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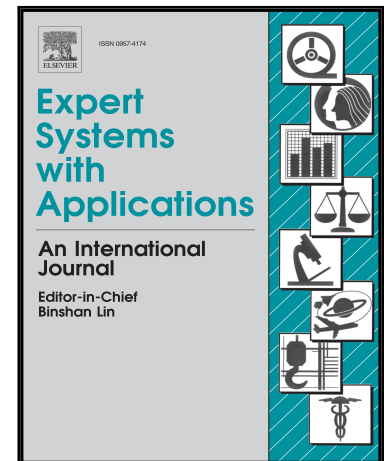
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Decision aid system founded on nonlinear valuation,
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selection in slope stability cable nets

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Highlights

- A decision aid system to address hierarchical decision-making problems is developed
- Levels of satisfaction are valuated using the Beta Cumulative Distribution Function
- Subcriteria are weighted based on their variability using measures of dispersion
- Dependencies between subcriteria are quantified through correlation coefficients
- The system is applied to the selection of wire rope in slope stability cable nets

ACCEPTED MANUSCRIPT

Decision aid system founded on nonlinear valuation, dispersion-based weighting and correlative aggregation for wire rope selection in slope stability cable nets

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Abstract

This paper presents a decision aid system to address hierarchically structured decision-making problems based on the determination of the satisfaction provided by a group of alternatives in relation to multiple conflicting subcriteria grouped into criteria. The system combines the action of three new methods related to the following concepts: nonlinear valuation, dispersion-based weighting and correlative aggregation. The first includes five value functions that allows the conversion of the ratings of the alternatives regarding the subcriteria into the satisfaction they produce in a versatile and simple manner through the Beta Cumulative Distribution Function. The use of measures of dispersion to weight the subcriteria by giving more importance to those factors that can make a difference due to their heterogeneity is revised to validate it when the values are not normally distributed. Dependencies between subcriteria are taken into account through the determination of their correlation coefficients, whose incorporation adjusts the results provided by the system to favour those alternatives having a balanced behaviour with respect to conflicting aspects. The overall satisfaction provided by each alternative is determined using a prioritisation operator to avoid compensation between criteria when aggregating the subcriteria. The system was tested through a novel field of application such as the selection of wire rope to form slope stability cable nets.

Keywords

Beta Cumulative Distribution Function; Correlation coefficient; Dispersion-based weighting; Prioritisation; Slope stability; Wire rope net

1. Introduction

Selecting the most preferred alternative from a group depending on the satisfaction degree they provide in relation to a set of conflicting and hierarchically structured aspects is a recurrent problem in many real-life applications. These problems are normally formulated in terms of a group of alternatives $A_l = \{A_1, \dots, A_p\}$ having different ratings x_{ki} regarding a set of subcriteria $SC_{ki} = \{SC_{11}, \dots, SC_{nm}\}$ belonging to several criteria $C_k = \{C_1, \dots, C_n\}$, so that the overall satisfaction s_l produced by each alternative in relation to that hierarchy made up of criteria and subcriteria is the final output being sought. A decision aid system consists of a set of interacting components forming a whole aimed at helping to solve decision-making problems under complex environments.

The need for several components stems from the need to solve each of the different phases that constitute this kind of problems. The first phase seeks the valuation of the ratings of the alternatives with respect to the subcriteria in terms of the satisfaction they generate. These ratings normally have different units of measurement, which suggests that scaling them into a standard range of values, e.g. $[0, 1]$, is desirable. The concept of satisfaction is beyond the basic normalisation step included in many decision-making methods, which assume linearity of variables (Opricovic & Tzeng, 2004; Teixeira de Almeida, 2007; Önüt & Soner, 2008). Other methods, based on the concepts of multi-attribute utility theory (MAUT) and multi-attribute value theory (MAVT) (Edwards, 1977; Keeney & Raiffa, 1976) derived from Utility Theory (Neumann & Morgenstern, 1953) and Value Engineering (Miles, 1961), respectively, represent the utility or value of an alternative l with regards to a subcriteria SC_{ki} through a function $f(x_{ki})$.

The Integrated Value Model for Sustainable Assessment (MIVES) and the Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) are the two most relevant methods that propose specific functions to model the value (preference degree in PROMETHEE terminology) associated with the performance of the alternatives in terms of the set of subcriteria. MIVES (Jato-Espino *et al.*, 2014; Pons & Aguado, 2012; Pons & De La Fuente, 2013; San-José Lombera & Garrucho Aprea, 2010) is based on an cumbersome equation that defines four different functions (concave, convex, linear, S-shape) according to three parameters (C_i , K_i , and P_i) and two bounds (x_{min} and x_{max}). Each of the nonlinear functions place the largest increase in satisfaction in three different sections (final, initial and central, respectively), which means that the method cannot model a variable whose increase in satisfaction is located

in both the initial and final sections of a function. This behaviour is typical in many real-life variables, wherein lower values represent the area to exceed the threshold of minimum satisfaction (initial section) and the excellence corresponds to the highest values (final section). PROMETHEE (Behzadian *et al.*, 2010; Dagdeviren, 2008; Herngren *et al.*, 2006; Wang & Yang, 2007) has six different preference functions to translate the difference between the evaluation of two actions for a certain criterion into a preference degree according to two parameters named the indifference and preference thresholds (q_j , p_j). Apart from two functions also present in MIVES (linear and S-shape, here known as gaussian), this method considers four additional shapes: usual, U-shape, V-shape and level. These functions are variants of constant and linear shapes with the only exception of considering different bounds. Therefore, PROMETHEE functions have insufficient flexibility to model nonlinear variables. These considerations prove the need for a new approach to value the degree of satisfaction provided by a group of alternatives in a versatile and simple manner.

The next phase to solve a decision-making problem formed by a series of hierarchical and conflicting factors is the aggregation of the elements in both levels of the hierarchy to determine the ranking of alternatives in terms of their overall degree of satisfaction. The relationship between the criteria is often of a form such that the aggregation process must not allow their compensation. The incorporation of the prioritisation operator developed by Yager (2008) into the decision aid system prevents that compensation from happening. Another key factor within the procedure is the calculation of the weights of the subcriteria. The standard deviation has been proposed by some authors (Wang *et al.*, 2007; Wang & Luo, 2010; Zardari *et al.*, 2014) as an objective weighting method that assigns small weights to those subcriteria having similar values across the alternatives. However, the application of this measure of spread in this context must be revised, since its validity depends on the distribution pattern of such values. The final step consists of the quantification of the conflicts between subcriteria. Despite its importance, no method has been developed for the characterisation of this operation, which is still excluded from decision-making processes.

Under these premises, the aim of this paper is twofold. First, to build a decision aid system capable of addressing all the operations required to solve hierarchical decision-making problems based on the valuation of the satisfaction degree provided by a set of alternatives in relation to multiple conflicting subcriteria grouped into several criteria. Such system seeks to overcome the deficiencies found in current decision-making approaches in terms of three main aspects in these problems (valuation, weighting and conflicting subcriteria) through the Beta Cumulative Distribution Function (CDF), the interquartile range and the statistical correlation. The second aim is to demonstrate the

applicability and usefulness of the decision aid system through a decision-making problem consisting of the selection of wire rope to form slope stability cable nets. This is a novel field of application defined by having prioritised criteria arranged into conflicting subcriteria with respect to which the satisfaction produced by some alternatives cannot be modelled using current valuation methods, which justifies the suitability of the proposed system to address it.

2. Methodology

A decision aid system based on the measurement of the satisfaction degree provided by a set of alternatives upon a group of hierarchically structured criteria and subcriteria can be designed through the combination of a series of methods as depicted in Figure 1.



Figure 1. Outline of the decision aid system proposed

First is the conversion of the performance of the alternatives under consideration into the satisfaction they produce using the value functions stemmed from the Beta CDF. The second operation consists of the prioritisation of criteria such that their compensation is avoided. Next, the set of subcriteria forming each criterion is weighted according to the degree of variability of the ratings of the alternatives in relation to them. Finally, the interactions between subcriteria are incorporated into the system through the concept of statistical correlation. The combination of these operations yields the final ranking of alternatives being sought. The following subsections delve into the working principles that characterise each of the four steps on which the decision aid system is based.

2.1. Valuation

The satisfaction s_{ki} provided by an alternative can be expressed as a function of its rating x_{ki} in relation to the subcriterion SC_{ki} under consideration ($s_{ki} = f(x_{ki})$). Since this rating is often not proportional to the satisfaction it generates, there is a need for a method that allows the modelling of nonlinear relationships.

The Beta CDF enables not only the characterisation of these areas where the satisfaction variations are more or less concentrated, but also the scaling of ratings measured in different units into the range $[0, 1]$. This is possible because this function has two shape parameters (α, β) and lower and upper bounds (A, B) , which makes it very versatile to fit a variety of different datasets. The generic formula for the probability density function of the Beta distribution is (Gupta & Nadarajah, 2004):

$$f(x; \alpha, \beta) = \frac{(x - A)^{\alpha-1} \cdot (B - x)^{\beta-1}}{B(x; \alpha, \beta)} = \frac{(x - A)^{\alpha-1} \cdot (B - x)^{\beta-1}}{\int_0^x t^{\alpha-1} (1 - t)^{\beta-1} dt} \quad (1)$$

where $B(x; \alpha, \beta)$ is the incomplete Beta function, which becomes the standard Beta function $B(\alpha, \beta)$ when $x = 1$. The Beta cumulative distribution function is formulated as follows (Gupta & Nadarajah, 2004):

$$F(x; \alpha, \beta) = I_x(\alpha, \beta) = \frac{B(x; \alpha, \beta)}{B(\alpha, \beta)} \quad (2)$$

where $I_x(\alpha, \beta)$ is the regularised incomplete Beta function. Up to five different value functions can be derived from the Beta CDF to model the satisfaction provided by an alternative regarding a subcriterion in a hierarchical decision-making problem, depending on how the parameters (α, β) are combined:

- Concave: the largest increase in satisfaction is located in the final section of the function ($0 < \alpha < 10$; $\beta = 1$).
- Convex: the largest increase in satisfaction is located in the initial section of the function ($\alpha = 1$; $0 < \beta < 10$).
- Linear: the satisfaction always increases at the same range regardless of the abscissa ($\alpha = 1$; $\beta = 1$).
- Logit: the largest increase in satisfaction is located in the initial and the final sections of the function ($0 < \alpha < 1$; $0 < \beta < 1$).

- Sigmoid: the largest increase in satisfaction is located in the central section of the function ($1 < \alpha < 10$; $1 < \beta < 10$).

Figure 2 shows the shapes associated with each of these value functions for a case example consisting of a variable whose ratings fluctuate in the interval $[0, 100]$. The ranges of α, β related to each value function provide the user with a large set of inflexions to model more or less pronounced shapes. The Beta CDF can also be inverted in case the rating of an alternative regarding some subcriterion and the satisfaction it produces are inversely related, i.e. if an increase in the rating results in a decrease in the satisfaction. Furthermore, it allows the establishment of constraints to take into account any limiting value from either a normative or technical point of view (i.e. if $x_{ij} \leq x_1 \rightarrow s_{ij} = 0$).

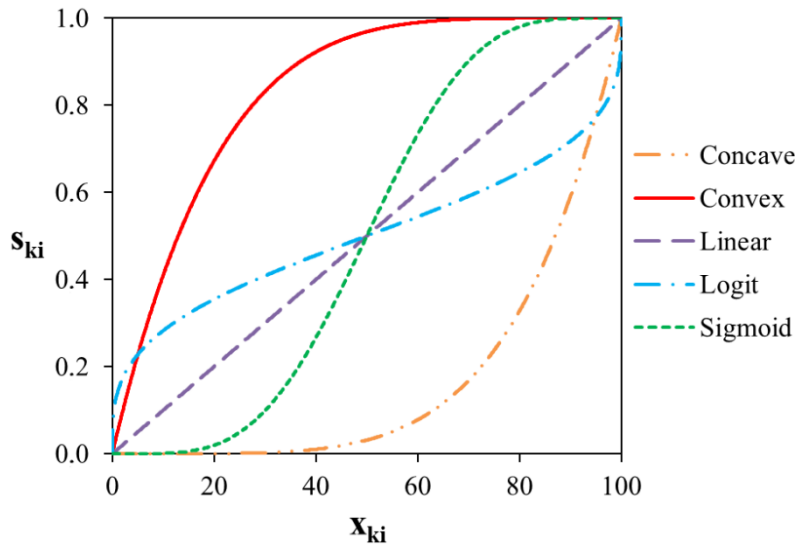


Figure 2. Value functions derived from the Beta cumulative distribution function

In addition to enable the modelling of a greater number of shapes than any other decision-making method based on value functions, the Beta CDF highlights by its automaticity, since it is integrated into common software packages such as MS Excel (Microsoft Corporation, 2013), Mathematica (Wolfram Research, 2014) or MATLAB (MathWorks, 2013) and therefore, does not need for complex equations based on a lot of parameters for its application.

2.2. Prioritisation

Common aggregation procedures to obtain the overall satisfaction s_l provided by an alternative l regarding a set of criteria $C_k = \{C_1, \dots, C_n\}$ are expressed as:

$$s_l = \sum_{k=1}^n w_k \cdot s_k \quad (3)$$

where the weights of criteria w_k satisfy two conditions: $w_k \in [0, 1]$ and $\sum_{k=1}^n w_k = 1$. These aggregation operators are monotonic (s_l does not decrease if any s_{ki} increases), bounded ($\min_k[s_{ki}] \leq s_l \leq \max_k[s_{ki}]$) and idempotent (if every $s_{ki} = a$ then $s_l = a$) (Yager, 2008). As a consequence, they allow the compensation between criteria, i.e. if Ψ denotes the relationship between criteria C_1 and C_2 , this type of operators allows a decrease Δ in C_1 to be compensated by an increase $\Psi \cdot \Delta$ in C_2 .

This kind of compensation is not always desired and sometimes a benefit in a criterion cannot lead to a loss in another. In other words, there is a prioritisation between those two criteria. Yager (2008) proposed a prioritised aggregation operator (see Eq. (4)) for the calculation of the overall satisfaction of an alternative l with respect to a set of criteria $C_k = \{C_1, \dots, C_n\}$ formed by several subcriteria $SC_{ki} = \{SC_{11}, \dots, SC_{nm}\}$, such that $SC_{ki} \in C_k$ and $C_1 > \dots > C_n$, which means that an increase in C_k cannot result in a decrease in C_{k-1} (compensation between criteria is not desired).

$$s_l = \sum_{k=1}^n \left(\sum_{i=1}^m w_{ki} \cdot s_{ki} \right) \quad (4)$$

where w_{ki} denotes the weight of $SC_{ki} \in C_k$. The prioritisation of criteria proceeds by first calculating the following expression:

$$l_k = \min_i s_{ki} \quad (5)$$

where l_k is the value of the least satisfied subcriterion in criterion C_k for an alternative l . This parameter can be used to relate each criterion C_k with a value v_k , which is the product of the least satisfied subcriterion in all criteria with higher priority than C_k and the highest priority criterion C_1 , beginning with $v_1 = 1$, and continuing progressively for the remaining criteria ($v_2 = l_1$; $v_3 = l_1 \cdot l_2$; $v_4 = l_1 \cdot l_2 \cdot l_3$ and so on). Hence, a weight w_k is obtained for each criterion C_k , such that $w_k = v_k$ and $w_k \in [0, 1]$. These weights can be normalised through Eq. (6) to fulfil $\sum_{k=1}^n w_k = 1$:

$$w_k = \frac{v_k}{\sum_{k=1}^n v_k} \quad (6)$$

2.3. Weighting of subcriteria

The application of the prioritised aggregation operator formulated in Eq. (4) entails the automatic determination of the weights of criteria w_k , but the calculation of the weights of subcriteria w_{ki} is still pending. Several methods have been developed throughout the years to carry out this operation, such as direct allocation, equitable weighting or elicitation of the opinions of a panel of experts according to pairwise comparisons (Nutt, 1980; Wang & Lee, 2009). Another approach consists of weighting the set of subcriteria based on their degree of dispersion, which implies giving more importance to those subcriteria for which the set of alternatives has more varying values. From another perspective, the goal is to reduce the weight of those subcriteria with respect to which the alternatives have homogeneous ratings.

This approach allows the preponderance of those subcriteria that are more diverse and consequently, those alternatives that can make a difference due to having good ratings in relation to heterogeneous aspects. In contrast, those alternatives performing well with regards to homogeneous subcriteria are not highly rewarded, since they are not adding relevant value to the overall degree of satisfaction achieved.

A pair of measures of dispersion are proposed to determine the weights of subcriteria, depending on whether they are normally distributed or not: standard deviation (σ) and interquartile range (IQR), respectively (see Eq. (7)).

$$D_{ki} = \begin{cases} \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} & \text{if } SC_{ki} \text{ is normally distributed} \\ IQR = Q_3 - Q_1 & \text{if } SC_{ki} \text{ is not normally distributed} \end{cases} \quad (7)$$

where x represents each value in the sample, \bar{x} is the mean of the sample, Q_3 is the third quartile and Q_1 is the first quartile. Similarly to Eq. (6), the normalised weight of a subcriterion $SC_{ki} \in C_k$ can be determined as:

$$w_{ki} = \frac{D_{ki}}{\sum_{i=1}^m D_{ki}} \quad (8)$$

The Kolmogorov-Smirnov (K-S) test (Smirnov, 1948), based on the calculation of the largest vertical difference between the theoretical and empirical distribution functions, can be used to check the hypothesis that the sample under analysis comes from a population with a normal distribution. This is determined through the p-value, which

represents the probability of wrongly rejecting the null hypothesis (H_0) if it is true, such that H_0 states that “the sample comes from a normally distributed population”. If the p-value is below the significance level (α), the probability to wrongly rejecting the null hypothesis is lower than a fixed value of α . A value of α equal to 0.05 is set to check the statistical significance of results.

2.4. Correlation

The reason behind the growth in the development of decision aid systems is the need for having support tools to help make better decisions to solve problems characterised by having multiple conflicting criteria. These conflicts or interactions can be positive or negative and refer to any relationship involving dependence. The strength of this dependence can be quantified through the correlation coefficients, which provide a statistical measure of the linear association between two or more variables.

A negative correlation implies that one variable increases as another variable decreases, whilst correlation is positive if both variables increase together. Values of negative and positive correlation coefficients between two subcriteria i and j have to be in the intervals $[-1, 0)$ and $(0, 1]$, respectively. Statistical correlations are measured through different coefficients (see Table 1) depending on the nature of the variables whose dependence is to be tested: quantitative, ordinal and nominal (Bachman, 2004). Nominal variables must first be dichotomised prior to determining their correlation with another variable.

Table 1. Correlation coefficients depending on the type of variables

SC_{i1} / SC_{i2}	Quantitative₂	Ordinal₂	Dichotomous₂
Quantitative₁	Pearson	Biserial	Point Biserial
Ordinal₁	Biserial	Spearman	Rank Biserial
Dichotomous₁	Point Biserial	Rank Biserial	Phi

The validity of a correlation coefficient is also determined through the p-value. In this case, H_0 is of the form that “there is not enough evidence to conclude that two subcriteria are correlated”. If the p-value is below 0.05 in 2-tailed tests, the correlation between the subcriteria is statistically significant.

The purpose of incorporating this concept into the decision aid system is to adjust the results derived from it, so that those alternatives having unbalanced ratings with respect to positively correlated subcriteria are penalised and those proving to be competitive regarding negatively correlated subcriteria are favoured. Again, this course of action

acts as a discriminatory tool for highlighting those solutions that can make a difference for reaching outstanding achievements in comparison with their competitors. The inclusion of the correlation coefficient u_{ki}^{kj} between two subcriteria i and j into the aggregation procedure adjusts the satisfaction s_k provided by an alternative l with respect to a criterion k as formulated in Eq. (9).

$$s'_k = \begin{cases} \sum_{i=1}^m w_{ki} \cdot s_{ki} \cdot \sum_{i \neq j} \frac{1}{2p} [1 - u_{ki}^{kj} \cdot (1 - \Delta s_{ki}^{kj})], & -1 \leq u_{ki}^{kj} < 0 \\ \sum_{i=1}^m w_{ki} \cdot s_{ki} \cdot \sum_{i \neq j} \frac{1}{p} [1 - u_{ki}^{kj} \cdot \Delta s_{ki}^{kj}], & 0 < u_{ki}^{kj} \leq 1 \end{cases} \quad (9)$$

where s'_k is the adjusted satisfaction degree for criterion k and Δs_{ki}^{kj} is the difference in the satisfaction provided by an alternative l in relation to two subcriteria $SC_{ki}, SC_{kj} \in C_k$. p denotes the number of subcriteria with which the subcriterion under analysis has a statistically significant correlation and causes Eq. (9) to satisfy $s'_k \in [0, 1]$. A negative correlation between subcriteria i and j increases s'_k as Δs_{ki}^{kj} decrease, whilst an increase in Δs_{ki}^{kj} leads to an increase in s'_k if SC_{ki} and SC_{kj} are positively correlated. The addition of this term further hinders achieving the concept of ideal alternative ($s_l = 1$), since the maximum satisfaction is reached when $u_{ki}^{kj} = -1 \parallel 1$ and $\Delta s_{ki}^{kj} = 0$.

3. Application to wire rope selection in slope stability cable nets

Wire rope is a type of cable consisting of several strands of steel wire twisted helically around a core. This element can adopt a great variety of configurations depending on the number and size of strands and wires and the way in which they are combined, which enable it to carry out very different tasks. One of them is to be weaved to form cable nets aimed at acting as protective systems for people and goods against slope instabilities, wherein wire rope is responsible for supporting loads and transmitting them to a series of bolts through which the net is anchored to the ground.

There are several types of slope instability such as landslide, avalanche, rockfall, rock slip and rotational slumps, which requires therefore different properties in the wire rope to deal with them all. Such fact, together with the wide range of existing types of wire rope, make this a complex problem characterised by having multiple conflicting criteria that can be approached through several different alternatives. Table 2 is a scheme of the decision-making problem in which the selection of wire rope net is framed.

Table 2. Hierarchical scheme of the decision-making problem of wire rope selection

A_k	ALTERNATIVE	C_i	CRITERIA	SC_{ij}	SUBCRITERIA
A_1	6 x 7 FC	C_1	Technical	SC_{11}	Tensile strength
A_2	6 x 19 FC				
A_3	6 x 19 Seale FC				
A_4	6 x 19 Seale IWRC			SC_{12}	Flexibility
A_5	6 x 19 Warrington FC				
A_6	6 x 19 Warrington IWRC				
A_7	6 x 37 FC	C_2	Economic	SC_{21}	Equivalent unit cost
A_8	6 x 37 IWRC				
A_9	7 x 7 WSC				
A_{10}	7 x 19 WSC			SC_{22}	Market share
A_{11}	8 x 7 FC				
A_{12}	19 x 7 WSC				

The first two columns list a set of available alternatives capable of dealing with this problem. Wire ropes are usually defined by two digits and an abbreviation. The first digit specifies the number of strands of the wire rope, whilst the second one indicates the number of wires forming each strand. The abbreviation denotes the core composition: fibre core (FC), wire strand core (WSC) or independent wire rope core (IWRC). A literature review on the wire rope and its application fields (Feyrer, 2007; Hipkins, 1896; Serrano-Núñez & Castro-Fresno, 2005) revealed that the structures with a diameter of 8 mm made by 6 outer strands of 7, 19 or 37 wires are those best suited to form cable nets for slope stabilisation. Within these types, there are special structures such as Seale or Warrington whose outer strands have different diameters. 8 x 7 FC and 19 x 7 WSC are alternatives that do not meet the recommendations in terms of number of strands and wires for slope stability, but are included in the analysis for comparative purposes. These alternatives are evaluated according to technical and economic criteria. Wire rope must both provide an adequate mechanical response against slope instabilities and be available at reasonable cost. These aspects are subject to a prioritisation rule: an economic benefit cannot compensate for a loss in the technical properties of the wire rope, i.e. $C_1 > C_2$ (see subsection 2.2).

The wiring forming a net anchored to slope surfaces is not subject to confinement conditions or torsional stresses and therefore, is not expected to suffer from phenomena such as crushing or tendency to twist, which are very common in other applications of wire rope (cranes, elevators, etc.). Tensile strength, flexibility and wear resistance are the three aspects that clearly highlight in this context in mechanical terms. Tensile strength can be measured through the minimum breaking load of the wire rope, which is the product of its calculated breaking load and wiring factor. The calculated breaking load of a wire rope is in turn the product of its metallic section by the unit tensile

strength of the wires, whilst the wiring factor stands for the loss of resistance due to the arrangement of the wires. The two other technical subcriteria are dependent on the structure of the wire rope. Thus, flexibility is enhanced when the number of outer strands and wires increases and the core is made of fibre, whilst wear resistance is directly proportional to the diameter of the outer wires.

The economic subcriteria, modelled from the information collected from nine Chilean companies specialised in selling wire rope, are limited by the impossibility of disclosing the prices they offer. Under this premise, SC_{21} is characterised by assuming that the cost per kg is constant regardless the type of wire rope. This assumption, although not fully realistic, brings out the fact that the unit price per meter for each type of wire rope can be expressed in terms of their weight. As for the market share, the availability of the set of alternatives listed in Table 2 is checked through the catalogues and websites of the nine Chilean companies under consideration: Distintce, Dolezych, Douglas y CIA, FENASA, IMDIFER, LIMACHE, Piolas y Cables Ltda., Prodinsa S.A. and TECNICABLES.

Table 3 summarises the modelling of subcriteria according to the Beta CDF parameters and bounds chosen in each case. SC_{11} and SC_{21} , which are directly quantified from the specifications catalogue of Tenso Unitex (2014), are modelled through sigmoid and negative linear functions, respectively. Increases in tensile strength at either the beginning or the end of the function do not result in great improvements in the satisfaction gained, whilst the unit cost shows an uninflected behaviour in all sections. SC_{12} is rated through the combination of three aspects related to flexibility: number of strands, number of wires and fibre core (yes = 2; no = 1). These aspects are simply added to obtain the overall performance of the alternatives with respect to flexibility, in order to give greater importance to those aspects having a wider range of scores. A logit shape is selected to characterise this subcriterion, since the structures of 6 x 19 are similar in terms of flexibility and the largest differences are located at the ends of the function (6 x 7 and 6 x 37 wire ropes). As for the wear resistance, the inflexion in satisfaction is related to those wire ropes with larger diameter in their outer wires, which justifies the choice of a concave function to favour such structures. Finally, SC_{22} is characterised according to a logit shape, which implies that at least one company is required to purchase the product (initial section) and the most favourable scenario is reached when the number of companies in which the wire rope is available approaches the maximum (final section).

Table 3. Modelling of subcriteria according to the Beta CDF

A_k	SC_{11}	No. strands	SC_{12}	FC	SC_{13}		SC_{21}	SC_{22}
	x_{11}		No. wires		x_{12}	x_{13}	x_{21}	x_{22}
A_1	3830	1	1	2	4	7	0.229	6
A_2	3540	1	3	2	6	3	0.221	4
A_3	3810	1	2	2	5	5	0.238	6
A_4	4120	1	2	1	4	5	0.262	6
A_5	3810	1	4	2	7	2	0.238	1
A_6	4120	1	4	1	6	2	0.262	1
A_7	3400	1	5	2	8	1	0.221	2
A_8	3670	1	5	1	7	1	0.244	1
A_9	4130	1	1	1	3	7	0.252	6
A_{10}	3820	1	3	1	5	3	0.244	4
A_{11}	3420	2	1	2	5	6	0.223	1
A_{12}	3780	3	1	1	5	4	0.257	6
α	2				0.40	2	1	0.50
β	2				0.45	1	1	0.75
A	3400				3	1	0.221	1
B	4130				8	7	0.262	6

The use of the Beta CDF for the specifics shown in Table 3 yielded the degrees of satisfaction provided by the alternatives with regards to each subcriterion (see Table 4). Normality of these datasets was checked through the Kolmogorov-Smirnov (K-S) test, which revealed that the null hypothesis was confirmed for all of them (p-values > 0.05). These results pointed to the standard deviation as the measure of dispersion to be used for weighting the subcriteria. The application of Eq. (7) resulted in the weight vector shown in Table 4, which demonstrated the preponderance of x_{11} , x_{13} and x_{22} over x_{12} and x_{21} , respectively. In other words, the values of satisfaction of SC_{11} , SC_{13} and SC_{22} were more dispersed than those of SC_{12} and SC_{21} .

Table 4. Satisfaction degrees and weights for the set subcriteria

A_k	s_{11}	s_{12}	s_{13}	s_{21}	s_{22}
A_1	0.632	0.346	1.000	0.805	1.000
A_2	0.096	0.590	0.111	1.000	0.690
A_3	0.592	0.476	0.444	0.585	1.000
A_4	0.999	0.346	0.444	0.000	1.000
A_5	0.592	0.714	0.028	0.585	0.000
A_6	0.999	0.590	0.028	0.000	0.000
A_7	0.000	1.000	0.000	1.000	0.380
A_8	0.309	0.714	0.000	0.439	0.000
A_9	1.000	0.000	1.000	0.244	1.000
A_{10}	0.612	0.476	0.111	0.439	0.690
A_{11}	0.002	0.476	0.694	0.951	0.000
A_{12}	0.531	0.476	0.250	0.122	1.000
K-S	0.890	0.815	0.545	0.988	0.454
w_{ki}	0.372	0.247	0.381	0.449	0.551

Table 5 shows the dependencies identified for each pair of subcriteria using the Pearson correlation coefficient, which is the appropriate coefficient to be applied in this case according to the nature of the variables (see Table 4). The results proved that the correlation of SC_{12} with both SC_{11} and SC_{13} was negative and statistically significant (p-value < 0.05). This is consistent from a point of view of the composition of a wire rope, since flexible wire ropes have some characteristics, such as fibre core or small wire diameters, which are detrimental to reach high values of tensile strength and wear resistance.

Table 5. Statistical correlation coefficients between subcriteria

	$SC_{11} * SC_{12}$	$SC_{11} * SC_{13}$	$SC_{12} * SC_{13}$	$SC_{21} * SC_{22}$
Pearson Correlation	-0.619*	0.274	-0.802*	-0.186
Sig. (2-tailed)	0.032	0.389	0.002	0.563
N	12	12	12	12

*. Correlation is significant at the 0.05 level (2-tailed).

The application of Eqs. (3) and (9) with the data shown in Table 4 and Table 5 allowed the intersection of the four operations depicted in Figure 1 to be accomplished, which resulted in the ranking of alternatives illustrated in Figure 3. The ranking was also determined without including the step described in subsection 2.4, in order to highlight the benefits provided by the consideration of the correlations between subcriteria.

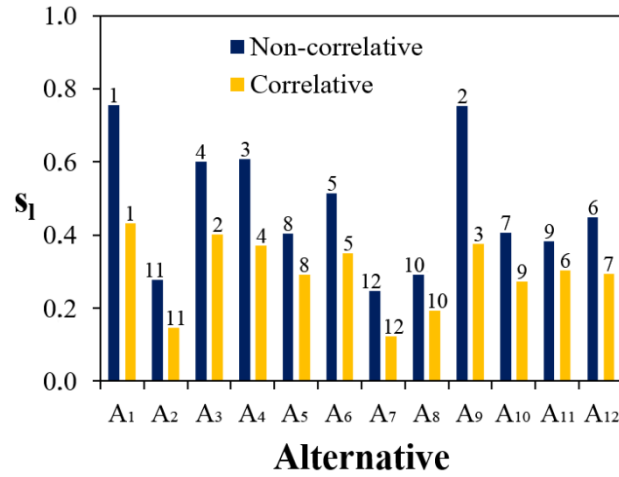


Figure 3. Overall satisfaction degrees and ranking of alternatives

The first position remained constant regardless of whether correlations were taken into account or not, due to the competence of A_1 regarding all subcriteria except flexibility. This characteristic was the reason behind the changes in positions 2, 3 and 4, since A_3 presented the most balanced behaviour with respect to technical subcriteria (see Table 4). In contrast, A_4 and especially A_9 had high values of Δs_{11}^{12} and Δs_{12}^{13} , which explained their drop in the ranking in favour of A_3 . A_9 was the most benefited alternative by the inclusion of the correlative aggregation due to the smaller value of Δs_{12}^{13} it reached in comparison with A_5 , A_{10} and A_{12} , since Δs_{12}^{13} indicated the difference in satisfaction in relation to the two more strongly correlated subcriteria (see Table 5). A_7 was the type of wire rope providing the lowest overall degree of satisfaction in both cases, because of its technical heterogeneity and poor response in mechanical terms. In summary, three wire rope structures were found to be the most suitable to form slope stability cable nets: 6 x 7 FC, 6 x 19 Seale FC, 7 x 7 WSC and 6 x 19 Seale IWRC. The first is a simple type of wire rope capable of withstanding the loads to which these cable nets are subject at reasonable cost, whilst Seale structures and 7 x 7 WSC involve a series of mechanical improvements that lead them to achieve top positions too.

4. Conclusions

This paper proposes and applies a decision aid system that combines three new methods founded on concepts such as nonlinear valuation, dispersion-based weighting and correlative aggregation to solve hierarchical decision-making problems characterised by having multiple conflicting subcriteria grouped into several criteria. The first consists of using the Beta Cumulative Distribution Function to transform the rating of the set of alternatives under consideration with regards to the criteria into the satisfaction they generate according to five different value functions, depending on where the inflexions

in satisfaction are concentrated: concave, convex, linear, logit and sigmoid. This approach outperforms existing valuation methods not only in terms of the number of shapes it allows to model, but also due to its simplicity, since only four parameters need to be chosen as inputs to automate its application.

The aggregation of the elements forming the decision-making problem is carried out using a prioritisation operator that avoids compensation between the criteria in the first level of the hierarchy. Subcriteria are weighted according to two measures of dispersion depending on whether they follow a normal distribution or not: standard deviation and interquartile range. This method increases the importance of those subcriteria that are more diverse and reduces the weight of homogeneous subcriteria, which leads to favour the selection of alternatives that perform well with respect to varying aspects. Finally, the dependence between subcriteria is studied through the calculation of their statistical correlations, which allows the adjustment of the results yielded by the system to enhance the performance of the alternatives in relation to their response to conflicting aspects. The combination of these operations results in a ranking of alternatives that rewards those solutions capable of making a difference and adding significant value to the overall degree of satisfaction achieved.

The usefulness of the decision aid system was tested through a novel application case study aimed at addressing the selection of wire rope to form slope stability cable nets. The versatility provided by the Beta CDF was demonstrated when modelling very diverse properties such as flexibility, wear resistance or market share. The incorporation of the correlation coefficients into the decision aid system proved to produce several changes in the ranking, which favoured those alternatives having a balanced behaviour in relation to dependent subcriteria. The structure with composition 6 x 7 FC was found to be the most suitable wire rope to deal with slope instabilities, due to its remarkable technical response and economic availability.

The flexibility of this decision aid system, which consists of a set of interacting methods, facilitates the use of their components either in isolation or as part of a different whole. Further research in this line should point to the design of software or web-based interfaces that enable the elements of the decision aid system to be linked and applied through manageable and interactive formats. Another future research direction might consider the incorporation of stochastic simulations and/or fuzzy logic into the valuation phase, in order to represent the uncertainty and vagueness that is often related to the modelling of some variables. Finally, the integration of the proposed weighting method with subjective approaches could also be contemplated to reflect both the objective information and the views of decision-makers.

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