Growth-rate function for the nonlinear analysis of the transient dynamics of microwave oscillators

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Abstract — In this paper, a new technique is presented for the analysis of the transient dynamics of microwave oscillators. The technique makes use of a nonlinear admittance function that can be identified in commercial Harmonic Balance software. This function is included in a time-frequency domain equation governing the transient dynamics. The equation provides the growth rate function of the first harmonic amplitude, which allows an exhaustive analysis of the transient speed from the neighborhood of the dc solution to the oscillation establishment, with no need for a numerical integration, as in time domain or envelope-transient methods. The technique has been applied to predict the length of the transient towards the oscillating state of a FET oscillator at 5 GHz.

Index Terms — microwave oscillator, frequency-domain analysis.

I. Introduction

The oscillator start-up time is a key specification in several applications. In the case of mobile communication devices, the faster the system can be powered up and down the less power consumed. Oscillators presenting short transients are also required in systems that modulate the oscillation amplitude such as ultra-wideband communications (UWB) or radar systems. Previous works [1]-[3] analyse the transient to oscillation dynamics by means of analytical expressions based on simple oscillator models that can fail in more complex oscillators. In [4], the transient dynamics is analysed through the derivatives of an admittance function, calculated about the unstable dc solution from which the oscillation originates, with the aid of an auxiliary generator (AG). The stability analysis of this complex equation provides the poles describing the initial transient speed.

In this work, a new technique is presented to analyse the whole transient dynamics. For this purpose, the AG used in [4] will be applied here to extract a two-variable admittance complex function. Then, a time-frequency model of the oscillator will be derived in terms of this admittance function. This model depends nonlinearly on the oscillation amplitude variable and, as a difference from [4], is valid along the whole transient, from the unstable dc solution to the steady-state oscillation. In addition, the formulation can be applied to obtain the oscillation amplitude growth rate function. This function allows an exhaustive analysis of the transient dynamics without the need of solving the oscillator transient using time domain or envelope-transient techniques. This makes the transient analysis faster and avoids the numerical errors inherent to these methods.

The paper is organized as follows. In Section II, the new formulation is presented and the instantaneous pole and growth rate concepts are introduced. In Section III, the technique is applied to describe the transient dynamics of a FET oscillator at 5 GHz. The dependence of the transient length on the varactor bias is also analyzed.

II. Time-Frequency Domain Formulation of the Oscillator Transient

A. Calculation of the admittance function

The technique will be applied to the microwave oscillator of Fig. 1. The model describing the oscillator transient behavior will be based on an admittance function that can be calculated in commercial HB by means of a voltage auxiliary generator (AG), connected in parallel at an observation node *q* (see Fig. 1). The AG is constituted by a single tone voltage source whose module, phase and frequency are respectively $(V_{AG}, \phi_{AG}, \omega_{AG})$, in series with an ideal bandpass filter that behaves as an open circuit for the first harmonic component and a closed circuit for the rest.

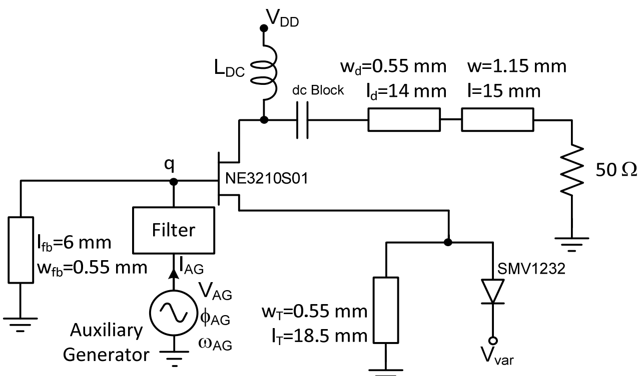

Fig. 1. Schematic of the FET oscillator at 5 GHz. The auxiliary generator is connected at the observation node *q*.

The total input admittance function describing the oscillator response is calculated performing a double sweep in the two variables (V_{AG}, ω_{AG}) of this independent generator, and solving the Harmonic Balance (HB) system at each sweep step. This provides an outer-tier function admittance function $Y[V_{AG}, \omega_{AG}]$ [5] that can be identified in commercial HB software by sweeping the AG values (V_{AG}, ω_{AG}):

$$Y[V_{AG}, \omega_{AG}] = \frac{I_{AG}\left[V_{AG}e^{j\phi AG}, \omega_{AG}\right]}{V_{AG}e^{j\phi AG}} = Y_{AG}[V_{AG}, \omega_{AG}] \quad (1)$$

The function $Y[V,\omega]$ provides the first harmonic total admittance at the observation node, with (V,ω) being the first harmonic amplitude and frequency. Note that at the steady state oscillation values (V_o,ω_o) the AG does not perturb the circuit and the admittance function vanishes: $Y[V_o,\omega_o]=0$. To obtain an envelope-transient formulation based on this outer-tier complex function, a modulation at much smaller time rate than the carrier frequency will be assumed. In this formulation, the instantaneous voltage at the observation node in the free-running regime is expressed as:

$$v(t)=\sum_{k=-N}^{N} V_k(t)e^{j[k\omega_o t+\phi_k(t)]} \tag{2}$$

Introducing expression (2) in the HB system in the absence of the AG, the total admittance equation at the observation node becomes:

$$\begin{aligned} &Y[V_1(t),\omega_o]V_1(t)e^{j\phi(t)}+\\ &+Y_\omega[V_1(t),\omega_o]\left[\dot{\phi}_1(t)V_1(t)-j\dot{V}_1(t)\right]e^{j\phi(t)}=0 \end{aligned} \tag{3}$$

where the subindex ω indicates differentiation with respect to the frequency. In this preliminary work, only the first derivative of the function $Y[V_1,\omega]$ has been considered, in a manner similar to what is done in previous envelope-transient formulation based on piecewise harmonic balance [6]. The inclusion of higher order derivatives would increase the accuracy of the analysis. The first-order differential equation (3), which should be integrated from the initial perturbation of the dc regime, at the initial time t_0, can be decoupled into two real equations for the phase and amplitude variables, respectively. Our purpose is to analyze the dynamics of the amplitude component, which is governed by the equation:

$$\begin{aligned} &\dot{V}_1(t)=\frac{Y^i(t)Y^r_\omega(t)-Y^r(t)Y^i_\omega(t)}{|Y_\omega(t)|^2}V_1(t)=g[V_1(t)]\\ &Y(t)\equiv Y^r[V_1(t),\omega_o]+jY^i[V_1(t),\omega_o]\\ &Y_\omega(t)\equiv Y^r_\omega[V_1(t),\omega_o]+jY^i_\omega[V_1(t),\omega_o] \end{aligned} \tag{4}$$

Note that, in contrast with previous works [4]-[5], equation (4) is nonlinear. This implies that the nonlinear function $g[V_1(t)]$ must be evaluated at each time step, interpolating the amplitude values in the previously calculated admittance function $Y[V_1,\omega_o]$. This nonlinear procedure allows the characterization of the whole transient dynamics, and is not restricted to the neighborhood of the steady state solutions as the linearized analyses [4]-[5]. Equation (4) can be linearized about any arbitrary amplitude value $V_1=V_p$, obtaining:

$$\begin{aligned} &\Delta\dot{V}_1(t)=g\left[V_p+\Delta V_1(t)\right]\approx\alpha(V_p)\Delta V_1(t)+g(V_p),\\ &\alpha(V_p)=g'(V_p) \end{aligned} \tag{5}$$

where $\alpha(V_p)$ will be called the *instantaneous pole*. The function $g(V_1)$ is the growth rate of the first harmonic amplitude. At the dc and oscillating values $V_1=0,V_o$ equation (4) provides $g(V_1)=0$, indicating that these are steady state values. At these values, $\alpha(V_1)$ agrees with the pole resulting from the stability analysis of each of these solutions. Indeed, by making $V_1=V_0$ in (4) it is seen that $\alpha(V_0)$ agrees with the expression for the pole determining the stability of the oscillating solution obtained in Kurokawa's analysis [5],[8].

III. Experimental and Numerical Validation

The technique has been applied to the oscillator of Fig. 1. In a first stage, the admittance function $Y[V_1,\omega]$ has been extracted from HB commercial software by means of the AG. This function provides the steady state (V_o,ω_o), the growth rate $g(V_1)$ and the instantaneous pole $\alpha(V_1)$ (see (4)-(5)). In Fig. 2, both the values $\alpha(0)$ and $\alpha(V_0)$ have been represented as a function of the voltage bias V_{DD}. As can be seen, at V_{DD} =0.62 V a Hopf bifurcation takes place giving rise to an autonomous oscillation, which is maintained for V_{DD} >0.62 V. The real part of the dc solution poles obtained through pole-zero identification [7] has been superimposed.

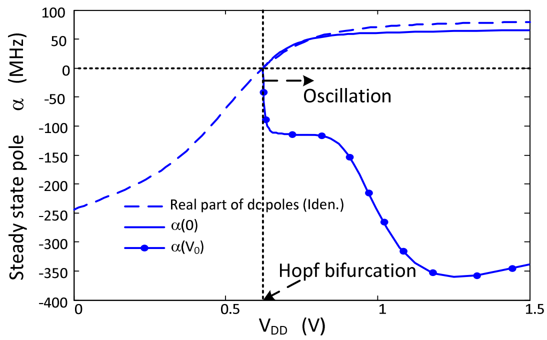

Fig. 2. Stability analysis in terms of the drain bias voltage V_{DD}.

At the intermediate values $V_1\in(0,V_o)$, the static functions $g(V_1)$ and $\alpha(V_1)$ provide information about the transient dynamics at each point. These functions are represented in Fig. 3 for V_{DD}= 1 V. As can be seen in this figure, at the dc value V_1=0, $\alpha(0)$ is positive, indicating that the dc solution is unstable and an oscillation can take place. The function $g(V_1)$ keeps positive until $V=V_o$, where it becomes zero, indicating that the first harmonic amplitude grows until this value is reached. The transient starts to slow down beyond $V_1=V_m$=0.33 V, where $\alpha(V_m)=0$ and $g(V_m)$ presents a maximum. Finally, the instantaneous pole $\alpha(V_0)$ agrees with the negative pole of the oscillating solution, indicating that this solution is stable.

This analysis has been validated in Fig. 4 through the simulation of the oscillator transient to the oscillating state. In this figure, the time integration of system (4) is represented together with the envelope transient simulation at circuit level in commercial software. The initial condition $V_1(0)=10^{-12}$ V has been considered. The discrepancy in the transient length is due to the fact that the commercial software considers all the circuit state variables in a separate manner, while equation (4) considers only the variable V_1. Note that the amplitude value V_m=0.33 V for the maximum growth rate $\dot{V}_1(t)$ is well predicted in both simulations by the static analysis of Fig. 4. This is a key parameter to determine the speed of the oscillator response.

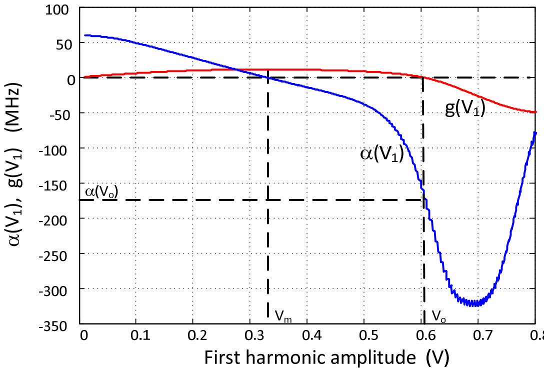

Fig. 3. Functions $\alpha(V_1)$ and $g(V_1)$ calculated from the admittance function $Y[V_1,\omega_o]$ by equations (4) and (5).

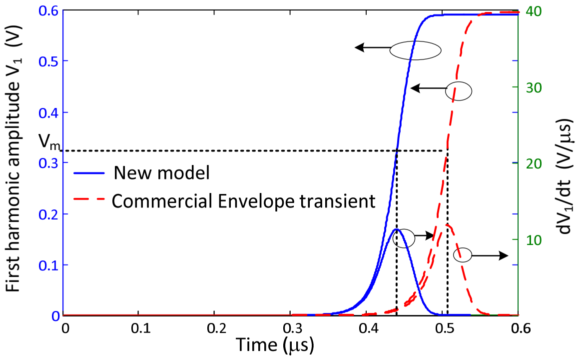

Fig. 4. Simulation of the transient to the oscillating state of the first harmonic amplitude.

In a second example, in Fig. 5 the transient length has been analyzed for two values of the varactor bias V_{var}. As can be seen, the transient length increases with this bias value. This qualitative behavior has been experimentally verified by means of an Infiniium 9000 Series DSO oscilloscope, with the results of Fig. 6.

IV. Conclusion

This paper presents a new technique to analyze the whole transient dynamics of the oscillator start-up. The technique provides the amplitude growth rate and the instantaneous pole functions, which allow an exhaustive analysis of the transient dynamics without the need of solving the oscillator transient using time domain or envelope-transient techniques. This makes the transient analysis faster and avoids the numerical errors inherent to these methods. In addition, this analysis provides a clear view of the oscillation growth rate as a function of the first harmonic amplitude. The technique has been validated in a FET oscillator at 5 GHz using both commercial HB and measurements.

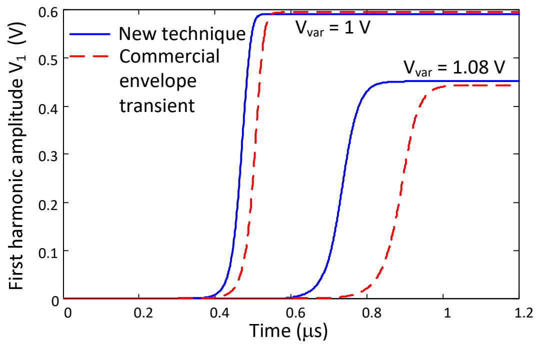

Fig. 5. Simulation of the transient to the oscillating state for two values of the varactor bias V_{var}=1, 1.08 V.

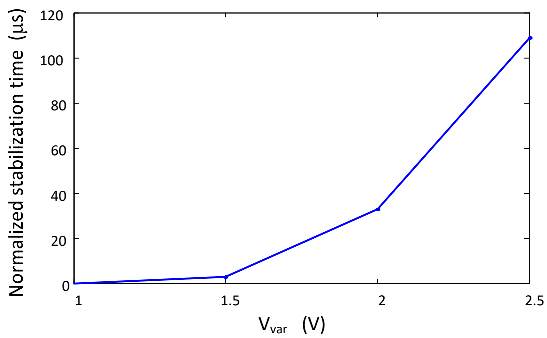

Fig. 6. Measured stabilization time as a function of the varactor bias V_{var}. The values are normalized representing the time increment from the minimum stabilization time.

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