

# Theoretical and experimental study of polarization switching in long-wavelength VCSELs subject to parallel optical injection

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## ABSTRACT

We report a theoretical and experimental analysis of the polarization switching found in a single-transverse mode VCSEL when subject to parallel optical injection. We have found a novel situation in which injection locking of the parallel polarization and excitation of the free-running orthogonal polarization of the VCSEL are simultaneously obtained. Analytical expressions for the power of both linear polarizations in the previous steady state are determined. We show that considering two linear polarization modes in a model of a VCSEL subject to parallel optical injection leads to simpler expressions than those found for a VCSEL with only a single linear polarization. We show that the power emitted in both linear polarizations depend linearly on the injected power. The stability region of this solution is measured in the plane injected power versus frequency detuning.

**Keywords:** Vertical-Cavity Surface-Emitting Laser (VCSEL), Polarization, Non-linear Dynamics, Injection-locking.

## 1. INTRODUCTION

Semiconductor lasers can be easily destabilized by the introduction of external perturbations such as optical injection, external optical feedback or modulation of laser parameters [1]. The availability and ease of operation of semiconductor lasers make them a convenient test-bed for exploring basic aspects of nonlinear dynamics [2]. Optical injection in semiconductor lasers has also been used to improve the performance of these devices without modifying their design [3-5]. Locking of the frequency of the injected laser to the one of the injecting laser has been used for the reduction of the laser linewidth or for an enhancement of its modulation bandwidth [3-5]. Nonlinear dynamics of optically injected semiconductor lasers has also been used for photonic microwave generation [6-8].

A lot of attention has been paid to the effects of optical injection on a special type of semiconductor laser: the vertical-cavity surface-emitting laser (VCSEL) [9-22]. VCSELs have demonstrated many impressive characteristics including low threshold currents, high coupling efficiency to optical fibers due to their circular output beam, on-wafer testing capability, integration, easy fabrication in large two-dimensional arrays, single-longitudinal mode operation, and high-speed modulation [9]. Enhancement of VCSEL characteristics can also be obtained by using optical injection [10]. For instance, injection locking [11], improvement of modulation bandwidth [12], and high-frequency microwave signal generation [13] can be obtained when the VCSEL is subjected to appropriate optical injection.

VCSELs also have additional degrees of freedom when compared to their edge-emitting counterparts like the direction of the emitted polarization. Due to their surface emission and cylindrical symmetry VCSELs lack strong polarization anisotropy and may undergo polarization switching (PS) when changing the temperature or bias current [9,23-26]. PS in VCSELs subject to optical injection has attracted considerable attention since the seminal work by Pan [14-22].

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Orthogonal optical injection is the usual technique for obtaining PS in optically-injected VCSELs [14-22]. In this technique linearly polarized light from a master laser is injected orthogonally to the linear polarization of the free-running VCSEL. Studies of PS in VCSELs subject to parallel optical injection, in which the directions of the linearly polarized light from the master laser and the slave VCSEL are parallel, are scarce: just a couple of experimental works have reported PS in VCSELs under these circumstances [21-22]. An analytical study of an optically injected two-polarization modes semiconductor laser has recently shown that pure and mixed-mode steady states can coexist [27]. Linear stability techniques have been used to analyze bistability between pure- and mixed-mode steady states [27].

In this work we make a theoretical and experimental analysis of the PS found in a single-transverse mode VCSEL subject to parallel optical injection. We report a state in which injection locking of the parallel polarization and excitation of the free-running orthogonal polarization of the VCSEL are simultaneously obtained. We will term this situation as IL+PS. This corresponds to the experimental observation in VCSELs of the mixed-mode steady states predicted in [27]. We obtain analytical expressions for the power of both linear polarizations in the IL+PS state. We show theoretically and experimentally that the power emitted in both linear polarizations depend linearly on the injected power. We also measure the stability region of the IL+PS state in the plane injected power versus frequency detuning.

The paper is organized as follows. In section 2, we describe the experimental setup. Section 3 and section 4 are devoted to the presentation of our experimental and theoretical results, respectively. Finally, in section 5, conclusions are presented.

## 2. EXPERIMENTAL SET-UP

We use an all-fiber experimental set-up shown in Fig. 1 to achieve parallel optical injection. Light from a tunable laser is injected into a 1550 nm quantum-well commercial VCSEL (Raycan Co.). The laser light is passed through a variable attenuator to adjust the injected power before the polarization is controlled with a polarization controller. We adjust polarization in a way that injection is achieved in the same polarization direction as the dominant VCSEL polarization. Optical injection is realized by a three port optical circulator. The power injected into the VCSEL is measured with a 50/50 optical coupler directing half of the injected power to a power meter. The bias current and temperature of the VCSEL are controlled by laser and temperature controllers, respectively. After port 3 of the circulator another polarization controller together with a polarization beam splitter are placed in order to measure power in both linear polarizations. Data acquisition is performed with either a power meter for each polarization or a high resolution (10 MHz) optical spectrum analyzer (BOSA).

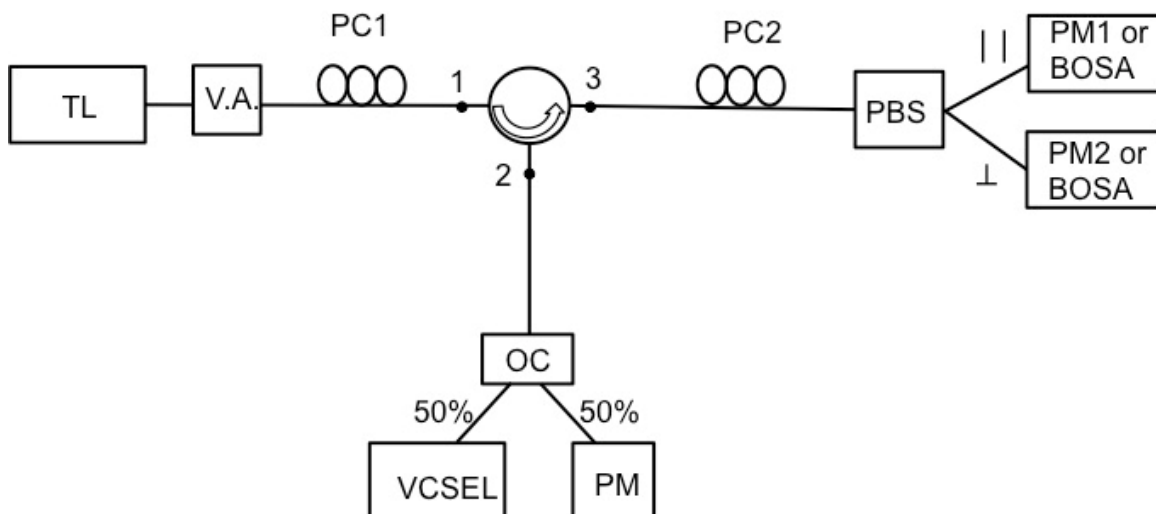


Figure 1. Experimental set-up for parallel optical injection. TL is a tunable laser; VA is a variable attenuator; PC1 and PC2 represent polarization controllers; PBS is a polarization beamsplitter; OC is a 50/50 optical coupler; PM, PM1 and PM2 are power meters; and BOSA is a high-resolution optical spectrum analyzer.

The free-running VCSEL emits in the fundamental transverse mode with a threshold current of  $I_{th} = 1.66$  mA at  $25^\circ\text{C}$ . The temperature and the bias current are held constant at  $25^\circ\text{C}$  and 3.05 mA, respectively. The solitary VCSEL emits in a linear polarization that we will call “x” or “parallel” polarization. The “orthogonal” or “y” polarization is 34 dB weaker than the parallel. The birefringence of the VCSEL is 33 GHz, corresponding to x and y wavelengths of  $\lambda_x = 1540.91$  nm and  $\lambda_y = 1540.65$  nm, respectively. Optical injection is characterized by its strength and by its frequency. The strength is given by the power measured in front of the VCSEL,  $P_i$ . The frequency is characterized by the frequency detuning,  $\nu_i$ , defined as the difference between the injected frequency and the frequency of the solitary VCSEL.

### 3. EXPERIMENTAL RESULTS

We have measured the output power at both ports of the polarization beam splitter as a function of  $P_i$ . Results are shown in Fig. 2. The power measured at the parallel port of the PBS,  $P_{x+r}$ , includes the contributions of both the light emitted by the VCSEL and the reflection of the injected light at the VCSEL mirror. The power measured at the orthogonal port,  $P_y$ , is directly proportional to the power emitted by the VCSEL at the y-polarization. Fig. 2 shows that at low values of  $P_i$  the VCSEL emits in the x-polarization, as in the free-running situation. PS is observed at  $P_i = 175 \mu\text{W}$  because at this value  $P_y$  becomes much larger than  $P_{x+r}$ . Excitation of the y-polarization is maintained while  $175 \mu\text{W} < P_i < 1045 \mu\text{W}$ . Both  $P_y$  and  $P_{x+r}$  depend linearly on  $P_i$ . We have also measured the power in the parallel polarization,  $P_x$ . This is done by turning the VCSEL off and measuring the power in the parallel port of the PBS,  $P_r$ , for different values of  $P_i$ .  $P_r$  is the reflected power at the VCSEL mirror. If we assume that the phase difference between the fields corresponding to the light emitted by the VCSEL in the parallel polarization and the reflection at the VCSEL mirror of the injected light is very close to  $-90^\circ$  or to  $90^\circ$ ,  $P_x$  can be obtained by using  $P_x = P_{x+r} - P_r$ . This assumption will be checked at section 4. Fig. 2 also shows  $P_x$  and  $P_{tot} = P_x + P_y$  as a function of  $P_i$ .  $P_x$  increases linearly in the interval  $175 \mu\text{W} < P_i < 1045 \mu\text{W}$ , in which PS has been observed. In this same interval we observe that  $P_{tot}$  is almost constant because it slightly increases, just a 3% from  $P_i = 175 \mu\text{W}$  to  $P_i = 1045 \mu\text{W}$ .

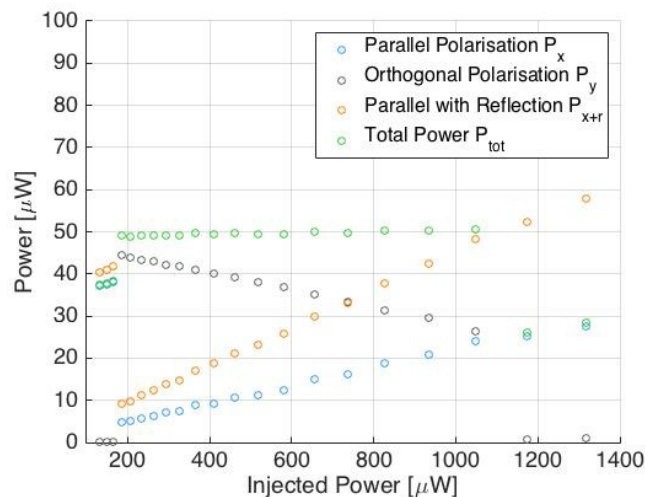


Figure 2. Experimental output powers as a function of the injected power,  $P_i$ . Results are shown for the power measured at the parallel port of the PBS ( $P_{x+r}$ ), power of y-polarization ( $P_y$ ), power of x-polarization ( $P_x$ ) and total power ( $P_{tot}$ ). The frequency detuning is  $\nu_i = -8.6$  GHz.

Figure 3 shows the experimental optical spectra of the total power as  $P_i$  is increased for a fixed value of  $\nu_i = -8.4$  GHz, very close to the detuning value considered in Fig. 2. The zero value of the frequency has been chosen to correspond to the x-polarization mode of the solitary VCSEL. Fig. 3(a) shows that the x-polarization mode experiences period-1 dynamics with a frequency equal to  $\nu_i$ . This spectrum corresponds in Fig. 2 to a value of  $P_i$  in which PS has not been observed yet.

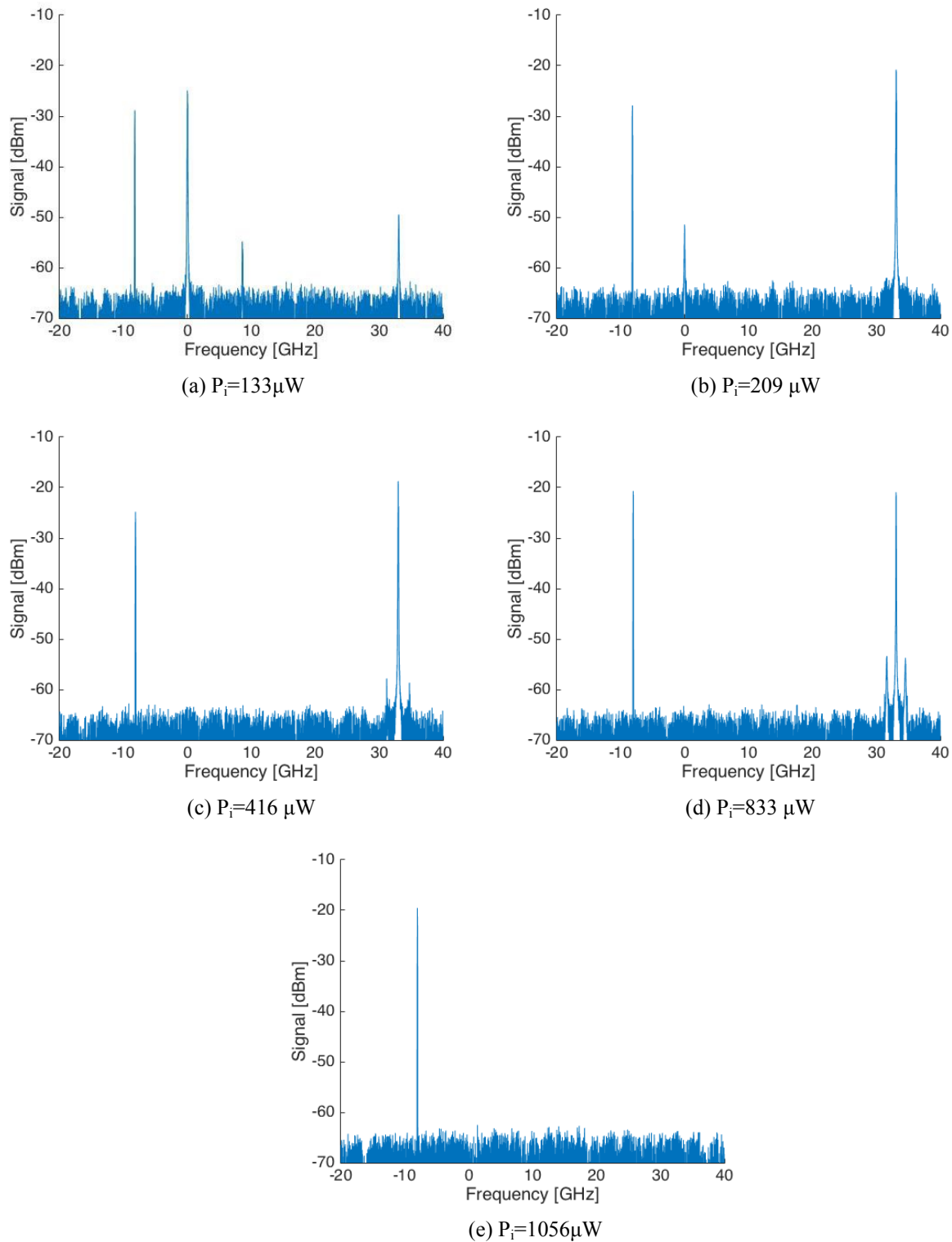


Figure 3. Experimental optical spectra corresponding to different values of  $P_i$  at a detuning of  $\nu_r = -8.4$  GHz.

Fig. 3(b) shows that when  $P_i$  increases PS is observed: the y (x) polarization mode is excited (suppressed) at its free-running frequency. Fig. 3(c) and Fig. 3(d) show injection locked emission in the x-polarization mode with a simultaneous excitation of the free-running y-polarization mode of the VCSEL. We denote this state as IL+PS.

Fig. 3(c) and Fig. 3(d) also show that the orthogonally polarized emission has two satellite peaks around the central peak. The frequency separation between the main and satellite peaks is slightly below the relaxation oscillation frequency, 2 GHz. Fig. 3(e) shows that injection locking in the parallel polarization without excitation of the orthogonal polarization mode is obtained at large enough values of  $P_i$ .

We now characterize in Fig. 4 the region of parameters space in which the IL+PS state is observed. For a fixed value of the frequency detuning we increase  $P_i$  from zero and record the range of injected powers in which the IL+PS is observed in the optical spectrum. Fig. 4 shows that there are two different regions, one at negative and another at positive values of  $\nu_i$ , in which IL+PS is observed. Results analyzed in Fig. 2 and Fig. 3 correspond to the region of negative  $\nu_i$ . The region that appears at positive  $\nu_i$  values appears at lower values of  $P_i$  than those observed at negative  $\nu_i$  values.

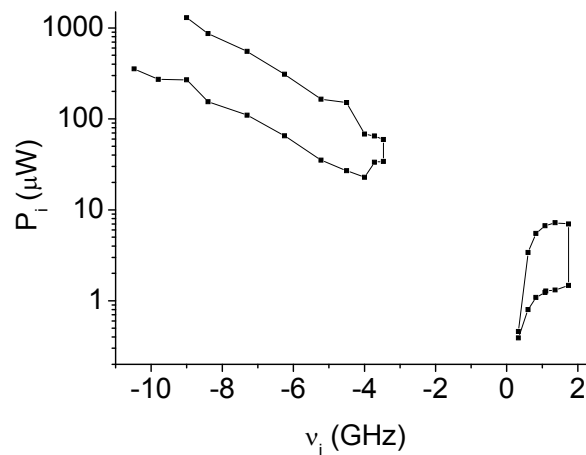


Figure 4. Experimental values of  $P_i$  for which IL+PS is observed as a function of  $\nu_i$ .

#### 4. THEORETICAL RESULTS

We now analyze the system from a theoretical point of view. We consider a rate equation model for the polarization modes of a single-transverse mode VCSEL based on the spin-flip model (SFM) [28]. In order to account for the parallel optical injection we introduce an injection term in the rate equation for the slowly varying component of the e-field in the x-direction. In this way the rate equations for SFM become:

$$\frac{dE_x}{dt} = -(\kappa + \gamma_a)E_x - i(\kappa\alpha + \gamma_p)E_x + \kappa(1 + i\alpha)(DE_x + inE_y) + \kappa E_{inj} e^{i2\pi\nu_{inj}t} \quad (1)$$

$$\frac{dE_y}{dt} = -(\kappa - \gamma_a)E_y - i(\kappa\alpha - \gamma_p)E_y + \kappa(1 + i\alpha)(DE_y - inE_x) \quad (2)$$

$$\frac{dD}{dt} = -\gamma \left[ D(1 + |E_x|^2 + |E_y|^2) - \mu + in(E_y E_x^* - E_x E_y^*) \right] \quad (3)$$

$$\frac{dn}{dt} = -\gamma_s n - \gamma \left[ n(|E_x|^2 + |E_y|^2) + iD(E_y E_x^* - E_x E_y^*) \right] \quad (4)$$

In these equations  $E_x, E_y$  are the two linearly polarized complex e-fields in the x and y directions, respectively,  $D$  is the total population inversion, and  $n$  is the difference between the population inversion for the spin-up and spin-down radiation channels. The parameters that appear in this model have been extracted for a similar free-running VCSEL [29-30]. Their meaning and values are the following: the field decay rate ( $\kappa=33\text{ns}^{-1}$ ), the linear dichroism ( $\gamma_a=-0.1\text{ ns}^{-1}$ ), the linewidth enhancement factor ( $\alpha=2.8$ ), the linear birefringence ( $\gamma_p=103.34\text{ ns}^{-1}$ ), the decay rate of  $D$  ( $\gamma=2.08\text{ ns}^{-1}$ ), the spin-flip relaxation rate ( $\gamma_s=2100\text{ ns}^{-1}$ ) and the normalized bias current ( $\mu=2.29$ ) that corresponds to 3.05 mA.  $E_{inj}$  is the amplitude of the injected light.  $\nu_{inj}$  is the detuning between the frequency of the injected light and the intermediate frequency between those of the x and y polarization,  $\nu_x$  and  $\nu_y$ , where  $2\pi\nu_x=\alpha\gamma_a-\gamma_p$  and  $2\pi\nu_y=\gamma_p-\alpha\gamma_a$ , and therefore  $\nu_i = \nu_{inj} - \nu_x$ . Spontaneous emission noise terms have also been included in the calculations as in [31].

Fig. 5 shows the optical spectrum obtained for a frequency detuning similar to that of Fig. 3 when  $E_{inj}=0.99$ . Optical spectra corresponding to x and y polarization are plotted with black and red colour, respectively. These spectra are similar to those of Fig. 3(c) and Fig. 3(d), that illustrate IL+PS state. The peak that appears in the spectrum of the x-polarization at zero frequency disappears when neglecting the spontaneous emission noise. Satellite peaks appear for both optical spectra. Amplitude of these peaks is larger for the y-polarization, in agreement with our experimental results in which only satellite peaks are observed for this polarization. The frequency separation between the central and satellite peaks is 1.8 GHz, a slightly smaller value than the theoretical relaxation oscillation frequency, 2.1 GHz.

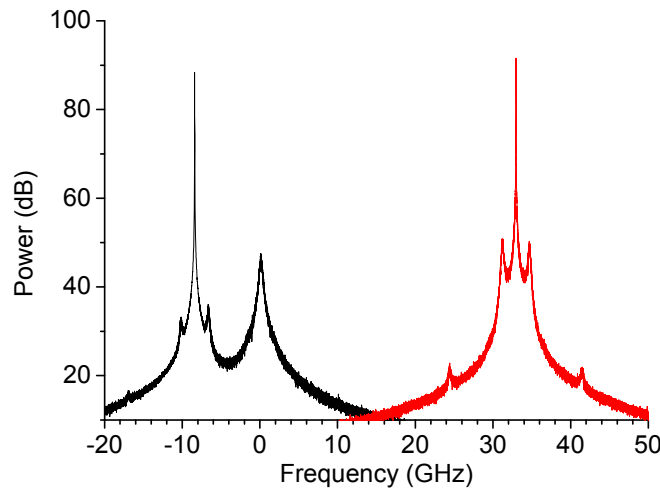


Figure 5. Theoretical optical spectrum illustrating the IL+PS state. Parameters are  $E_{inj}=0.99$  and  $\nu_i = -8.4$  GHz. Black and red colors represent the x and y polarizations, respectively.

Simple analytical expressions can be derived from Eqs. (1)-(4) for the case of very large values of spin-flip relaxation rate. In our case this value is large ( $\gamma_s=2100\text{ ns}^{-1}$ ) and therefore the  $n$  variable is very small. If we write  $E_x(t)=A_x(t)\exp(i(2\pi\nu_{inj}t+\phi_x(t)))$ ,  $E_y(t)=A_y(t)\exp(i(2\pi\nu_y t+\phi_y(t)))$ , and make the approximation  $n=0$  we obtain a set of rate equations for  $A_x, A_y, \phi_x, \phi_y$  and  $D$ . IL+PS solution is characterized by constant values of  $A_x$  and  $A_y$ , such that  $A_x>0$ , and  $A_y>0$ . Looking for this steady-state solution in the equation for  $A_y$  we get  $D=1-\gamma_a/\kappa$ , that is, the total population inversion is fixed to a value that does not depend on  $A_x$  nor  $\phi_x$ . In this way, the fact that the depressed polarization mode is lasing provides a simple expression for the total population inversion  $D$ . Notice, that when a single-polarization mode is considered the carrier density is given by  $D=1+\gamma_a/\kappa - E_{inj}\cos(\phi_x)/A_x$ , where  $A_x$  is found solving a third-order equation also involving  $\phi_x$ . In this way considering two linear polarization modes in a model of a VCSEL subject to parallel optical injection leads to simpler expressions than those found for a VCSEL with only a single linear polarization.

Steady-state values can be found by making the temporal derivatives in the equations for  $A_x, \phi_x$ , and  $D$  equal to zero and substituting  $D=1-\gamma_a/\kappa$ . We obtain the following simple expressions for the IL+PS state:

$$P_x = \left( \frac{\kappa}{2\gamma_a} \right)^2 \frac{P_{inj}}{1 + \left( \frac{\pi\nu_l}{\gamma_a} + \alpha \right)^2} \quad (5)$$

$$P_x + P_y = \frac{\mu}{1 - \gamma_a / \kappa} - 1 \quad (6)$$

$$\phi_x = -\arctan \left( \frac{\pi\nu_l}{\gamma_a} + \alpha \right) \quad (7)$$

where  $P_{inj}=E_{inj}^2$ ,  $P_x=A_x^2$ , and  $P_y=A_y^2$  are the power of the optical injection, x and y polarizations, respectively. Eq. (5) and Eq. (6) show that in the IL+PS state  $P_x$  and  $P_y$  depend linearly on  $P_{inj}$ , in agreement with the experimental results shown in Fig. 2. Substitution of the parameters of the model in Eq. (7) for the case of Fig. 2 gives  $\phi_x=-89.79^\circ$ . In this way we justify the approximation we did in Fig. 2 in order to obtain  $P_x$ . Eq. (6) also shows that  $P_x + P_y$  is constant, independent of the injection parameters,  $P_{inj}$  and  $\nu_l$ . This result is in agreement with the almost constant value of the experimental total power observed in Fig. 2.

## 5. CONCLUSIONS

In conclusion, we have investigated experimentally and theoretically the polarization switching found in a single-transverse mode VCSEL under parallel optical injection. We have obtained a novel state, IL+PS, characterized by simultaneous injection locking of the parallel polarization mode and excitation of the free-running orthogonal polarization mode of the VCSEL. This state has been experimentally characterized using the optical spectrum and the dependence of the power of both linear polarizations on the injected power. These powers depend linearly on the injected power in such a way that the total power is nearly constant. The stability region of IL+PS has been measured in the plane injected power versus frequency detuning. Theoretical results are obtained both from simulations of rate equations and from analytical expressions derived for the IL+PS. These expressions are very simple and describe well our experimental results.

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