Comparison of Model Order Reduction Techniques for Digital Predistortion of Power Amplifiers

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***Abstract*— This paper compares and discusses four techniques for model order reduction based on compressed sensing (CS), less relevant basis removal (LRBR), principal component analysis (PCA) and partial least squares (PLS). CS and PCA have already been used for reducing the order of power amplifier (PA) behavioral models for digital predistortion (DPD) purposes. While PLS, despite being popular in some signal processing areas, to the best author's knowledge, still has not been used in the PA linearization field. Finally, the LRBR is an iterative search algorithm proposed by the authors in this paper for the sake of comparison. Experimental results are presented and the advantages and drawbacks of each method discussed.**

Keywords—compressed sensing, digital predistortion, partial least squares, power amplifier, principal component analysis.

I. Introduction

With the increasing demand of data traffic, more spectrally efficient modulation schemes occupying wider bandwidths are being used. From the transmitter perspective, not only spectral efficiency is desirable, but also power efficiency. To cope with the well-known linearity versus efficiency trade-off of one of the main power hungry devices of the transmitter chain (i.e., the power amplifier (PA)), digital predistortion (DPD) linearization is probably the most common and spread solution.

When considering wide bandwidth signals the number of coefficients required in the DPD model to compensate for both nonlinear and memory effects of PAs can be significantly high. This has a negative impact in the least squares (LS) based DPD model extraction/adaptation because it increases the computational complexity and can drive to over-fitting and uncertainty. Reducing the order of the DPD model has beneficial effects in both the computational complexity and conditioning of the data matrices.

Several model order reduction techniques have been published in recent past years, mostly based on some kind of greedy or iterative search algorithm [2]-[4] that, given a minimization criterion, allows selecting the most relevant basis functions from the original data matrix. Alternatively, model order reduction can be achieved using techniques based on the transformation of the original data matrix into a new basis of orthogonal components [5],[6]. Because the components of the resulting transformed matrix are independent, the adaptation process is significantly simplified.

In this paper, we compare four different model order reduction techniques. Two of them fall in the category of direct reduction of the original data matrix through an intensive search algorithm (compressed sensing (CS) and less relevant basis removal (LRBR) techniques). The other two fall in the category of model reduction after a previous basis transformation (principal component analysis (PCA) and partial least squares (PLS)). The LRBR is a new method proposed in this paper by the authors for the sake of comparison. Moreover, to the best author's knowledge, the PLS technique is the first time that is used for model order reduction in the context of DPD linearization.

II. Digital Predistortion Linearization System

The DPD linearization is based on the direct learning approach, as depicted in Fig. 1. The model order reduction strategies compared in this paper are applied to the generalized memory polynomial (GMP) behavioral model [1].

$$\begin{aligned}\hat{y}[n] &= \sum_{l=0}^{L_A}\sum_{p=0}^{P_A} a_{pl}\cdot x[n-\tau_l^A]\cdot\left|x[n-\tau_l^A]\right|^p \\ &+\sum_{l=1}^{L_B}\sum_{m=1}^{M_B}\sum_{p=0}^{P_B} b_{pml}\cdot x[n-\tau_l^B]\cdot\left|x[n-\tau_l^B-\tau_m^B]\right|^p \\ &+\sum_{l=1}^{L_C}\sum_{m=1}^{M_C}\sum_{p=0}^{P_C} c_{pml}\cdot x[n-\tau_l^C]\cdot\left|x[n-\tau_l^C+\tau_m^C]\right|^p\end{aligned} \quad (1)$$

The input-output relationship of the DPD block is defined as

$$x[n]=u[n]-\mathbf{u}_n\mathbf{w} \quad (2)$$

where $\mathbf{w}=\left(w_0, w_1, \ldots, w_{M-1}\right)^T$ is a $Mx1$ vector of coefficients of the GMP model. Note that the original coefficients of the GMP in (1), a_{pl}, b_{pml} and c_{pml}, are mapped for simplicity into w_i coefficients.

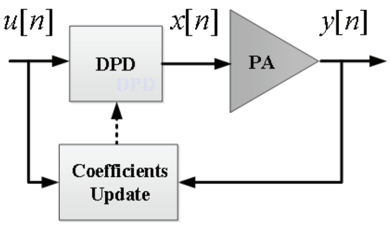

Fig. 1.Block diagram of the direct learning approach.

Moreover, $\mathbf{u}_n = (\varphi_0[n], \varphi_1[n], \cdots, \varphi_{M-1}[n])$ is the *1xM* data vector containing the GMP basis functions or waveforms. Following the direct learning approach, the DPD coefficients can be estimated iteratively using a weighted LS algorithm.

$$\mathbf{w}^{i+1} = \mathbf{w}^i + \mu\left(\mathbf{U^H U}\right)^{-1} \mathbf{U^H e} \tag{3}$$

with $\mathbf{U} = (\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{L-1})^T$ being the *LxM* data matrix (*n=0, 1,⋯ L-1*); with μ being a weighting factor and where $\mathbf{e}$ is the *Lx1* vector of the error defined as

$$e[n] = y[n]/G_0 - u[n] \tag{4}$$

where G_0 is the linear gain of the PA.

III. Model Order Reduction Strategies

In the following, four different model order reduction techniques are outlined and discussed. These techniques can be classified into two big groups: The ones based on a previous transformation of the original data matrix **U** to obtain a new orthogonal simplified basis (e.g. PCA and PLS); and the ones based on an intensive iterative search aimed at culling the redundant or unnecessary basis functions of the original data matrix **U** (e.g. CS and LRBR).

A. Compressed Sensing (CS) Technique

According to the compressed sensing (CS) theory, the sparsity of the behavioral model basis functions can be exploited to obtain an ordered sequence of the most significant components. Several compressed sensing (CS) techniques have been proposed for obtaining the estimates of the subset of active coefficients. The problem can be formulated as

$$\begin{aligned} &\min_{\mathbf{w}} \quad \|\mathbf{w}\|_0 \\ &\textit{subject to} \quad \|\mathbf{y} - \mathbf{Uw}\|_2 \le \varepsilon \end{aligned} \tag{5}$$

The solution is obtained by minimizing the number of active components (l_0-norm) subject to a constraint on the 2-norm of the identification error. Unfortunately, this is a combinatorial search problem which complexity class is non-deterministic polynomial-time hard (NP-hard). In [2] a suboptimal approach, based on the greedy method named orthogonal matching pursuit (OMP), is used to estimate the support set of the active basis functions. Moreover, the Bayesian information criterion (BIC) is applied to determine the most suitable number of basis functions.

B. Less Relevant Basis Removal (LRBR)

The proposed less relevant basis removal (LRBR) algorithm is an iterative (brute-force search) basis selection technique. Consists in, iteratively, evaluating the effect of culling one basis function on the overall normalized mean square error (NMSE) of identification. The algorithm is described as follows:

1) Remove, once at a time, each one of the M columns (corresponding to the basis functions) of matrix $\mathbf{U}$, and perform an LS fitting with the remaining basis (M-1 columns). At the end of the M fittings we obtain M values of the NMSE of identification.

2) Then, remove from matrix $\mathbf{U}$ the column (basis function) presenting the minimum NMSE (calculated in step 1), for being the one with less impact on the identification error.

3) Repeat steps 1) and 2) successively, eliminating one column at each iteration until having the desired number of remaining coefficients.

Finally, after 3) we obtain the subset of basis functions that survived the culling. Alternatively, you can run 3) until the very last coefficient to gather the information of the order of the columns (basis functions) according to their relevance (in terms of minimizing the NMSE of identification).

C. Principal Component Analysis (PCA)

The principal component analysis (PCA) is a statistical technique used to transform the original and possibly correlated data into an uncorrelated set or principal components. These principal components are ordered in such a way that the variance decreases along them. Thus, the first few components account for almost the totality of the variance present in the whole set. This can be used for eliminating redundancies and reducing the order of the model.

PCA finds combinations of the basis functions with large variance without any knowledge of the response values. That is, the transformation matrix is obtained from and to explain the data matrix **U**. Consequently, nothing guarantees that the principal components are valid to explain the response of the system. Taking into account the standard LS formulation in (3), the basis functions are described in the columns of the data matrix $\mathbf{U}$. As explained in [5], with PCA we can generate a new basis that results of a linear combination of the basis functions located in the columns of $\mathbf{U}$. The reduced-order *LxN* matrix $\tilde{\mathbf{U}}_{PCA}$ is defined as

$$\tilde{\mathbf{U}}_{PCA} = \mathbf{U}\mathbf{V}_{red} \tag{6}$$

with $\mathbf{U}$ being the *LxM* matrix and $\mathbf{V}_{red}$ a *MxN* matrix $\mathbf{V}_{red} = [\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_N]$ built selecting the *N* out of *M* eigenvectors (the ones with higher eigenvalue) of the original matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_M]$. $\mathbf{V}$ is the *MxM* matrix obtained finding the eigenvectors of the correlation matrix $\mathbf{U^H U}$ (whose eigenvalues are the same as the ones of the covariance matrix $\mathrm{cov}(\mathbf{U}) \approx \mathbf{UU^H}$).

D. Partial Least Squares (PLS)

As an alternative to PCA, partial least squares (PLS) regression constructs the new basis components while considering the observed response values (i.e., the output **y**), leading to a model with improved prediction capabilities. This method can be understood as a cross between linear regression and PCA [6]. That is, PLS regression finds components from

$\mathbf{U}$ that are also relevant for $\mathbf{y}$, calculating the new basis with high covariance with the response $\mathbf{y}$. The PLS solution can be obtained by applying the well-known SIMPLS algorithm described in [7]. As in the PCA case, with PLS a new orthogonal matrix is found ($\tilde{\mathbf{U}}_{PLS}$) by doing a linear transformation of the original one.

$$\tilde{\mathbf{U}}_{PLS} = \mathbf{UT} \quad (7)$$

$\mathbf{T}$ is the MxM PLS transformation matrix. The reduced matrix $\tilde{\mathbf{U}}_{red}^{PLS}$ is built selecting the N out of M columns of the transformed matrix $\tilde{\mathbf{U}}_{PLS}$.

In both PCA and PLS cases, despite the fact that the transformation matrices ($\mathbf{V}$ and $\mathbf{T}$, respectively) have been generated in a different way and have different properties, the resulting transformed (and eventually reduced) matrix is orthogonal,

$$\tilde{\mathbf{U}} = \begin{bmatrix} \gamma_1(0) & \gamma_2(0) & \cdots & \gamma_N(0) \\ \gamma_1(1) & \gamma_2(1) & \cdots & \gamma_N(1) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_1(\mathrm{L}-1) & \gamma_2(\mathrm{L}-1) & \cdots & \gamma_N(\mathrm{L}-1) \end{bmatrix} \quad (8)$$

where. $\gamma_i(\mathrm{n})$ are the new set of orthogonal basis functions (with $i=1, 2, ...N$). This implies that the DPD coefficients' extraction process can be significantly simplified. The LS solution in (3) can be solved without the need of inverting the correlation matrix, but solving each coefficient independently with simple dot products.

IV. EXPERIMENTAL RESULTS

The experimental test-bench is depicted in Fig. 2. For testing purposes, we used a broadband high efficiency continuous mode class-J power amplifier at 950 MHz, based on the CGH35030F packaged GaN HEMT from Cree Inc. The signal generation and measurement equipment consisted of: Texas Instruments boards (TSW1400EVM pattern generator + TSW30H84EVM DACs and I-Q modulator), and a Keysight Infinium DSO90404A oscilloscope for capturing the RF signals.

The test signal used is a LTE-like signal of 15 MHz bandwidth and around 9.4 dB of PAPR. The targeted mean output power was 28.3 dBm with ACLR levels below -45 dB to meet the LTE downlink specifications and a mean drain efficiency of 24.2 %. Fig. 3 shows the PA's output power spectra before and after DPD considering a GMP behavioral model with 64 coefficients.

As observed in Fig. 4 and Fig. 5 thanks to these model order reduction techniques we can halve the number of coefficients and still being compliant with the ACLR specifications and without degrading the NMSE. Moreover, as depicted in Fig. 4, the LRBR and PLS techniques show slightly better robustness against the reduction of the number of coefficients of the DPD function.

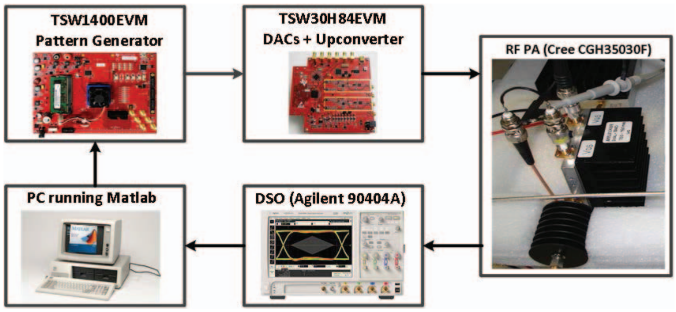

Fig. 2.Experimental set-up

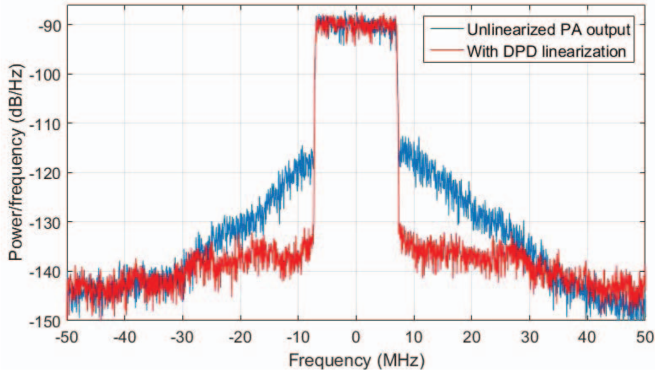

Fig. 3. Unlinearized and linearized (using a GMP-based DPD with 64 coefficients) output power spectra.

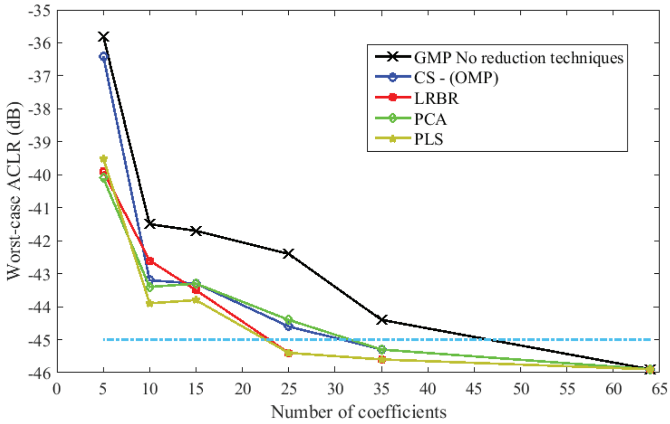

Fig. 4. Worst-ACLR versus number of coefficients for different model order reduction strategies.

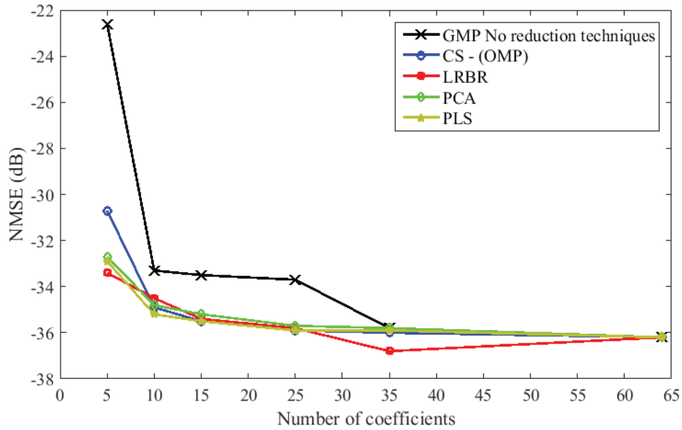

Fig. 5. NMSE versus number of coefficients for different model order reduction strategies.

TABLE I. COMPARISON OF MODEL ORDER REDUCTION TECHNIQUES

MODEL ORDER REDUCTION STRATEGY	# COEFF	ACLR (DB)		NMSE (DB)	RUNNING TIME UNITS
		IMD-3 L	IMD-3 U		
GMP No reduction	64	-46.9	-45.9	-36.2	--
	35	-45.7	-44.4	-35.8	
	25	-43.1	-42.4	-33.7	
GMP with CS (OMP) reduction	35	-46.7	-45.3	-36.0	47
	25	-46.1	-44.6	-35.9	
GMP with LRBR reduction	35	-47.0	-45.5	-36.8	990
	25	-46.7	-45.4	-35.8	
GMP with PCA reduction	35	-46.5	-45.3	-35.8	1
	25	-46.0	-44.4	-35.7	
GMP with PLS reduction	35	-46.7	-45.6	-35.9	17
	25	-47.0	-45.4	-35.9	

According to the BIC used in the CS technique, the suitable number of basis functions (previously ordered by the OMP algorithm) was 35. Table I compares the ACLR and NMSE after DPD when considering the GMP model with 35 and 25 coefficients for all the model order reduction techniques discussed in this paper. In addition, last column on Table I gives an estimate of the computational time (measured in units of time) consumed by each of these techniques for creating the new reduced order basis to be used along the DPD process.

V. CONCLUSION

The advantage of using the model order reduction methods that imply a basis transformation (i.e. PCA and PLS) is that significantly reduce the identification/adaptation process. Thanks to the orthogonality of the resulting transformed matrix, the coefficients' extraction can be done independently, with simple dot products and thus avoiding the matrix inversion of typical LS identification. On the other hand, the iterative brute-force search algorithms (i.e., CS and LRBR) directly apply the reduction to the matrix containing the original basis functions, and thus avoid the extra processing required to calculate the matrix transformation. The computational time to carry on these iterative searches is also relevant despite the fact that can be run off-line.

Experimental results have shown that all model order techniques under study show a similar trend regarding the loss of accuracy versus number of coefficients. However, slightly better results (better robustness against culling of coefficients) were obtained with LRBR and PLS techniques. The main drawback of LRBR is the required computational time to provide the list of basis functions sorted according to their relevance (or capacity to minimize the NMSE). On the other hand, PLS shows better performance than PCA (at the price of increasing the computational time) because takes into account the output data vector to compute the transformation matrix.

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