Prediction of odd-mode instabilities under output mismatch effects

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***Abstract*—A methodology is presented to predict odd-mode instability in power amplifiers under output mismatch effects, as in the case of amplifiers connected to an antenna. This kind of instability is often observed in power-combining configurations, due to their symmetry properties. Unlike the single-ended situation, there is a cancellation of odd multiples of the oscillation frequency at the circuit output, so there is no impact of the load impedance values at the sideband frequencies. The odd-mode instability depends on the terminations at the fundamental frequency and its harmonic terms, and can only be detected within the circuit unstable loop, instead of the output plane. Here a methodology for the prediction and suppression of odd-mode instabilities is presented. Low-pass filtering effects and the use of a shorted stub allow the stability analysis to be limited to the fundamental-frequency termination. Then, the stability boundaries are efficiently determined through bifurcation detection inside the unstable loop, using the magnitude and phase of the reflection coefficient as the analysis parameters. Results have been validated through pole-zero identification and experimental measurements.**

Keywords— Stability, bifurcation, mismatch effect, power amplifier

I. Introduction

The instability of power amplifiers (PAs) under termination conditions other than 50 Ω, usually due to antenna mismatch [1-2], can lead to serious malfunctioning, involving the observation of incommensurable oscillations and frequency divisions [2]. To guarantee a reliable operation in a variety of conditions, some applications impose stable operation even under highly reflective loads [3]. In order to achieve this requirement, the works [4-5] provide criteria for unconditional instability under output mismatch effects, which are applicable to single-ended circuits. This stability analysis must be carried out under unknown values of the load impedance. Due to its frequency dependence, this impedance will be different at the fundamental frequency and its various harmonic components, mf_{in}, and sideband frequencies, $mf_{in}+f$, where m is an integer and f is a perturbation frequency, to be swept in the stability analysis [5]. Under fulfilment of an equivalent of the Rollet's proviso established in [5], the potential instability can be detected at the circuit-output reference plane, and due to the low-pass filtering action of the output network, the consideration of mismatch effects can be limited to f_{in} and the lowest sideband frequencies f, $-f_{in}+f$ and $f_{in}+f$, with all the rest of components terminated in 50 Ω. The analysis in [5] predicts the possible observation of negative resistance at any of the three lowest sidebands under passive terminations at the other two sidebands, for all the possible values of the fundamental-frequency termination Γ_o. However, this method cannot be applied to circuit exhibiting odd-mode instabilities, often observed in circuits with symmetries, such as power combining PAs. This is because there is no observability of these instabilities at the circuit-output reference plane, due to their mathematical cancellations with right-hand side (RHS) zeroes [6]. They can only be detected through a stability analysis performed at the internal circuit nodes, under variation of the output impedance terminations at mf_{in}. The odd-mode instability often involves a subharmonic oscillation due to the influence of the input signal on the critical circuit frequencies, which are shifted to the divided-by-two frequency [7]. Here an efficient methodology for the detection of this kind of instability will be presented, taking into account the influence of the input power. The method will be illustrated through its application to a power-combining amplifier at f_{in} = 1.5 GHz, which has been manufactured and measured.

II. Analysis Method

Let a circuit exhibiting symmetries, such as the one in Fig. 1, be considered. Under an odd-mode oscillation at the frequency f_a, all the intermodulation products of the form $mf_{in}+(2n+1)f_a$, where m and n are integers, will exhibit 180º phase shift in equivalent nodes and branches of the two sub-amplifiers, and will inherently cancel out at the circuit output. Despite this, the antenna impedance can affect the circuit odd-mode stability since it will alter the termination impedance at mf_{in}, which will give rise to a change in the steady-state solution $\bar{X}_o$ about which the circuit is linearized in the stability analysis, where $\bar{X}_o$ represents the vector of harmonic components of the circuit state variables. This situation in which the stability properties depend on the termination at mf_{in} but are independent on the terminations at the sideband frequencies at $mf_{in}+f$ can also be interpreted as a failure of the proviso established in [5] (an extension of Rollet's proviso to the sideband-impedance problem). This is because the odd-mode instability will be observed even if the sideband frequencies are terminated in open or short circuits at the PA output [5].

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The odd-mode stability analysis under output mismatch effects will depend on the termination impedances at mf_{in}. However, due to inherent filtering effects, only the lower harmonic components will have an impact on these properties. To limit this impact to the fundamental frequency only, a 180º shorted stub at $2f_{in}$ will be introduced in parallel with the final 50 Ohm load (the load that will undergo changes under the mismatch effects), which will eliminate the influence of the impedance terminations at the even harmonic terms. On the other hand, in practical applications one can expect a limited influence of the termination at the third harmonic component $3f_{in}$, since the power level of this spectral line is usually much lower than that at f_{in}. Thus, the stability properties will mostly depend on the fundamental-frequency termination Γ_o. The analysis test-bench is shown in Fig. 1, where harmonics $|m|>1$ are terminated in 50 Ohms due to their limited influence.

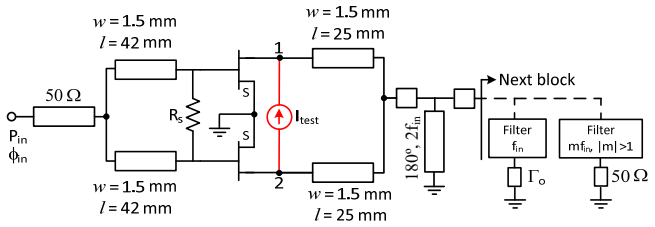

Fig. 1. Test-bench power amplifier based on a CLY5 transistor (RO4003C: ε_r=3.38, h=0.5 mm). The original value of the stabilization resistor is R_s = 170 Ω. The small-signal current source is introduced to evaluate the limit-oscillation conditions at an incommensurable frequency f or a subharmonic frequency $f_{in}/2$ (which needs the consideration of the phase ϕ_{in}).

Under the above assumptions, pole-zero identification [6] would be applicable to detect the odd-mode instability under variations of Γ_o. This requires a connection of the small-signal source in parallel between the two circuit branches, since, in any other position (between one node and ground, for instance), a constant complex impedance (Γ_o) at the sideband frequencies would not represent a physical behavior. Such an analysis would require a sweep in the perturbation frequency (going from 0 to f_{in} in periodic regimes), for each steady-state solution obtained through a double sweep in the amplitude and phase of Γ_o. Then, pole-zero identification should be applied to all the transfer functions resulting from this double sweep. The identification interval 0 to f_{in} should be, in general, divided into smaller intervals, so this analysis will be computationally demanding. Instead, the aim here will be to obtain directly the boundary between stable and unstable Γ_o values by tracing the bifurcation loci in the Smith Chart.

The Hopf-bifurcation locus [7-8] will provide the boundary of the load-impedance region for which the circuit exhibits an incommensurable oscillation. At the steady-state oscillation, the total admittance function Y_T, or current-to-voltage ratio, is equal to zero at all the circuit nodes. On the other hand, at the limit oscillation condition, the oscillation amplitude tends to zero. Taking these two conditions into account, the Hopf locus will be obtained by introducing a small-signal current source at the frequency f inside the potentially unstable loop (Fig. 1). Here it will be connected between equivalent device nodes of the two subcircuits. For each P_{in}, the stability boundary is given by:

$$\bar{H}(\bar{X}_o,\rho_o,\phi_o)=0, \qquad Y_T(\bar{X}_o,\rho_o,\phi_o,f_{AG})=0 \quad (1)$$

where $\bar{H}=0$ is the whole set of harmonic-balance (HB) equations, acting as an inner tier, and ρ_o,ϕ_o are the magnitude and phase of Γ_o. The steady-state solution $\bar{X}_o$ depends on Γ_o and the limit oscillation condition, $Y_T=0$, is evaluated with the conversion-matrix approach.

The analysis based on (1) should start with a global exploration of the Smith Chart, in order to provide a suitable initial value to the optimization/calculation procedure. This is done with a simple graphical technique that takes advantage of the bounded nature of ρ_o and ϕ_o. The perturbation frequency f is swept between 0 and f_{in} and, for each f, a double sweep is performed in ρ_o,ϕ_o, so as to cover the full Smith Chart. For each triplet f,ρ_o,ϕ_o, the total admittance Y_T is calculated as the ratio between the current delivered by the small-signal source and the voltage across its terminals $Y_T=I_{test}/(V_1-V_2)$. To fulfill $Y_T=0$, there must be changes of sign in both the real and imaginary parts of Y_T under variations of ρ_o,ϕ_o, which is easily evaluated by simple inspection. (An example is presented in Fig. 2). Initial values for the optimization should be close to $Y_T=0$, and this situation may be found in one or several regions of the Smith Chart. This initial value (or values) should be introduced in system (1), which will provide an initial Hopf-locus point ρ_o^i,ϕ_o^i,f^i. Then the whole locus will be efficiently traced through continuation, by sweeping ϕ_o around ϕ_o^i and solving (1) to obtain: $\rho_o(\phi_o), f(\phi_o)$. There will be one Hopf locus for each P_{in}.

One common case of odd-mode instability is the frequency division by 2, associated with flip bifurcations [7-8]. This phenomenon occurs when the input signal shifts the circuit natural frequency to one half the input frequency: $f_a \rightarrow f_{in}/2$, which is often associated with a parametric instability. This evolution involves the merging of a pair of complex-conjugate poles at f_a into two pairs of complex-conjugate poles at $f_{in}/2$, in order to preserve the system dimension [7]. At the division threshold, the subharmonic-oscillation amplitude will tend to zero, so the flip bifurcations can be detected by setting the frequency of the small-signal current source to $f_{in}/2$. Because this perturbation frequency ($f_{in}/2$) and the input frequency are commensurable, the phase shift between the input source and the current source is a relevant analysis variable [9]. For the bifurcation detection, one can set the phase of the current source to zero and consider the input-source phase ϕ_{in}. The mathematical conditions for the flip bifurcation are:

$$\bar{H}(\bar{X}_o,\rho_o,\phi_o,\phi_{in})=0, \qquad Y_T(\bar{X}_o,\rho_o,\phi_o,\phi_{in})=0 \quad (2)$$

Unless a modified conversion-matrix analysis [9,10] is applied, the above analysis must be carried out with HB at the fundamental frequency $f_{in}/2$, due to the frequency commensurability. The initial value(s) is obtained through 3 nested sweeps, in the input-source phase ϕ_{in}, varied between 0

and 360º, and in ρ_o, ϕ_o, to cover the full Smith Chart. Once an initial point $\rho_o{}^i, \phi_o{}^i, \phi_{in}{}^i$ has been obtained, the flip locus will be obtained through continuation, by sweeping the phase ϕ_o around $\phi_o{}^i$, and solving (2) to obtain: $\rho_o(\phi_o), \phi_{in}(\phi_o)$. In amplifiers composed of several stages, the above methodology should be applied at each stage, and covering all the possible oscillation modes resulting from the circuit symmetry.

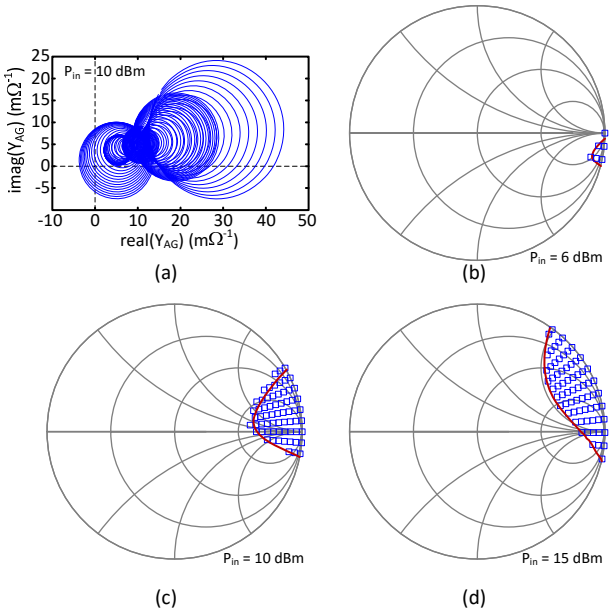

Fig. 2. Graphical method to obtain initial values. (a) Admittance diagram. (b) Region of points with Re(Y_T)<0 and |Im(Y_T)|≤ 10^{-3} Ω^{-1} for P_{in} = 6 dBm. (c) For P_{in} = 10 dBm. (d) For P_{in} = 15 dBm.

III. Application to a Mismatched Amplifier

The above method has been applied to the circuit in Fig. 1. In the absence of mismatch effects, this amplifier is stable for all the P_{in} values, as verified with pole-zero identification [1,3,6]. On the other hand, the amplifier does not exhibit even-mode instabilities under mismatch effects, as verified with the method in [5]. The purpose here will be to predict the possible odd-mode instability under output mismatch effects. The admittance plots $Y_T(\bar{X}_o, \rho_o, \phi_o, f)$ never cross the negative real semi-axis, so no incommensurable oscillation should be expected. The same analysis has been performed for the function $Y_T(\bar{X}_o, \rho_o, \phi_o, \phi_{in})$, which provides several crossings of the negative real semi-axis [Fig. 2(a)], indicating the possible fulfillment of the flip-bifurcation condition (2). Actually, processing the data in Fig. 2(a) it has been possible to obtain the Γ_o values giving negative conductance (Re(Y_T)<0), with a magnitude of the imaginary part |Im(Y_T)| below 10^{-3} Ω^{-1} at different P_{in} values, represented with squares in Fig. 2(b-d).

Using an initial value from Fig. 2(a), the flip bifurcation loci in Fig. 3 have been obtained, where each locus corresponds to a different P_{in}. Because the amplifier is stable in matched conditions, the stable region corresponds to the outside of the flip loci. For low P_{in}, the locus does not enter the Smith Chart, so there is unconditional stability. From $P_{in} \cong 5$ dBm, the locus crosses the Smith Chart, so the amplifier is potentially unstable under mismatch effects. Due to the natural reduction of the negative resistance from certain signal amplitude, one should expect the loci to escape from the Smith Chart from a given P_{in} value. The loci corresponding to the various P_{in} values considered in Fig. 2 have also been represented in that figure, in a red solid line. The unstable region contains a subset of the points with negative real part of Y_T and low magnitude of the imaginary part. Note that the negative real part and positive-slope resonance of Y_T do not constitute a general instability condition. However, the limit steady-state oscillation condition in (1) and (2) is rigorous and should be fulfilled at any circuit node at the stability boundary. Because this condition only depends on the load value Γ_o at f_{in}, all the possible implementation of this load should give rise to the same kind of behavior, either stable or unstable. This has been validated for two different Γ_o values, one at each side of the flip-bifurcation locus obtained for P_{in} = 15 dBm (in a solid red line in Fig. 3), indicated as Γ_{t1} and Γ_{t2} in Fig. 3. They are relatively close to the stability boundary to evaluate the degree of accuracy. Fig. 4(a) presents the results of an independent stability analysis based on pole-zero identification when Γ_{t1} and Γ_{t2} are implemented with an RL series network. Fig. 4(b) presents the results of the parallel-RL implementation. Poles of the Γ_{t1} (Γ_{t2}) load are represented with blue (red) crosses. With the two different implementations, the load Γ_{t1} is stable and the load Γ_{t2} is unstable, in agreement with results from (2).

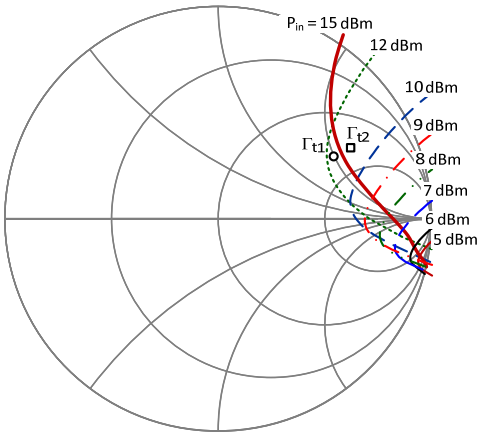

Fig. 3. Evolution of the flip locus obtained with (2) versus P_{in}. The loci only cross the Smith Chart in a certain P_{in} interval.

To determine the P_{in} interval with potential instability in an efficient manner, one can take into account the particular shape of the loci in Fig. 3. All the loci cross the boundary of the Smith Chart, so one can expect the locus to be tangent to this chart at the limits of the unstable P_{in} interval. This should be obtained for a magnitude of the reflection coefficient $\rho_o = 1$. The locus of P_{in} and ϕ_o values fulfilling the flip-bifurcation condition under $\rho_o = 1$ is expressed as:

$$\begin{aligned} \bar{H}(\bar{X}_o, \rho_o = 1, \phi_o, \phi_{in}, P_{in}) &= 0 \\ Y_T(\bar{X}_o, \rho_o = 1, \phi_o, \phi_{in}, P_{in}) &= 0 \end{aligned} \qquad (3)$$

For P_{in} values such that the flip locus in (2) crosses the unit Smith Chart, there will be at least two ϕ_o fulfilling (3) (Fig. 3). This is shown in Fig. 5, where the phase ϕ_o at the intersection points with the Smith Chart [calculated with (3)] has been represented versus P_{in}. At the boundaries of the unstable P_{in} interval there will only be one ϕ_o, since the locus is tangent to the Smith Chart. To stabilize the circuit under mismatch effects, the resistor R_s connected between the two amplifier branches will be reduced from its original value (170 Ω).As expected (Fig. 5) the locus (3) decreases in size with R_s and eventually vanishes, due to the damping effect of this parallel resistor. For R_s<120 Ω, the amplifier should be stable for all the P_{in} values.

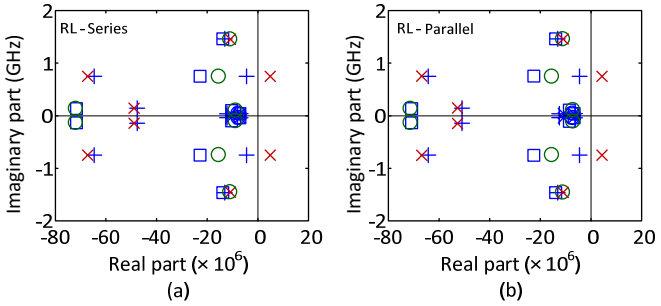

Fig. 4 Validation of the flip locus corresponding to P_{in} = 15 dBm with two different implementations of Γ_{t1} and Γ_{t2} in Fig. 3. (a) RL-*series* implementation. Poles of the Γ_{t1} (Γ_{t2}) load are represented with blue (red) crosses. (b) RL-*parallel* implementation. Poles of the Γ_{t1} (Γ_{t2}) load are represented with blue (red) crosses.

The PA has been measured for two R_s values (150 Ω and 100 Ω) and different positions of a triple-stub tuner, used to enable the load variation [Fig. 6(a)]. With R_s = 150 Ω, the circuit is stable for the measured loads A and B and exhibits a frequency division by two for the loads C and D. See the spectra corresponding to B and C in Fig. 6(b) and 6(c). The low amplitude of the subharmonic spectral line is due to the near cancellation of this frequency component at the circuit output, due its odd-mode nature. The region of the unstable loads is in very good correspondence with the analysis in Fig. 3. With R_s = 100 Ω the circuit is stable for all the load values [E,F,G,H are shown in Fig 6(a)] and all the P_{in} values, in agreement with Fig. 5.

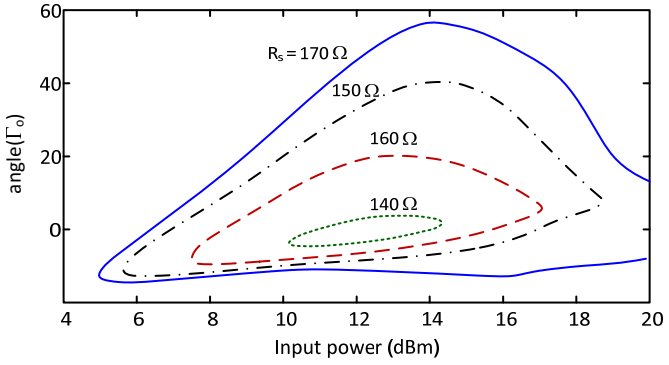

Fig. 5. Calculation of the unstable P_{in} interval using the locus in (3). The limit of this interval correspond to the edge points of the locus. The calculation has been repeated for different values of the stabilization resistor R_s.

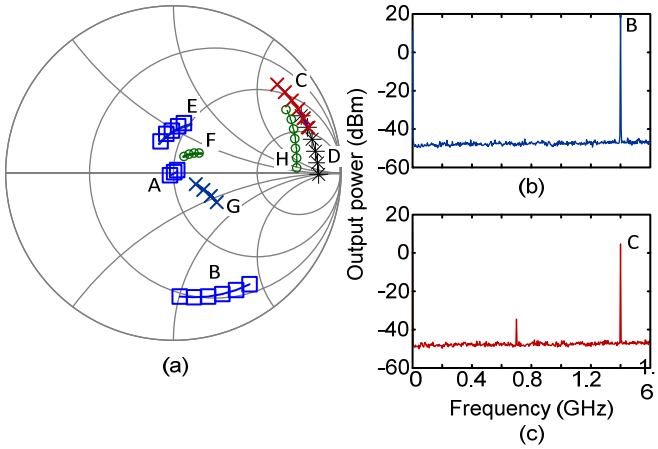

Fig. 6 Measurements for different positions of a triple-stub tuner. (a) The loads A,B,C,D correspond to tests under R_s = 150 Ω. The loads E,F,G,H correspond to tests under R_s = 100 Ω. (b) Spectrum for R_s = 150 Ω and load B. (c) Spectrum for R_s = 150 Ω and load C.

IV. Conclusion

A method has been presented to predict odd-mode instabilities in power amplifiers under output mismatch effects. It is based on tracing of the Hopf- and flip-bifurcation loci on the Smith Chart of the fundamental-frequency termination. The loci are calculated from a limit oscillation condition, evaluated at the unstable odd-mode loop. Initial values are efficiently obtained through a graphical method. Very good results have been obtained in the validation with pole-zero identification and with measurements.

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