The maximum circulation density in the center is fallen down to 15% of the original value due to the decrease of the concentration caused by diffusion. With increasing vorticity age this tendency continues with reduced gradient. On the whole the decrease of the circulation density in the center of the vorticity over the lowering according to (ref. 1) has more or less hyperbolic character.

A similar but less significant effect was obtained by Röttgermann et al. (ref. 5) with a linear vortex lattice method, where the vorticity diffusion was coupled to the expansion of the vorticity layer.

Krämer (ref. 1) compares the obtained contraction in the stationary final state with available hot-wire measurements, which Caradonna & Tung (ref. 4) have carried out for a some of the flow cases on the model rotor in addition to the pressure measurements.

In figure 3 the position of the vorticity in the presented case is marked by crosses.

Krämer (ref. 1) has also drawn the envelope of the trajectory of the blade-tip vortex in the representation of the calculated results as it is given by Kocurek's linear method (ref. 6) for this test case. Due to the high empirical part of the method in (ref. 6) Krämer considers the position of the vorticity obtained by this method for a validation of the presented algorithm as suitable.

The agreement of the numerically calculated contraction of the tip vortex structure with data obtained by Caradonna & Tung (ref. 4) and by Kocurek (ref. 6) is very good.

Figure 4 shows results obtained by the Wake-Capturing procedure (ref. 1) of a second test case compared with coupled models (refs. 7 and 8) that use a Navier-Stokes procedure. The results of Krämer's method (ref. 1) are substantially better at the inboard radial stations that is probably caused by the fact that the wake models neglect or do not properly respect the influence of the inner vortex and of the vortex sheet. These missing components of the downwash lead to an effective angle of attack that is too large. At $r/R = 0.8$ some differences especially in the recompression can be detected, which the coupled Navier-Stokes methods represent by a too flat gradient. The pressure rise is better reproduced by the method of (ref. 1). It should be mentioned, however, that the position of the shock is positioned farther to the rear because of the inviscid flow calculation.

For $r/R = 0.89$ corresponding results are missing in (refs. 7 and 9) so that a comparison is only possible with (ref. 8). There, the pressure loss close to the leading edge as well as the minimum pressure are better represented, whereas the pressure distribution on the lower side is less accurate than predicted by the method of (ref. 1). At the tip, results of both the present method and the one of (ref. 7) are not satisfactory. In the Krämer procedure the tip loss is overpredicted. The same is true for the coupled Navier-Stokes methods.

This comparison shows that the Wake-Capturing procedure (ref. 1) allows a remarkable increase in accuracy compared with Euler or Navier-Stokes procedures that are coupled with an additional wake model.

This is also true for Wake-Capturing methods of other authors, e.g. (refs. 10, 11, 12) who also demonstrate a remarkable improvement compared with coupled methods.

At a calculation of a flow around a helicopter rotor by a Euler method in blade-fixed coordinate system the beginning of the numerical iteration can be compared with the real situation of a rotor which is accelerated by a sudden start up to the given rotational velocity.

Figure 5 shows the time dependant development of the wake at different times of the iteration process. As an example the distribution of circulation is shown in a reference plane that is positioned 30 degrees behind the blade in plane normal to the rotor disc. The sequence of figures shows very clearly the development of the two tip vortex systems. The numerically calculated starting process of the rotor shows the typical behaviour that is also obsereved in experiments.

4. SUMMARY

The method developed by E. Krämer (ref. 1) to calculate the transonic flow field of a helicopter rotor was presented here. This method shows that the examining of the wake as a part of the solution is a particular feature so that the blade–wake–interference, which is typical for rotor aerodynamics, is implicitly given that is to say without the help of a separate vortex model (Wake–Capturing method).

The basic system of equations was established in a blade–fixed, rotating coordinate system. However, the radially varying contributions of the free stream velocity steming from transport velocity and from terms of the conservative solution vector were eliminated. The results obtained by the present method for steady hovering show a very good agreement with measurements not only concerning the pressure distribution but also the contraction of the tip vortex trajectory.

5. LITERATURE

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